# StarNEig User's Guide master branch

Generated by Doxygen 1.8.17

Thu Nov 19 2020 18:32:52

| 1 Introduction                                | 1  |
|---|----|
| 2 Installation                                | 5  |
| 3 Basic usage                                 | 9  |
| 4 Distributed memory                          | 11 |
| 5 Standard eigenvalue problem                 | 17 |
| 6 Generalized eigenvalue problem              | 21 |
| 7 Expert functions                            | 25 |
| 8 Test program                                | 27 |
| 9 Known problems and changelog                | 33 |
| 10 License, authors and literature            | 37 |
| 11 Todo List                                  | 41 |
| 12 Deprecated List                            | 43 |
| 13 Module Documentation                       | 45 |
| 13.1 Library configuration                    | 45 |
| 13.1.1 Detailed Description                   | 45 |
| 13.1.2 Macro Definition Documentation         | 46 |
| 13.1.2.1 STARNEIG_ENABLE_MPI                  | 46 |
| 13.1.2.2 STARNEIG_ENABLE_CUDA                 | 46 |
| 13.1.2.3 STARNEIG_ENABLE_BLACS                | 46 |
| 13.1.2.4 STARNEIG_SEP_DM_HESSENBERG           | 46 |
| 13.1.2.5 STARNEIG_GEP_DM_HESSENBERGTRIANGULAR | 46 |
| 13.1.2.6 STARNEIG_SEP_DM_REDUCE               | 47 |
| 13.1.2.7 STARNEIG_GEP_DM_REDUCE               | 47 |
| 13.2 Error codes                              | 48 |
| 13.2.1 Detailed Description                   | 48 |
| 13.2.2 Macro Definition Documentation         | 48 |
| 13.2.2.1 STARNEIG_SUCCESS                     | 48 |
| 13.2.2.2 STARNEIG_GENERIC_ERROR               | 49 |
| 13.2.2.3 STARNEIG_NOT_INITIALIZED             | 49 |
| 13.2.2.4 STARNEIG_INVALID_CONFIGURATION       | 49 |
| 13.2.2.5 STARNEIG_INVALID_ARGUMENTS           | 49 |
| 13.2.2.6 STARNEIG_INVALID_DISTR_MATRIX        | 49 |
| 13.2.2.7 STARNEIG_DID_NOT_CONVERGE            | 49 |
| 13.2.2.8 STARNEIG_PARTIAL_REORDERING          | 50 |
| 13.2.2.9 STARNEIG_CLOSE_EIGENVALUES           | 50 |

| 13.3 Intra-node execution environment           | 51 |
|---|----|
| 13.3.1 Detailed Description                     | 52 |
| 13.3.2 Macro Definition Documentation           | 52 |
| 13.3.2.1 STARNEIG_USE_ALL                       | 52 |
| 13.3.2.2 STARNEIG_DEFAULT                       | 52 |
| 13.3.2.3 STARNEIG_HINT_SM                       | 52 |
| 13.3.2.4 STARNEIG_HINT_DM                       | 53 |
| 13.3.2.5 STARNEIG_FXT_DISABLE                   | 53 |
| 13.3.2.6 STARNEIG_AWAKE_WORKERS                 | 53 |
| 13.3.2.7 STARNEIG_AWAKE_MPI_WORKER              | 53 |
| 13.3.2.8 STARNEIG_FAST_DM                       | 54 |
| 13.3.2.9 STARNEIG_NO_VERBOSE                    | 54 |
| 13.3.2.10 STARNEIG_NO_MESSAGES                  | 54 |
| 13.3.3 Function Documentation                   | 54 |
| 13.3.3.1 starneig_node_init()                   | 54 |
| 13.3.3.2 starneig_node_initialized()            | 55 |
| 13.3.3.3 starneig_node_get_cores()              | 55 |
| 13.3.3.4 starneig_node_set_cores()              | 55 |
| 13.3.3.5 starneig_node_get_gpus()               | 56 |
| 13.3.3.6 starneig_node_set_gpus()               | 56 |
| 13.3.3.7 starneig_node_enable_pinning()         | 56 |
| 13.3.3.8 starneig_node_disable_pinning()        | 56 |
| 13.4 Shared Memory / Standard EVP               | 57 |
| 13.4.1 Detailed Description                     | 57 |
| 13.4.2 Function Documentation                   | 58 |
| 13.4.2.1 starneig_SEP_SM_Hessenberg()           | 58 |
| 13.4.2.2 starneig_SEP_SM_Schur()                | 58 |
| 13.4.2.3 starneig_SEP_SM_ReorderSchur()         | 59 |
| 13.4.2.4 starneig_SEP_SM_Reduce()               | 60 |
| 13.4.2.5 starneig_SEP_SM_Eigenvectors()         | 61 |
| 13.4.2.6 starneig_SEP_SM_Select()               | 62 |
| 13.4.2.7 starneig_SEP_SM_Hessenberg_expert()    | 62 |
| 13.4.2.8 starneig_SEP_SM_Schur_expert()         | 63 |
| 13.4.2.9 starneig_SEP_SM_ReorderSchur_expert()  | 64 |
| 13.4.2.10 starneig_SEP_SM_Eigenvectors_expert() | 65 |
| 13.5 Shared Memory / Generalized EVP            | 66 |
| 13.5.1 Detailed Description                     | 66 |
| 13.5.2 Function Documentation                   | 67 |
| 13.5.2.1 starneig_GEP_SM_HessenbergTriangular() | 67 |
| 13.5.2.2 starneig_GEP_SM_Schur()                | 67 |
| 13.5.2.3 starneig_GEP_SM_ReorderSchur()         | 68 |
| 13.5.2.4 starneig_GEP_SM_Reduce()               | 70 |

| 13.5.2.5 starneig_GEP_SM_Eigenvectors()         |  | 71 |
|---|--|----|
| 13.5.2.6 starneig_GEP_SM_Select()               |  | 72 |
| 13.5.2.7 starneig_GEP_SM_Schur_expert()         |  | 72 |
| 13.5.2.8 starneig_GEP_SM_ReorderSchur_expert()  |  | 73 |
| 13.5.2.9 starneig_GEP_SM_Eigenvectors_expert()  |  | 74 |
| 13.6 Distributed Memory / Distributed matrices  |  | 76 |
| 13.6.1 Detailed Description                     |  | 77 |
| 13.6.2 Data Structure Documentation             |  | 77 |
| 13.6.2.1 struct starneig_distr_block            |  | 77 |
| 13.6.3 Enumeration Type Documentation           |  | 77 |
| 13.6.3.1 starneig_distr_order_t                 |  | 77 |
| 13.6.3.2 starneig_datatype_t                    |  | 79 |
| 13.6.4 Function Documentation                   |  | 79 |
| 13.6.4.1 starneig_distr_init()                  |  | 79 |
| 13.6.4.2 starneig_distr_init_mesh()             |  | 79 |
| 13.6.4.3 starneig_distr_init_func()             |  | 80 |
| 13.6.4.4 starneig_distr_duplicate()             |  | 80 |
| 13.6.4.5 starneig_distr_destroy()               |  | 81 |
| 13.6.4.6 starneig_distr_matrix_create()         |  | 81 |
| 13.6.4.7 starneig_distr_matrix_create_local()   |  | 82 |
| 13.6.4.8 starneig_distr_matrix_destroy()        |  | 83 |
| 13.6.4.9 starneig_distr_matrix_copy()           |  | 83 |
| 13.6.4.10 starneig_distr_matrix_copy_region()   |  | 83 |
| 13.6.4.11 starneig_distr_matrix_get_blocks()    |  | 84 |
| 13.6.4.12 starneig_distr_matrix_get_distr()     |  | 84 |
| 13.6.4.13 starneig_distr_matrix_get_datatype()  |  | 85 |
| 13.6.4.14 starneig_distr_matrix_get_elemsize()  |  | 85 |
| 13.6.4.15 starneig_distr_matrix_get_rows()      |  | 86 |
| 13.6.4.16 starneig_distr_matrix_get_cols()      |  | 86 |
| 13.6.4.17 starneig_distr_matrix_get_row_blksz() |  | 86 |
| 13.6.4.18 starneig_distr_matrix_get_col_blksz() |  | 87 |
| 13.7 Distributed Memory / Helper functions      |  | 88 |
| 13.7.1 Detailed Description                     |  | 88 |
| 13.7.2 Function Documentation                   |  | 88 |
| 13.7.2.1 starneig_mpi_set_comm()                |  | 88 |
| 13.7.2.2 starneig_mpi_get_comm()                |  | 88 |
| 13.7.2.3 starneig_mpi_broadcast()               |  | 89 |
| 13.8 Distributed Memory / Standard EVP          |  | 90 |
| 13.8.1 Detailed Description                     |  | 90 |
| 13.8.2 Function Documentation                   |  | 91 |
| 13.8.2.1 starneig_SEP_DM_Hessenberg()           |  | 91 |
| 13.8.2.2 starneig_SEP_DM_Schur()                |  | 91 |

| 13.8.2.3 starneig_SEP_DM_ReorderSchur()         | 92  |
|---|-----|
| 13.8.2.4 starneig_SEP_DM_Reduce()               | 92  |
| 13.8.2.5 starneig_SEP_DM_Eigenvectors()         | 93  |
| 13.8.2.6 starneig_SEP_DM_Select()               | 94  |
| 13.8.2.7 starneig_SEP_DM_Hessenberg_expert()    | 94  |
| 13.8.2.8 starneig_SEP_DM_Schur_expert()         | 95  |
| 13.8.2.9 starneig_SEP_DM_ReorderSchur_expert()  | 96  |
| 13.8.2.10 starneig_SEP_DM_Eigenvectors_expert() | 96  |
| 13.9 Distributed Memory / Generalized EVP       | 98  |
| 13.9.1 Detailed Description                     | 98  |
| 13.9.2 Function Documentation                   | 99  |
| 13.9.2.1 starneig_GEP_DM_HessenbergTriangular() | 99  |
| 13.9.2.2 starneig_GEP_DM_Schur()                | 99  |
| 13.9.2.3 starneig_GEP_DM_ReorderSchur()         | 100 |
| 13.9.2.4 starneig_GEP_DM_Reduce()               | 101 |
| 13.9.2.5 starneig_GEP_DM_Eigenvectors()         | 102 |
| 13.9.2.6 starneig_GEP_DM_Select()               | 102 |
| 13.9.2.7 starneig_GEP_DM_Schur_expert()         | 103 |
| 13.9.2.8 starneig_GEP_DM_ReorderSchur_expert()  | 104 |
| 13.9.2.9 starneig_GEP_DM_Eigenvectors_expert()  | 105 |
| 13.10 Expert configuration structures           | 106 |
| 13.10.1 Detailed Description                    | 108 |
| 13.10.2 Data Structure Documentation            | 108 |
| 13.10.2.1 struct starneig_hessenberg_conf       | 108 |
| 13.10.2.2 struct starneig_schur_conf            | 108 |
| 13.10.2.3 struct starneig_reorder_conf          | 110 |
| 13.10.2.4 struct starneig_eigenvectors_conf     | 112 |
| 13.10.3 Enumeration Type Documentation          | 112 |
| 13.10.3.1 starneig_reorder_plan_t               | 112 |
| 13.10.3.2 starneig_reorder_blueprint_t          | 115 |
| 13.10.4 Function Documentation                  | 116 |
| 13.10.4.1 starneig_hessenberg_init_conf()       | 116 |
| 13.10.4.2 starneig_schur_init_conf()            | 116 |
| 13.10.4.3 starneig_reorder_init_conf()          | 116 |
| 13.10.4.4 starneig_eigenvectors_init_conf()     | 117 |
| 13.11 ScaLAPACK compatibility / BLACS matrices  | 118 |
| 13.11.1 Detailed Description                    | 118 |
| 13.11.2 Data Structure Documentation            | 118 |
| 13.11.2.1 struct starneig_blacs_descr           | 118 |
| 13.11.3 Function Documentation                  | 119 |
| 13.11.3.1 starneig_distr_to_blacs_context()     | 119 |
| 13.11.3.2 starneig blacs context to distr()     | 110 |

| 13.11.3.3 starneig_distr_is_blacs_compatible()        | <br>120               |
|---|-----------------------|
| 13.11.3.4 starneig_distr_is_compatible_with()         | <br>120               |
| 13.11.3.5 starneig_blacs_create_matrix()              | <br>120               |
| 13.11.3.6 starneig_blacs_destroy_matrix()             | <br>121               |
| 13.11.3.7 starneig_distr_matrix_to_blacs_descr()      | <br>121               |
| 13.11.3.8 starneig_blacs_descr_to_distr_matrix()      | <br>122               |
| 13.11.3.9 starneig_distr_matrix_is_blacs_compatible() | <br>122               |
| 13.11.3.10 starneig_distr_matrix_is_compatible_with() | <br>123               |
| 13.12 ScaLAPACK compatibility / BLACS helpers         | <br>124               |
| 13.12.1 Detailed Description                          | <br>124               |
| 13.12.2 Function Documentation                        | <br>124               |
| 13.12.2.1 starneig_blacs_pinfo()                      | <br>124               |
| 13.12.2.2 starneig_blacs_get()                        | <br>125               |
| 13.12.2.3 starneig_blacs_gridinit()                   | <br>125               |
| 13.12.2.4 starneig_blacs_gridinfo()                   | <br>126               |
| 13.12.2.5 starneig_blacs_pcoord()                     | <br>126               |
| 13.12.2.6 starneig_blacs_gridexit()                   | <br>126               |
| 13.12.2.7 starneig_blacs_exit()                       | <br>127               |
| 13.12.2.8 starneig_blacs_numroc()                     | <br>127               |
| 13.12.2.9 starneig_blacs_descinit()                   | <br>127               |
| 14 File Documentation                                 | 129                   |
|   |                       |
| 14.1 blacs_helpers.h File Reference                   |                       |
| 14.1.1 Detailed Description                           |                       |
| 14.2 Diacs_Matrix.fi File Reletence                   |                       |
| 14.3 configuration.h File Reference                   |                       |
| 14.3.1 Detailed Description                           | 131<br>131            |
| 14.4 distr_helpers.h File Reference                   | 132                   |
| 14.4.1 Detailed Description                           | 132                   |
| 14.5 distr_matrix.h File Reference                    | 132                   |
| 14.5.1 Detailed Description                           | 134                   |
| 14.6 error.h File Reference                           | 134                   |
| 14.6.1 Detailed Description                           | 135                   |
| 14.7 expert.h File Reference                          | 135                   |
| 14.7.1 Detailed Description                           | 137                   |
| 14.8 gep_dm.h File Reference                          | 137                   |
| 14.8.1 Detailed Description                           | 138                   |
|   |                       |
| 14.9 gep_sm.h File Reference                          | <br>138               |
| 14.9.1 Detailed Description                           | 120                   |
| 14.9.1 Detailed Description                           | 139                   |
| 14.9.1 Detailed Description                           | <br>139<br>139<br>141 |

| 14.11 sep_dm.h File Reference   | 41 |
|---------------------------------|----|
| 14.11.1 Detailed Description    | 42 |
| 14.12 sep_sm.h File Reference   | 42 |
| 14.12.1 Detailed Description    | 43 |
| 14.13 starneig.h File Reference | 43 |
| 14.13.1 Detailed Description    | 43 |
| 15 Example Documentation 1      | 45 |
| 15.1 gep_dm_full_chain.c        | 45 |
| 15.2 gep_sm_eigenvectors.c      | 47 |
| 15.3 gep_sm_full_chain.c        | 48 |
| 15.4 sep_dm_full_chain.c        | 49 |
| 15.5 sep_sm_eigenvectors.c      | 51 |
| 15.6 sep_sm_full_chain.c        | 51 |
| Index 1                         | 53 |

## Introduction

StarNEig library aims to provide a complete task-based software stack for solving **dense nonsymmetric** (generalized) eigenvalue problems. The library is built on top of the StarPU runtime system and targets both **shared memory** and **distributed memory** machines. Some components of the library support **GPUs**.

The four main components of the library are:

- Hessenberg(-triangular) reduction: A dense matrix (or a dense matrix pair) is reduced to upper Hessenberg (or Hessenberg-triangular) form.
- Schur reduction (QR/QZ algorithm): A upper Hessenberg matrix (or a Hessenberg-triangular matrix pair) is reduced to (generalized) Schur form. The (generalized) eigenvalues can be determined from the diagonal blocks.
- Eigenvalue reordering and deflating subspaces: Reorders a user-selected set of (generalized) eigenvalues to the upper left corner of an updated (generalized) Schur form.
- **Computation of eigenvectors**: Computes (generalized) eigenvectors for a user-selected set of (generalized) eigenvalues.

The library has been developed as a part of the NLAFET project. The project has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No. 671633. Support has also been received from eSSENCE, a collaborative e-Science programme funded by the Swedish Government via the Swedish Research Council (VR), and VR Grant E0485301. The development and performance evaluations were performed on resources provided by the Swedish National Infrastructure for Computing (SNIC) at HPC2N partially funded by VR through grant agreement No. 2016-07213. The library is published under open-source BSD 3-Clause license.

Please cite the following article when referring to StarNEig:

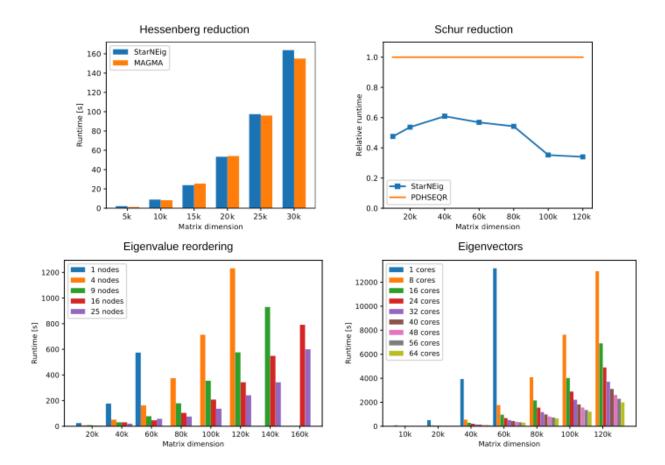
Mirko Myllykoski, Carl Christian Kjelgaard Mikkelsen: *Task-based, GPU-accelerated and Robust Library for Solving Dense Nonsymmetric Eigenvalue Problems*, Concurrency and Computation: Practice and Experience, 2020, doi: 10.1002/cpe.5915

Please see publications and authors.

2 Introduction

#### **Performance**

Performance comparisons against MAGMA (GPU) and ScaLAPACK (distributed memory), and strong scalability on shared and distributed memory machines:



Also, see following publications:

- Mirko Myllykoski: A Task-based Multi-shift QR/QZ Algorithm with Aggressive Early Deflation, arXiv
   — :2007.03576
- Mirko Myllykoski, Carl Christian Kjelgaard Mikkelsen: Task-based, GPU-accelerated and Robust Library for Solving Dense Nonsymmetric Eigenvalue Problems, Concurrency and Computation: Practice and Experience, 2020, doi: 10.1002/cpe.5915
- Mirko Myllykoski, Carl Christian Kjelgaard Mikkelsen, Angelika Schwarz, Bo Kågström: D2.7 Eigenvalue solvers for nonsymmetric problems, public NLAFET deliverable, 2019 ( download)

### **Current status (development series)**

The development series is under continuous development and considered **unstable**. See stable 0.1-series.

The library currently supports only real arithmetic (real input and output matrices but **real and/or complex** eigenvalues and eigenvectors). In addition, some interface functions are implemented as LAPACK and ScaLAPACK wrapper functions.

Standard eigenvalue problems:

| Component             | Shared memory | Distributed memory | CUDA         |  |
|-----------------------|---------------|--------------------|--------------|--|
| Hessenberg reduction  | Complete      | Incomplete         | Incomplete   |  |
| Schur reduction       | Complete      | Complete           | Experimental |  |
| Eigenvalue reordering | Complete      | Complete           | Experimental |  |
| Eigenvectors          | Complete      | _                  | _            |  |

Generalized eigenvalue problems:

| Component             | Shared memory | Distributed memory | CUDA         |
|-----------------------|---------------|--------------------|--------------|
| HT reduction          | LAPACK        | 3rd party          | _            |
| Schur reduction       | Complete      | Complete           | Experimental |
| Eigenvalue reordering | Complete      | Complete           | Experimental |
| Eigenvectors          | Complete      | _                  | _            |

Please see changelog and known problems.

#### **Documentation:**

The StarNEig User's Guide is available in both HTML and PDF formats at https://nlafet.github.io/← StarNEig. The PDF version is also available under releases.

#### Installation

Prebuild binary packages (Ubuntu 18.04 and Ubuntu 20.04) are available under releases and can be installed with the following command:

\$ sudo apt install ./StarNEig-v0.xx.yy-ubuntu-vv.uu.deb

The binary packages rely on mainstream StarPU packages and do not necessary provide full functionality.

For full functionality, it is recommended that StarNEig (and StarPU) are compiled from the source code, see below and/or the StarNEig User's Guide. Please consider using the stable 0.1 series; preferably one of the tested release versions.

#### **Dependencies**

Library dependencies:

- Linux
- · CMake 3.3 or newer
- Portable Hardware Locality (hwloc)
- Starpu 1.2 or 1.3
  - Newer versions require the user set the STARPU\_LIBRARIES, STARPU\_MPI\_LIBRARIES and STARPU\_INCLUDE\_PATH environmental variables.

4 Introduction

- · OpenBLAS, MKL, GotoBLAS or single-threaded BLAS library
- LAPACK
- · MPI (optional)
- · CUDA + cuBLAS (optional)
- ScaLAPACK + BLACS (optional)

Test program and example code dependencies:

- · pkg-config
- · GNU Scientific Library (optional)
- · MAGMA (optional)

#### Configure, build and install

To compile, validate and install StarNEig, execute in the same directory as this README . md file:

```
$ mkdir build
$ cd build/
$ cmake ../
$ make
$ make test
$ sudo make install
```

## **Example**

The following example demonstrates how a dense matrix A is reduced to real Schur form:

```
// my_program.c
#include <starneig/starneig.h>
#include <stdlib.h>
#include <time.h>
int main()
     int n = 3000;
     srand((unsigned) time(NULL));
     // generate a random matrix A
     int 1dA = ((n/8)+1)*8;
double *A = malloc(n*1dA*sizeof(double));
     for (int j = 0; j < n; j++)

for (int i = 0; i < n; i++)
               A[j*ldA+i] = 2.0*rand()/RAND_MAX - 1.0;
     // generate an identity matrix Q int 1dQ = ((n/8)+1)*8;
     double *Q = malloc(n*dA*sizeof(double));
for (int j = 0; j < n; j++)
    for (int i = 0; i < n; i++)</pre>
              Q[j*ldQ+i] = i == j ? 1.0 : 0.0;
     // allocate space for the eigenvalues
double *real = malloc(n*sizeof(double));
     double *imag = malloc(n*sizeof(double));
     // initialize the StarNEig library starneig_node_init(STARNEIG_USE_ALL, STARNEIG_USE_ALL, STARNEIG_HINT_SM);
     // reduce matrix A to real Schur form S = Q^T A Q
     starneig_SEP_SM_Reduce(
          n, A, ldA, Q, ldQ, real, imag, NULL, NULL, NULL, NULL);
     // de-initialize the StarNEig library
starneig_node_finalize();
     free(A); free(Q); free(real); free(imag);
     return 0;
Compile:
$ gcc -o my_program my_program.c -lstarneig
```

Generated on Thu Nov 19 2020 18:32:52 for StarNEig User's Guide by Doxygen

\$ gcc -o my\_program my\_program.c \$(pkg-config --cflags starneig) \$(pkg-config --libs starneig)

## Installation

The prebuild Ubuntu packages are available under releases and can be installed with the following command: \$ sudo apt install ./StarNEig-v0.xx.yy-ubuntu-vv.uu.deb

These packages rely on mainstream StarPU packages and do not necessary provide full functionality.

For full functionality, it is recommended that StarNEig (and StarPU) are compiled from the source code, see instructions below.

## **Dependencies**

Library dependencies:

- Linux
- · CMake 3.3 or newer
- · Portable Hardware Locality (hwloc)
- Starpu 1.2 or 1.3
  - Newer versions require the user set the STARPU\_LIBRARIES, STARPU\_MPI\_LIBRARIES and STARPU\_INCLUDE\_PATH environmental variables.
- · OpenBLAS, MKL, GotoBLAS or single-threaded BLAS library
- LAPACK
- · MPI (optional)
- · CUDA + cuBLAS (optional)
- ScaLAPACK + BLACS (optional)

Test program and example code dependencies:

- pkg-config
- · GNU Scientific Library (optional)
- MAGMA (optional)

#### Attention

StarNEig supports OpenBLAS, MKL and GotoBLAS. For optimal performance, a multi-threaded variant of one of the listed BLAS libraries must be provided. StarNEig will automatically set the BLAS library to single-threaded more when necessary. If a different BLAS library is provided, then the user is responsible for setting the BLAS library to *single-threaded* mode. However, the use of a non-supported BLAS library can still impact the performance negatively.

6 Installation

#### StarPU 1.3.4 installation

```
1. Download StarPU 1.3.4 (or newer) from http://starpu.gforge.inria.fr/files/
```

2. Unzip the package and create/enter directory starpu-1.3.4/build

```
3. Configure: $ ../configure
```

4. Compile: \$ make

5. Install: \$ sudo make install

The default installation path is /usr/local but this can be changed during the configuration phase (\$ ../configure --prefix=...). It is something necessary to append the CPATH, LIBRARY\_PATH, and LD\_LIBRARY\_PATH environmental variables by adding the following to  $\sim$ /.profile:

```
export CPATH=$CPATH:/usr/local/include/
export LIBRARY_PATH=$LIBRARY_PATH:/usr/local/lib/
export LD_LIBRARY_PATH=$LD_LIBRARY_PATH:/usr/local/lib/
```

See the StarPU handbook for further instructions: http://starpu.gforge.inria.fr/doc/html/← BuildingAndInstallingStarPU.html

## Configuration

It is recommended that a user builds the library in a separate build directory:

```
$ cd path_to_the_top_directory/
$ mkdir build
$ cd build
```

The library is configured with the cmake command. In most cases, it is not necessary to give this command any additional arguments:

```
$ cmake ../
...
-- Configuring done
-- Generating done
-- Build files have been written to: /.../build
```

However, the library can be customized with various options. For example, the example codes and documentation generation can be enabled by setting the <code>STARNEIG\_ENABLE\_EXAMPLES</code> and <code>STARNEIG\_ENABLE\_DOCS</code> options:

```
$ cmake -DSTARNEIG_ENABLE_EXAMPLES=ON -DSTARNEIG_ENABLE_DOCS=ON ../
```

The installation path can be changed during the configuration phase:

```
$ cmake -DCMAKE_INSTALL_PREFIX=/path/to/somewhere/ .../
```

#### Remarks

The library can be compiled separately from the other software components:

```
$ mkdir build-src
$ cd build-src/
$ cmake ../src/
$ make
```

It may sometimes be necessary to compile CUDA source files with a different compiler than what <code>cmake</code> uses by default. For example, some CUDA version do not support GCC compilers that are newer than GCC 5 release series. In that case <code>cmake</code> can be configured to use a GCC 5 release series compiler:

```
$ cmake -DCUDA_HOST_COMPILER=/usr/bin/gcc-5 -DCUDA_PROPAGATE_HOST_FLAGS=OFF ../
```

List of StarNEig library specific configuration options:

• STARNEIG\_ENABLE\_OPTIMIZATION: Enables extra compiler optimizations (ON by default).

