Research Review

Game Tree Searching by Min/Max Approximation

Ron Rivest, April 1987, https://people.csail.mit.edu/rivest/pubs/Riv87c.pdf

Summary:

This paper introduces the concept of "min/max approximation", a technique in which the "min" and "max" operators are approximated using generalized mean-value operators, allowing the node that is expected to have the most effect on the root value to always be expanded, rather than always expanding nodes in a "right to left traversal" pattern. Because the mean-value operation has continuous derivatives in respect to all arguments, the node which has the most effect on the root value can be defined in a non-trivial manner, and nodes can be expanded in the order of "most effect" to "least effect" using the chain rule.

The technique proposed is an example of an "iterative" heuristic, which is a heuristic that has these properties (as defined in the paper):

1. It "grows" the search tree one step at a time
2. At each step, a "tip" node of the current tree is chosen, expanded, and added to the tree
3. The values at the new leaves are propagated through their ancestors to the root of the tree
4. The tree does not need to be searched to a uniform depth

The author states that implementing this technique is computationally "difficult" however, requiring a large computational cost to take powers and roots, when implemented in it's most accurate form. The "reverse approximation" variation skips the computation of the generalized mean values, and instead uses the appropriate min or max values. This works without introducing very much error because the the point of using the generalized mean values was for their derivatives, not for the values themselves.

Results:

The "min/max approximation" method is demonstrated to have superior results compared to those produced by minimax search with alpha-beta pruning when calls to the move operator is the limiting resource. However, when CPU time is the limiting resource, minimax search with alpha-beta pruning produces superior results. Of note, when a time bound was in effect, the number of distinct positions considered by minimax search with alpha-beta was approximately 3 times larger than those considered by the min/max approximation method.