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Operational Research

PROJECT 6: HOUSES BUILDER

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Introduction

This report presents the solution to a problem faced by a developer who has acquired a plot of land measuring 80,000 m² and intends to build two types of residential units on it. The first type consists of maisonettes, and the second type consists of apartments. The total available capital is €40,000,000. However, if the initial capital is not sufficient, the developer can receive an advance payment of 30% from the sale of each maisonette and 25% from the sale of each apartment.

The table below shows the construction costs, the profit from the sale of each type of residence, as well as the land area required for construction.

Type	Maisonette	Apartment
Quantity	x	y
Area per unit	400 m^2	240 m^2
Construction cost per unit	420.000€	250.000€
Profit per unit	350.000€	170.000€
Selling price	770.000€	420.000€

Additional Constraints

The developer must also comply with certain constraints. First, the total number of residential units must not exceed 150. Additionally, the total area used for construction must not exceed 60% of the available land. If this limit is exceeded, there is a 30% chance that a fine of €1,000 per square meter of excess area will be imposed.

Furthermore, at least 15 units of each type must be constructed, or none at all. Lastly, if the developer decides to construct both types of residences, the number of apartments must be at least double the number of maisonettes.

Question 1:

Formulate the problem as a Linear Programming (LP) model.

Decision Variables:

1) **c1**: Binary variable equal to 1 if maisonettes are built, 0 otherwise.

2) **c2**: Binary variable equal to 1 if apartments are built, 0 otherwise.

3) **x**: Number of maisonettes to be built.

4) **y**: Number of maisonettes to be built.

Auxiliary Variable:

1) Let s = 60%*80.000-400x-240y

   If $s > 0$, we are within legal land use limits (remaining area = s m^2).
   If $s < 0$, we have exceeded the allowed coverage by $|s|$ m^2.

We also define:

2) $s = s^{+} - s^{-}$

Objective Function

We aim to maximize the net profit. If the developer does not comply with the constraint that limits land use to 60% of the available area, the expected fine will be:

$$0.3 \times 1000 \times s^{-}$$

This is because there is a 30% probability of a random inspection, and the fine is €1,000 per excess square meter (s^{-}).

Constraints

1. c1, c2 are binary variables.
2. x, y are integer variables.
3. x ≤ M * c1 and x ≥ 15 * c1
   - (Either no maisonettes are built, or at least 15 are built.)
4. y ≤ M * c2 and y ≥ 15 * c2
   - (Either no apartments are built, or at least 15 are built.)
5. x + y ≤ 150

- (The total number of housing units cannot exceed 150.)

6. $400x + 240y \leq 80{,}000$
   - (The total available land area is 80,000 m².)
7. $2x - y \leq M * (2 - (c1 + c2))$
   - (If both types of housing are constructed — i.e., if c1 = c2 = 1 — then the number of apartments y must be at least twice the number of maisonettes x. This constraint is only active when both c1 and c2 equal 1.)
8. $420{,}000x + 250{,}000y \leq 40{,}000{,}000$
   - (The construction cost cannot exceed the available capital, unless prepayment is allowed.)
9. $s = s^{+} - s^{-}$
   - (The land usage deviation is expressed as the difference of two non-negative variables.)
10. $x, y, s^{+}, s^{-} \geq 0$

- (Non-negativity constraints.)

11) Prepayment Constraint (Alternative to Constraint 8):

If prepayments are considered, constraint 8 is replaced by:

$420{,}000x+250{,}000y\leq40{,}000{,}000+0.30\cdot770{,}000x+0.25\cdot420{,}000y$

Since:

- Selling price of each maisonette = 420,000 (cost) + 350,000 (profit) = €770,000
- Selling price of each apartment = 250,000 + 170,000 = €420,000

Question 2:

Solve the problem using Excel SOLVER.