- Can break portability.
- STARNEIG\_ENABLE\_EXAMPLES: Enables examples (OFF by default).
- STARNEIG\_ENABLE\_DOCS: Enables documentation generation (OFF by default).
- STARNEIG\_ENABLE\_TESTS:: Enables test program (ON by default).
- STARNEIG\_ENABLE\_FULL\_TESTS: Enables additional tests (OFF by default).
- STARNEIG\_ENABLE\_REFERENCE:: Enables reference MPI implementations (OFF by default).
  - A user must initialize and update the included GIT submodules.
- STARNEIG\_DISABLE\_MPI: Explicitly disables the MPI support even when the system would support it (OFF by default).
  - Must be set before cmake is run for the first time.
- STARNEIG\_DISABLE\_CUDA: Explicitly disables the CUDA support even when the system would support it (OFF by default).
  - Must be set before cmake is run for the first time.
- STARNEIG\_DISABLE\_BLACS: Explicitly disables the ScaLAPACK/BLACS support even when the system would support it (OFF by default).
  - Must be set before cmake is run for the first time.
- STARNEIG ENABLE MESSAGES: Enable basic verbose messages (ON by default).
- STARNEIG\_ENABLE\_VERBOSE: Enable additional verbose messages (OFF by default).
- STARNEIG\_ENABLE\_EVENTS: Enable event traces (OFF by default).
- STARNEIG\_ENABLE\_EVENT\_PARSER: Enable event parser (OFF by default).
- STARNEIG\_ENABLE\_SANITY\_CHECKS: Enables additional satiny checks. (OFF by default).
  - These checks are very expensive and should not be enabled unless absolutely necessary.
- STARNEIG\_ENABLE\_PRUNING: Enable task graph pruning (ON by default).
- STARNEIG\_ENABLE\_MRM: Enable multiple linear regression performance models (OFF by default).
- STARNEIG\_ENABLE\_CUDA\_REORDER\_WINDOW: Enable CUDA-based reorder\_window codelet (OFF by default).
- STARNEIG\_ENABLE\_INTEGER\_SCALING: Enable integer-based scaling factors (ON by default).

The following **environmental variables** can be used to configure the used libraries:

- BLAS\_LIBRARIES: BLAS library.
- LAPACK\_LIBRARIES: LAPACK library.
- HWLOC\_LIBRARIES: Portable Hardware Locality (hwloc) library.
- MPI\_LIBRARIES: C MPI library.
- ${\tt MPI\_Fortran\_LIBRARIES:} \textbf{Fortran MPI library}.$
- SCALAPACK\_LIBRARIES: ScaLAPACK library.
- BLACS\_LIBRARIES: BLACS library.
- STARPU\_LIBRARIES: StarPU library.

8 Installation

- STARPU\_MPI\_LIBRARIES: StarPU-MPI library.
- GSL\_LIBRARIES: GNU Scientific Library.
- MAGMA\_LIBRARIES: MAGMA library.
- MISC\_LIBRARIES: Miscellaneous libraries.

For example, if a user has a custom build ATLAS BLAS library and a matching LAPACK library that are not detected by the build system, then the user might define BLAS\_LIBRARIES=/usr/local/atlas/lib/libsatlas.so and LAPACK\_LIBRARIES=/usr/local/atlas/lib/liblapack.so before calling cmake.

The following environmental variables can be used to configure include paths for the used libraries:

- OMP\_INCLUDE\_PATH: OpenMP include path.
- BLAS\_INCLUDE\_PATH: BLAS include path.
- MKL\_INCLUDE\_PATH: MKL include path.
- HWLOC\_INCLUDE\_PATH: Portable Hardware Locality (hwloc) include path.
- MPI\_INCLUDE\_PATH: MPI include path.
- STARPU\_INCLUDE\_PATH: StarPU include path.
- GSL\_INCLUDE\_PATH: GNU Scientific Library include path.
- MAGMA\_INCLUDE\_PATH: MAGMA include path.
- MISC\_INCLUDE\_PATH: Miscellaneous include paths.

## Compile

The library (and other software components) are compiled with the make command:

```
$ make
...
[99%] Building C object test/CMakeFiles/starneig-test.dir/3rdparty/matrixmarket/mmio.c.o
[100%] Linking C executable ../starneig-test
[100%] Built target starneig-test
```

#### **Test**

The automated tests can be executed as follows:

The STARNEIG\_ENABLE\_FULL\_TESTS cmake option can be used to enable additional tests.

#### Install

The library and the related header files are installed by executing:

```
$ sudo make install
```

## Basic usage

#### **Header files**

The principal interface functions can be found from the starneig/node.h header file. The library provides separate header files for shared memory (starneig/sep\_sm.h, starneig/gep\_sm.h) and distributed memory (starneig/sep\_dm.h, starneig/gep\_dm.h). In most cases, a user can simply include all header files as follows: #include <starneig/starneig.h>

Certain header files and interface functions exist only when the library is compiled with MPI and ScaLAPACK / BLACS support. The configuration of the installed library can be found from the starneig/configuration.h header file. See module Library configuration for further information.

## Library initialization

Each node must call the starneig\_node\_init() interface function to initialize the library and the starneig\_node\_finalize() interface function to shutdown the library:

```
starneig_node_init(cores, gpus, flags);
...
starneig_node_finalize();
```

The starneig\_node\_init() interface function initializes StarPU (and cuBLAS) and pauses all worker threads. The cores argument specifies the total number of used CPU cores. In distributed memory mode, one of these C $\leftarrow$ PU cores is automatically allocated for the StarPU-MPI communication thread. The gpus argument specifies the total number of used GPUs. One or more CPU cores are automatically allocated for GPU devices. The flags (starneig\_flag\_t) argument can provide additional configuration information.

A node can also be configured with default values: starneig\_node\_init(STARNEIG\_USE\_ALL, STARNEIG\_USE\_ALL, STARNEIG\_DEFAULT);

This tells the library to use all available CPU cores and GPUs.

See module Intra-node execution environment for further information.

10 Basic usage

## **Error handling**

Most interface functions return one of the following values:

- STARNEIG\_SUCCESS (0): The interface function was executed successfully.
- A negative number -i: The i'th interface function argument was invalid.
- A positive number i: The interface function encountered an error or a warning was raised. See module Error codes for further information.

All return values (starneig\_error\_t) are defined in the starneig/error.h header file.

#### Remarks

The library may call the exit() and abort() functions if an interface function encounters a fatal error from which it cannot recover.

#### Performance models

The StarPU performance models must be calibrated before the software can function efficiently on heterogeneous platforms (CPUs + GPUs). The calibration is triggered automatically if the models are not calibrated well enough for a given problem size. This may impact the execution time negatively during the first run. Please see the StarPU handbook for further information:  $http://starpu.gforge.inria.fr/doc/html/Scheduling. \leftarrow html$ 

## **Compilation and linking**

During compilation, the starneig library library must be linked with the user's software: \$ gcc -o my\_program my\_program.c -lstarneig

StarNEig provides a pkg-config file for easing the compilation. A user may integrate it to their Makefile:

```
CFLAGS += $$(pkg-config --cflags starneig)
LDLIBS += $$(pkg-config --libs starneig)
%.o: %.c
$(CC) -c -o $@ $< $(CFLAGS)

my_program: my_program.c
$(CC) -o $@ $^ $(LDLIBS)
```

#### Or their CMakeLists.txt file:

```
find_package(PkgConfig REQUIRED)
pkg_search_module(STARNEIG REQUIRED starneig)
include_directories (${STARNEIG_INCLUDE_DIRS})
link_directories (${STARNEIG_LIBRARY_DIRS})
set (CMAKE_C_FLAGS "${STARNEIG_C_FLAGS} ${CMAKE_C_FLAGS}")
add_executable (my_program my_program.c)
target_link_libraries (my_program ${STARNEIG_LIBRARIES})
```

## **Distributed memory**

The STARNEIG\_HINT\_DM initialization flag tells the library to configure itself for distributed memory computation. The flag is indented to be only a hint and the library will automatically reconfigure itself for the correct computation mode. A user is allowed to mix shared memory and distributed memory functions without reninitializing the library. The library is intended to be run in a **hybrid configuration** (each MPI rank is mapped to several CPU cores). Failing to do so leads to CPU core oversubscription. It is generally a good idea to map each MPI rank to a full node or a NUMA island / CPU socket:

```
# OpenMPI / one rank per node:
$ mpirun -n RANKS --map-by ppr:1:node --bind-to none ...
# OpenMPI / one rank per socket:
$ mpirun -n RANKS --map-by ppr:1:socket --bind-to socket ...
# Intel MPI / one rank per node:
$ mpirun -n RANKS -binding "pin=on;domain=node" ...
# Intel MPI / one rank per socket:
$ mpirun -n RANKS -binding "pin=on;domain=socket" ...
```

#### Attention

StarPU attempts to bind the worker threads to the available CPU cores. This may sometimes conflict with the MPI library and/or the batch system CPU core allocation. StarNEig library attempts to correct for by factoring in the CPU core binding mask. However, if neither the MPI library nor the batch system enforces such a binding mask, it is possible that several StarPU worker threads end up bound to a same CPU core. In such a situation, it is recommended that a user disables the StarPU thread binding explicitly:

STARPU\_WORKERS\_NOBIND=1 mpirun ...

This is particularly important when several ranks / processes are mapped to a same node.

The library assumes that the MPI library is already initialized when the starneig\_node\_init() interface function is called with the STARNEIG\_HINT\_DM flag or when the library reconfigures itself for distributed memory after a user has called a distributed memory interface function. The MPI library should be initialized either in the serialized mode:

```
int thread_support;
MPI_Init_thread(
    &argc, (char ***) &argv, MPI_THREAD_SERIALIZED, &thread_support);
if (thread_support < MPI_THREAD_SERIALIZED) {</pre>
    fprintf(stderr,
        "MPI_THREAD_SERIALIZED is not supported. Aborting...\n");
Or in the multi-threaded mode:
int thread_support;
MPI_Init_thread(
    &argc, (char ***)&argv, MPI_THREAD_MULTIPLE, &thread_support);
if (thread_support < MPI_THREAD_SERIALIZED) {</pre>
    fprintf(stderr,
        "MPI_THREAD_SERIALIZED is not supported. Aborting...\n");
    abort();
} else if (thread_support < MPI_THREAD_MULTIPLE) {</pre>
    fprintf(stderr
        "Warning: MPI_THREAD_MULTIPLE is not supported.\n");
```

A user is allowed to change the library MPI communicator with the starneig\_mpi\_set\_comm() interface function. This interface function should be called **before** the library is initialized.

12 Distributed memory

#### **Data distribution**

Distributed matrices are represented using two opaque objects:

- Data distribution (starneig\_distr\_t)
- Distributed matrix (starneig\_distr\_matrix\_t)

Each matrix is divided into rectangular blocks of uniform size (excluding the last block row and column):

| (0,0) | (0,1) | (0,2) | (0,3) | (0,4) | (0,5) | (0, | 6) |
|-------|-------|-------|-------|-------|-------|-----|----|
| (1,0) | (1,1) | (1,2) | (1,3) | (1,4) | (1,5) | (1, | 6) |
| (2,0) | (2,1) | (2,2) | (2,3) | (2,4) | (2,5) | (2, | 6) |
| (3,0) | (3,1) | (3,2) | (3,3) | (3,4) | (3,5) | (3, | 6) |
| (4,0) | (4,1) | (4,2) | (4,3) | (4,4) | (4,5) | (4, | 6) |

Figure 4.1 A matrix divided into rectangular blocks of uniform size.

The blocks are indexed using a two-dimensional index space. A data distribution encapsulates an arbitrary mapping from this two-dimensional block index space to the one-dimensional MPI rank space:

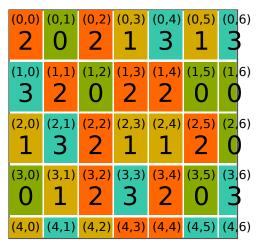


Figure 4.2 An example of a block mapping.

In the above example, the rank 0 owns the blocks (0,1), (1,2), (1,5), (1,6), (2,6), (3,0) and (3,5). Naturally, a data distribution can describe a two-dimensional block cyclic distribution that is very common with ScaLAPACK subroutines:

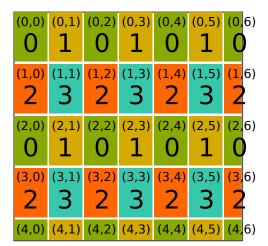


Figure 4.3 An example of a row-major ordered two-dimensional block cyclic mapping.

A data distribution can be created using one of the following interface functions:

- starneig\_distr\_init() creates a default distribution (row-major ordered two-dimensional block cyclic distribution with a squarish mesh).
- starneig\_distr\_init\_mesh() creates a row-major or column-major ordered two-dimensional block cyclic distribution with desired number of rows and columns in the mesh.
- starneig\_distr\_init\_func() creates an arbitrary distribution defined by a function.

#### Fox example,

```
starneig_distr_t distr = starneig_distr_init_mesh(4, 6, STARNEIG_ORDER_DEFAULT);
```

would create a two-dimensional block cyclic distribution with 4 rows and 6 columns in the mesh. Alternatively, a user can create an equivalent data distribution using the starneig\_distr\_init\_func() interface function:

A data distribution is destroyed with the starneig\_distr\_destroy() interface function.

#### Remarks

Certain interface functions (starneig\_GEP\_DM\_HessenbergTriangular() and starneig\_GEP\_DM\_Reduce()) are wrappers for / use several ScaLAPACK subroutines. The involved matrices should thus have a two-dimensional block cyclic data distribution. The library will automatically convert the matrices to a compatible format but this requires extra memory.

14 Distributed memory

#### Distributed matrix

A distributed matrix is created using the starneig\_distr\_matrix\_create() interface function. The function call will automatically allocate the required local resources. For example,

```
starneig_distr_t distr =
    starneig_distr_init_mesh(4, 6, STARNEIG_ORDER_DEFAULT);
starneig_distr_matrix_t dA =
    starneig_distr_matrix_create(m, n, bm, bn, STARNEIG_REAL_DOUBLE, distr);
```

would create a  $m \times n$  double-precision real matrix that is distributed in a two-dimensional block cyclic fashion in  $bm \times bn$  blocks. Or,

```
starneig_distr_matrix_t dB =
    starneig_distr_matrix_create(n, n, -1, -1, STARNEIG_REAL_DOUBLE, NULL);
```

would create a  $n \times n$  double-precision real matrix with a default data distribution (NULL argument) and a default block size (-1, -1).

#### Attention

StarNEig library is designed to use much larger distributed blocks than ScaLAPACK. Selecting a too small distributed block size will be detrimental to the performance.

A user may access the locally owned blocks using the starneig\_distr\_matrix\_get\_blocks() interface function. A distributed matrix is destroyed using the starneig\_distr\_matrix\_destroy() interface function. This will deallocate all local resources. See module Distributed Memory / Distributed matrices for further information.

#### Remarks

Certain interface functions (starneig\_GEP\_DM\_HessenbergTriangular() and starneig\_GEP\_DM\_Reduce()) are wrappers for / use several ScaLAPACK subroutines. The involved matrices should thus be distributed in square blocks. In addition, the ScaLAPACK subroutines usually perform better when the block size is relatively small. The library will automatically convert the matrices to a compatible format but this requires extra memory.

## Copy, scatter and gather

An entire distributed matrix can be copied with the starneig\_distr\_matrix\_copy() interface function:

```
starneig_distr_matrix_t dA, dB;
...
starneig_distr_matrix_copy(dB, dA);
```

This copies distributed matrix dB to a distributed matrix dA. A region (submatrix) of a distributed matrix can be copied to a second distributed matrix using the starneig\_distr\_matrix\_copy\_region() interface function.

A local matrix can be converted to a "single owner" distributed matrix with the starneig\_distr\_matrix\_create\_local() interface function:

```
int owner = 0; // MPI rank that owns the local matrix
double *A; // local pointer
int ldA; // matching leading dimension
...
starneig_distr_matrix_t lA = starneig_distr_matrix_create_local(
    n, n, STARNEIG_REAL_DOUBLE, owner, A, ldA);
```

This creates a wrapper object, i.e., the pointer A and the distributed matrix 1A point to the same data on the owner node. The created distributed matrix is associated with a data distribution that indicated that the whole matrix is owned by the node owner. The used block size is  $n \times n$ .

Copying from a "single owner" distributed matrix to a distributed matrix performs a *scatter* operation and copying from a distributed matrix to a "single owner" distributed matrix performs a *gather* operation.

### ScaLAPACK compatibility layer

The library provides a ScaLAPACK compatibility layer:

- BLACS contexts are encapsulated inside starneig\_blacs\_context\_t objects.
- BLACS descriptors are encapsulated inside starneig blacs descr t objects.

A two-dimensional block cyclic data distribution can be converted to a BLACS context and vice versa using the starneig\_distr\_to\_blacs\_context() and starneig\_blacs\_context\_to\_distr() interface functions, respectively. Similarly, a distributed matrix that uses a two-dimensional block cyclic data distribution can be converted to a BLACS descriptor (and a local buffer) and vice versa using the starneig\_distr\_matrix\_to\_blacs\_descr() and starneig\_blacs\_descr\_to\_distr\_matrix() interface functions, respectively. The conversion is performed in-place and a user is allowed to mix StarNEig interface functions with ScaLAPACK style subroutines/functions without reconversion.

#### For example,

converts a distributed matrix dA to a BLACS descriptor descr\_a and a local pointer local\_a. The descriptor and the local array are then fed to a ScaLAPACK subroutine. A user must make sure that the live time of the distributed matrix dA is at least as long as the live time of the matching BLACS descriptor descr\_a. See modules ScaLAPACK compatibility / BLACS helpers and ScaLAPACK compatibility / BLACS matrices for further information.

16 **Distributed memory** 

## Standard eigenvalue problem

Given a matrix  $A \in \mathbb{R}^{n \times n}$ , the standard eigenvalue problem (SEP) consists of finding eigenvalues  $\lambda_i \in \mathbb{C}$  and associated eigenvectors  $0 \neq v_i \in \mathbb{C}^n$  such that

$$Av_i = \lambda_i v_i$$
, for  $i = 1, 2, \dots, n$ .

The eigenvalues are the n (potentially complex) roots of the polynomial  $\det(A-\lambda I)=0$  of degree n. There is often a full set of n linearly independent eigenvectors, but if there are *multiple* eigenvalues (i.e., if  $\lambda_i=\lambda_j$  for some  $i\neq j$ ) then there might not be a full set of independent eigenvectors.

The library provides 12 interface functions for the standard case.

## Hessenberg reduction

Given a general matrix A, the starneig\_SEP\_SM\_Hessenberg() and starneig\_SEP\_DM\_Hessenberg() interface functions compute a Hessenberg decomposition

$$A = U * H * U^T,$$

where H is upper Hessenberg and U is orthogonal. On exit, A is overwritten by H and Q (which is an orthogonal matrix on entry) is overwritten by

$$Q \leftarrow Q * U$$
.

This is done in order to greatly accelerate the subsequent computation of a Schur decomposition since when working on H of size  $n \times n$ , the amount of work in each iteration of the QR algorithm is reduced from  $\mathcal{O}(n^3)$  to  $\mathcal{O}(n^2)$  flops.

#### Schur reduction

Given a Hessenberg decomposition

$$A = Q * H * Q^T,$$

of a general matrix A, the starneig\_SEP\_SM\_Schur() and starneig\_SEP\_DM\_Schur() interface functions compute a Schur decomposition

$$A = Q * (U * S * U^T) * Q^T$$

where S is upper quasi-triangular with  $1\times 1$  and  $2\times 2$  blocks on the diagonal (Schur matrix) and U is orthogonal. The  $1\times 1$  blocks correspond to the real eigenvalues and each  $2\times 2$  block corresponds to a pair of complex conjugate eigenvalues. On exit, H is overwritten by S and Q is overwritten by

$$Q \leftarrow Q * U$$
.

### Eigenvalue reordering

Given a Schur decomposition

$$A = Q * S * Q^T$$

of a general matrix A and a selection of eigenvalues, the starneig\_SEP\_SM\_ReorderSchur() and starneig\_SEP\_DM\_ReorderSchur() interface functions attempt to compute an updated Schur decomposition

$$A = Q * \left( U * \begin{bmatrix} \hat{S}_{11} & \hat{S}_{12} \\ 0 & \hat{S}_{22} \end{bmatrix} * U^T \right) * Q^T,$$

where the selected eigenvalues appear in  $\hat{S}_{11}$ . On exit, S is overwritten by  $\hat{S}$  and Q is overwritten by

$$Q \leftarrow Q * U$$
.

Reordering may in rare cases fail. In such cases the output is guaranteed to be a Schur decomposition and all (if any) selected eigenvalues that are correctly placed are marked in the selection array on exit. Reordering may perturb the eigenvalues and the eigenvalues after reordering are returned.

## Combined reduction to Schur form and eigenvalue reordering

Given a general matrix A, the starneig\_SEP\_SM\_Reduce() and starneig\_SEP\_DM\_Reduce() interface functions compute a (reordered) Schur decomposition

$$A = U * S * U^T.$$

Optionally, the interface functions attempt to reorder selected eigenvalues to the top left corner of the Schur matrix S. On exit, A is overwritten by S and Q (which is an orthogonal matrix on entry) is overwritten by

$$Q \leftarrow Q * U$$
.

Reordering may in rare cases fail. In such cases the output is guaranteed to be a Schur decomposition and all (if any) selected eigenvalues that are correctly placed are marked in the selection array on exit. Reordering may perturb the eigenvalues and the eigenvalues after reordering are returned.

### **Eigenvectors**

Given a subset consisting of  $m \leq n$  of the eigenvalues  $\lambda_i$  for  $i=1,2,\ldots,m$  and a Schur decomposition  $A=QSQ^H$ , we can compute for each  $\lambda_i$  an eigenvector  $v_i \neq 0$  such that  $Av_i = \lambda_i v_i$  by first computing an eigenvector  $w_i$  of S and then transform it back to the original basis by pre-multiplication with Q.

Given a Schur decomposition

$$A = Q * S * Q^T$$

of a general matrix A and a selection of eigenvalues, the starneig\_SEP\_SM\_Eigenvectors() and starneig\_SEP\_DM\_Eigenvectors() interface functions compute and return an eigenvector for each of the selected eigenvalues.

The eigenvectors are stored as columns in the output matrix X in the same order as their corresponding eigenvalues appear in the selection array. A real eigenvector is stored as a single column. The real and imaginary parts of a complex eigenvector are stored as consecutive columns.

For a selected pair of complex conjugate eigenvalues, an eigenvector is computed only for the eigenvalue with positive imaginary part. Thus, every selected eigenvalue contributes one column to the output matrix and thus the number of selected eigenvalues is equal to the number of columns of X.

## Eigenvalue selection helper

Given a Schur matrix and a predicate function, the starneig\_SEP\_SM\_Select() and starneig\_SEP\_DM\_Select() interface functions conveniently generate a correct selection array and count the number of selected eigenvalues. The count is useful when allocating storage for the eigenvector matrix computed by the starneig SEP\_SM\_Eigenvectors() and starneig SEP\_DM\_Eigenvectors() interface functions.

```
// a predicate function that selects all eigenvalues that have a real
// part that is larger than a given value
static int predicate(double real, double imag, void *arg)
{
    double value = * (double *) arg;
    if (value < real)
        return 1;
    return 0;
}
void func(...)
{
    double *S; int ldS;
    ...
    double value = 0.5;
    int num_selected, *selected = malloc(n*sizeof(int));
    starneig_SEP_SM_Select(
        n, S, ldS, &predicate, &value, selected, &num_selected);
    ...
}</pre>
```

See modules Shared Memory / Standard EVP and Distributed Memory / Standard EVP for further information. See also examples sep\_sm\_full\_chain.c, sep\_dm\_full\_chain.c and sep\_sm\_eigenvectors.c.

| Standard  | eigenva  | lue | nroh | lem   |
|-----------|----------|-----|------|-------|
| Stariuaru | cigeriva | uc  | PIUD | ICIII |

## Generalized eigenvalue problem

Given a matrix pair (A,B), where  $A,B\in\mathbb{R}^{n\times n}$ , the generalized eigenvalue problem (GEP) consists of finding generalized eigenvalues  $\lambda_i\in\mathbb{C}$  and associated generalized eigenvectors  $0\neq v_i\in\mathbb{C}^n$  such that

$$Av_i = \lambda_i Bv_i$$
, for  $i = 1, 2, \dots, n$ .

The eigenvalues are the n (potentially complex) roots of the polynomial  $\det(A - \lambda B) = 0$  of degree n. There is often a full set of n linearly independent generalized eigenvectors, but if there are *multiple eigenvalues* (i.e., if  $\lambda_i = \lambda_j$  for some  $i \neq j$ ) then there might not be a full set of independent eigenvectors.

At least in principle, a GEP can be transformed into a SEP provided that B is invertible, since

$$Av = \lambda Bv \Leftrightarrow (B^{-1}A)v = \lambda v.$$

However, in finite precision arithmetic this practice is not recommended.

The library provides 12 interface functions for the generalized case.

### Hessenberg-triangular reduction

Given a general matrix pair (A,B), the starneig\_GEP\_SM\_HessenbergTriangular() and starneig\_GEP\_DM\_HessenbergTriangular() interface functions compute a Hessenberg-triangular decomposition

$$(A,B) = U_1 * (H,R) * U_2^T,$$

where H is upper Hessenberg, R is upper triangular, and  $U_1$  and  $U_2$  are orthogonal. On exit, A is overwritten by H, B is overwritten by R, and Q and Z (which are orthogonal matrices on entry) are overwritten by

$$Q \leftarrow Q * U_1 \text{ and } Z \leftarrow Z * U_2.$$

This is done in order to greatly accelerate the subsequent computation of a generalized Schur decomposition.

#### **Generalized Schur reduction**

Given a Hessenberg-triangular decomposition

$$(A,B) = Q * (H,R) * Z^T$$

of a general matrix pair (A,B), the starneig\_GEP\_SM\_Schur() and starneig\_GEP\_DM\_Schur() interface functions function compute a generalized Schur decomposition

$$(A,B) = Q * (U_1 * (S,T) * U_2^T) * Z^T,$$

where S is upper quasi-triangular with  $1 \times 1$  and  $2 \times 2$  blocks on the diagonal, T is a upper triangular matrix, and  $U_1$  and  $U_2$  are orthogonal. The  $1 \times 1$  blocks on the diagonal of (S,T) correspond to the real generalized eigenvalues and each  $2 \times 2$  block corresponds to a pair of complex conjugate generalized eigenvalues.

On exit, H is overwritten by S, R is overwritten by T, and Q and Z are overwritten by

$$Q \leftarrow Q * U_1$$
 and  $Z \leftarrow Z * U_2$ .

The computed generalized eigenvalues are returned as a pair of values  $(\alpha, \beta)$  such that  $\alpha/\beta$  gives the actual generalized eigenvalue. The quantity  $\alpha/\beta$  may overflow.

## Generalized eigenvalue reordering

Given a generalized Schur decomposition

$$(A,B) = Q * (S,T) * Z^T$$

of a general matrix pair (A,B) and a selection of generalized eigenvalues, the starneig\_GEP\_SM\_ReorderSchur() and starneig\_GEP\_DM\_ReorderSchur() interface functions attempt to compute an updated generalized Schur decomposition

$$(A,B) = Q * \left( U_1 * \left( \begin{bmatrix} \hat{S}_{11} & \hat{S}_{12} \\ 0 & \hat{S}_{22} \end{bmatrix}, \begin{bmatrix} \hat{T}_{11} & \hat{T}_{12} \\ 0 & \hat{T}_{22} \end{bmatrix} \right) * U_2^T \right) * Z^T.$$

where the selected generalized eigenvalues appear in  $(\hat{S}_{11},\hat{T}_{11})$ .

On exit, S is overwritten by  $\hat{S}$ , T is overwritten by  $\hat{T}$ , and Q and Z are overwritten by

$$Q \leftarrow Q * U_1$$
 and  $Z \leftarrow Z * U_2$ .

Reordering may in rare cases fail. In such cases the output is guaranteed to be a Schur decomposition and all (if any) selected generalized eigenvalues that are correctly placed are marked in the selection array on exit.

Reordering may perturb the generalized eigenvalues and the generalized eigenvalues after reordering are returned. The computed generalized eigenvalues are returned as a pair of values  $(\alpha, \beta)$  such that  $\alpha/\beta$  gives the actual generalized eigenvalue. The quantity  $\alpha/\beta$  may overflow.

## Combined reduction to generalized Schur form and eigenvalue reordering

Given a general matrix pair (A,B), the starneig\_GEP\_SM\_Reduce() and starneig\_GEP\_DM\_Reduce() interface functions compute a (reordered) generalized Schur decomposition

$$(A,B) = U_1 * (S,T) * U_2^T.$$

Optionally, the interface functions attempt to reorder selected generalized eigenvalues to the top left corner of the generalized Schur decomposition.

