

Assignment 1

You are given the sequence x_0, \dots, x_{10} :

2.000 1.000 -0.500 0.950 0.935 -1.115 0.442 0.705 0.099 0.224 0.481.

We make the hypothesis is that the sequence is generated by a linear recursion of the form

$$x_n = \theta_0 + \theta_1 x_{n-1} + \theta_2 x_{n-2} + \dots + \theta_K x_{n-K}$$

Here K is the “order” of the model, and $\theta_0, \dots, \theta_K$ are the parameters.

Your goal is to identify the model of minimal order that can explain the observed sequence.

Please use computer to solve the least-square problems below.

- 1) Suppose K=1. Use least-square regression to identify the best value of θ_0 and θ_1 that would explain the data. Report the mean-square error (MSE) over the data set.
- 2) Suppose K=2. Use least-square regression to identify the best value of θ_0 , θ_1 , θ_2 that would explain the data. What is the mean-square error (MSE) over the data set?
- 3) Same for K=3,4,5.

Please use this table to submit your responses to 1), 2), 3) :

	θ_0	θ_1	θ_2	θ_3	θ_4	θ_5	MSE
K=1	0.4421	-0.0348					0.2237
K=2	0.8622	-0.5077	-0.4786				0.0400
K=3	0.0010	0.2991	-0.1007	0.5996			0.0000
K=4	0.0018	-0.5216	0.1446	0.5167	0.4921		0.0000
K=5	0.0045	-1.2775	-0.4796	0.6939	0.8589	0.5093	0.0000

- 4) What is the minimal value of K for which the residual is “close enough” to 0 ?

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- 5) Predict the 10 next elements of the sequence by using the system model of minimal order that you have identified. Report numbers with 3 decimal points.

x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	x_{18}	x_{19}	x_{20}
0.182	0.141	0.313	0.189	0.111	0.203	0.164	0.096	0.135	0.130