Solution:

1st case without down payment

Variables				
	Built = 1 Not Built= 0 (c1 , c2	Units (x , y)		
Maisonettes	1	95		
Apartments	0	0		
Extent to which the land coverage constraint was exceeded (s+, s-)	s+	10000	Cell C7 (s+) shows how many square meters remained unused, while cell C8 (s-) shows by how many square	
	s-	0		

Data				
	m^2 per unit	cost in € per unit (in thousands)	profit in € per unit (in thousands)"	
Maisonettes	400	420	350	
Apartments	240	250	170	
"Total land area in m^2	80000	Total land area in m² actually used	38000	
Total available capital in € (in thousands)	40000			
s =s+ - s- = 60%*80.000 -400x-240y =	10000	s- =	0	
Penalty imposed (only if s⁺ - s⁻ > 0) in € (in thousands)	30%*1*s-=	0		
If cell B15 is positive, then s > 0, meaning the area constraint has been exceeded by s m². If s < 0, then the area constraint has not been exceeded, and s m² remain unused. The total m² actually used is shown in cell D13.				

Objective Function - Maximizing profit	
Total Profit in € (in thousands)	33250

Restrictions

Non-negativity	x	95	>=	0
	y	0	>=	0
Binary variables	c1	1	=	binary (0 ή 1)
	c2	0	=	binary (0 ή 1)

In the following, M will refer to a very large number, say 100,000.

Maisonettes to be built: either none or at least 15	$x \leq M*c1$	95	<=	100000
	$x \geq 15*c1$	95	>=	15
Apartments to be built: either none or at least 15	$y \leq M*c2$	0	<=	0
	$y \geq 15*c2$	0	>=	0
A maximum of 150 houses can be built in total.	$x+y \leq 150$	95	<=	150
Total available land area (cannot be exceeded)	$400m^2\,x + 240m^2\,y \leq 80.000\ m^2$	38000	<=	80000
Whether or not the 60% of the available land area has been exceeded	s = s+ - s- =	10000	=	10000
If c1 = c2 = 1, meaning both types are built, the number of apartments must be at least twice the number of townhouses.	$2x-y \leq M\{ 2-(c1+c2) \}$	190	<=	100000

The constraint is activated only if c1 = c2 = 1, while for any other combination of c1 and c2, it becomes inactive, as it will apply to any x, y since M is a very large number.

Total available capital constraint (in thousands)	$420x + 250y \leq 40.000$	39900	<=	45000

Microsoft Excel 16.0 Answer Report
Worksheet: [excel final.xlsx]Solution
Report Created: 7/1/2024 3:58:44 μμ
Result: Solver found a solution. All Constraints and optimality conditions are satisfied.
Solver Engine
Engine: Simplex LP
Solution Time: 0,171 Seconds.
Iterations: 3 Subproblems: 6
Solver Options
Max Time Unlimited, Iterations Unlimited, Precision 0,000001
Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 1%, Assume NonNegative

Objective Cell (Max)

Cell	Name	Original Value	Final Value
$B$23	Total Profit in € (in thousands)	0	33250

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$B$4	Maisonettes (Built = 1, Not Built = 0)	0	1	Binary
$C$4	Maisonettes Units	0	95	Integer
$B$5	Apartments (Built = 1, Not Built = 0)	0	0	Binary
$C$5	Apartments Units	0	0	Integer
$C$7	s+	0	10000	Contin
$C$8	s-	0	0	Contin

Name	Cell Value	Formula	Status	Slack
x ≤ M*c1	95	$C$37<=$E$37	Not Binding	99905
x ≥ 15*c1	95	$C$38>=$E$38	Not Binding	80
y ≤ M*c2	0	$C$39<=$E$39	Binding	0
y ≥ 15*c2	0	$C$40>=$E$40	Binding	0
x+y ≤ 150	95	$C$42<=$E$42	Not Binding	55
400m² x +240m² y ≤ 80.000 m²	38000	$C$44<=$E$44	Not Binding	42000
s= s+ - s- =	10000	$C$45=$E$45	Binding	0
2x-y ≤ M{ 2-(c1+c2) }	190	$C$47<=$E$47	Not Binding	99810
420x + 250y ≤ 40.000	39900	$C$50<=$E$50	Not Binding	100
r				
r				