On exit, A is overwritten by S, B is overwritten by T, and Q and Z (which are orthogonal matrices on entry) are overwritten by

$$Q \leftarrow Q * U_1$$
 and  $Z \leftarrow Z * U_2$ .

The computed generalized eigenvalues are returned as a pair of values  $(\alpha, \beta)$  such that  $\alpha/\beta$  gives the actual generalized eigenvalue. The quantity  $\alpha/\beta$  may overflow.

Reordering may in rare cases fail. In such cases the output is guaranteed to be a Schur-triangular decomposition and all (if any) selected generalized eigenvalues that are correctly placed are marked in the selection array on exit.

## **Generalized eigenvectors**

Given a subset consisting of  $m \leq n$  of the eigenvalues  $\lambda_i$  for  $i=1,2,\ldots,m$  and a generalized Schur decomposition  $(A,B)=Q(S,T)Z^H$ , we can compute for each  $\lambda_i$  a generalized eigenvector  $v_i \neq 0$  such that  $Av_i=\lambda_i Bv_i$  by first computing a generalized eigenvector  $w_i$  of  $S-\lambda_i T$  and then transform it back to the original basis by pre-multiplication with Z.

Given a generalized Schur decomposition

$$(A,B) = Q * (S,T) * Z^T$$

of a general matrix pair (A,B) and a selection of generalized eigenvalues, the starneig\_GEP\_SM\_Eigenvectors() and starneig\_GEP\_DM\_Eigenvectors() interface functions compute and return a generalized eigenvector for each of the selected generalized eigenvalues.

The generalized eigenvectors are stored as columns in the output matrix X in the same order as their corresponding generalized eigenvalues appear in the selection array. A real generalized eigenvector is stored as a single column. The real and imaginary parts of a complex generalized eigenvector are stored as consecutive columns.

For a selected pair of complex conjugate generalized eigenvalues, a generalized eigenvector is computed only for the generalized eigenvalue with positive imaginary part. Thus, every selected generalized eigenvalue contributes one column to the output matrix and thus the number of selected generalized eigenvalues is equal to the number of columns of X.

## Eigenvalue selection helper

Given a Schur-triangular matrix pair (S,T) and a predicate function, the starneig\_GEP\_SM\_Select() and starneig\_GEP\_DM\_Select() interface functions conveniently generate a correct selection array and count the number of selected generalized eigenvalues. The count is useful when allocating storage for the generalized eigenvector matrix computed by starneig GEP\_DM\_Eigenvectors().

```
// a predicate function that selects all finite generalized eigenvalues that
// have a real part that is larger than a given value
static int predicate(double real, double imag, double beta, void *arg)
{
    double value = * (double *) arg;
    if (beta != 0.0 && value < real/beta)
        return 1;
    return 0;
}
void func(...)
{
    ...
    double value = 0.5;
    int num_selected, *selected = malloc(n*sizeof(int));
    starneig_GEP_SM_Select(
        n, S, ldS, T, ldT, &predicate, &value, selected, &num_selected);
    ...
}</pre>
```

See modules Shared Memory / Generalized EVP and Distributed Memory / Generalized EVP for further information. See also examples gep\_sm\_full\_chain.c, gep\_dm\_full\_chain.c and gep\_sm\_eigenvectors.c.

## **Expert functions**

The library provides a set of configuration structures:

- starneig\_hessenberg\_conf : A configuration structure for Hessenberg reduction related expert interface functions.
- starneig\_schur\_conf : A configuration structure for Schur reduction related expert interface functions.
- starneig\_reorder\_conf: A configuration structure for eigenvalue reordering related interface functions.
- starneig\_eigenvectors\_conf : A configuration structure for eigenvector computation related interface functions.

The default parameters can generated with the following interface functions:

- starneig\_hessenberg\_init\_conf(): Generates default parameters for Hessenberg reduction related expert interface functions.
- starneig\_schur\_init\_conf(): Generates default parameters for Schur reduction related expert interface functions.
- starneig\_reorder\_init\_conf(): Generates default parameters for eigenvalue reordering related interface functions
- starneig\_eigenvectors\_init\_conf(): Generates default parameters for eigenvector computation related interface functions.

A user is allowed to modify these default values before passing them to the expert interface function.

Only certain interface functions have expert version:

- starneig\_SEP\_SM\_Hessenberg\_expert()
- starneig\_SEP\_SM\_Schur\_expert()
- starneig\_SEP\_SM\_ReorderSchur\_expert()
- starneig\_SEP\_SM\_Eigenvectors\_expert()
- starneig\_SEP\_DM\_Hessenberg\_expert()
- starneig\_SEP\_DM\_Schur\_expert()

26 Expert functions

- starneig\_SEP\_DM\_ReorderSchur\_expert()
- starneig\_GEP\_SM\_Schur\_expert()
- starneig\_GEP\_SM\_ReorderSchur\_expert()
- starneig\_GEP\_SM\_Eigenvectors\_expert()
- starneig\_GEP\_DM\_Schur\_expert()
- starneig\_GEP\_DM\_ReorderSchur\_expert()

See module Expert configuration structures for further information.

## Test program

The test program provides an unified interface for testing the entire library. Most command line arguments have default values so only few of them have to be set in most situations. General usage information can be printed as follows:

```
$ ./starneig-test
Usage: ./starneig-test (options)
Global options:
    --mpi -- Enable MPI
    --mpi-mode [serialized,multiple] -- MPI mode
    --seed (num) -- Random number generator seed
    --experiment (experiment) -- Experiment module
    --test-workers [(num),default] -- Test program StarPU worker count
    --blas-threads [(num),default] -- Test program BLAS thread count
    --lapack-threads [(num),default] -- LAPACK solver thread count
    --scalapack-threads [(num),default] -- ScaLAPACK solver thread count
Available experiment modules:
    'hessenberg' : Hessenberg reduction experiment
    'reorder' : Eigenvalue reordering experiment
    'reorder' : Eigenvectors experiment
    'eigenvectors' : Eigenvectors experiment
    'partial-hessenberg' : Partial Hessenberg reduction experiment
    'validator' : Validation experiment
```

The --mpi option enables the MPI support and --seed (num) option initializes the random number generator with a given seed. Available experiment modules are listed below the global options and the desired experiment module is selected with the --experiment (experiment) option. For example, the Hessenberg reduction specific experiment module usage information can be printed as follows:

```
./starneig-test --experiment hessenberg
Usage: ./starneig-test (options)
Global options:
   --mpi -- Enable MPI
   --mpi-mode [serialized, multiple] -- MPI mode
  --seed (num) -- Random number generator seed
   --experiment (experiment) -- Experiment module
  --test-workers [(num),default] -- Test program StarPU worker count

--blas-threads [(num),default] -- Test program BLAS thread count

--lapack-threads [(num),default] -- LAPACK solver thread count
     scalapack-threads [(num),default] -- ScaLAPACK solver thread count
Available experiment modules:
      'hessenberg' : Hessenberg reduction experiment
     'schur' : Schur reduction experiment
'reorder' : Eigenvalue reordering experiment
     'eigenvectors': Eigenvectors experiment
'full-chain': Full chain experiment
     'partial-hessenberg' : Partial Hessenberg reduction experiment
     ^{\prime} validator ^{\prime} : Validation experiment
Experiment module (hessenberg) specific options:
  --data-format (format) -- Data format
--init (initializer) -- Initialization module
   --solver (solver) -- Solver module
   --hooks (hook:mode, ...) -- Hooks and modes
  --no-reinit -- Do not reinitialize after each repetition
  --repeat (num) -- Repeated experiment --warmup (num) -- Perform "warmups"
  --keep-going -- Try to recover from a solver failure

--abort -- Call abort() in failure
```

28 Test program

The overall design of the test program is modular. Each experiment module is built on *initializers*, *solvers* and *hooks*. Each experiment module contains several of each, thus allowing a user to initialize the inout data in various ways and compare different solvers with each other. Each of these building blocks can be configured with various command line arguments. However, in most situations only the problem dimension --n (num) needs to be specified.

Hooks are used to test and validate the output of the solver. For example, ——hooks hessenberg residual print option enables hooks that

- · check whether the output matrix is in upper Hessenberg form;
- computes the related residuals using Frobenius norm (reported in terms of unit roundoff error) and checks that they are within the permissible limits; and
- · prints the output matrices.

Certain general purpose initializers allow a user to read the input data from a disk (read-mtx and read-raw) and output data can be stored to a disk using a suitable post-processing hook (store-raw).

The test program supports various data formats. For example, shared memory experiments are usually performed using the pencil-local data format which stores the matrices continuously in the main memory. Distributed memory experiments are performed using either a StarNEig library specific distributed data format (pencil-starneig) and the BLACS data format (pencil-starneig-blacs). The test program will detect the correct data format automatically. The test program is also is capable of converting the data between various data formats. The related data converter modules can be in most cases configured using additional command line arguments. For example, the distributed data formats distribute the data in rectangular sections and the section size can be set with command line arguments --section-height (num) and --section-width (num)).

#### Performance models

The StarPU performance models must be calibrated before the software can function efficiently on heterogeneous platforms (CPUs+GPUs). The calibration is triggered automatically if the models are not calibrated well enough for a given problem size. This can impact the execution time negatively. The test program provides an easy-to-use solution for this problem:

```
$ ./starneig-test ... --warmup 3
```

The --warmup (number) argument causes the test program to perform a number of warm-up runs before the actual execution time measurement takes place.

Please see the StarPU handbook for further instructions:  $http://starpu.gforge.inria. \leftarrow fr/doc/html/Scheduling.html$ 

### **Examples**

• Reorder a 4000 x 4000 matrix using the StarNEig implementation:

```
$ ./starneig-test --experiment reorder --n 4000

TEST: --seed 1585762840 --experiment reorder --test-workers default --blas-threads default --lapack-threads default --scalapack-threads default --data-format pencil-local --init default --n 4000
--complex-distr uniform --complex-ratio 0.500000 --zero-ratio 0.010000 --inf-ratio 0.010000
--data-distr default --section-height default --section-width default --select-ratio 0.350000
--solver starneig --cores default --gpus default --tile-size default --window-size default
--values-per-chain default --small-window-size default --small-window-threshold default
--update-width default --update-height default --plan default --blueprint default --hooks
schur:normal eigenvalues:normal analysis:normal reordering:normal residual:normal
--eigenvalues-fail-threshold 10000 --eigenvalues-warn-threshold 1000 --reordering-fail-threshold
10000 --reordering-warn-threshold 1000 --residual-fail-threshold 10000 --residual-warn-threshold 500
--repeat 1 --warmup 0

THREADS: Using 6 StarPU worker threads during initialization and validation.
```

```
THREADS: Using 6 BLAS threads during initialization and validation.
THREADS: Using 6 BLAS threads in LAPACK solvers.
THREADS: Using 1 BLAS threads in Scalapack solvers.
INIT...
PREPARE...
PROCESS..
[starneig] [message] Setting tile size to 192.
[starneig][message] Using multi-part task insertion plan.
[starneig][message] Using two-pass backward dummy blueprint.
[starneig][message] Using "rounded" window size.
EXPERIMENT TIME = 1701 MS
FINALIZE...
EIGENVALUES CHECK: mean = 0 u, min = 0 u, max = 0 u
EIGENVALUES ANALYSIS: zeros = 36, infinities = 0, indefinites = 0
EIGENVALUES ANALYSIS: close zeros = 0, close infinities = 0, close indefinites = 0
REORDERING CHECK: Checking selected eigenvalues...
REORDERING CHECK: Checking other eigenvalues...
REORDERING CHECK: mean = 71 u, min = 0 u, max = 526 u | Q ~A Q^T - A| / |A| = 314 u
|Q Q^T - I| / |I| = 140 u
TIME = 1701 MS [avg 1701 MS, cv 0.00, min 1701 MS, max 1701 MS]
NO FAILED SCHUR FORM TESTS
EIGENVALUES CHECK (WARNINGS): 0 runs effected [avg 0.0, cv 0.00, min 0, max 0]
EIGENVALUES CHECK (FAILS): 0 runs effected [avg 0.0, cv 0.00, min 0, max 0]
EIGENVALUES CHECK (MEANS): [avg 0 u, cv 0.00, min 0 u, max 0 u]
EIGENVALUES CHECK (MIN): [avg 0 u, cv 0.00, min 0 u, max 0 u]
EIGENVALUES CHECK (MAX): [avg 0 u, cv 0.00, min 0 u, max 0 u]
EIGENVALUES ANALYSIS (ZEROS): [avg 36.0, cv 0.00, min 36, max 36]
EIGENVALUES ANALYSIS (CLOSE ZEROS): [avg 0.0, cv 0.00, min 0, max 0]
EIGENVALUES ANALYSIS (INFINITIES): [avg 0.0, cv 0.00, min 0, max 0]
EIGENVALUES ANALYSIS (CLOSE INFINITIES): [avg 0.0, cv 0.00, min 0, max 0]
EIGENVALUES ANALYSIS (INDEFINITES): [avg 0.0, cv 0.00, min 0, max 0]
EIGENVALUES ANALYSIS (CLOSE INDEFINITES): [avg 0.0, cv 0.00, min 0, max 0]
REORDERING CHECK (WARNINGS): 0 runs effected [avg 0.0, cv 0.00, min 0, max 0] REORDERING CHECK (FAILS): 0 runs effected [avg 0.0, cv 0.00, min 0, max 0]
REORDERING CHECK (MEANS): [avg 71 u, cv 0.00, min 71 u, max 71 u]
REORDERING CHECK (MIN): [avg 0 u, cv 0.00, min 0 u, max 0 u]
REORDERING CHECK (MAX): [avg 526 u, cv 0.00, min 526 u, max 526 u]
\begin{array}{l|l} |Q \sim A \ Q \cap T - A| \ / \ |A| = [avg \ 314 \ u, \ cv \ 0.00, \ min \ 314 \ u, \ max \ 314 \ u] \\ |Q \ Q \cap T - I| \ / \ |I| = [avg \ 140 \ u, \ cv \ 0.00, \ min \ 140 \ u, \ max \ 140 \ u] \end{array}
```

• Reorder a 4000 x 4000 matrix stencil (A,B) using the StarNEig implementation, initialize the random number generator using the seed 1480591971, fortify the matrix stencil (A,B) against failed swaps, and disable GPUs:

```
\$ ./starneig-test --experiment reorder --n 4000 --generalized --seed 1480591971 --fortify --gpus 0
TEST: --seed 1480591971 --experiment reorder --test-workers default --blas-threads default --lapack-threads
        default --scalapack-threads default --data-format pencil-local --init default --n 4000 --generalized --complex-distr uniform --fortify --data-distr default --section-height default --section-width default --select-ratio 0.350000 --solver starneig --cores default --gpus 0 --tile-size default --window-size default --small-window-threshold
        default --update-width default --update-height default --plan default --blueprint default --hooks
        schur:normal eigenvalues:normal analysis:normal reordering:normal residual:normal
         --eigenvalues-fail-threshold 10000 --eigenvalues-warn-threshold 1000 --reordering-fail-threshold
        10000 --reordering-warn-threshold 1000 --residual-fail-threshold 10000 --residual-warn-threshold 500
         --repeat 1 --warmup 0
THREADS: Using 6 StarPU worker threads during initialization and validation.
THREADS: Using 6 BLAS threads during initialization and validation.
THREADS: Using 6 BLAS threads in LAPACK solvers.
THREADS: Using 1 BLAS threads in ScaLAPACK solvers.
TNTT...
PREPARE...
PROCESS..
[starneig] [message] Setting tile size to 192.
[starneig][message] Using multi-part task insertion plan.
[starneig][message] Using two-pass backward dummy blueprint.
[starneig][message] Using "rounded" window size.
EXPERIMENT TIME = 7472 MS
FINALIZE...
EIGENVALUES CHECK: mean = 0 u, min = 0 u, max = 0 u
EIGENVALUES ANALYSIS: zeros = 0, infinities = 0, indefinites = 0
EIGENVALUES ANALYSIS: close zeros = 0, close infinities = 0, close indefinites = 0
REORDERING CHECK: Checking selected eigenvalues...
REORDERING CHECK: Checking other eigenvalues...
REORDERING CHECK: mean = 23 u, min = 0 u, max = 169 u
|Q \sim A \times T - A| / |A| = 46 u
|Q \sim B \times T - B| / |B| = 63 u
|Q Q^T - I| / |I| = 44 u
|Z Z^T - I| / |I| = 43 u
TIME = 7472 MS [avg 7472 MS, cv 0.00, min 7472 MS, max 7472 MS]
NO FAILED SCHUR FORM TESTS
EIGENVALUES CHECK (WARNINGS): 0 runs effected [avg 0.0, cv 0.00, min 0, max 0]
```

30 Test program

```
EIGENVALUES CHECK (FAILS): 0 runs effected [avg 0.0, cv 0.00, min 0, max 0] 
EIGENVALUES CHECK (MEANS): [avg 0 u, cv 0.00, min 0 u, max 0 u] 
EIGENVALUES CHECK (MIN): [avg 0 u, cv 0.00, min 0 u, max 0 u] 
EIGENVALUES CHECK (MIN): [avg 0 u, cv 0.00, min 0 u, max 0 u] 
EIGENVALUES CHECK (MX): [avg 0 u, cv 0.00, min 0 u, max 0 u] 
EIGENVALUES ANALYSIS (ZEROS): [avg 0.0, cv 0.00, min 0, max 0] 
EIGENVALUES ANALYSIS (CLOSE ZEROS): [avg 0.0, cv 0.00, min 0, max 0] 
EIGENVALUES ANALYSIS (INFINITIES): [avg 0.0, cv 0.00, min 0, max 0] 
EIGENVALUES ANALYSIS (INDEFINITES): [avg 0.0, cv 0.00, min 0, max 0] 
EIGENVALUES ANALYSIS (INDEFINITES): [avg 0.0, cv 0.00, min 0, max 0] 
EIGENVALUES ANALYSIS (CLOSE INDEFINITES): [avg 0.0, cv 0.00, min 0, max 0] 
REORDERING CHECK (WARNINGS): 0 runs effected [avg 0.0, cv 0.00, min 0, max 0] 
REORDERING CHECK (MEANS): [avg 23 u, cv 0.00, min 23 u, max 23 u] 
REORDERING CHECK (MIN): [avg 0 u, cv 0.00, min 169 u, max 169 u] 
|Q ~A Z^T - A| / |A| = [avg 46 u, cv 0.00, min 46 u, max 46 u] 
|Q ~B Z^T - B| / |B| = [avg 63 u, cv 0.00, min 63 u, max 63 u] 
|Q Q^T - I| / |II = [avg 44 u, cv 0.00, min 44 u, max 44 u] 
|Z Z^T - I| / |II = [avg 43 u, cv 0.00, min 43 u, max 43 u]
```

 Reduce a dense matrix to upper Hessenberg form, validate the output and store the output matrices to the disk:

```
$ ./starneig-test --experiment hessenberg --n 4000 --hooks hessenberg residual store-raw --store-raw-output
         hessenberg_%s.dat
TEST: --seed 1585762935 --experiment hessenberg --test-workers default --blas-threads default
         --lapack-threads default --scalapack-threads default --data-format pencil-local --init default --n 4000 --data-distr default --section-height default --section-width default --solver starneig --cores
         default --gpus default --tile-size default --panel-width default --hooks hessenberg:normal residual:normal store-raw:normal --residual-fail-threshold 10000 --residual-warn-threshold 500
          --store-raw-output hessenberg_%s.dat --repeat 1 --warmup 0
THREADS: Using 6 StarPU worker threads during initialization and validation.
THREADS: Using 6 BLAS threads during initialization and validation.
THREADS: Using 6 BLAS threads in LAPACK solvers.
THREADS: Using 1 BLAS threads in ScaLAPACK solvers.
INIT...
PREPARE.
PROCESS..
[starneig] [message] Setting tile size to 336.
[starneig][message] Setting panel width to 288.
EXPERIMENT TIME = 13121 MS
FINALIZE...
|Q \sim A Q^T - A| / |A| = 15 u
|Q Q^T - I| / |I| = 11 u
WRITING TO hessenberg_A.dat...
WRITING TO hessenberg_Q.dat...
WRITING TO hessenberg CA.dat...
TIME = 13121 MS [avg 13121 MS, cv 0.00, min 13121 MS, max 13121 MS]
NO FAILED HESSENBERG FORM TESTS
\begin{array}{l|l} |Q \sim A \ Q^T - A| \ / \ |A| = [avg \ 15 \ u, \ cv \ 0.00, \ min \ 15 \ u, \ max \ 15 \ u] \\ |Q \ Q^T - I| \ / \ |I| = [avg \ 11 \ u, \ cv \ 0.00, \ min \ 11 \ u, \ max \ 11 \ u] \end{array}
```

Read an upper Hessenberg matrix from the disk, reduce it to Schur form and set tile size to 128:

```
--experiment schur --init read-raw --input hessenberg_%s.dat --tile-size 128
TEST: --seed 1585762972 --experiment schur --test-workers default --blas-threads default --lapack-threads default --scalapack-threads default --data-format pencil-local --init read-raw --input
        hessenberg_%s.dat --data-distr default --section-height default --section-width default --solver
        starneig --cores default --gpus default --tile-size 128 --small-limit
        default --aed-window-size default --aed-shift-count default --aed-nibble default --aed-parallel-soft-limit default --aed-parallel-hard-limit default --window-size default
        --shifts-per-window default --update-width default --update-height default --left-threshold default --right-threshold default --hooks schur:normal eigenvalues:normal
        known-eigenvalues:normal analysis:normal residual:normal --eigenvalues-fail-threshold 10000
        --eigenvalues-warn-threshold 1000 --known-eigenvalues-fail-threshold 100000 --known-eigenvalues-warn-threshold 100000 --residual-fail-threshold 10000 --residual-warn-threshold
        500 --repeat 1 --warmup 0
THREADS: Using 6 StarPU worker threads during initialization and validation.
THREADS: Using 6 BLAS threads during initialization and validation.
THREADS: Using 6 BLAS threads in LAPACK solvers.
THREADS: Using 1 BLAS threads in ScaLAPACK solvers.
INIT...
READING FROM hessenberg_A.dat...
READING A 4000 X 4000 MATRIX ...
READING FROM hessenberg_Q.dat...
READING A 4000 X 4000 MATRIX ...
READING FROM hessenberg_CA.dat...
READING A 4000 X 4000 MATRIX ...
PREPARE...
[starneig] [message] Using AED windows size 320.
```

```
[starneig][message] Using 240 shifts.
EXPERIMENT TIME = 9479 MS
FINALTZE
EIGENVALUES CHECK: mean = 0 u, min = 0 u, max = 0 u
KNOWN EIGENVALUES CHECK: The stored pencil does not contain the known eigenvalues. Skipping.
EIGENVALUES ANALYSIS: zeros = 0, infinities = 0, indefinites = 0
EIGENVALUES ANALYSIS: close zeros = 0, close infinities = 0, close indefinites = 0
|Q \sim A Q^T - A| / |A| = 68 u
|Q Q^T - I| / |I| = 89 u
TIME = 9479 MS [avg 9479 MS, cv 0.00, min 9479 MS, max 9479 MS]
NO FAILED SCHUR FORM TESTS
EIGENVALUES CHECK (WARNINGS): 0 runs effected [avg 0.0, cv 0.00, min 0, max 0]
EIGENVALUES CHECK (FAILS): 0 runs effected [avg 0.0, cv 0.00, min 0, max 0]
EIGENVALUES CHECK (MEANS): [avg 0 u, cv 0.00, min 0 u, max 0 u]
EIGENVALUES CHECK (MIN): [avg 0 u, cv 0.00, min 0 u, max 0 u] EIGENVALUES CHECK (MAX): [avg 0 u, cv 0.00, min 0 u, max 0 u]
EIGENVALUES ANALYSIS (ZEROS): [avg 0.0, cv 0.00, min 0, max 0]
EIGENVALUES ANALYSIS (CLOSE ZEROS): [avg 0.0, cv 0.00, min 0, max 0]
EIGENVALUES ANALYSIS (INFINITIES): [avg 0.0, cv 0.00, min 0, max 0]
EIGENVALUES ANALYSIS (CLOSE INFINITIES): [avg 0.0, cv 0.00, min 0, max 0]
EIGENVALUES ANALYSIS (INDEFINITES): [avg 0.0, cv 0.00, min 0, max 0]
EIGENVALUES ANALYSIS (CLOSE INDEFINITES): [avg 0.0, cv 0.00, min 0, max 0] |Q \sim A Q \sim T - A| / |A| = [avg 68 u, cv 0.00, min 68 u, max 68 u] |Q <math>\sim T - I| / |I| = [avg 89 u, cv 0.00, min 89 u, max 89 u]
```

# Distributed memory example

• Reorder a 4000 x 4000 matrix using the StarNEig implementation, use two MPI ranks, use three CPU cores per rank, distribute the matrix in 1024 x 1024 sections, and use tile size 256:

```
$ mpirun -n 2 --map-by :PE=3 ./starneig-test --mpi --experiment reorder --n 4000 --section-height 1024
        -section-width 1024 --tile-size 256
```

#### Rank 0 output:

```
MPI INIT..
              --mpi-mode serialized --seed 1585763077 --experiment reorder --test-workers default
        --blas-threads default --lapack-threads default --scalapack-threads default --data-format pencil-starneig-blacs --init default --n 4000 --complex-distr uniform --complex-ratio 0.500000
        -zero-ratio 0.010000 --inf-ratio 0.010000 --data-distr default --section-height 1024 --section-width 1024 --select-ratio 0.350000 --solver starneig --cores default --gpus default --tile-size 256
          -window-size default --values-per-chain default --small-window-size default --small-window-threshold
        default --update-width default --update-height default --plan default --blueprint default --hooks
        schur:normal eigenvalues:normal analysis:normal reordering:normal residual:normal
         --eigenvalues-fail-threshold 10000 --eigenvalues-warn-threshold 1000 --reordering-fail-threshold
        10000 --reordering-warn-threshold 1000 --residual-fail-threshold 10000 --residual-warn-threshold 500
          -repeat 1 --warmup 0
THREADS: Using 3 StarPU worker threads during initialization and validation.
THREADS: Using 3 BLAS threads during initialization and validation.
THREADS: Using 3 BLAS threads in LAPACK solvers.
THREADS: Using 1 BLAS threads in ScaLAPACK solvers.
INIT...
PREPARE...
PROCESS...
[starneig][message] Attempting to set tile size to 256.
[starneig] [message] Setting tile size to 256.
[starneig][message] Using multi-part task insertion plan.
[starneig][message] Using two-pass backward dummy blueprint. [starneig][message] Using "rounded" window size.
EXPERIMENT TIME = 3320 MS
EIGENVALUES CHECK: mean = 0 u, min = 0 u, max = 0 u
EIGENVALUES ANALYSIS: zeros = 37, infinities = 0, indefinites = 0
EIGENVALUES ANALYSIS: close zeros = 0, close infinities = 0, close indefinites = 0
REORDERING CHECK: Checking selected eigenvalues...
REORDERING CHECK: Checking other eigenvalues...
REORDERING CHECK: mean = 70 u, min = 0 u, max = 527 u
|Q \sim A Q^T - A| / |A| = 314 u
|Q Q^T - I| / |I| = 139 u
TIME = 3319 MS [avg 3320 MS, cv 0.00, min 3320 MS, max 3320 MS]
NO FAILED SCHUR FORM TESTS
EIGENVALUES CHECK (WARNINGS): 0 runs effected [avg 0.0, cv 0.00, min 0, max 0]
EIGENVALUES CHECK (FAILS): 0 runs effected [avg 0.0, cv 0.00, min 0, max 0]
EIGENVALUES CHECK (MEANS): [avg 0 u, cv 0.00, min 0 u, max 0 u]
EIGENVALUES CHECK (MIN): [avg 0 u, cv 0.00, min 0 u, max 0 u]
EIGENVALUES CHECK (MAX): [avg 0 u, cv 0.00, min 0 u, max 0 u]
EIGENVALUES ANALYSIS (ZEROS): [avg 37.0, cv 0.00, min 37, max 37]
EIGENVALUES ANALYSIS (CLOSE ZEROS): [avg 0.0, cv 0.00, min 0, max 0]
EIGENVALUES ANALYSIS (INFINITIES): [avg 0.0, cv 0.00, min 0, max 0]
```