Second Case (with Advance Payment)

Variables			
	Built = 1 Not Built= 0 (c1 , c2)	Units (x , y)	
Maisonettes	1	150	
Apartments	0	0	
Extent to which the land coverage constraint was exceeded (s+, s-)	s+	0	Cell C7 (s+) shows how many square meters remained unused, while cell C8 (s-) shows by how many square meters the constraint was
	s-	12000	

Data			
	m^2 per unit	cost in € per unit (in thousands)	profit in € per unit (in thousands)"
Maisonettes	400	420	350
Apartments	240	250	170
"Total land area in m^2	80000	Total land area in m² actually used	60000
Total available capital in € (in thousands)	40000	Advance	34650
Total Available Capital in € (in thousands)		74650	
s =s+ - s- = 60%*80.000 -400x-240y =	-12000	s- =	12000
Penalty imposed (only if $s^+ - s^- > 0$) in € (in thousands)	30%*1*s-=	3600	

If cell B15 is positive, then s > 0, meaning the area constraint has been exceeded by s m². If s < 0, then the area constraint has not been exceeded, and s m² remain unused. The total m² actually used is shown in cell D13.

Objective Function - Maximizing profit	
Total Profit in € (in thousands)	48900

Restrictions				
Non-negativity	x	150	>=	0
	y	0	>=	0
Binary variables	c1	1	=	binary (0 ή 1)
	c2	0	=	binary (0 ή 1)
In the following, M will refer to a very large number, say 100,000.				
Maisonettes to be built: either none or at least 15	$x \leq M*c1$	150	<=	100000
	$x \geq 15*c1$	150	>=	15
Apartments to be built: either none or at least 15	$y \leq M*c2$	0	<=	0
	$y \geq 15*c2$	0	>=	0
A maximum of 150 houses can be built in total.	$x+y \leq 150$	150	<=	150
Total available land area (cannot be exceeded)	$400m^2\ x + 240m^2\ y \leq$ 80.000 m^2	60000	<=	80000
Whether or not the 60% of the available land area has been exceeded	s = s+ - s- =	-12000	=	-12000
If c1 = c2 = 1, meaning both types are built, the number of apartments must be at least twice the number of townhouses.	$2x-y \leq M\{ 2-(c1+c2) \}$	300	<=	100000
The constraint is activated only if c1 = c2 = 1, while for any other combination of c1 and c2, it becomes inactive, as it will apply to any x, y since M is a very large number.				
Total available capital constraint (in thousands)	$420x + 250y \leq$ 40.000	63000	<=	74650

Microsoft Excel 16.0 Answer Report
Worksheet: [excel final.xlsx]Solution
Report Created: 07/01/2024 14:53:44
Result: Solver found a solution. All Constraints and optimality conditions are satisfied.
Solver Engine
Engine: Simplex LP
Solution Time: 0.047 Seconds.
Iterations: 2 Subproblems: 2
Solver Options
Max Time Unlimited, Iterations Unlimited, Precision 0.000001, Use Automatic Scaling
Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 1%, Assume NonNegative

Objective Cell (Max)

Cell	Name	Original Value	Final Value
$B$24	Total Profit in € (in thousands)	0	48900

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$B$4	Maisonettes (Built = 1, Not Built = 0)	0	1	Binary
$C$4	Maisonettes Units	0	150	Integer
$B$5	Apartments (Built = 1, Not Built = 0)	0	0	Binary
$C$5	Apartments Units	0	0	Integer
$C$7	s+	0	0	Contin
$C$8	s-	0	12000	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$C$38	x ≤ M*c1 s- =	150	$C$38<=$E$38	Not Binding	99850
$C$43	x+y ≤ 150 s- =	150	$C$43<=$E$43	Binding	0
$C$51	420x + 250y ≤ 40.000 s- =	63000	$C$51<=$E$51	Not Binding	11650
$C$46	s = s+ - s- =	-12000	$C$46=$E$46	Binding	0
$C$39	x ≥ 15*c1 s- =	150	$C$39>=$E$39	Not Binding	135
$C$48	2x-y ≤ M{ 2-(c1+c2) } s- =	300	$C$48<=$E$48	Not Binding	99700
$C$41	y ≥ 15*c2 s- =	0	$C$41>=$E$41	Binding	0
$C$40	y ≤ M*c2 s- =	0	$C$40<=$E$40	Binding	0
$C$45	400m² x +240m² y ≤ 80.000 m² s- =	60000	$C$45<=$E$45	Not Binding	20000
$B$5=Binary					
$B$4=Binary					
$C$4=Integer					
$C$5=Integer					

Question 3:

Provide a detailed formulation of the solution

Solution:

We initially solved the problem without making use of advance payments. As a result, our total available capital was €40,000,000. By following all the constraints, as shown in our model, the solution provided by Excel's Solver was to construct 95 maisonettes, which yields a net profit of €33,250,000. As observed, the remaining budget is €100,000, which is not enough to construct an additional maisonette or apartment. Furthermore, the total land area used was 38,000 m², meaning none of the urban planning or land coverage constraints were violated, and therefore no penalty will be imposed.

As previously stated, the optimal solution involves constructing only maisonettes, since they yield higher revenues and thus maximize net profit. Additionally, the available capital is sufficient to achieve a feasible solution under these conditions.

However, in the case where the developer chooses to receive an advance payment from property sales, specifically 30% of the selling price of each maisonette (€231,000 per maisonette) and 25% of the selling price of each apartment (€105,000 per apartment), and we once again apply Excel's Solver, the optimal solution is to construct 150 maisonettes with a total expected profit of €48,900,000.

In this case, the land coverage limit is exceeded by 12,000 m², which means there is a 30% chance that a fine of €12,000,000 will be imposed. If the penalty is applied, the profit would be €40,500,000, and if not, the profit would be €52,000,000. As shown in our model, the objective function subtracts the expected penalty value from the profit.

We observe that if the developer receives the advance payment and constructs 150 maisonettes, the profit is higher than the profit without the advance payment, regardless of whether the penalty is imposed or not. As demonstrated, we received €34,650,000 from the advance payment, which allowed us to maximize the number of units constructed—specifically, 150 maisonettes.

In this case, €11,650,000 remains from the available capital, which cannot be used to construct additional housing, as it is not permitted by urban planning constraints.

Question 4:

Assume that, due to increased demand, the profit from selling a single maisonette has increased by €40,000. Without resolving the problem again, determine whether the developer should consider the option of constructing more maisonettes.

Answer:

If the profit from the sale of a maisonette increases by €40,000, the new profit per maisonette will be €390,000. However, as we have already demonstrated, it was in the developer's best interest from the outset to build only maisonettes. Therefore, this increase in the selling price does not affect the structure of the optimal solution, but it does increase the total profit for the developer.

In both cases—whether the developer receives the advance payment or not—it is not possible to build more maisonettes:

- Without the advance payment, there is no remaining capital to fund the construction of additional units.
- With the advance payment, the urban planning constraint limiting the number of residences to a maximum of 150 has already been reached.

Thus, if no advance payment is received and the selling price of each maisonette rises to €390,000, the profit from building 95 maisonettes increases to €37,050,000.

If the advance payment is received and 150 maisonettes are constructed, then:

- If a planning inspection occurs and a penalty is imposed, the profit will be €46,500,000.
- If no penalty is imposed, the profit will be €58,500,000.