32 Test program

```
EIGENVALUES ANALYSIS (CLOSE INFINITIES): [avg 0.0, cv 0.00, min 0, max 0] EIGENVALUES ANALYSIS (INDEFINITES): [avg 0.0, cv 0.00, min 0, max 0] EIGENVALUES ANALYSIS (CLOSE INDEFINITES): [avg 0.0, cv 0.00, min 0, max 0] REORDERING CHECK (WARNINGS): 0 runs effected [avg 0.0, cv 0.00, min 0, max 0] REORDERING CHECK (FAILS): 0 runs effected [avg 0.0, cv 0.00, min 0, max 0] REORDERING CHECK (MEANS): [avg 70 u, cv 0.00, min 70 u, max 70 u] REORDERING CHECK (MIN): [avg 0 u, cv 0.00, min 0 u, max 0 u] REORDERING CHECK (MAX): [avg 527 u, cv 0.00, min 527 u, max 527 u] |Q ~A Q^T - A| / |A| = [avg 314 u, cv 0.00, min 314 u, max 314 u] |Q Q^T - I| / |I| = [avg 139 u, cv 0.00, min 139 u, max 139 u]
```

```
Rank 1 output:
MPI INIT..
TEST: --mpi
               --mpi-mode serialized --seed 1585763077 --experiment reorder --test-workers default
        -blas-threads default --lapack-threads default --scalapack-threads default --data-format pencil-starneig-blacs --init default --n 4000 --complex-distr uniform --complex-ratio 0.500000
        -zero-ratio 0.010000 --inf-ratio 0.010000 --data-distr default --section-height 1024 --section-width 1024 --select-ratio 0.350000 --solver starneig --cores default --gpus default --tile-size 256 --window-size default --values-per-chain default --small-window-size default --small-window-threshold
        default --update-width default --update-height default --plan default --blueprint default --hooks
        schur:normal eigenvalues:normal analysis:normal reordering:normal residual:normal
         --eigenvalues-fail-threshold 10000 --eigenvalues-warn-threshold 1000 --reordering-fail-threshold
        10000 --reordering-warn-threshold 1000 --residual-fail-threshold 10000 --residual-warn-threshold 500
        --repeat 1 --warmup 0
THREADS: Using 3 StarPU worker threads during initialization and validation.
THREADS: Using 3 BLAS threads during initialization and validation.
THREADS: Using 3 BLAS threads in LAPACK solvers.
THREADS: Using 1 BLAS threads in Scalapack solvers.
INIT...
PREPARE...
PROCESS...
[starneig][message] Attempting to set tile size to 256.
[starneig][message] Setting tile size to 256.
[starneig][message] Using multi-part task insertion plan.
[starneig][message] Using two-pass backward dummy blueprint.
[starneig][message] Using "rounded" window size.
EXPERIMENT TIME = 3320 MS
FINALIZE...
EIGENVALUES CHECK: mean = 0 \text{ u}, min = 0 \text{ u}, max = 0 \text{ u}
EIGENVALUES ANALYSIS: zeros = 37, infinities = 0, indefinites = 0
EIGENVALUES ANALYSIS: close zeros = 0, close infinities = 0, close indefinites = 0
REORDERING CHECK: Checking selected eigenvalues...
REORDERING CHECK: Checking other eigenvalues...
REORDERING CHECK: mean = 70 u, min = 0 u, max = 527 u
|Q ~A Q^T - A| / |A| = 314 u
|Q Q^T - I| / |I| = 139 u
TIME = 3319 MS [avg 3320 MS, cv 0.00, min 3320 MS, max 3320 MS]
NO FAILED SCHIR FORM TESTS
EIGENVALUES CHECK (WARNINGS): 0 runs effected [avg 0.0, cv 0.00, min 0, max 0]
EIGENVALUES CHECK (FAILS): 0 runs effected [avg 0.0, cv 0.00, min 0, max 0]
EIGENVALUES CHECK (MEANS): [avg 0 u, cv 0.00, min 0 u, max 0 u]
EIGENVALUES CHECK (MIN): [avg 0 u, cv 0.00, min 0 u, max 0 u]
EIGENVALUES CHECK (MAX): [avg 0 u, cv 0.00, min 0 u, max 0 u]
EIGENVALUES ANALYSIS (ZEROS): [avg 37.0, cv 0.00, min 37, max 37]
EIGENVALUES ANALYSIS (CLOSE ZEROS): [avg 0.0, cv 0.00, min 0, max 0] EIGENVALUES ANALYSIS (INFINITIES): [avg 0.0, cv 0.00, min 0, max 0]
EIGENVALUES ANALYSIS (CLOSE INFINITIES): [avg 0.0, cv 0.00, min 0, max 0]
EIGENVALUES ANALYSIS (INDEFINITES): [avg 0.0, cv 0.00, min 0, max 0]
EIGENVALUES ANALYSIS (CLOSE INDEFINITES): [avg 0.0, cv 0.00, min 0, max 0]
REORDERING CHECK (WARNINGS): 0 runs effected [avg 0.0, cv 0.00, min 0, max 0]
REORDERING CHECK (FAILS): 0 runs effected [avg 0.0, cv 0.00, min 0, max 0] REORDERING CHECK (MEANS): [avg 70 u, cv 0.00, min 70 u, max 70 u] REORDERING CHECK (MIN): [avg 0 u, cv 0.00, min 0 u, max 0 u]
REORDERING CHECK (MAX): [avg 527 u, cv 0.00, min 527 u, max 527 u]
```

# **Chapter 9**

# Known problems and changelog

# **Known problems**

- If the CUDA support is enabled, then the starneig\_SEP\_SM\_Hessenberg() interface function cannot always handle problems that do not fit into GPU's memory. The cause of this problem is not known.
- The outputs of the starneig\_GEP\_SM\_Schur() and starneig\_GEP\_DM\_Schur() interface functions are not always in the so-called standard format. It is possible that some diagonal entries in the right-hand side output matrix are negative.
- The starneig GEP SM Eigenvectors() interface function may scale the input matrices.

## Known compatibility problems

#### **BLAS**

- With some OpenBLAS versions, it is necessary to set the OPENBLAS\_NUM\_THREADS environmental variable to value 1 (export OPENBLAS\_NUM\_THREADS=1).
- Some MKL versions can cause poor scalability. The problem appears to be related to Intel's OpenMP library. Setting the KMP\_AFFINITY environmental variable to value disabled fixes the problem (export K MP\_AFFINITY=disabled).
- OpenBLAS version 0.3.1 has a bug that can cause an incorrect result.
- OpenBLAS versions 0.3.3-0.3.5 can cause poor scalability.

#### MPI

- Some older OpenMPI versions (<= 2.1.1) have a bug that can cause a segmentation fault during a parallel AED.
- The library has an unsolved memory leak problem with OpenMPI. Only large problem sizes are effected. It is not known whether this problem is related to StarNEig, StarPU, OpenMPI or something else. The problem is known to occur with PMIx 2.2.1, UCX 1.5.0, OpenMPI 3.1.3, and StarPU 1.2.8. The memory leak is sometimes accompanied by the following warning:
  - mpool.c:38 UCX WARN object 0x2652000 was not returned to mpool ucp\_requests
- The test program can trigger the following bug in UCX 1.6.1: https://github.com/openucx/ucx/issues/4525

#### **StarPU**

- For optimal CUDA performance, StarPU version that is newer than 1.3.3 is recommended.
- StarPU versions 1.2.4 1.2.8 and some StarPU 1.3 snapshots cav cause poor CUDA performance. The problem can be fixed by compiling StarPU with --disable-cuda-memcpy-peer. It is possible that newer versions of StarPU are also effected by this problem.
- The STARPU\_MINIMUM\_AVAILABLE\_MEM and STARPU\_TARGET\_AVAILABLE\_MEM environmental variables can be used to fix some GPU-related memory allocation problems:

  STARPU\_MINIMUM\_AVAILABLE\_MEM=10 STARPU\_TARGET\_AVAILABLE\_MEM=15 ...

# Changelog

# Planned for v0.2.0-beta.1:

- · New experimental Hessenberg reduction code with distributed memory and multi-GPU support.
- · Fix a bug that may cause the code to hang in distributed memory Schur reduction.
- · Remove deprecated interface functions.
- · Improved performance models.
- · Disable OpenCL workers.
- Rename aed\_shift\_count parameter as shift\_count. Rename the default value STARNEIG\_S ← CHUR\_DEFAULT\_AED\_SHIFT\_COUNT as STARNEIG\_SCHUR\_DEFAULT\_SHIFT\_COUNT.
- Rename STARPU\_LIBRARIES\_BASE and STARPU\_LIBRARIES\_MPI environmental variables as S

  TARPU\_LIBRARIES and STARPU\_MPI\_LIBRARIES, respectively.
- · Updates to the documentation.
- · Add deb packages.

## v0.1.0:

· First stable release of the library.

#### v0.1-beta.6:

- Fix starneig\_node\_enable\_pinning() and starneig\_node\_disable\_pinning() functions.
- · Built pdgghrd as a separate library.
- Deprecate several precompiler defines and interface functions. Add <starneig/distr\_helpers.h> header file.

## v0.1-beta.5:

- Improve the performance of the Hessenberg reduction phase by limiting the number of submitted tasks. This should reduce the task scheduling overhead.
- Allocate pinned memory by default when CUDA support is enabled. Add starneig\_enable\_
   pinning() and starneig\_disable\_pinning().

# v0.1-beta.4:

- Fix a problem where infinite eigenvalues were detected too late.
- Add an option to choose between the norm stable deflation condition (STARNEIG\_SCHUR\_NORM\_STAB ← LE\_THRESHOLD) and and the LAPACK style deflation condition (STARNEIG\_SCHUR\_LAPACK\_THRE ← SHOLD).

## v0.1-beta.3:

• Re-implemented Hessenberg reduction.

# v0.1-beta.2:

- Fix an installation-related bug.
- Fix a MPI-related compile error.
- · Remove unused code.

# v0.1-beta.1:

· First beta release of the library.

# **Chapter 10**

# License, authors and literature

## License

Copyright (c) 2019-2020, Umeå Universitet

Redistribution and use in source and binary forms, with or without modification, are permitted provided that the following conditions are met:

- 1. Redistributions of source code must retain the above copyright notice, this list of conditions and the following disclaimer.
- 2. Redistributions in binary form must reproduce the above copyright notice, this list of conditions and the following disclaimer in the documentation and/or other materials provided with the distribution.
- 3. Neither the name of the copyright holder nor the names of its contributors may be used to endorse or promote products derived from this software without specific prior written permission.

THIS SOFTWARE IS PROVIDED BY THE COPYRIGHT HOLDERS AND CONTRIBUTORS "AS IS" AND ANY EXPRESS OR IMPLIED WARRANTIES, INCLUDING, BUT NOT LIMITED TO, THE IMPLIED WARRANTIES OF MERCHANTABILITY AND FITNESS FOR A PARTICULAR PURPOSE ARE DISCLAIMED. IN NO EVENT SHALL THE COPYRIGHT HOLDER OR CONTRIBUTORS BE LIABLE FOR ANY DIRECT, INDIRECT, INCIDENTAL, SPECIAL, EXEMPLARY, OR CONSEQUENTIAL DAMAGES (INCLUDING, BUT NOT LIMITED TO, PROCUREMENT OF SUBSTITUTE GOODS OR SERVICES LOSS OF USE, DATA, OR PROFITS OR BUSINESS INTERRUPTION) HOWEVER CAUSED AND ON ANY THEORY OF LIABILITY, WHETHER IN CONTRACT, STRICT LIABILITY, OR TORT (INCLUDING NEGLIGENCE OR OTHERWISE) ARISING IN ANY WAY OUT OF THE USE OF THIS SOFTWARE, EVEN IF ADVISED OF THE POSSIBILITY OF SUCH DAMAGE.

## **Authors**

The following people have contributed to the StarNEig library:

- Angelika Schwarz ( angies@cs.umu.se)
  - Standard eigenvectors
- Bo Kågström ( bokg@cs.umu.se)

- Coordinator and scientific director for the NLAFET project
- Documentation
- Carl Christian Kjelgaard Mikkelsen ( spock@cs.umu.se)
  - Generalized eigenvectors
- Lars Karlsson ( larsk@cs.umu.se)
  - Miscellaneous user interface functions
  - Documentation
- Mirko Myllykoski ( mirkom@cs.umu.se)
  - Hessenberg reduction
  - Schur reduction (standard and generalized)
  - Eigenvalue reordering (standard and generalized)
  - Miscellaneous user interface functions
  - Test program
  - Documentation

The distributed memory Hessenberg-triangular reduction is implemented as a ScaLAPACK wrapper and PDGGHRD subroutine (Lars Karlsson, Björn Adlerborn) is included with the library. The test program includes Matrix Market I/O library for ANSI C ( http://math.nist.gov/MatrixMarket).

# Related publications

## Research papers

- Mirko Myllykoski: A Task-based Multi-shift QR/QZ Algorithm with Aggressive Early Deflation, arXiv←:2007.03576
- Mirko Myllykoski, Carl Christian Kjelgaard Mikkelsen: Task-based, GPU-accelerated and Robust Library for Solving Dense Nonsymmetric Eigenvalue Problems, Concurrency and Computation: Practice and Experience, 2020, doi: 10.1002/cpe.5915
- Mirko Myllykoski, Carl Christian Kjelgaard Mikkelsen: Introduction to StarNEig A Task-based Library for Solving Nonsymmetric Eigenvalue Problems, In Parallel Processing and Applied Mathematics, 13th International Conference, PPAM 2019, Bialystok, Poland, September 8–11, 2019, Revised Selected Papers, Part I, Lecture Notes in Computer Science, Vol. 12043, Wyrzykowski R., Deelman E., Dongarra J., Karczewski K. (eds), Springer International Publishing, pp. 70-81, 2020, doi: 10.1007/978-3-030-43229-4\_7
- Carl Christian Kjelgaard Mikkelsen, Mirko Myllykoski: Parallel Robust Computation of Generalized Eigenvectors of Matrix Pencils, presented at PPAM 2019, In Parallel Processing and Applied Mathematics, 13th International Conference, PPAM 2019, Bialystok, Poland, September 8–11, 2019, Revised Selected Papers, Part I, Lecture Notes in Computer Science, Vol. 12043, Wyrzykowski R., Deelman E., Dongarra J., Karczewski K. (eds), Springer International Publishing, pp. 58-69, 2020, doi: 10.1007/978-3-030-43229-4\_6
- Carl Christian Kjelgaard Mikkelsen, Angelika Schwarz, Lars Karlsson: *Parallel Robust Solution of Triangular Linear Systems*, Concurrency and Computation: Practice and Experience, 31 (19), 2019, doi: 10.← 1002/cpe.5064
- Angelika Schwarz, Lars Karlsson: Scalable eigenvector computation for the non-symmetric eigenvalue problem, Parallel Computing, 85, pp. 131-140, 2019, doi: 10.1016/j.parco.2019.04.001

- Mirko Myllykoski: A Task-Based Algorithm for Reordering the Eigenvalues of a Matrix in Real Schur Form, In Parallel Processing and Applied Mathematics, 12th International Conference, PPAM 2017, Lublin, Poland, September 10-13, 2017, Revised Selected Papers, Part I, Lecture Notes in Computer Science, Vol. 10777, Wyrzykowski R., Dongarra J., Deelman E., Karczewski K. (eds), Springer International Publishing, pp. 207-216, 2018, doi: 10.1007/978-3-319-78024-5\_19
- Carl Christian Kjelgaard Mikkelsen, Lars Karlsson. Blocked Algorithms for Robust Solution of Triangular Linear Systems, In Parallel Processing and Applied Mathematics, 12th International Conference, PPAM 2017, Lublin, Poland, September 10-13, 2017, Revised Selected Papers, Part I, Lecture Notes in Computer Science, Vol. 10777, Wyrzykowski R., Dongarra J., Deelman E., Karczewski K. (eds), Springer International Publishing, pp. 207-216, 2018, doi: 10.1007/978-3-319-78024-5\_7

# Reports, deliverables etc

- Angelika Schwarz, Carl Christian Kjelgaard Mikkelsen, Lars Karlsson: Robust Parallel Eigenvector Computation For the Non-Symmetric Eigenvalue Problem, Report UMINF 20.02, Department of Computing Science, Umeå University, SE-901 87 Umeå, Sweden, 2020 ( download)
- Angelika Schwarz: Towards efficient overflow-free solvers for systems of triangular type, Licentiate thesis, Department of computing science, Umeå University, ISSN: 0348-0542, 2019
- Mirko Myllykoski, Carl Christian Kjelgaard Mikkelsen, Angelika Schwarz, Bo Kågström: D2.7 Eigenvalue solvers for nonsymmetric problems, public NLAFET deliverable, 2019 ( download)
- Lars Karlsson, Mahmoud Eljammaly, Mirko Myllykoski: D6.5 Evaluation of auto-tuning techniques, public NLAFET deliverable, 2019 ( download)
- Bo Kågström et al.: D7.8 Release of the NLAFET library, public NLAFET deliverable, 2019 ( download)
- Mirko Myllykoski, Lars Karlsson, Bo Kågström, Mahmoud Eljammaly, Srikara Pranesh, Mawussi Zounon: D2.6 Prototype Software for Eigenvalue Problem Solvers, public NLAFET deliverable, 2018 ( download)
- Mirko Myllykoski, Carl Christian Kjelgaard Mikkelsen, Lars Karlsson, Bo Kågström: Task-Based Parallel Algorithms for Reordering of Matrices in Real Schur Forms, NLAFET Working Note WN-11, 2017. Also as Report UMINF 17.11, Department of Computing Science, Umeå University, SE-901 87 Umeå, Sweden (download)

#### Youtube videos

- Task-based Schur reduction, N = 20 000, shared memory, 28 cores
- Task-based Hessenberg reduction, N = 20 000, shared memory, 6 cores

| License, authors and lite | teratu | re |
|---------------------------|--------|----|
|---------------------------|--------|----|

# **Chapter 11**

# **Todo List**

Global starneig\_GEP\_DM\_Eigenvectors (int selected[], starneig\_distr\_matrix\_t S, starneig\_distr\_matrix\_t T, starneig\_distr\_matrix\_t Z, starneig\_distr\_matrix\_t X)

This interface function is not implemented.

Global starneig\_GEP\_DM\_Eigenvectors\_expert (struct starneig\_eigenvectors\_conf \*conf, int selected[], starneig\_distr\_matrix\_t S, starneig\_distr\_matrix\_t T, starneig\_distr\_matrix\_t Z, starneig\_distr\_matrix\_t X)

This interface function is not implemented.

Global starneig\_SEP\_DM\_Eigenvectors (int selected[], starneig\_distr\_matrix\_t S, starneig\_distr\_matrix\_t Q, starneig\_distr\_matrix\_t X)

This interface function is not implemented.

Global starneig\_SEP\_DM\_Eigenvectors\_expert (struct starneig\_eigenvectors\_conf \*conf, int selected[], starneig\_distr\_matrix\_t S, starneig\_distr\_matrix\_t X)

This interface function is not implemented.

42 **Todo List** 

# **Chapter 12**

# **Deprecated List**

# Global STARNEIG\_SEP\_DM\_HESSENBERG

The STARNEIG\_SEP\_DM\_HESSENBERG precompiler define is deprecated and will be removed in the next release of the library.

# Global STARNEIG\_SEP\_DM\_REDUCE

The STARNEIG\_SEP\_DM\_REDUCE precompiler define is deprecated and will be removed in the next release of the library.

44 **Deprecated List** 

# **Chapter 13**

# **Module Documentation**

# 13.1 Library configuration

Configuration of the installed library.

#### **Macros**

- #define STARNEIG\_VERSION\_MAJOR
  - StarNEig major version.
- #define STARNEIG\_VERSION\_MINOR
  - StarNEig minor version.
- #define STARNEIG\_VERSION\_PATCH
  - StarNEig patch level.
- #define STARNEIG\_ENABLE\_MPI
  - MPI support enabled.
- #define STARNEIG\_ENABLE\_CUDA
  - CUDA support enabled.
- #define STARNEIG\_ENABLE\_BLACS
  - BLACS support enabled.
- #define STARNEIG\_SEP\_DM\_HESSENBERG
  - Distributed memory Hessenberg reduction enabled.
- #define STARNEIG\_GEP\_DM\_HESSENBERGTRIANGULAR
  - Distributed memory Hessenberg-triangular reduction enabled.
- #define STARNEIG\_SEP\_DM\_REDUCE
  - Full distributed memory reduction enabled.
- #define STARNEIG\_GEP\_DM\_REDUCE

Full distributed memory generalized reduction enabled.

# 13.1.1 Detailed Description

Configuration of the installed library.

## 13.1.2 Macro Definition Documentation

# 13.1.2.1 STARNEIG\_ENABLE\_MPI

#define STARNEIG\_ENABLE\_MPI

MPI support enabled.

Defined if the library was compiled with MPI support.

## 13.1.2.2 STARNEIG\_ENABLE\_CUDA

#define STARNEIG\_ENABLE\_CUDA

CUDA support enabled.

Defined if the library was compiled with CUDA support.

## 13.1.2.3 STARNEIG\_ENABLE\_BLACS

#define STARNEIG\_ENABLE\_BLACS

BLACS support enabled.

Defined if the library was compiled with ScaLAPACK compatibility layer.

#### 13.1.2.4 STARNEIG\_SEP\_DM\_HESSENBERG

#define STARNEIG\_SEP\_DM\_HESSENBERG

Distributed memory Hessenberg reduction enabled.

Defined if the starneig\_SEP\_DM\_Hessenberg() function exists.

**Deprecated** The STARNEIG\_SEP\_DM\_HESSENBERG precompiler define is deprecated and will be removed in the next release of the library.

#### 13.1.2.5 STARNEIG\_GEP\_DM\_HESSENBERGTRIANGULAR

#define STARNEIG\_GEP\_DM\_HESSENBERGTRIANGULAR

Distributed memory Hessenberg-triangular reduction enabled.

Defined if the starneig\_GEP\_DM\_HessenbergTriangular() function exists.

# 13.1.2.6 STARNEIG\_SEP\_DM\_REDUCE

#define STARNEIG\_SEP\_DM\_REDUCE

Full distributed memory reduction enabled.

Defined if the starneig\_SEP\_DM\_Reduce() function exists.

**Deprecated** The STARNEIG\_SEP\_DM\_REDUCE precompiler define is deprecated and will be removed in the next release of the library.

# 13.1.2.7 STARNEIG\_GEP\_DM\_REDUCE

#define STARNEIG\_GEP\_DM\_REDUCE

Full distributed memory generalized reduction enabled.

Defined if the starneig\_GEP\_DM\_Reduce() function exists.

# 13.2 Error codes

Interface function return values and error codes.

#### **Macros**

• #define STARNEIG\_SUCCESS 0

Success.

• #define STARNEIG\_GENERIC\_ERROR 1

Reneric error.

• #define STARNEIG NOT INITIALIZED 2

Not initialized.

#define STARNEIG\_INVALID\_CONFIGURATION 3

Invalid configuration.

• #define STARNEIG INVALID ARGUMENTS 4

Invalid argument.

#define STARNEIG\_INVALID\_DISTR\_MATRIX 5

Invalid distributed matrix.

• #define STARNEIG\_DID\_NOT\_CONVERGE 6

Did not converge.

#define STARNEIG\_PARTIAL\_REORDERING 7

Partial reordering.

• #define STARNEIG\_CLOSE\_EIGENVALUES 8

Close eigenvalues.

# **Typedefs**

· typedef int starneig\_error\_t

Interface function return value data type.

# 13.2.1 Detailed Description

Interface function return values and error codes.

## 13.2.2 Macro Definition Documentation

# 13.2.2.1 STARNEIG\_SUCCESS

#define STARNEIG\_SUCCESS 0

Success.

The interface function was executed successfully.

13.2 Error codes 49

## 13.2.2.2 STARNEIG\_GENERIC\_ERROR

#define STARNEIG\_GENERIC\_ERROR 1

Reneric error.

The interface function encountered a generic error.

## 13.2.2.3 STARNEIG\_NOT\_INITIALIZED

#define STARNEIG\_NOT\_INITIALIZED 2

Not initialized.

The library was not initialized when the interface function was called.

## 13.2.2.4 STARNEIG\_INVALID\_CONFIGURATION

#define STARNEIG\_INVALID\_CONFIGURATION 3

Invalid configuration.

The interface function encountered an invalid configuration argument.

## 13.2.2.5 STARNEIG\_INVALID\_ARGUMENTS

#define STARNEIG\_INVALID\_ARGUMENTS 4

Invalid argument.

The interface function encountered an invalid argument.

# 13.2.2.6 STARNEIG\_INVALID\_DISTR\_MATRIX

#define STARNEIG\_INVALID\_DISTR\_MATRIX 5

Invalid distributed matrix.

One or more of the involved distributed matrices have an invalid distribution, invalid dimensions and/or an invalid distributed block size.

## 13.2.2.7 STARNEIG\_DID\_NOT\_CONVERGE

#define STARNEIG\_DID\_NOT\_CONVERGE 6

Did not converge.

The interface function encountered a situation where the QR/QZ algorithm did not converge. The matrix (pair) may be partially in (generalized) Schur form.

# 13.2.2.8 STARNEIG\_PARTIAL\_REORDERING

#define STARNEIG\_PARTIAL\_REORDERING 7

Partial reordering.

The interface function failed to reorder the (generalized) Schur form. The (generalized) Schur form may be partially reordered

# 13.2.2.9 STARNEIG\_CLOSE\_EIGENVALUES

#define STARNEIG\_CLOSE\_EIGENVALUES 8

Close eigenvalues.

The interface function encountered a situation where two selected eigenvalues were close to each other. The computed result may be inaccurate.

## 13.3 Intra-node execution environment

Interface to configure the intra-node execution environment.

#### **Functions**

• void starneig\_node\_init (int cores, int gpus, starneig\_flag\_t flags)

Initializes the intra-node execution environment.

int starneig\_node\_initialized ()

Checks whether the intra-node execution environment is initialized.

int starneig\_node\_get\_cores ()

Returns the number of cores (threads) per MPI rank.

· void starneig node set cores (int cores)

Changes the number of CPUs cores (threads) to use per MPI rank.

int starneig\_node\_get\_gpus ()

Returns the number of GPUs per MPI rank.

void starneig\_node\_set\_gpus (int gpus)

Changes the number of GPUs to use per MPI rank.

void starneig\_node\_finalize ()

Deallocates resources associated with the intra-node configuration.

# Library initialization flags

• typedef unsigned starneig\_flag\_t

Library initialization flag data type.

• #define STARNEIG USE ALL -1

Use all resources.

• #define STARNEIG\_DEFAULT 0x0

Default mode.

• #define STARNEIG HINT SM 0x0

Shared memory mode.

#define STARNEIG\_HINT\_DM 0x1

Distributed memory mode.

#define STARNEIG\_FXT\_DISABLE 0x2

No FxT traces mode.

#define STARNEIG\_AWAKE\_WORKERS 0x4

Awake worker mode.

• #define STARNEIG\_AWAKE\_MPI\_WORKER 0x8

Awake MPI worker mode.

• #define STARNEIG\_FAST\_DM (STARNEIG\_HINT\_DM | STARNEIG\_AWAKE\_WORKERS | STARNEIG\_AWAKE\_MPI\_WOR Fast distributed memory mode.

#define STARNEIG\_NO\_VERBOSE 0x10

No verbose mode.

• #define STARNEIG NO MESSAGES (STARNEIG NO VERBOSE | 0x20)

No messages mode.

# **Pinned host memory**

void starneig\_node\_enable\_pinning ()
 Enable CUDA host memory pinning.

• void starneig\_node\_disable\_pinning ()

Disables CUDA host memory pinning.

# 13.3.1 Detailed Description

Interface to configure the intra-node execution environment.

## 13.3.2 Macro Definition Documentation

## 13.3.2.1 STARNEIG\_USE\_ALL

```
#define STARNEIG_USE_ALL -1
```

Use all resources.

Tells StarNEig to use all available CPU cores / GPUs.

## **Examples**

gep\_dm\_full\_chain.c, gep\_sm\_eigenvectors.c, gep\_sm\_full\_chain.c, sep\_dm\_full\_chain.c, sep\_sm\_eigenvectors.c, and sep\_sm\_full\_chain.c.

## 13.3.2.2 STARNEIG\_DEFAULT

#define STARNEIG\_DEFAULT 0x0

Default mode.

As a default, the library configures itself to shared memory mode.

#### 13.3.2.3 STARNEIG\_HINT\_SM

#define STARNEIG\_HINT\_SM 0x0

Shared memory mode.

Initializes the library for shared memory computation. The library will automatically reconfigure itself for distributed memory computation if necessary

## **Examples**

gep sm eigenvectors.c, gep sm full chain.c, sep sm eigenvectors.c, and sep sm full chain.c.

## 13.3.2.4 STARNEIG\_HINT\_DM

#define STARNEIG\_HINT\_DM 0x1

Distributed memory mode.

Initializes the library for distributed memory computation. The library will automatically reconfigure itself for shared memory computation if necessary

#### **Examples**

sep\_dm\_full\_chain.c.

## 13.3.2.5 STARNEIG\_FXT\_DISABLE

#define STARNEIG\_FXT\_DISABLE 0x2

No FxT traces mode.

Disables FXT traces.

Attention

This flag does not work reliably with all StarPU versions.

# 13.3.2.6 STARNEIG\_AWAKE\_WORKERS

#define STARNEIG\_AWAKE\_WORKERS 0x4

Awake worker mode.

Keeps the StarPU worker threads awake between interface function calls. Improves the performance in certain situations but can interfere with other software.

## **Examples**

gep\_sm\_full\_chain.c.

## 13.3.2.7 STARNEIG\_AWAKE\_MPI\_WORKER

#define STARNEIG\_AWAKE\_MPI\_WORKER 0x8

Awake MPI worker mode.

Keeps the StarPU-MPI communication thread awake between interface function calls. Improves the performance in certain situations but can interfere with other software.

## 13.3.2.8 STARNEIG\_FAST\_DM

```
#define STARNEIG_FAST_DM (STARNEIG_HINT_DM | STARNEIG_AWAKE_WORKERS | STARNEIG_AWAKE_MPI_WORKER)
```

Fast distributed memory mode.

Keeps the worker threads and StarPU-MPI communication thread awake between interface function calls. Improves the performance in certain situations but can interfere with other software.

**Examples** 

```
gep_dm_full_chain.c.
```

## 13.3.2.9 STARNEIG\_NO\_VERBOSE

```
#define STARNEIG_NO_VERBOSE 0x10
```

No verbose mode.

Disables all additional verbose messages.

## 13.3.2.10 STARNEIG\_NO\_MESSAGES

```
#define STARNEIG_NO_MESSAGES (STARNEIG_NO_VERBOSE | 0x20)
```

No messages mode.

Disables all messages (including verbose messages).

## 13.3.3 Function Documentation

## 13.3.3.1 starneig\_node\_init()

Initializes the intra-node execution environment.

The interface function initializes StarPU (and cuBLAS) and pauses all worker The cores argument specifies the **total number of used CPU cores**. In distributed memory mode, one CPU core is automatically allocated for the StarPU-MPI communication thread. One or more CPU cores are automatically allocated for GPU devices.

#### **Parameters**

| in | cores | The number of cores (threads) to use per MPI rank. Can be set to STARNEIG_USE_ALL in                                      |
|----|-------|---|
|    |       | which case the library uses all available cores.  |
| in | gpus  | The number of GPUs to use per MPI rank. Can be set to STARNEIG_USE_ALL in which case the library uses all available GPUs. |
| in | flags | Initialization flags.   |

## **Examples**

gep\_dm\_full\_chain.c, gep\_sm\_eigenvectors.c, gep\_sm\_full\_chain.c, sep\_dm\_full\_chain.c, sep\_sm\_eigenvectors.c, and sep\_sm\_full\_chain.c.

#### 13.3.3.2 starneig\_node\_initialized()

```
int starneig_node_initialized ( )
```

Checks whether the intra-node execution environment is initialized.

## Returns

Non-zero if the environment is initialized, 0 otherwise.

## 13.3.3.3 starneig\_node\_get\_cores()

```
int starneig_node_get_cores ( )
```

Returns the number of cores (threads) per MPI rank.

#### Returns

The number of cores (threads) per MPI rank.

# 13.3.3.4 starneig\_node\_set\_cores()

Changes the number of CPUs cores (threads) to use per MPI rank.

#### **Parameters**

cores The number of CPUs to use per MPI rank.

# 13.3.3.5 starneig\_node\_get\_gpus()

```
int starneig_node_get_gpus ( )
```

Returns the number of GPUs per MPI rank.

#### Returns

The number of GPUs per MPI rank.

#### 13.3.3.6 starneig\_node\_set\_gpus()

Changes the number of GPUs to use per MPI rank.

## **Parameters**

gpus The number of GPUs to use per MPI rank.

# 13.3.3.7 starneig\_node\_enable\_pinning()

```
void starneig_node_enable_pinning ( )
```

Enable CUDA host memory pinning.

Should be called before any memory allocations are made.

## 13.3.3.8 starneig\_node\_disable\_pinning()

```
void starneig_node_disable_pinning ( )
```

Disables CUDA host memory pinning.

Should be called before any memory allocations are made.

# 13.4 Shared Memory / Standard EVP

Functions for solving non-symmetric standard eigenvalue problems on shared memory systems.

# **Computational functions**

• starneig\_error\_t starneig\_SEP\_SM\_Hessenberg (int n, double A[], int IdA, double Q[], int IdQ)

Computes a Hessenberg decomposition of a general matrix.

• starneig\_error\_t starneig\_SEP\_SM\_Schur (int n, double H[], int ldH, double Q[], int ldQ, double real[], double imag[])

Computes a Schur decomposition given a Hessenberg decomposition.

• starneig\_error\_t starneig\_SEP\_SM\_ReorderSchur (int n, int selected[], double S[], int ldS, double Q[], int ldQ, double real[], double imag[])

Reorders selected eigenvalues to the top left corner of a Schur decomposition.

Computes a (reordered) Schur decomposition of a general matrix.

• starneig\_error\_t starneig\_SEP\_SM\_Eigenvectors (int n, int selected[], double S[], int ldS, double Q[], int ldQ, double X[], int ldX)

Computes an eigenvector for each selected eigenvalue.

#### **Helper functions**

• starneig\_error\_t starneig\_SEP\_SM\_Select (int n, double S[], int ldS, int(\*predicate)(double real, double imag, void \*arg), void \*arg, int selected[], int \*num\_selected)

Generates a selection array for a Schur matrix using a user-supplied predicate function.

## **Expert computational functions**

• starneig\_error\_t starneig\_SEP\_SM\_Hessenberg\_expert (struct starneig\_hessenberg\_conf \*conf, int n, int begin, int end, double A[], int IdA, double Q[], int IdQ)

Computes a Hessenberg decomposition of a general matrix.

• starneig\_error\_t starneig\_SEP\_SM\_Schur\_expert (struct starneig\_schur\_conf \*conf, int n, double H[], int ldH, double Q[], int ldQ, double real[], double imag[])

Computes a Schur decomposition given a Hessenberg decomposition.

• starneig\_error\_t starneig\_SEP\_SM\_ReorderSchur\_expert (struct starneig\_reorder\_conf \*conf, int n, int selected[], double S[], int ldS, double Q[], int ldQ, double real[], double imag[])

Reorders selected eigenvalues to the top left corner of a Schur decomposition.

• starneig\_error\_t starneig\_SEP\_SM\_Eigenvectors\_expert (struct starneig\_eigenvectors\_conf \*conf, int n, int selected[], double S[], int ldS, double Q[], int ldQ, double X[], int ldX)

Computes an eigenvector for each selected eigenvalue.

## 13.4.1 Detailed Description

Functions for solving non-symmetric standard eigenvalue problems on shared memory systems.

# 13.4.2 Function Documentation

# 13.4.2.1 starneig\_SEP\_SM\_Hessenberg()

```
starneig_error_t starneig_SEP_SM_Hessenberg ( int n, double A[\ ], int 1dA, double Q[\ ], int 1dQ )
```

Computes a Hessenberg decomposition of a general matrix.

#### **Parameters**

| in     | n   | The order of $A$ and $Q$ .  |
|--------|-----|---|
| in,out | Α   | On entry, the general matrix $A.$ On exit, the upper Hessenberg matrix $H.$   |
| in     | ldA | The leading dimension of $A$ .  |
| in,out | Q   | On entry, the orthogonal matrix $Q$ . On exit, the product matrix $Q\ast U$ . |
| in     | ldQ | The leading dimension of $Q$ .  |

## Returns

STARNEIG\_SUCCESS (0) on success. Negative integer -i when i'th argument is invalid. Positive error code otherwise.

## **Examples**

```
sep_sm_full_chain.c.
```

## 13.4.2.2 starneig\_SEP\_SM\_Schur()

```
starneig_error_t starneig_SEP_SM_Schur (
    int n,
    double H[],
    int ldH,
    double Q[],
    int ldQ,
    double real[],
    double imag[])
```

Computes a Schur decomposition given a Hessenberg decomposition.

| in     | n | The order of $H$ and $Q$ .  |
|--------|---|---|
| in,out | Н | On entry, the upper Hessenberg matrix $H.$ On exit, the Schur matrix $S.$ |

#### **Parameters**

| in     | ldH  | The leading dimension of $H$ .  |
|--------|------|---|
| in,out | Q    | On entry, the orthogonal matrix $Q$ . On exit, the product matrix $Q\ast U$ .                       |
| in     | ldQ  | The leading dimension of $\mathcal{Q}$ .  |
| out    | real | An array of the same size as ${\cal H}$ containing the real parts of the computed eigenvalues.      |
| out    | imag | An array of the same size as ${\cal H}$ containing the imaginary parts of the computed eigenvalues. |

#### Returns

STARNEIG\_SUCCESS (0) on success. Negative integer -i when i'th argument is invalid. Positive error code otherwise. STARNEIG\_DID\_NOT\_CONVERGE if the QR algorithm failed to converge.

# Examples

sep\_sm\_full\_chain.c.

# 13.4.2.3 starneig\_SEP\_SM\_ReorderSchur()

```
starneig_error_t starneig_SEP_SM_ReorderSchur (
    int n,
    int selected[],
    double S[],
    int ldS,
    double Q[],
    int ldQ,
    double real[],
    double imag[])
```

Reorders selected eigenvalues to the top left corner of a Schur decomposition.

| in     | n        | The order of $S$ and $Q$ .  |
|--------|----------|---|
| in,out | selected | The selection array. On entry, the initial positions of the selected eigenvalues. On exit, the final positions of all correctly placed selected eigenvalues. In case of failure, the number of 1's in the output may be less than the number of 1's in the input. |
| in,out | S        | On entry, the Schur matrix $S$ . On exit, the updated Schur matrix $\hat{S}$ .  |
| in     | ldS      | The leading dimension of $S$ .  |
| in,out | Q        | On entry, the orthogonal matrix $Q$ . On exit, the product matrix $Q\ast U$ .   |
| in     | ldQ      | The leading dimension of $Q$ .  |
| out    | real     | An array of the same size as ${\cal S}$ containing the real parts of the computed eigenvalues.  |
| out    | imag     | An array of the same size as ${\cal S}$ containing the imaginary parts of the computed eigenvalues.   |

#### Returns

STARNEIG\_SUCCESS (0) on success. Negative integer -i when i'th argument is invalid. Positive error code otherwise. STARNEIG\_PARTIAL\_REORDERING if the Schur form is not fully reordered.

#### See also

```
starneig_SEP_SM_Select
```

## **Examples**

```
sep_sm_full_chain.c.
```

## 13.4.2.4 starneig\_SEP\_SM\_Reduce()

```
starneig_error_t starneig_SEP_SM_Reduce (
    int n,
    double A[],
    int ldA,
    double Q[],
    int ldQ,
    double real[],
    double imag[],
    int(*) (double real, double imag, void *arg) predicate,
    void * arg,
    int selected[],
    int * num_selected )
```

Computes a (reordered) Schur decomposition of a general matrix.

| in     | п            | The order of $A$ and $Q$ .  |
|--------|--------------|---|
| in,out | Α            | On entry, the general matrix $A$ . On exit, the Schur matrix $S$ .  |
| in     | IdA          | The leading dimension of $A$ .  |
| in,out | Q            | On entry, the orthogonal matrix $Q$ . On exit, the product matrix $Q\ast U$ .   |
| in     | ldQ          | The leading dimension of $Q$ .  |
| out    | real         | An array of the same size as ${\cal A}$ containing the real parts of the computed eigenvalues.  |
| out    | imag         | An array of the same size as ${\cal A}$ containing the imaginary parts of the computed eigenvalues.   |
| in     | predicate    | A function that takes a (complex) eigenvalue as input and returns non-zero if it should be selected. For complex conjugate pairs of eigenvalues, the predicate is called only for the eigenvalue with positive imaginary part and the corresponding $2\times 2$ block is either selected or deselected. The reordering step is skipped if the argument is a NULL pointer. |
| in     | arg          | An optional argument for the predicate function.  |
| out    | selected     | The final positions of all correctly placed selected eigenvalues.   |
| out    | num_selected | The number of selected eigenvalues (a complex conjugate pair is counted as two selected eigenvalues).   |

#### Returns

STARNEIG\_SUCCESS (0) on success. Negative integer -i when i'th argument is invalid. Positive error code otherwise. STARNEIG\_DID\_NOT\_CONVERGE if the QR algorithm failed to converge. STARNEIG\_PARTIAL\_REORDERING if the Schur form is not fully reordered.

# Examples

sep\_sm\_eigenvectors.c.

## 13.4.2.5 starneig\_SEP\_SM\_Eigenvectors()

```
starneig_error_t starneig_SEP_SM_Eigenvectors (
    int n,
    int selected[],
    double S[],
    int ldS,
    double Q[],
    int ldQ,
    double X[],
    int ldX)
```

Computes an eigenvector for each selected eigenvalue.

#### **Parameters**

| in  | n        | The order of $S$ and $Q$ and the number of rows of $X$ .                                    |
|-----|----------|---|
| in  | selected | The selection array specifying the locations of the selected eigenvalues. The number of 1's |
|     |          | in the array is the same as the number of columns in $X$ .                                  |
| in  | S        | The Schur matrix $S$ .  |
| in  | ldS      | The leading dimension of $S$ .  |
| in  | Q        | The orthogonal matrix $Q$ .   |
| in  | ldQ      | The The leading dimension of $Q$ .  |
| out | Χ        | A matrix with $n$ rows and one column for each selected eigenvalue. The columns             |
|     |          | represent the computed eigenvectors as previously described.                                |
| in  | ldX      | The leading dimension of $X$ .  |

#### Returns

STARNEIG\_SUCCESS (0) on success. Negative integer -i when i'th argument is invalid. Positive error code otherwise.

# See also

```
starneig_SEP_SM_Select
```

# **Examples**

sep\_sm\_eigenvectors.c.

# 13.4.2.6 starneig\_SEP\_SM\_Select()

```
starneig_error_t starneig_SEP_SM_Select (
    int n,
    double S[],
    int ldS,
    int(*) (double real, double imag, void *arg) predicate,
    void * arg,
    int selected[],
    int * num_selected )
```

Generates a selection array for a Schur matrix using a user-supplied predicate function.

#### **Parameters**

| in  | n            | The order of $S$ .   |
|-----|--------------|--|
| in  | S            | The Schur matrix $S$ .   |
| in  | ldS          | The leading dimension of $S$ .   |
| in  | predicate    | A function that takes a (complex) eigenvalue as input and returns non-zero if it should be selected. For complex conjugate pairs of eigenvalues, the predicate is called only for the eigenvalue with positive imaginary part and the corresponding $2 \times 2$ block is either selected or deselected. |
| in  | arg          | An optional argument for the predicate function.   |
| out | selected     | The selection array. Both elements of a selected complex conjugate pair are set to 1.  |
| out | num_selected | The number of selected eigenvalues (a complex conjugate pair is counted as two selected eigenvalues).  |

#### Returns

STARNEIG\_SUCCESS (0) on success. Negative integer -i when i'th argument is invalid. Positive error code otherwise.

## **Examples**

sep\_sm\_eigenvectors.c, and sep\_sm\_full\_chain.c.

# 13.4.2.7 starneig\_SEP\_SM\_Hessenberg\_expert()

Computes a Hessenberg decomposition of a general matrix.

#### **Parameters**

| in     | conf  | Configuration structure.  |
|--------|-------|---|
| in     | n     | The order of $A$ and $Q$ .  |
| in     | begin | First column to be reduced.   |
| in     | end   | Last column to be reduced + 1.  |
| in,out | Α     | On entry, the general matrix $A.$ On exit, the upper Hessenberg matrix $H.$   |
| in     | ldA   | The leading dimension of $A$ .  |
| in,out | Q     | On entry, the orthogonal matrix $Q$ . On exit, the product matrix $Q\ast U$ . |
| in     | ldQ   | The leading dimension of $Q$ .  |

#### Returns

STARNEIG\_SUCCESS (0) on success. Negative integer -i when i'th argument is invalid. Positive error code otherwise.

#### See also

```
starneig_SEP_SM_Hessenberg
starneig_hessenberg_conf
starneig_hessenberg_init_conf
```

## 13.4.2.8 starneig\_SEP\_SM\_Schur\_expert()

Computes a Schur decomposition given a Hessenberg decomposition.

| in     | conf | Configuration structure.  |
|--------|------|---|
| in     | n    | The order of $H$ and $Q$ .  |
| in,out | Н    | On entry, the upper Hessenberg matrix $H.$ On exit, the Schur matrix $S.$                           |
| in     | ldH  | The leading dimension of $H$ .  |
| in,out | Q    | On entry, the orthogonal matrix $Q$ . On exit, the product matrix $Q\ast U$ .                       |
| in     | ldQ  | The leading dimension of $\mathcal{Q}$ .  |
| out    | real | An array of the same size as ${\cal H}$ containing the real parts of the computed eigenvalues.      |
| out    | imag | An array of the same size as ${\cal H}$ containing the imaginary parts of the computed eigenvalues. |

#### Returns

STARNEIG\_SUCCESS (0) on success. Negative integer -i when i'th argument is invalid. Positive error code otherwise.

#### See also

```
starneig_SEP_SM_Schur
starneig_schur_conf
starneig_schur_init_conf
```

## 13.4.2.9 starneig\_SEP\_SM\_ReorderSchur\_expert()

Reorders selected eigenvalues to the top left corner of a Schur decomposition.

## Parameters

| in     | conf     | Configuration structure.  |
|--------|----------|---|
| in     | n        | The order of $S$ and $Q$ .  |
| in,out | selected | The selection array.  |
| in,out | S        | On entry, the Schur matrix $S$ . On exit, the updated Schur matrix $\hat{S}$ .                      |
| in     | ldS      | The leading dimension of $S$ .  |
| in,out | Q        | On entry, the orthogonal matrix $Q$ . On exit, the product matrix $Q \ast U$ .                      |
| in     | ldQ      | The leading dimension of $Q$ .  |
| out    | real     | An array of the same size as ${\cal S}$ containing the real parts of the computed eigenvalues.      |
| out    | imag     | An array of the same size as ${\cal S}$ containing the imaginary parts of the computed eigenvalues. |

#### Returns

STARNEIG\_SUCCESS (0) on success. Negative integer -i when i'th argument is invalid. Positive error code otherwise.

#### See also

```
starneig_SEP_SM_ReorderSchur
starneig_SEP_SM_Select
starneig_reorder_conf
starneig_reorder_init_conf
```

## 13.4.2.10 starneig\_SEP\_SM\_Eigenvectors\_expert()

Computes an eigenvector for each selected eigenvalue.

### **Parameters**

| in  | conf     | Configuration structure.   |  |
|-----|----------|--|--|
| in  | n        | The order of $S$ and $Q$ and the number of rows of $X$ .   |  |
| in  | selected | The selection array specifying the locations of the selected eigenvalues. The number of 1's in the array is the same as the number of columns in $X$ . |  |
| in  | S        | The Schur matrix $S$ .   |  |
| in  | ldS      | The leading dimension of $S$ .   |  |
| in  | Q        | The orthogonal matrix $Q$ .  |  |
| in  | ldQ      | The The leading dimension of $Q$ .   |  |
| out | X        | A matrix with $n$ rows and one column for each selected eigenvalue. The columns represent the computed eigenvectors as previously described.           |  |
| in  | ldX      | The leading dimension of $X$ .   |  |

## Returns

STARNEIG\_SUCCESS (0) on success. Negative integer -i when i'th argument is invalid. Positive error code otherwise.

## See also

starneig\_SEP\_SM\_Select

# 13.5 Shared Memory / Generalized EVP

Functions for solving non-symmetric generalized eigenvalue problems on shared memory systems.

## Computational functions

• starneig\_error\_t starneig\_GEP\_SM\_HessenbergTriangular (int n, double A[], int IdA, double B[], int IdB, double Q[], int IdQ, double Z[], int IdZ)

Computes a Hessenberg-triangular decomposition of a general matrix pencil.

• starneig\_error\_t starneig\_GEP\_SM\_Schur (int n, double H[], int ldH, double R[], int ldR, double Q[], int ldQ, double Z[], int ldZ, double real[], double imag[], double beta[])

Computes a generalized Schur decomposition given a Hessenberg-triangular decomposition.

• starneig\_error\_t starneig\_GEP\_SM\_ReorderSchur (int n, int selected[], double S[], int ldS, double T[], int ldT, double Q[], int ldQ, double Z[], int ldZ, double real[], double imag[], double beta[])

Reorders selected generalized eigenvalues to the top left corner of a generalized Schur decomposition.

• starneig\_error\_t starneig\_GEP\_SM\_Reduce (int n, double A[], int IdA, double B[], int IdB, double Q[], int IdQ, double Z[], int IdZ, double real[], double imag[], double beta[], int(\*predicate)(double real, double imag, double beta, void \*arg), void \*arg, int selected[], int \*num selected)

Computes a (reordered) generalized Schur decomposition given a general matrix pencil.

• starneig\_error\_t starneig\_GEP\_SM\_Eigenvectors (int n, int selected[], double S[], int ldS, double T[], int ldT, double Z[], int ldZ, double X[], int ldX)

Computes a generalized eigenvector for each selected generalized eigenvalue.

### **Helper functions**

• starneig\_error\_t starneig\_GEP\_SM\_Select (int n, double S[], int ldS, double T[], int ldT, int(\*predicate)(double real, double imag, double beta, void \*arg), void \*arg, int selected[], int \*num\_selected)

Generates a selection array for a Schur-triangular matrix pencil using a user-supplied predicate function.

### Expert computational functions

- starneig\_error\_t starneig\_GEP\_SM\_Schur\_expert (struct starneig\_schur\_conf \*conf, int n, double H[], int ldH, double R[], int ldR, double Q[], int ldQ, double Z[], int ldZ, double real[], double imag[], double beta[])
  - Computes a generalized Schur decomposition given a Hessenberg-triangular decomposition.
- starneig\_error\_t starneig\_GEP\_SM\_ReorderSchur\_expert (struct starneig\_reorder\_conf \*conf, int n, int selected[], double S[], int ldS, double T[], int ldT, double Q[], int ldQ, double Z[], int ldZ, double real[], double imag[], double beta[])

Reorders selected eigenvalues to the top left corner of a generalized Schur decomposition.

• starneig\_error\_t starneig\_GEP\_SM\_Eigenvectors\_expert (struct starneig\_eigenvectors\_conf \*conf, int n, int selected[], double S[], int ldS, double T[], int ldT, double Z[], int ldX)

Computes a generalized eigenvector for each selected generalized eigenvalue.

### 13.5.1 Detailed Description

Functions for solving non-symmetric generalized eigenvalue problems on shared memory systems.

## 13.5.2 Function Documentation

## 13.5.2.1 starneig\_GEP\_SM\_HessenbergTriangular()

```
starneig_error_t starneig_GEP_SM_HessenbergTriangular (
    int n,
    double A[],
    int ldA,
    double B[],
    int ldB,
    double Q[],
    int ldQ,
    double Z[],
    int ldZ)
```

Computes a Hessenberg-triangular decomposition of a general matrix pencil.

### Remarks

This function is a wrapper for several LAPACK subroutines.

#### **Parameters**

| in     | n   | The order of $A, B, Q$ and $Z$ .   |  |
|--------|-----|--|--|
| in,out | Α   | On entry, the general matrix $A$ . On exit, the upper Hessenberg matrix $H$ .    |  |
| in     | ldA | The leading dimension of $A$ .   |  |
| in,out | В   | On entry, the general matrix $B$ . On exit, the upper triangular matrix $R$ .    |  |
| in     | ldB | The leading dimension of $B$ .   |  |
| in,out | Q   | On entry, the orthogonal matrix $Q$ . On exit, the product matrix $Q \ast U_1$ . |  |
| in     | ldQ | The leading dimension of $Q$ .   |  |
| in,out | Z   | On entry, the orthogonal matrix $Z$ . On exit, the product matrix $Z \ast U_2$ . |  |
| in     | IdZ | The leading dimension of $Z$ .   |  |

# Returns

STARNEIG\_SUCCESS (0) on success. Negative integer -i when i'th argument is invalid. Positive error code otherwise.

### **Examples**

```
gep_sm_full_chain.c.
```

## 13.5.2.2 starneig\_GEP\_SM\_Schur()

```
starneig_error_t starneig_GEP_SM_Schur (  \qquad \qquad \text{int } n,
```

```
double H[],
int 1dH,
double R[],
int 1dR,
double Q[],
int 1dQ,
double Z[],
int 1dZ,
double real[],
double imag[],
double beta[])
```

Computes a generalized Schur decomposition given a Hessenberg-triangular decomposition.

### **Parameters**

| in     | n    | The order of $H, T, Q$ and $Z$ .  |  |
|--------|------|---|--|
| in,out | Н    | On entry, the upper Hessenberg matrix $H$ . On exit, the Schur matrix $S$ .   |  |
| in     | ldH  | The leading dimension of $H$ .  |  |
| in,out | R    | On entry, the upper triangular matrix $R$ . On exit, the upper triangular matrix $T$ .  |  |
| in     | ldR  | The leading dimension of $R$ .  |  |
| in,out | Q    | On entry, the orthogonal matrix $Q$ . On exit, the product matrix $Q \ast U_1$ .  |  |
| in     | ldQ  | The leading dimension of $Q$ .  |  |
| in,out | Z    | On entry, the orthogonal matrix $Z$ . On exit, the product matrix $Z*U_2$ .   |  |
| in     | ldZ  | The leading dimension of $Z$ .  |  |
| out    | real | An array of the same size as $H$ containing the real parts of the $\alpha$ values of the computed generalized eigenvalues.      |  |
| out    | imag | An array of the same size as $H$ containing the imaginary parts of the $\alpha$ values of the computed generalized eigenvalues. |  |
| out    | beta | An array of the same size as $H$ containing the $\beta$ values of computed generalized eigenvalues.                             |  |

#### Returns

STARNEIG\_SUCCESS (0) on success. Negative integer -i when i'th argument is invalid. Positive error code otherwise. STARNEIG\_DID\_NOT\_CONVERGE if the QZ algorithm failed to converge.

# **Examples**

gep\_sm\_full\_chain.c.

## 13.5.2.3 starneig\_GEP\_SM\_ReorderSchur()

```
starneig_error_t starneig_GEP_SM_ReorderSchur (
    int n,
    int selected[],
    double S[],
    int ldS,
    double T[],
    int ldT,
```

```
double Q[],
int 1dQ,
double Z[],
int 1dZ,
double real[],
double imag[],
double beta[])
```

Reorders selected generalized eigenvalues to the top left corner of a generalized Schur decomposition.