In all cases, the developer's total profit increases due to this price increase, reaching:

- €46,500,000 to €58,500,000 (depending on whether the penalty is imposed), if the advance is received, and
- €37,050,000 if no advance is received.

Question 5

A different type of detached house, featuring a garden and a private swimming pool, has a construction cost of €1,000,000, yields a profit of €550,000, and requires 500 square meters of land. Examine whether it is financially advantageous to construct such houses, without resolving the original optimization problem from scratch. For these detached houses, there is no constraint requiring either zero units or at least fifteen units to be built—it is permitted to construct even a single unit.

Solution:

From the solution obtained earlier using the solver, we observe that the optimal outcome involves constructing 95 maisonettes and no apartments, at a total cost of €39,900,000 out of the available €40,000,000, yielding a profit of €33,250,000. The total area occupied by the 95 maisonettes is 38,000 m², which is within the allowable 48,000 m² limit (as we are permitted to build on up to 60% of the total 80,000 m² land without incurring any penalty).

Next, we will evaluate whether it is advantageous to construct detached houses with gardens and private pools. The construction cost, profit, and land required per unit for each housing type are summarized below:

Housing Type	Cost per Unit (€)	Profit per Unit (€)	Land per Unit (m²)
Detached House	1,000,000	550,000	500
Maisonette	420,000	350,000	400
Apartment	250,000	170,000	240

Case 1: Constructing only detached houses

We first assess how many detached houses could be built under the €40,000,000 capital constraint. Since each house costs €1,000,000, up to 40 units could be built.

We then check if constructing 40 detached houses violates the land-use limit:

$40 \times 500m^2 = 20.000m^2 < 48.000m^2$

Since the land constraint is still satisfied, we could indeed build all 40 units. The total profit would then be:

$$40 \times 550.000€ = 22.000.000€$$

This is significantly lower than the profit of €33,250,000 from building 95 maisonettes. Therefore, constructing only detached houses is not financially optimal.

Case 2: Constructing one detached house

Suppose we wish to build just one detached house. We only have €100,000 of capital remaining, so we need an additional €900,000. We could free up this capital by choosing not to build a certain number of maisonettes:

$$\frac{900.000}{420.000} \approx 2{,}14\ \mu\varepsilon\zeta o\nu\acute{\varepsilon}\tau\varepsilon\varsigma$$

Since the number of maisonettes must be an integer, we would need to not build 3 maisonettes, reducing the total to 92 maisonettes.

This would free up:

$$3 \times 420.000€ = 1.260.000€$$

Which allows us to build one detached house for €1,000,000, leaving €260,000 unallocated.

Land usage in this case:

$$92 \times 400m^2 + 1 \times 500m^2 = 37.300m^2 < 48.000m^2$$

So there is no land penalty. The total profit becomes:

$$92 \times 350.000€ + 1 \times 550.000€ = 32.750.000€ < 33.250.000€$$

Που όπως βλέπουμε και πάλι δεν μας συμφέρει.

Thus, even this case is less profitable than the original scenario.

Note: We could potentially use the leftover €260,000 to build one apartment, which costs €250,000 and yields a profit of €170,000. This is feasible based on the remaining land. In that case, total profit becomes:

€32,750,000+€170,000=€32,920,000,

which is still less than the original €33,250,000.

Case 3: Constructing two detached houses

Suppose we build two detached houses. We would need an additional €1,900,000 in capital (since only €100,000 is available). Again, we could achieve this by building fewer maisonettes:

$$\frac{1.900.000}{420.000} \approx 4{,}5\ \mu\varepsilon\zeta o\nu\acute{\varepsilon}\tau\varepsilon\varsigma$$

Thus, we reduce the maisonettes to 90. This frees up:

$$5 \times 420.000€ = 2.100.000€$$

allowing for two detached houses with €100,000 left over.