### **Parameters**

| in     | n        | The order of $H, T, Q$ and $Z$ .  |  |
|--------|----------|---|--|
| in,out | selected | The selection array. On entry, the initial positions of the selected generalized eigenvalues. On exit, the final positions of all correctly placed selected generalized eigenvalues. In case of failure, the number of 1's in the output may be less than the number of 1's in the input. |  |
| in,out | S        | On entry, the Schur matrix $S$ . On exit, the updated Schur matrix $\hat{S}$ .  |  |
| in     | ldS      | The leading dimension of $S$ .  |  |
| in,out | T        | On entry, the upper triangular $T.$ On exit, the updates upper triangular matrix $\hat{T}.$   |  |
| in     | ldT      | The leading dimension of $T$ .  |  |
| in,out | Q        | On entry, the orthogonal matrix $Q$ . On exit, the product matrix $Q \ast U_1$ .  |  |
| in     | ldQ      | The leading dimension of $Q$ .  |  |
| in,out | Z        | On entry, the orthogonal matrix $Z$ . On exit, the product matrix $Z*U_2$ .   |  |
| in     | ldZ      | The leading dimension of $Z$ .  |  |
| out    | real     | An array of the same size as $S$ containing the real parts of the $\alpha$ values of the computed generalized eigenvalues.  |  |
| out    | imag     | An array of the same size as $S$ containing the imaginary parts of the $\alpha$ values of the computed generalized eigenvalues.   |  |
| out    | beta     | An array of the same size as $S$ containing the $\beta$ values of computed generalized eigenvalues.   |  |

### Returns

STARNEIG\_SUCCESS (0) on success. Negative integer -i when i'th argument is invalid. Positive error code otherwise. STARNEIG\_PARTIAL\_REORDERING if the generalized Schur form is not fully reordered.

## See also

starneig\_GEP\_SM\_Select

## Examples

gep\_sm\_full\_chain.c.

## 13.5.2.4 starneig\_GEP\_SM\_Reduce()

```
starneig\_error\_t starneig\_GEP\_SM\_Reduce (
             int n_{,}
             double A[],
             int 1dA,
             double B[],
             int ldB,
             double Q[],
             int 1dQ,
             double Z[],
             int 1dZ,
             double real[],
             double imag[],
             double beta[],
             int(*)(double real, double imag, double beta, void *arg) predicate,
             void * arg,
             int selected[],
             int * num_selected )
```

Computes a (reordered) generalized Schur decomposition given a general matrix pencil.

#### **Parameters**

| in     | п            | The order of $A, B, Q$ and $Z$ .  |  |
|--------|--------------|---|--|
| in,out | Α            | On entry, the general matrix $A$ . On exit, the Schur matrix $S$ .  |  |
| in     | ldA          | The leading dimension of $A$ .  |  |
| in,out | В            | On entry, the general matrix $B$ . On exit, the upper triangular matrix $T$ .   |  |
| in     | ldB          | The leading dimension of $B$ .  |  |
| in,out | Q            | On entry, the orthogonal matrix $Q$ . On exit, the product matrix $Q \ast U_1$ .  |  |
| in     | ldQ          | The leading dimension of $Q$ .  |  |
| in,out | Z            | On entry, the orthogonal matrix $Z$ . On exit, the product matrix $Z \ast U_2$ .  |  |
| in     | ldZ          | The leading dimension of $Z$ .  |  |
| out    | real         | An array of the same size as $A$ containing the real parts of the $\alpha$ values of the computed generalized eigenvalues.  |  |
| out    | imag         | An array of the same size as $A$ containing the imaginary parts of the $\alpha$ values of the computed generalized eigenvalues.   |  |
| out    | beta         | An array of the same size as $A$ containing the $\beta$ values of computed generalized eigenvalues.   |  |
| in     | predicate    | A function that takes a (complex) generalized eigenvalue as input and returns non-zero if it should be selected. For complex conjugate pairs of generalized eigenvalues, the predicate is called only for the generalized eigenvalue with positive imaginary part and the corresponding $2\times 2$ block is either selected or deselected. The reordering step is skipped if the argument is a NULL pointer. |  |
| in     | arg          | An optional argument for the predicate function.  |  |
| out    | selected     | The final positions of all correctly placed selected generalized eigenvalues.   |  |
| out    | num_selected | The number of selected generalized eigenvalues (a complex conjugate pair is counted as two selected generalized eigenvalues).   |  |

## Returns

STARNEIG\_SUCCESS (0) on success. Negative integer -i when i'th argument is invalid. Positive error code otherwise. STARNEIG\_DID\_NOT\_CONVERGE if the QZ algorithm failed to converge. STARNEIG\_PARTIAL\_REORDERING if the generalized Schur form is not fully reordered.

# Examples

gep\_sm\_eigenvectors.c.

# 13.5.2.5 starneig\_GEP\_SM\_Eigenvectors()

```
starneig_error_t starneig_GEP_SM_Eigenvectors (
    int n,
    int selected[],
    double S[],
    int ldS,
    double T[],
    int ldT,
    double Z[],
    int ldZ,
    double X[],
    int ldX)
```

Computes a generalized eigenvector for each selected generalized eigenvalue.

#### **Parameters**

| in  | n        | The order of $S$ and $Q$ and the number of rows of $X$ .                                  |  |
|-----|----------|---|--|
| TII | n        | The order of S and Q and the number of rows of A.   |  |
| in  | selected | The selection array specifying the locations of the selected generalized eigenvalues. The |  |
|     |          | number of 1's in the array is the same as the number of columns in $X$ .                  |  |
|     |          | •   |  |
| in  | S        | The Schur matrix $S$ .  |  |
| in  | ldS      | The leading dimension of $S$ .  |  |
| in  | T        | The upper triangular matrix $T$ .   |  |
| in  | ldT      | The leading dimension of $T$ .  |  |
| in  | Z        | The orthogonal matrix $Z$ .   |  |
| in  | ldZ      | The leading dimension of $Z$ .  |  |
| out | Χ        | A matrix with $n$ rows and one column for each selected generalized eigenvalue. The       |  |
|     |          | columns represent the computed generalized eigenvectors as previously described.          |  |
| in  | ldX      | The leading dimension of $X$ .  |  |

# Returns

STARNEIG\_SUCCESS (0) on success. Negative integer -i when i'th argument is invalid. Positive error code otherwise.

### See also

```
starneig_GEP_SM_Select
```

### **Examples**

gep\_sm\_eigenvectors.c.

## 13.5.2.6 starneig\_GEP\_SM\_Select()

```
starneig_error_t starneig_GEP_SM_Select (
    int n,
    double S[],
    int ldS,
    double T[],
    int ldT,
    int(*) (double real, double imag, double beta, void *arg) predicate,
    void * arg,
    int selected[],
    int * num_selected )
```

Generates a selection array for a Schur-triangular matrix pencil using a user-supplied predicate function.

#### **Parameters**

| in  | n            | The order of $S$ and $T$ .   |  |
|-----|--------------|--|--|
| in  | S            | The Schur matrix $S$ .   |  |
| in  | ldS          | The leading dimension of $S$ .   |  |
| in  | Т            | The upper triangular matrix $T$ .  |  |
| in  | ldT          | The leading dimension of $T$ .   |  |
| in  | predicate    | A function that takes a (complex) generalized eigenvalue as input and returns non-zero if it should be selected. For complex conjugate pairs of generalized eigenvalues, the predicate is called only for the generallized eigenvalue with positive imaginary part and the corresponding $2\times 2$ block is either selected or deselected. |  |
| in  | arg          | An optional argument for the predicate function.   |  |
| out | selected     | The selection array. Both elements of a selected complex conjugate pair are set to 1.  |  |
| out | num_selected | The number of selected generalized eigenvalues (a complex conjugate pair is counted as two selected generalized eigenvalues).  |  |

### Returns

STARNEIG\_SUCCESS (0) on success. Negative integer -i when i'th argument is invalid. Positive error code otherwise.

### **Examples**

gep\_sm\_eigenvectors.c, and gep\_sm\_full\_chain.c.

# 13.5.2.7 starneig\_GEP\_SM\_Schur\_expert()

```
int 1dQ,
double Z[],
int 1dZ,
double real[],
double imag[],
double beta[])
```

Computes a generalized Schur decomposition given a Hessenberg-triangular decomposition.

#### **Parameters**

| in     | conf | Configuration structure.  |  |
|--------|------|---|--|
| in     | n    | The order of $H,T,Q$ and $Z$ .  |  |
| in,out | Н    | On entry, the upper Hessenberg matrix $H.$ On exit, the Schur matrix $S.$   |  |
| in     | ldH  | The leading dimension of $H$ .  |  |
| in,out | R    | On entry, the upper triangular matrix $R$ . On exit, the upper triangular matrix $T$ .  |  |
| in     | ldR  | The leading dimension of $R$ .  |  |
| in,out | Q    | On entry, the orthogonal matrix $Q$ . On exit, the product matrix $Q \ast U_1$ .  |  |
| in     | ldQ  | The leading dimension of $Q$ .  |  |
| in,out | Z    | On entry, the orthogonal matrix $Z$ . On exit, the product matrix $Z*U_2$ .   |  |
| in     | IdZ  | The leading dimension of $Z$ .  |  |
| out    | real | An array of the same size as $H$ containing the real parts of the $\alpha$ values of the computed generalized eigenvalues.      |  |
| out    | imag | An array of the same size as $H$ containing the imaginary parts of the $\alpha$ values of the computed generalized eigenvalues. |  |
| out    | beta | An array of the same size as $H$ containing the $\beta$ values of computed generalized eigenvalues.                             |  |

## Returns

STARNEIG\_SUCCESS (0) on success. Negative integer -i when i'th argument is invalid. Positive error code otherwise.

### See also

```
starneig_GEP_SM_Schur
starneig_schur_conf
starneig_schur_init_conf
```

### 13.5.2.8 starneig\_GEP\_SM\_ReorderSchur\_expert()

```
int ldT,
double Q[],
int ldQ,
double Z[],
int ldZ,
double real[],
double imag[],
double beta[])
```

Reorders selected eigenvalues to the top left corner of a generalized Schur decomposition.

### **Parameters**

| in     | conf     | Configuration structure.  |  |
|--------|----------|---|--|
| in     | n        | The order of $H, T, Q$ and $Z$ .  |  |
| in,out | selected | The selection array.  |  |
| in,out | S        | On entry, the Schur matrix $S$ . On exit, the updated Schur matrix $\hat{S}$ .  |  |
| in     | ldS      | The leading dimension of $S$ .  |  |
| in,out | Т        | On entry, the upper triangular $T$ . On exit, the updates upper triangular matrix $\hat{T}$ .                                   |  |
| in     | ldT      | The leading dimension of $T$ .  |  |
| in,out | Q        | On entry, the orthogonal matrix $Q$ . On exit, the product matrix $Q*U_1$ .   |  |
| in     | ldQ      | The leading dimension of $Q$ .  |  |
| in,out | Z        | On entry, the orthogonal matrix $Z$ . On exit, the product matrix $Z \ast U_2$ .  |  |
| in     | ldZ      | The leading dimension of $Z$ .  |  |
| out    | real     | An array of the same size as $S$ containing the real parts of the $\alpha$ values of the computed generalized eigenvalues.      |  |
| out    | imag     | An array of the same size as $S$ containing the imaginary parts of the $\alpha$ values of the computed generalized eigenvalues. |  |
| out    | beta     | An array of the same size as $S$ containing the $\beta$ values of computed generalized eigenvalues.                             |  |

### Returns

STARNEIG\_SUCCESS (0) on success. Negative integer -i when i'th argument is invalid. Positive error code otherwise.

## See also

```
starneig_GEP_SM_ReorderSchur
starneig_GEP_SM_Select
starneig_reorder_conf
starneig_reorder_init_conf
```

## 13.5.2.9 starneig\_GEP\_SM\_Eigenvectors\_expert()

```
int n,
int selected[],
double S[],
int ldS,
double T[],
int ldT,
double Z[],
int ldZ,
double X[],
int ldX)
```

Computes a generalized eigenvector for each selected generalized eigenvalue.

### **Parameters**

| in  | conf     | Configuration structure.   |  |
|-----|----------|--|--|
| in  | n        | The order of $S$ and $Q$ and the number of rows of $X$ .   |  |
| in  | selected | The selection array specifying the locations of the selected generalized eigenvalues. The number of 1's in the array is the same as the number of columns in $X$ .   |  |
| in  | S        | The Schur matrix $S$ .   |  |
| in  | ldS      | The leading dimension of $S$ .   |  |
| in  | T        | The upper triangular matrix $T$ .  |  |
| in  | ldT      | The leading dimension of $T$ .   |  |
| in  | Z        | The orthogonal matrix $Z$ .  |  |
| in  | ldZ      | The leading dimension of $Z$ .   |  |
| out | X        | A matrix with $n$ rows and one column for each selected generalized eigenvalue. The columns represent the computed generalized eigenvectors as previously described. |  |
| in  | ldX      | The leading dimension of $X$ .   |  |

# Returns

STARNEIG\_SUCCESS (0) on success. Negative integer -i when i'th argument is invalid. Positive error code otherwise.

# See also

starneig\_GEP\_SM\_Select

# 13.6 Distributed Memory / Distributed matrices

Data types and functions for distributed matrices.

### **Data Structures**

· struct starneig\_distr\_block

Distributed block. More ...

### **Data distributions**

enum starneig\_distr\_order\_t { STARNEIG\_ORDER\_DEFAULT, STARNEIG\_ORDER\_ROW\_MAJOR, STARNEIG\_ORDER\_COL\_MAJOR }

Process mapping order.

typedef struct starneig distr \* starneig distr t

Data distribution.

• starneig\_distr\_t starneig\_distr\_init ()

Creates a default data distribution.

starneig\_distr\_t starneig\_distr\_init\_mesh (int rows, int cols, starneig\_distr\_order\_t order)

Creates a two-dimensional block cyclic data distribution.

• starneig\_distr\_t starneig\_distr\_init\_func (int(\*func)(int row, int col, void \*arg), void \*arg, size\_t arg\_size)

Creates a distribution using a data distribution function.

starneig\_distr\_t starneig\_distr\_duplicate (starneig\_distr\_t distr)

Duplicates a data distribution.

void starneig\_distr\_destroy (starneig\_distr\_t distr)

Destroys a data distribution.

### **Distributed matrices**

enum starneig\_datatype\_t { STARNEIG\_REAL\_DOUBLE }

Distributed matrix element data type.

typedef struct starneig\_distr\_matrix \* starneig\_distr\_matrix\_t

Distributed matrix.

starneig\_distr\_matrix\_t starneig\_distr\_matrix\_create (int rows, int cols, int row\_blksz, int col\_blksz, starneig\_datatype\_t type, starneig\_distr\_t distr)

Creates a distributed matrix with uninitialized matrix elements.

starneig\_distr\_matrix\_t starneig\_distr\_matrix\_create\_local (int rows, int cols, starneig\_datatype\_t type, int owner, double \*A, int ldA)

Creates a single-owner distributed matrix from a local matrix.

• void starneig\_distr\_matrix\_destroy (starneig\_distr\_matrix\_t matrix)

Destroys a distributed matrix.

• void starneig\_distr\_matrix\_copy (starneig\_distr\_matrix\_t source, starneig\_distr\_matrix\_t dest)

Copies the contents of a distributed matrix to a second distributed matrix.

void starneig\_distr\_matrix\_copy\_region (int sr, int sc, int dr, int dc, int rows, int cols, starneig\_distr\_matrix\_t source, starneig\_distr\_matrix\_t dest)

Copies region of a distributed matrix to a second distributed matrix.

## **Query functions**

void starneig\_distr\_matrix\_get\_blocks (starneig\_distr\_matrix\_t matrix, struct starneig\_distr\_block \*\*blocks, int \*num\_blocks)

Returns the locally owned distributed blocks.

starneig\_distr\_t starneig\_distr\_matrix\_get\_distr (starneig\_distr\_matrix\_t matrix)

Returns the distribution that is associated with a distributed matrix.

starneig datatype t starneig distr matrix get datatype (starneig distr matrix t matrix)

Returns the matrix element data type.

size\_t starneig\_distr\_matrix\_get\_elemsize (starneig\_distr\_matrix\_t matrix)

Returns the matrix element size.

int starneig\_distr\_matrix\_get\_rows (starneig\_distr\_matrix\_t matrix)

Returns the number of (global) rows.

int starneig\_distr\_matrix\_get\_cols (starneig\_distr\_matrix\_t matrix)

Returns the number of (global) columns.

int starneig\_distr\_matrix\_get\_row\_blksz (starneig\_distr\_matrix\_t matrix)

Returns the number of rows in a distribution block.

int starneig\_distr\_matrix\_get\_col\_blksz (starneig\_distr\_matrix\_t matrix)

Returns the number of columns in a distribution block.

# 13.6.1 Detailed Description

Data types and functions for distributed matrices.

# 13.6.2 Data Structure Documentation

# 13.6.2.1 struct starneig\_distr\_block

Distributed block.

### **Data Fields**

| int    | row_blksz  | The number of rows in the block.          |  |
|--------|--|---|--|
| int    | int col_blksz The number of columns in the block.            |   |  |
| int    | glo_row The topmost global row that belong to the block.     |   |  |
| int    | glo_col The leftmost global column that belong to the block. |   |  |
| int    | ld   | The leading dimension of the local array. |  |
| void * | ptr  | A pointer to the local array.             |  |

## 13.6.3 Enumeration Type Documentation

## 13.6.3.1 starneig\_distr\_order\_t

enum starneig\_distr\_order\_t

78 **Module Documentation** Process mapping order.

### Enumerator

| STARNEIG_ORDER_DEFAULT   | Default ordering.              |
|--------------------------|--------------------------------|
| STARNEIG_ORDER_ROW_MAJOR | Row-major natural ordering.    |
| STARNEIG_ORDER_COL_MAJOR | Column-major natural ordering. |

## 13.6.3.2 starneig\_datatype\_t

```
enum starneig_datatype_t
```

Distributed matrix element data type.

#### Enumerator

| STARNEIG_REAL_DOUBLE   Double precision real numbers. |
|---|
|---|

# 13.6.4 Function Documentation

# 13.6.4.1 starneig\_distr\_init()

```
starneig_distr_t starneig_distr_init ()
```

Creates a default data distribution.

### Returns

A new data distribution.

## 13.6.4.2 starneig\_distr\_init\_mesh()

Creates a two-dimensional block cyclic data distribution.

## Parameters

| in        | rows      | The number of rows in the mesh. Can be set to -1 in which case the library decides the value.    |  |
|-----------|-----------|--|--|
| in        | cols      | The number of columns in the mesh. Can be set to -1 in which case the library decides the value. |  |
| Generated | on Thu No | v The process mapping order Guide by Doxygen   |  |

#### Returns

A new data distribution.

### **Examples**

gep\_dm\_full\_chain.c, and sep\_dm\_full\_chain.c.

## 13.6.4.3 starneig\_distr\_init\_func()

```
starneig_distr_t starneig_distr_init_func (
          int(*)(int row, int col, void *arg) func,
          void * arg,
          size_t arg_size )
```

Creates a distribution using a data distribution function.

The distribution function maps each block to it's owner. The function takes three arguments: block's row index, blocks's column index and an optional user defined argument.

### **Parameters**

| in | func   | The data distribution function.                  |  |
|----|--|--|--|
| in | arg  | An optional data distribution function argument. |  |
| in | in arg_size The size of the optional data distribution function argume |  |  |

### Returns

A new data distribution.

### 13.6.4.4 starneig\_distr\_duplicate()

Duplicates a data distribution.

### **Parameters**

| in | distr | The data distribution to be duplicated. |
|----|-------|---|
|----|-------|---|

### Returns

A duplicated data distribution.

### 13.6.4.5 starneig\_distr\_destroy()

Destroys a data distribution.

### **Parameters**

| in,out <i>dist</i> | The data distribution to be destroyed. |
|--------------------|--|
|--------------------|--|

### **Examples**

gep\_dm\_full\_chain.c, and sep\_dm\_full\_chain.c.

## 13.6.4.6 starneig\_distr\_matrix\_create()

```
starneig_distr_matrix_t starneig_distr_matrix_create (
    int rows,
    int cols,
    int row_blksz,
    int col_blksz,
    starneig_datatype_t type,
    starneig_distr_t distr )
```

## Creates a distributed matrix with uninitialized matrix elements.

```
// create a m X n double-precision real matrix that is distributed in a
// two-dimensional block cyclic fashion in bm X bn blocks
starneig_distr_t distr = starneig_distr_init();
starneig_distr_matrix_t dA =
    starneig_distr_matrix_create(m, n, bm, bn, STARNEIG_REAL_DOUBLE, distr);
```

### Attention

StarNEig library is designed to use much larger distributed blocks than ScaLAPACK. Selecting a too small distributed block size will be detrimental to the performance.

#### **Parameters**

| in | rows      | The number of (global) rows in the matrix.   |  |
|----|-----------|--|--|
| in | cols      | The number of (global) columns in the matrix.  |  |
| in | row_blksz | The number of rows in a distribution block. Can be set to -1 in which case the library         |  |
|    |           | decides the value.   |  |
| in | col_blksz | The number of columns in a distribution block. Can be set to -1 in which case the library      |  |
|    |           | decides the value.   |  |
| in | type      | The matrix element data type.  |  |
| in | distr     | The data distribution. Can be left to NULL in which case the library decides the distribution. |  |

#### Returns

A new distributed matrix.

### **Examples**

gep\_dm\_full\_chain.c, and sep\_dm\_full\_chain.c.

### 13.6.4.7 starneig\_distr\_matrix\_create\_local()

```
starneig_distr_matrix_t starneig_distr_matrix_create_local (
    int rows,
    int cols,
    starneig_datatype_t type,
    int owner,
    double * A,
    int ldA )
```

Creates a single-owner distributed matrix from a local matrix.

This creates a wrapper. The contents of the local matrix may be modified by the functions that use the wrapper. The starneig\_distr\_matrix\_destroy() function does not free the local matrix.

```
int m = 1000, n = 1000;
double *A = NULL; size_t ldA = 0;
// rank 3 initialized the local matrix
if (my_rank = 3) {
    A = initialize_matrix(m, n, &ldA);
}
// all ranks initialize the distributed matrix
starneig_distr_matrix_t lA = starneig_distr_matrix_create_local(
    m, n, STARNEIG_REAL_DOUBLE, 3, A, ldA);
```

#### **Parameters**

| in | rows       | The number of rows in the matrix.   |  |  |
|----|------------|---|--|--|
| in | cols       | The number of columns in the matrix.  |  |  |
| in | type       | latrix element data type.   |  |  |
| in | owner      | MPI rank that owns the distributed matrix.  |  |  |
| in | Α          | A pointer to the local matrix. This argument is ignored the calling rank is not the same as the |  |  |
|    |            | owner.  |  |  |
| in | <i>ldA</i> | The leading dimension of the local matrix. This argument is ignored the calling rank is not the |  |  |
|    |            | same as the owner.  |  |  |

### Returns

A new distributed matrix.

### **Examples**

gep\_dm\_full\_chain.c, and sep\_dm\_full\_chain.c.

## 13.6.4.8 starneig\_distr\_matrix\_destroy()

Destroys a distributed matrix.

#### **Parameters**

|  | in,out | matrix | The distributed matrix to be destroyed. |
|--|--------|--------|---|
|--|--------|--------|---|

### **Examples**

gep\_dm\_full\_chain.c, and sep\_dm\_full\_chain.c.

## 13.6.4.9 starneig\_distr\_matrix\_copy()

Copies the contents of a distributed matrix to a second distributed matrix.

## Parameters

| in  | source | The source matrix.      |
|-----|--------|-------------------------|
| out | dest   | The destination matrix. |

# **Examples**

gep\_dm\_full\_chain.c, and sep\_dm\_full\_chain.c.

### 13.6.4.10 starneig\_distr\_matrix\_copy\_region()

```
int sc,
int dr,
int dc,
int rows,
int cols,
starneig_distr_matrix_t source,
starneig_distr_matrix_t dest)
```

Copies region of a distributed matrix to a second distributed matrix.

#### **Parameters**

| in  | sr     | The first source matrix row to be copied.    |  |
|-----|--------|--|--|
| in  | sc     | The first source matrix column to be copied. |  |
| in  | dr     | The first destination matrix row.            |  |
| in  | dc     | The first destination matrix column.         |  |
| in  | rows   | The number of rows to copy.                  |  |
| in  | cols   | The number of columns to copy.               |  |
| in  | source | The source matrix.                           |  |
| out | dest   | The destination matrix.                      |  |

### 13.6.4.11 starneig\_distr\_matrix\_get\_blocks()

Returns the locally owned distributed blocks.

## Attention

A user is allowed to modify the contents of the locally owned blocks but the the returned array itself should not be modified.

#### **Parameters**

| in matrix The distributed matrix.                 |     | The distributed matrix. |  |
|---|-----|-------------------------|--|
| out blocks An array that contains all locally own |     | blocks                  | An array that contains all locally owned distributed blocks. |
|   | out | num_blocks              | The total number of locally owned distributed blocks.        |

# 13.6.4.12 starneig\_distr\_matrix\_get\_distr()

Returns the distribution that is associated with a distributed matrix.

### Attention

The distributed matrix maintains the ownership of the returned data distribution. A user must duplicate the data distribution if necessary.

### **Parameters**

| in matrix The distributed matrix |
|----------------------------------|
|----------------------------------|

### Returns

The associated distribution.

### 13.6.4.13 starneig\_distr\_matrix\_get\_datatype()

Returns the matrix element data type.

### **Parameters**

| in | matrix | The distributed matrix. |
|----|--------|-------------------------|
|----|--------|-------------------------|

### Returns

The matrix element data type.

# 13.6.4.14 starneig\_distr\_matrix\_get\_elemsize()

Returns the matrix element size.

#### **Parameters**

| in | matrix | The distributed matrix. |
|----|--------|-------------------------|

### Returns

The matrix element size.

## 13.6.4.15 starneig\_distr\_matrix\_get\_rows()

Returns the number of (global) rows.

### **Parameters**

| in | matrix | The distributed matrix. |
|----|--------|-------------------------|
|----|--------|-------------------------|

### Returns

The number of (global) rows.

## 13.6.4.16 starneig\_distr\_matrix\_get\_cols()

Returns the number of (global) columns.

### **Parameters**

| in  | matrix | The distributed matrix. |
|-----|--------|-------------------------|
| T11 | maurix | The distributed matrix. |

#### Returns

The number of (global) columns.

## 13.6.4.17 starneig\_distr\_matrix\_get\_row\_blksz()

Returns the number of rows in a distribution block.

### **Parameters**

| in | matrix | The distributed matrix. |
|----|--------|-------------------------|

## Returns

The number of rows in a distribution block.

## 13.6.4.18 starneig\_distr\_matrix\_get\_col\_blksz()

Returns the number of columns in a distribution block.

### **Parameters**

| in <i>matrix</i> | The distributed matrix. |
|------------------|-------------------------|
|------------------|-------------------------|

### Returns

The number of columns in a distribution block.

# 13.7 Distributed Memory / Helper functions

Distributed memory helper functions.

### **MPI** communicator

• void starneig\_mpi\_set\_comm (MPI\_Comm comm)

Sets a MPI communicator for the library.

• MPI\_Comm starneig\_mpi\_get\_comm ()

Returns the library MPI communicator.

### **Broadcast**

void starneig\_mpi\_broadcast (int root, size\_t size, void \*buffer)
 Broadcast a buffer.

# 13.7.1 Detailed Description

Distributed memory helper functions.

### 13.7.2 Function Documentation

### 13.7.2.1 starneig\_mpi\_set\_comm()

Sets a MPI communicator for the library.

Should be called before the starneig\_node\_init() interface function.

#### **Parameters**

|  | in | comm | The library MPI communicator. |  |
|--|----|------|-------------------------------|--|
|--|----|------|-------------------------------|--|

## 13.7.2.2 starneig\_mpi\_get\_comm()

```
MPI_Comm starneig_mpi_get_comm ( )
```

Returns the library MPI communicator.

### Returns

The library MPI communicator.

# 13.7.2.3 starneig\_mpi\_broadcast()

## Broadcast a buffer.

### **Parameters**

| in     | root   | The rank that is going to broadcast the buffer. |
|--------|--------|---|
| in     | size   | The size of the buffer.                         |
| in,out | buffer | A pointer to the buffer.                        |

# 13.8 Distributed Memory / Standard EVP

Functions for solving non-symmetric standard eigenvalue problems on distributed memory systems.

# **Computational functions**

- starneig\_error\_t starneig\_SEP\_DM\_Hessenberg (starneig\_distr\_matrix\_t A, starneig\_distr\_matrix\_t Q)

  Computes a Hessenberg decomposition of a general matrix.
- starneig\_error\_t starneig\_SEP\_DM\_Schur (starneig\_distr\_matrix\_t H, starneig\_distr\_matrix\_t Q, double real[], double imag[])

Computes a Schur decomposition given a Hessenberg decomposition.

starneig\_error\_t starneig\_SEP\_DM\_ReorderSchur (int selected[], starneig\_distr\_matrix\_t S, starneig\_distr\_matrix\_t
 Q, double real[], double imag[])

Reorders selected eigenvalues to the top left corner of a Schur decomposition.

• starneig\_error\_t starneig\_SEP\_DM\_Reduce (starneig\_distr\_matrix\_t A, starneig\_distr\_matrix\_t Q, double real[], double imag[], int(\*predicate)(double real, double imag, void \*arg), void \*arg, int selected[], int \*num\_selected)

Computes a (reordered) Schur decomposition of a general matrix.

starneig\_error\_t starneig\_SEP\_DM\_Eigenvectors (int selected[], starneig\_distr\_matrix\_t S, starneig\_distr\_matrix\_t X)

Computes an eigenvector for each selected eigenvalue.

### **Helper functions**

• starneig\_error\_t starneig\_SEP\_DM\_Select (starneig\_distr\_matrix\_t S, int(\*predicate)(double real, double imag, void \*arg), void \*arg, int selected[], int \*num\_selected)

Generates a selection array for a Schur matrix using a user-supplied predicate function.

### Expert computational functions

• starneig\_error\_t starneig\_SEP\_DM\_Hessenberg\_expert (struct starneig\_hessenberg\_conf \*conf, int begin, int end, starneig\_distr\_matrix\_t A, starneig\_distr\_matrix\_t Q)

Computes a Hessenberg decomposition of a general matrix.

• starneig\_error\_t starneig\_SEP\_DM\_Schur\_expert (struct starneig\_schur\_conf \*conf, starneig\_distr\_matrix\_t H, starneig\_distr\_matrix\_t Q, double real[], double imag[])

Computes a Schur decomposition given a Hessenberg decomposition.

• starneig\_error\_t starneig\_SEP\_DM\_ReorderSchur\_expert (struct starneig\_reorder\_conf \*conf, int selected[], starneig\_distr\_matrix\_t S, starneig\_distr\_matrix\_t Q, double real[], double imag[])

Reorders selected eigenvalues to the top left corner of a Schur decomposition.

• starneig\_error\_t starneig\_SEP\_DM\_Eigenvectors\_expert (struct starneig\_eigenvectors\_conf \*conf, int selected[], starneig\_distr\_matrix\_t S, starneig\_distr\_matrix\_t Q, starneig\_distr\_matrix\_t X)

Computes an eigenvector for each selected eigenvalue.

## 13.8.1 Detailed Description

Functions for solving non-symmetric standard eigenvalue problems on distributed memory systems.

### 13.8.2 Function Documentation

## 13.8.2.1 starneig\_SEP\_DM\_Hessenberg()

Computes a Hessenberg decomposition of a general matrix.

#### **Parameters**

|        |   | On entry, the general matrix $A$ . On exit, the upper Hessenberg matrix $H$ .  |
|--------|---|--|
| in,out | Q | On entry, the orthogonal matrix $Q$ . On exit, the product matrix $Q \ast U$ . |

#### Returns

STARNEIG\_SUCCESS (0) on success. Negative integer -i when i'th argument is invalid. Positive error code otherwise

## Examples

sep\_dm\_full\_chain.c.

## 13.8.2.2 starneig\_SEP\_DM\_Schur()

Computes a Schur decomposition given a Hessenberg decomposition.

#### **Parameters**

| in,out | Н    | On entry, the upper Hessenberg matrix $H$ . On exit, the Schur matrix $S$ .                    |
|--------|------|--|
| in,out | Q    | On entry, the orthogonal matrix $Q$ . On exit, the product matrix $Q \ast U$ .                 |
| out    | real | An array of the same size as ${\cal H}$ containing the real parts of the computed eigenvalues. |
| out    | imag | An array of the same size as $H$ containing the imaginary parts of the computed eigenvalues.   |

### Returns

STARNEIG\_SUCCESS (0) on success. Negative integer -i when i'th argument is invalid. Positive error code otherwise. STARNEIG\_DID\_NOT\_CONVERGE if the QR algorithm failed to converge.

### **Examples**

```
sep_dm_full_chain.c.
```

## 13.8.2.3 starneig\_SEP\_DM\_ReorderSchur()

```
starneig_error_t starneig_SEP_DM_ReorderSchur (
    int selected[],
    starneig_distr_matrix_t S,
    starneig_distr_matrix_t Q,
    double real[],
    double imag[])
```

Reorders selected eigenvalues to the top left corner of a Schur decomposition.

#### **Parameters**

| in,out | selected | The selection array. On entry, the initial positions of the selected eigenvalues. On exit, the final positions of all correctly placed selected eigenvalues. In case of failure, the number of 1's in the output may be less than the number of 1's in the input. |
|--------|----------|---|
| in,out | S        | On entry, the Schur matrix $S$ . On exit, the updated Schur matrix $\hat{S}$ .  |
| in,out | Q        | On entry, the orthogonal matrix $Q$ . On exit, the product matrix $Q\ast U$ .   |
| out    | real     | An array of the same size as ${\cal S}$ containing the real parts of the computed eigenvalues.  |
| out    | imag     | An array of the same size as ${\cal S}$ containing the imaginary parts of the computed eigenvalues.   |

### Returns

STARNEIG\_SUCCESS (0) on success. Negative integer -i when i'th argument is invalid. Positive error code otherwise. STARNEIG\_PARTIAL\_REORDERING if the Schur form is not fully reordered.

#### See also

```
starneig_SEP_DM_Select
```

## Examples

```
sep_dm_full_chain.c.
```

# 13.8.2.4 starneig\_SEP\_DM\_Reduce()

Computes a (reordered) Schur decomposition of a general matrix.

### **Parameters**

| in,out | A            | On entry, the general matrix $A.$ On exit, the Schur matrix $S.$   |
|--------|--------------|--|
| in,out | Q            | On entry, the orthogonal matrix $Q$ . On exit, the product matrix $Q\ast U$ .  |
| out    | real         | An array of the same size as ${\cal A}$ containing the real parts of the computed eigenvalues.   |
| out    | imag         | An array of the same size as ${\cal A}$ containing the imaginary parts of the computed eigenvalues.  |
| in     | predicate    | A function that takes a (complex) eigenvalue as input and returns non-zero if it should be selected. For complex conjugate pairs of eigenvalues, the predicate is called only for the eigenvalue with positive imaginary part and the corresponding $2 \times 2$ block is either selected or deselected. The reordering step is skipped if the argument is a NULL pointer. |
| in     | arg          | An optional argument for the predicate function.   |
| out    | selected     | The final positions of all correctly placed selected eigenvalues.  |
| out    | num_selected | The number of selected eigenvalues (a complex conjugate pair is counted as two selected eigenvalues).  |

### Returns

STARNEIG\_SUCCESS (0) on success. Negative integer -i when i'th argument is invalid. Positive error code otherwise. STARNEIG\_DID\_NOT\_CONVERGE if the QR algorithm failed to converge. STARNEIG\_PARTIAL\_REORDERING if the Schur form is not fully reordered.

# 13.8.2.5 starneig\_SEP\_DM\_Eigenvectors()

```
starneig_error_t starneig_SEP_DM_Eigenvectors (
    int selected[],
    starneig_distr_matrix_t S,
    starneig_distr_matrix_t Q,
    starneig_distr_matrix_t X)
```

Computes an eigenvector for each selected eigenvalue.

### **Parameters**

| in  | selected | The selection array specifying the locations of the selected eigenvalues. The number of 1's in the array is the same as the number of columns in $X$ . |
|-----|----------|--|
| in  | S        | The Schur matrix $S$ .   |
| in  | Q        | The orthogonal matrix $Q$ .  |
| out | X        | A matrix with $n$ rows and one column for each selected eigenvalue. The columns represent the computed eigenvectors as previously described.           |

### Returns

STARNEIG\_SUCCESS (0) on success. Negative integer -i when i'th argument is invalid. Positive error code otherwise.

### See also

```
starneig_SEP_DM_Select
```

**Todo** This interface function is not implemented.

### 13.8.2.6 starneig\_SEP\_DM\_Select()

Generates a selection array for a Schur matrix using a user-supplied predicate function.

#### **Parameters**

| in  | S            | The Schur matrix $S$ .   |
|-----|--------------|--|
| in  | predicate    | A function that takes a (complex) eigenvalue as input and returns non-zero if it should be selected. For complex conjugate pairs of eigenvalues, the predicate is called only for the eigenvalue with positive imaginary part and the corresponding $2 \times 2$ block is either selected or deselected. |
| in  | arg          | An optional argument for the predicate function.   |
| out | selected     | The selection array. Both elements of a selected complex conjugate pair are set to 1.  |
| out | num_selected | The (global) number of selected eigenvalues (a complex conjugate pair is counted as two selected eigenvalues).   |

# Returns

STARNEIG\_SUCCESS (0) on success. Negative integer -i when i'th argument is invalid. Positive error code otherwise.

### **Examples**

```
sep_dm_full_chain.c.
```

## 13.8.2.7 starneig\_SEP\_DM\_Hessenberg\_expert()

Computes a Hessenberg decomposition of a general matrix.

#### **Parameters**

| in     | conf        | Configuration structure.  |
|--------|-------------|---|
| in     | begin       | First column to be reduced.   |
| in     | end         | Last column to be reduced + 1.  |
| in, ou | ıt 🛮 🖊      | On entry, the general matrix $A.$ On exit, the upper Hessenberg matrix $H.$   |
| in, ou | ıt <b>Q</b> | On entry, the orthogonal matrix $Q$ . On exit, the product matrix $Q\ast U$ . |

### Returns

STARNEIG\_SUCCESS (0) on success. Negative integer -i when i'th argument is invalid. Positive error code otherwise.

## 13.8.2.8 starneig\_SEP\_DM\_Schur\_expert()

Computes a Schur decomposition given a Hessenberg decomposition.

## **Parameters**

| in     | conf | Configuration structure.  |
|--------|------|---|
| in,out | Н    | On entry, the upper Hessenberg matrix $H.$ On exit, the Schur matrix $S.$                           |
| in,out | Q    | On entry, the orthogonal matrix $Q$ . On exit, the product matrix $Q\ast U$ .                       |
| out    | real | An array of the same size as ${\cal H}$ containing the real parts of the computed eigenvalues.      |
| out    | imag | An array of the same size as ${\cal H}$ containing the imaginary parts of the computed eigenvalues. |

# Returns

STARNEIG\_SUCCESS (0) on success. Negative integer -i when i'th argument is invalid. Positive error code otherwise.

### See also

```
starneig_SEP_DM_Schur
starneig_schur_conf
starneig_schur_init_conf
```

## 13.8.2.9 starneig\_SEP\_DM\_ReorderSchur\_expert()

Reorders selected eigenvalues to the top left corner of a Schur decomposition.

#### **Parameters**

| in     | conf     | Configuration structure.  |
|--------|----------|---|
| in,out | selected | The selection array.  |
| in,out | S        | On entry, the Schur matrix $S$ . On exit, the updated Schur matrix $\hat{S}$ .                      |
| in,out | Q        | On entry, the orthogonal matrix $Q$ . On exit, the product matrix $Q\ast U$ .                       |
| out    | real     | An array of the same size as ${\cal S}$ containing the real parts of the computed eigenvalues.      |
| out    | imag     | An array of the same size as ${\cal S}$ containing the imaginary parts of the computed eigenvalues. |

### Returns

STARNEIG\_SUCCESS (0) on success. Negative integer -i when i'th argument is invalid. Positive error code otherwise.

## See also

```
starneig_SEP_DM_ReorderSchur
starneig_SEP_DM_Select
starneig_reorder_conf
starneig_reorder_init_conf
```

## 13.8.2.10 starneig\_SEP\_DM\_Eigenvectors\_expert()

Computes an eigenvector for each selected eigenvalue.

### **Parameters**

| in             | conf     | Configuration structure.   |
|----------------|----------|--|
| in             | selected | The selection array specifying the locations of the selected eigenvalues. The number of 1's        |
|                |          | in the array is the same as the number of columns in $X$ .   |
| in             | S        | The Schur matrix $S$ .  Generated on Thu Nov 19 2020 18:32:52 for StarNEig User's Guide by Doxygen |
| in Q The ortho |          | The orthogonal matrix $Q$ .  |
| out            | X        | A matrix with $n$ rows and one column for each selected eigenvalue. The columns                    |
|                |          | represent the computed eigenvectors as previously described.                                       |

### Returns

STARNEIG\_SUCCESS (0) on success. Negative integer -i when i'th argument is invalid. Positive error code otherwise.

See also

starneig\_SEP\_DM\_Select

**Todo** This interface function is not implemented.

# 13.9 Distributed Memory / Generalized EVP

Functions for solving non-symmetric generalized eigenvalue problems on distributed memory systems.

## **Computational functions**

starneig\_error\_t starneig\_GEP\_DM\_HessenbergTriangular (starneig\_distr\_matrix\_t A, starneig\_distr\_matrix\_t B, starneig\_distr\_matrix\_t Q, starneig\_distr\_matrix\_t Z)

Computes a Hessenberg-triangular decomposition of a general matrix pencil.

starneig\_error\_t starneig\_GEP\_DM\_Schur (starneig\_distr\_matrix\_t H, starneig\_distr\_matrix\_t R, starneig\_distr\_matrix\_t Z, double real[], double imag[], double beta[])

Computes a generalized Schur decomposition given a Hessenberg-triangular decomposition.

• starneig\_error\_t starneig\_GEP\_DM\_ReorderSchur (int selected[], starneig\_distr\_matrix\_t S, starneig\_distr\_matrix\_t T, starneig\_distr\_matrix\_t Z, double real[], double imag[], double beta[])

Reorders selected generalized eigenvalues to the top left corner of a generalized Schur decomposition.

• starneig\_error\_t starneig\_GEP\_DM\_Reduce (starneig\_distr\_matrix\_t A, starneig\_distr\_matrix\_t B, starneig\_distr\_matrix\_t Q, starneig\_distr\_matrix\_t Z, double real[], double imag[], double beta[], int(\*predicate)(double real, double imag, double beta, void \*arg), void \*arg, int selected[], int \*num\_\cup selected]

Computes a (reordered) generalized Schur decomposition given a general matrix pencil.

starneig\_error\_t starneig\_GEP\_DM\_Eigenvectors (int selected[], starneig\_distr\_matrix\_t S, starneig\_distr\_matrix\_t T, starneig\_distr\_matrix\_t Z, starneig\_distr\_matrix\_t X)

Computes a generalized eigenvector for each selected generalized eigenvalue.

## **Helper functions**

• starneig\_error\_t starneig\_GEP\_DM\_Select (starneig\_distr\_matrix\_t S, starneig\_distr\_matrix\_t T, int(\*predicate)(double real, double imag, double beta, void \*arg), void \*arg, int selected[], int \*num\_selected)

Generates a selection array for a Schur-triangular matrix pencil using a user-supplied predicate function.

### **Expert computational functions**

starneig\_error\_t starneig\_GEP\_DM\_Schur\_expert (struct starneig\_schur\_conf \*conf, starneig\_distr\_matrix\_t H, starneig\_distr\_matrix\_t R, starneig\_distr\_matrix\_t Z, double real[], double imag[], double beta[])

Computes a generalized Schur decomposition given a Hessenberg-triangular decomposition.

• starneig\_error\_t starneig\_GEP\_DM\_ReorderSchur\_expert (struct starneig\_reorder\_conf \*conf, int selected[], starneig\_distr\_matrix\_t S, starneig\_distr\_matrix\_t T, starneig\_distr\_matrix\_t Q, starneig\_distr\_matrix\_t Z, double real[], double imag[], double beta[])

Reorders selected generalized eigenvalues to the top left corner of a generalized Schur decomposition.

starneig\_error\_t starneig\_GEP\_DM\_Eigenvectors\_expert (struct starneig\_eigenvectors\_conf \*conf, int selected[], starneig\_distr\_matrix\_t S, starneig\_distr\_matrix\_t T, starneig\_distr\_matrix\_t Z, starneig\_distr\_matrix\_t X)

Computes a generalized eigenvector for each selected generalized eigenvalue.

## 13.9.1 Detailed Description

Functions for solving non-symmetric generalized eigenvalue problems on distributed memory systems.

### 13.9.2 Function Documentation

## 13.9.2.1 starneig\_GEP\_DM\_HessenbergTriangular()

```
starneig\_error\_t \ starneig\_GEP\_DM\_HessenbergTriangular \ ( starneig\_distr\_matrix\_t \ A, starneig\_distr\_matrix\_t \ B, starneig\_distr\_matrix\_t \ \mathcal{Q}, starneig\_distr\_matrix\_t \ \mathcal{Z} \ )
```

Computes a Hessenberg-triangular decomposition of a general matrix pencil.

#### Attention

This function is a wrapper for several ScaLAPACK subroutines. The function exists if STARNEIG\_GEP\_DM\_HESSENBERGTRIA is defined.

#### **Parameters**

| in,out | Α | On entry, the general matrix $A$ . On exit, the upper Hessenberg matrix $H$ .    |
|--------|---|--|
|        |   | On entry, the general matrix $B$ . On exit, the upper triangular matrix $R$ .    |
|        |   | On entry, the orthogonal matrix $Q$ . On exit, the product matrix $Q \ast U_1$ . |
| in,out | Z | On entry, the orthogonal matrix $Z$ . On exit, the product matrix $Z \ast U_2$ . |

## Returns

STARNEIG\_SUCCESS (0) on success. Negative integer -i when i'th argument is invalid. Positive error code otherwise.

## **Examples**

```
gep_dm_full_chain.c.
```

## 13.9.2.2 starneig\_GEP\_DM\_Schur()

Computes a generalized Schur decomposition given a Hessenberg-triangular decomposition.

### **Parameters**

| in,out | Н    | On entry, the upper Hessenberg matrix $H.$ On exit, the Schur matrix $S.$   |
|--------|------|---|
| in,out | R    | On entry, the upper triangular matrix $R$ . On exit, the upper triangular matrix $T$ .  |
| in,out | Q    | On entry, the orthogonal matrix $Q$ . On exit, the product matrix $Q*U_1$ .   |
| in,out | Z    | On entry, the orthogonal matrix $Z$ . On exit, the product matrix $Z*U_2$ .   |
| out    | real | An array of the same size as $H$ containing the real parts of the $\alpha$ values of the computed generalized eigenvalues.      |
| out    | imag | An array of the same size as $H$ containing the imaginary parts of the $\alpha$ values of the computed generalized eigenvalues. |
| out    | beta | An array of the same size as $H$ containing the $\beta$ values of computed generalized eigenvalues.                             |

## Returns

STARNEIG\_SUCCESS (0) on success. Negative integer -i when i'th argument is invalid. Positive error code otherwise. STARNEIG\_DID\_NOT\_CONVERGE if the QZ algorithm failed to converge.

### **Examples**

gep\_dm\_full\_chain.c.

## 13.9.2.3 starneig\_GEP\_DM\_ReorderSchur()

```
starneig_error_t starneig_GEP_DM_ReorderSchur (
    int selected[],
    starneig_distr_matrix_t S,
    starneig_distr_matrix_t T,
    starneig_distr_matrix_t Q,
    starneig_distr_matrix_t Z,
    double real[],
    double imag[],
    double beta[])
```

Reorders selected generalized eigenvalues to the top left corner of a generalized Schur decomposition.

### **Parameters**

| in,out | selected | The selection array. On entry, the initial positions of the selected generalized eigenvalues. On exit, the final positions of all correctly placed selected generalized eigenvalues. In case of failure, the number of 1's in the output may be less than the number of 1's in the input. |
|--------|----------|---|
| in,out | S        | On entry, the Schur matrix $S$ . On exit, the updated Schur matrix $\hat{S}$ .  |
| in,out | T        | On entry, the upper triangular $T$ . On exit, the updates upper triangular matrix $\hat{T}$ .   |
| in,out | Q        | On entry, the orthogonal matrix $Q$ . On exit, the product matrix $Q st U_1$ .  |
| in,out | Z        | On entry, the orthogonal matrix $Z$ . On exit, the product matrix $Z \ast U_2$ .  |
| out    | real     | An array of the same size as $S$ containing the real parts of the $\alpha$ values of the computed generalized eigenvalues.  |
| out    | imag     | An array of the same size as $S$ containing the imaginary parts of the $\alpha$ values of the computed generalized eigenvalues.   |
| out    | beta     | An array of the same size assamentaining thee 18 2020 ues 2022 consecuted users caliceed boxygen eigenvalues.   |

#### Returns

STARNEIG\_SUCCESS (0) on success. Negative integer -i when i'th argument is invalid. Positive error code otherwise. STARNEIG\_PARTIAL\_REORDERING if the generalized Schur form is not fully reordered.

#### See also

```
starneig_GEP_DM_Select
```

#### **Examples**

gep\_dm\_full\_chain.c.

### 13.9.2.4 starneig\_GEP\_DM\_Reduce()

```
starneig_error_t starneig_GEP_DM_Reduce (
    starneig_distr_matrix_t A,
    starneig_distr_matrix_t B,
    starneig_distr_matrix_t Q,
    starneig_distr_matrix_t Z,
    double real[],
    double imag[],
    double beta[],
    int(*) (double real, double imag, double beta, void *arg) predicate,
    void * arg,
    int selected[],
    int * num_selected )
```

Computes a (reordered) generalized Schur decomposition given a general matrix pencil.

#### Attention

This function uses several ScaLAPACK subroutines. The function exists if STARNEIG\_GEP\_DM\_REDUCE is defined.

#### **Parameters**

| Parameters       |                        |   |  |
|------------------|------------------------|---|--|
| in,out           | Α                      | On entry, the general matrix $A$ . On exit, the Schur matrix $S$ .  |  |
| in,out           | В                      | On entry, the general matrix $B$ . On exit, the upper triangular matrix $T$ .   |  |
| in,out           | Q                      | On entry, the orthogonal matrix $Q$ . On exit, the product matrix $Q*U_1$ .   |  |
| in,out           | Z                      | On entry, the orthogonal matrix $Z$ . On exit, the product matrix $Z \ast U_2$ .  |  |
| out              | real                   | An array of the same size as $A$ containing the real parts of the $\alpha$ values of the computed generalized eigenvalues.  |  |
| out              | imag                   | An array of the same size as $A$ containing the imaginary parts of the $\alpha$ values of the computed generalized eigenvalues.   |  |
| out              | beta                   | An array of the same size as $A$ containing the $\beta$ values of computed generalized eigenvalues.   |  |
| in               | predicate              | A function that takes a (complex) generalized eigenvalue as input and returns non-zero if it should be selected. For complex conjugate pairs of generalized eigenvalues, the predicate is called only for the generalized eigenvalue with positive imaginary part and the corresponding $2\times 2$ block is either selected or deselected. The reordering step is skipped if the argument is a NULL pointer. |  |
| Gengrated on Thu | N@1709 2020 18:32:52 1 | or Amarupitional នេះឲ្យបានសម្រាប់ស្នែនៅne predicate function.   |  |
| out              | selected               | The final positions of all correctly placed selected generalized eigenvalues.   |  |
| out              | num_selected           | The number of selected generalized eigenvalues (a complex conjugate pair is   |  |

counted as two selected generalized eigenvalues).

#### Returns

STARNEIG\_SUCCESS (0) on success. Negative integer -i when i'th argument is invalid. Positive error code otherwise. STARNEIG\_DID\_NOT\_CONVERGE if the QZ algorithm failed to converge. STARNEIG\_PARTIAL\_REORDERING if the generalized Schur form is not fully reordered.

### 13.9.2.5 starneig\_GEP\_DM\_Eigenvectors()

```
starneig_error_t starneig_GEP_DM_Eigenvectors (
    int selected[],
    starneig_distr_matrix_t S,
    starneig_distr_matrix_t T,
    starneig_distr_matrix_t Z,
    starneig_distr_matrix_t X )
```

Computes a generalized eigenvector for each selected generalized eigenvalue.

#### **Parameters**

| in  | selected | The selection array specifying the locations of the selected generalized eigenvalues. The number of 1's in the array is the same as the number of columns in $X$ .   |
|-----|----------|--|
| in  | S        | The Schur matrix $S$ .   |
| in  | T        | The upper triangular matrix $T$ .  |
| in  | Z        | The orthogonal matrix $Z$ .  |
| out | X        | A matrix with $n$ rows and one column for each selected generalized eigenvalue. The columns represent the computed generalized eigenvectors as previously described. |

### Returns

STARNEIG\_SUCCESS (0) on success. Negative integer -i when i'th argument is invalid. Positive error code otherwise.

### See also

```
starneig_GEP_DM_Select
```

Todo This interface function is not implemented.

### 13.9.2.6 starneig\_GEP\_DM\_Select()

Generates a selection array for a Schur-triangular matrix pencil using a user-supplied predicate function.

#### **Parameters**

| in  | S            | The Schur matrix $S$ .   |
|-----|--------------|--|
| in  | T            | The upper triangular matrix $T$ .  |
| in  | predicate    | A function that takes a (complex) generalized eigenvalue as input and returns non-zero if it should be selected. For complex conjugate pairs of generalized eigenvalues, the predicate is called only for the generallized eigenvalue with positive imaginary part and the corresponding $2\times 2$ block is either selected or deselected. |
| in  | arg          | An optional argument for the predicate function.   |
| out | selected     | The selection array. Both elements of a selected complex conjugate pair are set to 1.  |
| out | num_selected | The number of selected generalized eigenvalues (a complex conjugate pair is counted as two selected generalized eigenvalues).  |

#### Returns

STARNEIG\_SUCCESS (0) on success. Negative integer -i when i'th argument is invalid. Positive error code otherwise.

### **Examples**

gep\_dm\_full\_chain.c.

### 13.9.2.7 starneig\_GEP\_DM\_Schur\_expert()

```
starneig_error_t starneig_GEP_DM_Schur_expert (
    struct starneig_schur_conf * conf,
    starneig_distr_matrix_t H,
    starneig_distr_matrix_t R,
    starneig_distr_matrix_t Q,
    starneig_distr_matrix_t Z,
    double real[],
    double imag[],
    double beta[])
```

Computes a generalized Schur decomposition given a Hessenberg-triangular decomposition.

### **Parameters**

| in     | conf | Configuration structure.  |
|--------|------|---|
| in,out | Н    | On entry, the upper Hessenberg matrix $H.$ On exit, the Schur matrix $S.$   |
| in,out | R    | On entry, the upper triangular matrix $R$ . On exit, the upper triangular matrix $T$ .  |
| in,out | Q    | On entry, the orthogonal matrix $Q$ . On exit, the product matrix $Q \ast U_1$ .  |
| in,out | Z    | On entry, the orthogonal matrix $Z$ . On exit, the product matrix $Z*U_2$ .   |
| out    | real | An array of the same size as $H$ containing the real parts of the $\alpha$ values of the computed generalized eigenvalues.      |
| out    | imag | An array of the same size as $H$ containing the imaginary parts of the $\alpha$ values of the computed generalized eigenvalues. |
| out    | beta | An array of the same size as $H$ containing the $\beta$ values of computed generalized eigenvalues.                             |

#### Returns

STARNEIG\_SUCCESS (0) on success. Negative integer -i when i'th argument is invalid. Positive error code otherwise.

#### See also

```
starneig_GEP_DM_Schur
starneig_schur_conf
starneig_schur_init_conf
```

#### 13.9.2.8 starneig GEP DM ReorderSchur expert()

Reorders selected generalized eigenvalues to the top left corner of a generalized Schur decomposition.

#### **Parameters**

| in     | conf     | Configuration structure.  |
|--------|----------|---|
| in,out | selected | The selection array.  |
| in,out | S        | On entry, the Schur matrix $S$ . On exit, the updated Schur matrix $\hat{S}$ .  |
| in,out | T        | On entry, the upper triangular $T$ . On exit, the updates upper triangular matrix $\hat{T}$ .                                   |
| in,out | Q        | On entry, the orthogonal matrix $Q$ . On exit, the product matrix $Q \ast U_1$ .  |
| in,out | Z        | On entry, the orthogonal matrix $Z$ . On exit, the product matrix $Z \ast U_2$ .  |
| out    | real     | An array of the same size as $S$ containing the real parts of the $\alpha$ values of the computed generalized eigenvalues.      |
| out    | imag     | An array of the same size as $S$ containing the imaginary parts of the $\alpha$ values of the computed generalized eigenvalues. |
| out    | beta     | An array of the same size as $S$ containing the $\beta$ values of computed generalized eigenvalues.                             |

### Returns

STARNEIG\_SUCCESS (0) on success. Negative integer -i when i'th argument is invalid. Positive error code otherwise.