Land usage becomes:

$$90 \times 400m^2 + 2 \times 500m^2 = 37.000m^2 < 48.000m^2$$

So again, no penalty. Total profit:

$$90 \times 350.000€ + 2 \times 550.000€ = 32.600.000€ < 33.250.000€$$

Again, this is less profitable, so not advantageous.

General Case: Constructing *n* detached houses

Suppose we wish to construct n detached houses (with n < 150).

We would require additional capital:

€1,000,000 × n − €100,000

To obtain this, we must reduce the number of maisonettes by approximately:

(1,000,000 × n − 100,000) / 420,000 ≈ 2.38n − 0.24

The capital saved would be:

(2.38n − 0.24) × 420,000 = €999,600 × n − €100,800

This would allow us to build n detached houses.

We then calculate the profit to determine whether it's greater than the original:

(95 − 2.38n + 0.24) × €350,000 + n × €550,000 > €33,250,000

Solving:

(550,000−833,000)×n+84,000>0
→−283,000×n+84,000>0
→ n < 0.3

Thus, for profit to exceed the original, n must be less than 0.3, which is not feasible as n must be a whole number.

Conclusion:

Constructing detached houses with gardens and private pools is not financially beneficial under the current conditions. In all tested cases, including building one or two units, the total profit falls short of the profit obtained by constructing 95 maisonettes. Even with optimal reallocation of capital and land, the original solution remains the most profitable.

Question 6

If the developer manages to secure additional funding of up to €5,000,000, examine whether it is in their interest to accept all or part of this additional financing. For the amount of additional funding the developer chooses to utilize, determine the maximum acceptable cost of capital (i.e., the maximum borrowing interest rate) that would justify this additional investment.

The proposal should be evaluated under the assumption that only maisonettes and apartments will be constructed (i.e., no detached houses) and that all regulatory constraints must be fully respected.

Solution:

We begin by examining the scenario in which the entire additional funding of €5,000,000 is accepted. Thus, the total available capital would rise to €45,000,000.

We resolve the optimization problem using the Solver, this time with a capital limit of €45,000,000.

The resulting solution is as follows:

Maisonettes	400	420	350
Apartments	240	250	170
"Total land area in m^2	80000	Total land area in m² actually used	60000
Total available capital in € (in thousands)	45000		
s =s+ - s- = 60%*80.000 -400x-240y =	5200	s- =	0
Penalty imposed (only if $s^+ - s^- > 0$) in € (in thousands)	30%*1*s-=	0	

We observe that the land usage constraint has not been exceeded, since the value of the variable *s* is positive. This indicates that there is no risk of a penalty being imposed by the urban planning authority. Therefore, as the amount of borrowed capital increases, the potential for profit also increases. It is, thus, in our interest to utilize the entire amount of available additional funding in order to achieve the maximum possible financial benefit.

Based on the updated solution from the Solver, the total revenue from the sale of the residential units that would be built using the additional €5,000,000 in funding would be:

T′ = 45,000,000 + 37,450,000 = €82,450,000

Without the additional funding, the total amount would be:

T = 40,000,000 + 33,250,000 = €73,250,000

We calculate the difference in revenue between the two cases:

T′ − T = €9,200,000

This means that the additional revenue generated by utilizing the funding amounts to €9,200,000. Out of this, at least €5,000,000 must be repaid. Therefore, the real additional profit, assuming zero interest, would be:

€9,200,000 − €5,000,000 = €4,200,000

We now examine different interest rates to calculate the repayment amount and the net profit in each case:

Interest Rate	Repayment Amount (€)	Net Profit (€)
10%	5,500,000	3,700,000
20%	6,000,000	3,200,000
30%	6,500,000	2,700,000
40%	7,000,000	2,200,000
50%	7,500,000	1,700,000
60%	8,000,000	1,200,000
70%	8,500,000	700,000
80%	9,000,000	200,000
84%	9,200,000	0

We observe that the threshold interest rate at which the developer would break even (i.e., earn no additional profit from the funding) is 84%, which is very high.

Therefore, even under realistic or moderately high interest rates (e.g., 10%), the net profit justifies the developer's decision to obtain the additional funding.

Question 7:

The developer is negotiating the purchase of an adjacent plot of land with an area of 9,000 square meters. What is the maximum amount the developer would be willing to pay for acquiring the additional land? It is clarified that the plot can only be purchased in its entirety—partial acquisition is not an option. The proposal should be evaluated under the condition that only maisonettes and apartments will be constructed and that all planning regulations must be strictly followed.

Solution:

If the developer acquires the adjacent plot, the available land area increases from 80,000 m² to 89,000 m². As a reminder, in the solution to Question 1, the maximum profit was €33,250,000, achieved by building 95 maisonettes (without considering the down payment). To construct these 95 maisonettes, the developer used €39,900,000 of the available €40,000,000, and 38,000 m² of the 48,000 m² permitted for development (based on the 60% land-use restriction).

Thus, the limiting factor was not the land area, but the available capital, since 10,000 m² of land remained unused. Therefore, without additional capital, the extra plot would be unnecessary. We then examine the case where the initial capital is increased to determine whether additional land would be beneficial in that context.

For the 80,000 m² plot, the available investment capital was €40,000,000. Proportionally, for a plot of 89,000 m², the corresponding capital would be €44,500,000. In Question 6, we examined the scenario where the investment capital was €45,000,000 and found that the utilized land area was 42,800 m². This means that even with increased capital, the land-use restriction is not exceeded, and in fact, 5,200 m² remains unused. Hence, the additional plot still wouldn't be utilized effectively unless the capital is significantly increased.

A significant capital increase, however, may be achieved through receiving down payments for the homes to be constructed.

We therefore examine the scenario in which the developer receives these down payments, as in Question 3. In this case, the available land becomes 89,000 m². For this area, we assume the housing unit limit of 150 can be proportionally increased to 166. We adjust the data in Excel and resolve the optimization problem accordingly.

As shown below, the optimal solution involves constructing all 166 maisonettes—fully utilizing the housing limit—and exceeding the land-use restriction by 13,000 m², since the new limit is 60% of 89,000 m². The expected profit amounts to €54,200,000. Compared to Question 3, where the available land was 80,000 m² and expected profit was €48,900,000, we observe an increase of €5,300,000.

Therefore, for any purchase price below €5,300,000, the developer makes a profit. However, to justify the investment and ensure that the return is worthwhile, it is assumed that the plot should not be purchased for a price exceeding 30% of the profit it would generate. Accordingly, the maximum purchase price is:

✅ €1,590,000,
yielding a corresponding net profit of €3,710,000.

Variables			
	Built = 1 Not Built= 0 (c1 , c2)	Units (x , y)	
Maisonettes	1	166	
Apartments	0	0	
Extent to which the land coverage constraint was exceeded (s+, s-)	s+	0	Cell C7 (s+) shows how many square meters remained unused, while cell C8 (s-) shows by how many square meters the constraint was
	s-	13000	

Data				
	m^2 per unit	cost in € per unit (in thousands)	profit in € per unit (in thousands)"	
Maisonettes	400	420	350	
Apartments	240	250	170	
"Total land area in m^2	89000	Total land area in m^2 actually used	66400	
Total available capital in € (in thousands)	40000	Advance	38346	
Total Available Capital in € (in thousands)		78346		
s =s+ - s- = 60%*80.000 -400x-240y =	-13000	s- =	13000	
Penalty imposed (only if $s^+ - s^- > 0$) in € (in thousands)	30%*1*s-=	3900		

If cell B15 is positive, then s > 0, meaning the area constraint has been exceeded by s m^2. If s < 0, then the area constraint has not been exceeded, and s m^2 remain unused. The total m^2 actually used is shown in cell D13.

Objective Function - Maximizing profit

Total Profit in € (in thousands)	54200