#### See also

```
starneig_GEP_DM_ReorderSchur
starneig_GEP_DM_Select
starneig_reorder_conf
starneig_reorder_init_conf
```

### 13.9.2.9 starneig\_GEP\_DM\_Eigenvectors\_expert()

Computes a generalized eigenvector for each selected generalized eigenvalue.

#### **Parameters**

| in  | conf     | Configuration structure.   |
|-----|----------|--|
| in  | selected | The selection array specifying the locations of the selected generalized eigenvalues. The number of 1's in the array is the same as the number of columns in $X$ .   |
| in  | S        | The Schur matrix $S$ .   |
| in  | T        | The upper triangular matrix $T$ .  |
| in  | Z        | The orthogonal matrix $Z$ .  |
| out | X        | A matrix with $n$ rows and one column for each selected generalized eigenvalue. The columns represent the computed generalized eigenvectors as previously described. |

#### Returns

STARNEIG\_SUCCESS (0) on success. Negative integer -i when i'th argument is invalid. Positive error code otherwise.

### See also

```
starneig_GEP_DM_Select
```

**Todo** This interface function is not implemented.

# 13.10 Expert configuration structures

Configuration structures and functions for the expert interface functions.

#### **Data Structures**

· struct starneig\_hessenberg\_conf

Hessenberg reduction configuration structure. More...

· struct starneig\_schur\_conf

Schur reduction configuration structure. More...

· struct starneig\_reorder\_conf

Eigenvalue reordering configuration structure. More...

struct starneig\_eigenvectors\_conf

Eigenvector computation configuration structure. More...

### **Hessenberg reduction**

void starneig\_hessenberg\_init\_conf (struct starneig\_hessenberg\_conf \*conf)

Initializes a Hessenberg reduction configuration structure with default parameters.

#define STARNEIG HESSENBERG DEFAULT TILE SIZE -1

Default tile size.

#define STARNEIG\_HESSENBERG\_DEFAULT\_PANEL\_WIDTH -1

Default panel width.

### **Schur reduction**

• void starneig\_schur\_init\_conf (struct starneig\_schur\_conf \*conf)

Initializes a Schur reduction configuration structure with default parameters.

#define STARNEIG\_SCHUR\_DEFAULT\_INTERATION\_LIMIT -1

Default iteration limit.

#define STARNEIG\_SCHUR\_DEFAULT\_TILE\_SIZE -1

Default tile size.

#define STARNEIG\_SCHUR\_DEFAULT\_SMALL\_LIMIT -1

Default sequential QR limit.

#define STARNEIG\_SCHUR\_DEFAULT\_AED\_WINDOW\_SIZE -1

Default AED window size.

#define STARNEIG\_SCHUR\_DEFAULT\_AED\_NIBBLE -1

Default nibble value.

#define STARNEIG\_SCHUR\_DEFAULT\_AED\_PARALLEL\_SOFT\_LIMIT -1

Default soft sequential AED limit.

• #define STARNEIG\_SCHUR\_DEFAULT\_AED\_PARALLEL\_HARD\_LIMIT -1

Default hard sequential AED limit.

• #define STARNEIG SCHUR DEFAULT SHIFT ORIGIN -1

Default shift origin.

#define STARNEIG\_SCHUR\_DEFAULT\_SHIFT\_COUNT -1

Default shift count.

• #define STARNEIG SCHUR DEFAULT WINDOW SIZE -1

Default bulge chasing window size.

• #define STARNEIG\_SCHUR\_ROUNDED\_WINDOW\_SIZE -2

Rounded bulge chasing window.

• #define STARNEIG SCHUR DEFAULT SHIFTS PER WINDOW -1

Default number of shifts per bulge chasing window.

#define STARNEIG SCHUR DEFAULT UPDATE WIDTH -1

Default left-hand side update width.

#define STARNEIG\_SCHUR\_DEFAULT\_UPDATE\_HEIGHT -1

Default right-hand side update height.

#define STARNEIG SCHUR DEFAULT THRESHOLD -1

Default deflation threshold.

#define STARNEIG\_SCHUR\_NORM\_STABLE\_THRESHOLD -2

Norm stable deflation threshold.

#define STARNEIG SCHUR LAPACK THRESHOLD -3

LAPACK-style deflation threshold.

#### Eigenvalue reordering

enum starneig\_reorder\_plan\_t { STARNEIG\_REORDER\_DEFAULT\_PLAN = 1, STARNEIG\_REORDER\_ONE\_PART\_PLAN = 2, STARNEIG\_REORDER\_MULTI\_PART\_PLAN = 3 }

Reordering plan enumerator.

enum starneig\_reorder\_blueprint\_t {

```
\label{eq:starneig_reorder_default_blueprint = 1, Starneig_reorder_dummy_insert_A = 2, Starneig_reorder_dummy_insert_B = 3, Starneig_reorder_chain_insert_A = 4, Starneig_reorder_chain_insert_B = 5, Starneig_reorder_chain_insert_C = 6, Starneig_reorder_chain_insert_D = 7, Starneig_reorder_chain_insert_E = 8, Starneig_reorder_chain_insert_F = 9,
```

Task insertion blueprint.

void starneig reorder init conf (struct starneig reorder conf \*conf)

Initializes an eigenvalue reordering configuration structure with default parameters.

• #define STARNEIG\_REORDER\_DEFAULT\_UPDATE\_WIDTH -1

Default left-hand side update task width.

#define STARNEIG\_REORDER\_DEFAULT\_UPDATE\_HEIGHT -1

Default right-hand side update task height.

#define STARNEIG\_REORDER\_DEFAULT\_TILE\_SIZE -1

Default tile size.

#define STARNEIG\_REORDER\_DEFAULT\_VALUES\_PER\_CHAIN -1

Default number of selected eigenvalues per window.

#define STARNEIG\_REORDER\_DEFAULT\_WINDOW\_SIZE -1

Default default window size.

• #define STARNEIG REORDER ROUNDED WINDOW SIZE -2

Default rounded window size.

• #define STARNEIG\_REORDER\_DEFAULT\_SMALL\_WINDOW\_SIZE -1

Default small window size.

#define STARNEIG\_REORDER\_DEFAULT\_SMALL\_WINDOW\_THRESHOLD -1

Default small window threshold.

### **Eigenvectors**

void starneig\_eigenvectors\_init\_conf (struct starneig\_eigenvectors\_conf \*conf)

Initializes an eigenvectors configuration structure with default parameters.

#define STARNEIG\_EIGENVECTORS\_DEFAULT\_TILE\_SIZE -1

Default tile size.

# 13.10.1 Detailed Description

Configuration structures and functions for the expert interface functions.

### 13.10.2 Data Structure Documentation

### 13.10.2.1 struct starneig\_hessenberg\_conf

Hessenberg reduction configuration structure.

### **Data Fields**

| int | tile_size   | The matrices are divided into square tiles. This parameter defines the used tile size. If the parameter is set to STARNEIG_HESSENBERG_DEFAULT_TILE_SIZE, then the   |
|-----|-------------|---|
|     |             | implementation will determine a suitable tile size automatically.   |
| int | panel_width | The reduction is performed one panel at a time. This parameter defines the used panel width. If the parameter is set to STARNEIG_HESSENBERG_DEFAULT_PANEL_WIDTH, then the implementation will determine a suitable panel width automatically. |

### 13.10.2.2 struct starneig\_schur\_conf

Schur reduction configuration structure.

### **Data Fields**

| int | iteration_limit | The QR/QZ is an iterative algorithm. This parameter defines the maximum number of iterations the algorithm is allowed to perform. If the parameter is STARNEIG_SCHUR_DEFAULT_INTERATION_LIMIT, then the implementation will determine a suitable iteration limit automatically.  |
|-----|-----------------|--|
| int | tile_size       | The matrices are divided into square tiles. This parameter defines the used tile size. If the parameter is set to STARNEIG_SCHUR_DEFAULT_TILE_SIZE, then the implementation will determine a suitable tile size automatically.   |
| int | small_limit     | As the QR/QZ algorithm progresses, the size of the active region shrinks. Once the size of the active region is small enough, then the remaining problem is solved in a sequential manner. This parameter defines the transition point where the implementation switches to a sequential QR algorithm. If the parameter is set to STARNEIG_SCHUR_DEFAULT_SMALL_LIMIT, then the implementation will determine a suitable switching point automatically. |
| int | aed_window_size | The implementation relies on a so-called Aggressive Early Deflation (AED) technique to accelerate the convergence of the algorithm. Each AED is performed inside a small diagonal window. This parameter defines used AED window size. If the parameter is set to STARNEIG_SCHUR_DEFAULT_AED_WINDOW_SIZE, then the implementation will determine a suitable AED window size automatically.   |
| int | aed_nibble      | The implementation relies on a so-called Aggressive Early Deflation (AED) technique to accelerate the convergence of the algorithm. Each AED is performed inside a small diagonal window. If the number deflated (converged) eigenvalues is larger than (aed_nibble / 100) × size of AED window, then the next bulge chasing step is skipped. If the parameter is set to STARNEIG_SCHUR_DEFAULT_AED_NIBBLE,  |
|     |                 | the parameter is set to STARNEIG_SCHUR_DEFAULT_AED_NIBBLE then the implementation will determine a suitable value automatically generated on the Nov 19 2020 18:32:52 for starking user's Guide by Default.  |

### **Data Fields**

| int | aed parallel soft limit   | The implementation relies on a second Aggregative Early Deflation   |
|-----|---------------------------|---|
| int | aeo_parallel_soit_lifflit | The implementation relies on a so-called Aggressive Early Deflation (AED) technique to accelerate the convergence of the algorithm. Each AED is performed inside a small diagonal window. An AED can be performed sequentially or in parallel. This parameter defines the transition point where the implementation allowed to switch to a sequential AED algorithm. The decision is made based on the size of the AED window. If the parameter is set to STARNEIG_SCHUR_DEFAULT_AED_PARALLEL_SOFT_LIMIT, then the implementation will determine a suitable switching point automatically.  |
| int | aed_parallel_hard_limit   | The implementation relies on a so-called Aggressive Early Deflation (AED) technique to accelerate the convergence of the algorithm. Each AED is performed inside a small diagonal window. An AED can be performed sequentially or in parallel. This parameter defines the transition point where the implementation switches to a sequential AED algorithm. The decision is made based on the size of the AED window. If the parameter is set to STARNEIG_SCHUR_DEFAULT_AED_PARALLEL_HARD_LIMIT, then the implementation will determine a suitable switching point automatically.   |
| int | shift_origin              | The QR/QZ algorithm chases a set of $3 \times 3$ bulges across the diagonal of the Hessenberg(-triangular) decomposition. Two shifts (eigenvalue estimates) are required to generate each bulge. This parameter defines how the shifts are computed. If the parameter is set to STARNEIG_SCHUR_AED_SHIFT_ORIGIN, then the implementation will use the failed eigenvalue candidates from an AED as shifts. If the parameter is set to STARNEIG_SCHUR_SEPARATE_SHIFT_ORIGIN, then the implementation will compute the shift separately. If the parameter is set to STARNEIG_SCHUR_DEFAULT_SHIFT_ORIGIN, then the implementation will determine a suitable shift origin automatically.   |
| int | shift_count               | The QR/QZ algorithm chases a set of $3\times3$ bulges across the diagonal of the Hessenberg(-triangular) decomposition. Two shifts (eigenvalue estimates) are required to generate each bulge. This parameter defines the number of shifts to use. If the parameter is set to STARNEIG_SCHUR_DEFAULT_SHIFT_COUNT, then the implementation will determine a suitable shift count automatically.  |
| int | window_size               | The QR/QZ algorithm chases a set of 3 × 3 bulges across the diagonal of the Hessenberg(-triangular) decomposition. The bulges are chased in batches. The related similarity transformations are initially restricted to inside a small diagonal window and the accumulated transformation are applied only later as BLAS-3 updates. This parameter defines the used bulge chasing window size. If the parameter is set to STARNEIG_SCHUR_ROUNDED_WINDOW_SIZE, then  • maximum window size is set to 2 * tile_size and  • the windows are placed such that their lower right corners respect the boundaries of the underlying data tiles.  If the parameter is set to STARNEIG_SCHUR_DEFAULT_WINDOW_SIZE, then the implementation will determine a quitable window size outcome tight. |
| int | shifts_per_window         | implementation will determine a suitable window size automatically. The QR/QZ algorithm chases a set of $3\times 3$ bulges across the diagonal of the Hessenberg(-triangular) decomposition. The bulges are chased in batches. This parameter defines the used batch size. If the parameter is set to STARNEIG_SCHUR_DEFAULT_SHIFTS_PER_WINDOW then the implementation will determine a suitable batch size automatically.  |

### **Data Fields**

| int    | update_width    | The similarity similarity transformations are initially restricted to inside a small diagonal window and the accumulated transformation are applied only later as BLAS-3 updates. This parameter defines the width of each left-hand side update task. The value should be multiple of the tile size. If the parameter is set to STARNEIG_SCHUR_DEFAULT_UPDATE_WIDTH, then the implementation will determine a suitable width automatically.  |
|--------|-----------------|---|
| int    | update_height   | The similarity similarity transformations are initially restricted to inside a small diagonal window and the accumulated transformation are applied only later as BLAS-3 updates. This parameter defines the height of each right-hand side update task. The value should be multiple of the tile size. If the parameter is set to STARNEIG_SCHUR_DEFAULT_UPDATE_HEIGHT, then the implementation will determine a suitable height automatically.  |
| double | left_threshold  | The QR/QZ algorithm is allowed to set tiny matrix entires to zero as long as their magnitudes are smaller that a given threshold. This parameter defines the threshold for the left-hand side matrix ( $H$ ). If the parameter is set to STARNEIG_SCHUR_DEFAULT_THRESHOLD, then the implementation will determine a suitable threshold automatically. If the parameter is set to STARNEIG_SCHUR_NORM_STABLE_THRESHOLD, then the implementation will use the threshold $u H _F$ , where $u$ is the unit roundoff and $ H _F$ is the Frobenius norm of the matrix $H$ . If the parameter is set to STARNEIG_SCHUR_LAPACK_THRESHOLD, then the implementation will use a deflation threshold that is compatible with LAPACK.                    |
| double | right_threshold | The QZ algorithm is allowed to set tiny matrix entires to zero as long as their magnitudes are smaller that a given threshold. This parameter defines the threshold for the right-hand side matrix ( $R$ ) off-diagonal entires. If the parameter is set to STARNEIG_SCHUR_DEFAULT_THRESHOLD, then the implementation will determine a suitable threshold automatically. If the parameter is set to STARNEIG_SCHUR_NORM_STABLE_THRESHOLD, then the implementation will use the threshold $u R _F$ , where $u$ is the unit roundoff and $ H _F$ is the Frobenius norm of the matrix $R$ . If the parameter is set to STARNEIG_SCHUR_LAPACK_THRESHOLD, then the implementation will use a deflation threshold that is compatible with LAPACK. |
| double | inf_threshold   | The QZ algorithm is allowed to set tiny matrix entires to zero as long as their magnitudes are smaller that a given threshold. This parameter defines the threshold for the right-hand side matrix ( $R$ ) diagonal entries. If the parameter is set to STARNEIG_SCHUR_DEFAULT_THRESHOLD, then the implementation will determine a suitable threshold automatically. If the parameter is set to STARNEIG_SCHUR_NORM_STABLE_THRESHOLD, then the implementation will use the threshold $u R _F$ , where $u$ is the unit roundoff and $ R _F$ is the Frobenius norm of the matrix $R$ .  |

### 13.10.2.3 struct starneig\_reorder\_conf

Eigenvalue reordering configuration structure.

### **Data Fields**

| starneig_reorder_plan_t starneig_reorder_blueprint_t | plan              | This parameter plan defines the used reordering plan. If the parameter is set to STARNEIG_REORDER_DEFAULT_PLAN, then the implementation will determine a suitable reordering plan automatically.  This parameter defines the used task insertion   |          |
|--|-------------------|--|----------|
| starrietg_reorder_bideprint_t                        | Биерин            | blueprint. If the parameter is set to STARNEIG_REORDER_DEFAULT_BLUEPRINT, then the implementation will determine a suitable task insertion blueprint automatically.  |          |
| int  | tile_size         | The matrices are divided into square tiles. This parameter defines the used tile size. If the parameter is set to STARNEIG_REORDER_DEFAULT_TILE_SIZE, then the implementation will determine a suitable tile size automatically.   |          |
| int  | values_per_chain  | The selected eigenvalues are processed in batches and each batch is assigned a window chain. This parameter defines the number of selected eigenvalues processed by each window chain. If the parameter is set to STARNEIG_REORDER_DEFAULT_VALUES_PER_then the implementation will determine a suitable value automatically. | CHAIN,   |
| int  | window_size       | The similarity similarity transformations are initially restricted to inside a small diagonal window and the accumulated transformation are applied only later as BLAS-3 updates. This parameter defines the size of the window. If the parameter is set to STARNEIG_REORDER_ROUNDED_WINDOW_SIZ then                         | Ε,       |
|  |                   | <ul> <li>maximum window size is set to 2 * tile_size,</li> <li>the windows are placed such that their upper left corners respect the boundaries of the underlying data tiles, and</li> </ul>   |          |
|  |                   | the parameter values_per_chain is ignored.  If the parameter is set to  STARNEIG_REORDER_DEFAULT_WINDOW_SIZE then the implementation will determine a suitable window size automatically.  | ,        |
| int  | small_window_size | Larger diagonal window are processed using even smaller diagonal windows in a recursive manner. This parameter defines the used small window size. If the parameter is set to STARNEIG_REORDER_DEFAULT_SMALL_WINDO then the implementation will determine a suitable small window size automatically.                        | oW_SIZE, |

### **Data Fields**

| int | small_window_threshold | Larger diagonal window are processed using even smaller diagonal windows in a recursive manner. This parameter defines the largest diagonal window that is processed in a scalar manner. If the parameter is set to STARNEIG_REORDER_DEFAULT_SMALL_WINDOWN then the implementation will determine a suitable threshold automatically.   | )W_THRESHOLD |
|-----|------------------------|---|--------------|
| int | update_width           | The similarity similarity transformations are initially restricted to inside a small diagonal window and the accumulated transformation are applied only later as BLAS-3 updates. This parameter defines the width of each left-hand side update task. The value should be multiple of the tile size. If the parameter is set to STARNEIG_REORDER_DEFAULT_UPDATE_WIDT then the implementation will determine a suitable width automatically.    | Ή,           |
| int | update_height          | The similarity similarity transformations are initially restricted to inside a small diagonal window and the accumulated transformation are applied only later as BLAS-3 updates. This parameter defines the height of each right-hand side update task. The value should be multiple of the tile size. If the parameter is set to STARNEIG_REORDER_DEFAULT_UPDATE_HEIG then the implementation will determine a suitable height automatically. | НТ,          |

### 13.10.2.4 struct starneig\_eigenvectors\_conf

Eigenvector computation configuration structure.

### Data Fields

| int | tile_size | The matrices are divided into tiles. This parameter defines the used tile size. If the parameter |  |
|-----|-----------|--|--|
|     |           | is set to STARNEIG_EIGENVECTORS_DEFAULT_TILE_SIZE, then the implementation will                  |  |
|     |           | determine a suitable tile size automatically.  |  |

# 13.10.3 Enumeration Type Documentation

### 13.10.3.1 starneig\_reorder\_plan\_t

enum starneig\_reorder\_plan\_t

Reordering plan enumerator.

Eigenvalues that fall within a diagonal computation *window* are reordered such that all selected eigenvalues are moved to the upper left corner of the window. The corresponding orthogonal transformations are accumulated to separate accumulator matrix / matrices.

A window chain comprises from multiple overlapping diagonal computation windows that are intended to be processed in a particular order. More precisely, the windows are placed such that the overlap between two windows is big enough to accommodate all selected eigenvalues that fall within the preceding windows. In this way, the windows can be processed in sequential order, starting from the bottom window, such that the reordering that takes place in one window always moves the preceding selected eigenvalues to the lower right corner of the next window. In the end, all selected that fall within the combined computation area of the chain are moved to the upper left corner of the topmost window.

An example showing how an eigenvalue can be moved six entries upwards by using three diagonal windows:

| ++           | ++           | ++                                     | ++   |
|--------------|--------------|--|--|
| x x x   ===> | x x x   ===> | x x x   ===>                           | #<+  |
| x-x-++       | X-X-++       | x-x-++                                 | ¤- -++   |
| x x x        | x x x        | #<+                                    |  |
| ++x-x-++     | ++x-x-++     | $++ {\color{myallow} \alpha}-\mid -++$ | $++\!$ |
| x x x        | #<+          |  |  |
| ++ x x       | ++           | ++ ¤                                   | ++a ¤  |
| #            | ¤            | ¤                                      | ¤  |
| ++           | ++           | ++                                     | ++   |

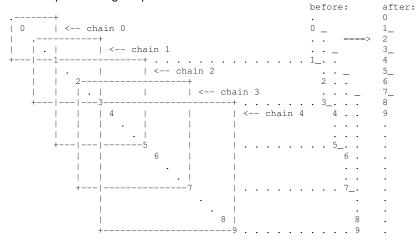
The number of selected eigenvalues that can be moved by a single window chain is limited by the windows size. Thus, the whole reordering procedure usually involves multiple chains that must be be processed in a particular order. A *chain list* describes a list of chains that are intended to be processed together. Window chains that belong to different chain lists are processed separately.

A plan consists from one or more chain lists that are intended to be processed in a particular order.

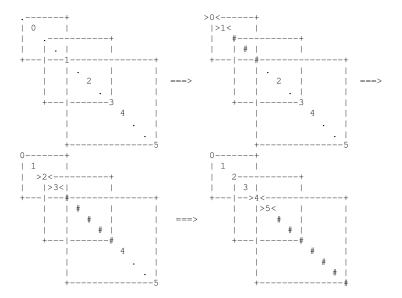
### STARNEIG\_REORDER\_ONE\_PART\_PLAN:

The first chain is placed in the upper left corner of the matrix and its size is chosen such that it contains a desired number of selected eigenvalues (starneig\_reorder\_conf::values\_per\_chain parameter). The next chain is places such that its upper left corner is located one entry after the location where the last selected eigenvalue, that falls within the first chain, would be after the reordering. The chain is sized such that the part of the chain, that does not intersect the first chain, contain the desired number of selected eigenvalues. This same procedure is repeated until all selected eigenvalues have been accounted for. All chains belong to the same chain lists and are intended to be processed sequentially.

An example showing the placement of the chains in a case where each chain wields two selected eigenvalues:



An example showing what happens when the first three chains are processed:



If necessary, each chain is re-sized to avoid splitting any  $2\times 2$  tiles.

Windows are placed such that the first window is located in the lower right corner of the computation area of the window chain. The last window is correspondingly placed in the upper left corner of the computation area.

If necessary, each window is re-sized to avoid splitting any  $2 \times 2$  tiles.

### STARNEIG\_REORDER\_MULTI\_PART\_PLAN:

A multi-part reordering plan is derived from an one-part reordering plan by splitting the chains into sub-chains as shown below:

```
Initial one-part plan:
 Chain 0: aaaaaa
 Chain 1: bbbbbbbbbb
                              a,b,c,d,e diagonal computation window
 Chain 2:
          ccccccccccc
           ddddddddddddddd
 Chain 4:
            eeeeeeeeeeeeeee
Resulting multi-part plan:
 Chain 0: aaaaaa
 Chain 1: .....bbbb
 Chain 2:
                                   chain list 0
          Chain 3:
          .....dddd
            ....eeee
 Chain 0: bbbbbb....
 Chain 1:
          .....cccc....
                                   chain list 1
 Chain 2:
           .....dddd....
 Chain 3:
            .....eeee....
 Chain 0:
 Chain 1:
           .....dddd.....
                                   chain list 2
 Chain 2:
            .....eeee.....
 Chain 0:
                                   chain list 3
           dddddd......
            .....eeee.......
 Chain 0:
            eeeeee.......
                                   chain list 4
```

Note that the chains that belong to the same chain list are independent from each other and can therefore be processed in an arbitrary order.

#### Enumerator

| STARNEIG_REORDER_DEFAULT_PLAN    | Default plan.    |
|----------------------------------|------------------|
| STARNEIG_REORDER_ONE_PART_PLAN   | One part plan.   |
| STARNEIG_REORDER_MULTI_PART_PLAN | Multi part plan. |

### 13.10.3.2 starneig\_reorder\_blueprint\_t

enum starneig\_reorder\_blueprint\_t

Task insertion blueprint.

A task insertion blueprint defines how a reordering plan is carried out.

#### Enumerator

| STARNEIG_REORDER_DEFAULT_BLUEPRINT | Default blueprint.  |
|------------------------------------|---|
| STARNEIG_REORDER_DUMMY_INSERT_A    | One-pass forward dummy blueprint. Processes the window chains in order starting from the topmost chain. All update tasks are inserted right after each window reordering task.  |
| STARNEIG_REORDER_DUMMY_INSERT_B    | Two-pass backward dummy blueprint. Processes the window chains in two phases starting from the bottommost chain. The window reordering tasks and the right-hand side update tasks are inserted during the first phase. Other update tasks are inserted during the second phase.   |
| STARNEIG_REORDER_CHAIN_INSERT_A    | One-pass forward chain blueprint. Processes the window chains in order starting from the topmost chain. The window reordering tasks and high priority right-hand side update tasks are inserted first. Other update tasks are inserted after them.  |
| STARNEIG_REORDER_CHAIN_INSERT_B    | Two-pass forward chain blueprint. Processes the window chains in two phases starting from the topmost chain. The window reordering tasks and high priority right-hand side update tasks are inserted first. The left- hand side updates are inserted after them. Other updates are inserted during the second phase.  |
| STARNEIG_REORDER_CHAIN_INSERT_C    | One-pass backward chain blueprint. Processes the window chains in order starting from the bottommost chain. The window reordering tasks and high priority right-hand side update tasks are inserted first. Other update tasks are inserted later.   |
| STARNEIG_REORDER_CHAIN_INSERT_D    | Two-pass backward chain blueprint. Processes the window chains in two phases starting from the bottommost chain. The window reordering tasks and high priority right-hand side update tasks are inserted first. Update tasks that are related to the Schur matrix are inserted later. Update tasks that are related to the orthogonal matrices are inserted during the second phase.                          |
| STARNEIG_REORDER_CHAIN_INSERT_E    | Two-pass delayed backward chain blueprint. Processes the window chains in order starting from the bottommost chain. The window reordering tasks and high priority right-hand side update tasks are inserted first. Update tasks that are related to the Schur matrix are inserted later. Update tasks that are related to the orthogonal matrices are inserted only after all chain list have been processed. |

#### Enumerator

| STARNEIG_REORDER_CHAIN_INSERT_F | Three-pass delayed backward chain blueprint.            |
|---------------------------------|---|
|                                 | Processes the window chains in two phases starting      |
|                                 | from the bottommost chain. The window reordering        |
|                                 | tasks and high priority right-hand side update tasks    |
|                                 | are inserted during the first phase. Update tasks that  |
|                                 | are related to the Schur matrix are inserted during the |
|                                 | second phase. Update tasks that are related to the      |
|                                 | orthogonal matrices are inserted only after all chain   |
|                                 | list have been processed.                               |
|                                 | list have been processed.                               |

#### 13.10.4 Function Documentation

### 13.10.4.1 starneig\_hessenberg\_init\_conf()

Initializes a Hessenberg reduction configuration structure with default parameters.

#### **Parameters**

|  | out | conf | The Hessenberg reduction configuration structure. |
|--|-----|------|---|
|--|-----|------|---|

### 13.10.4.2 starneig\_schur\_init\_conf()

Initializes a Schur reduction configuration structure with default parameters.

### **Parameters**

|  | out | conf | The Schur reduction configuration structure. |
|--|-----|------|--|
|--|-----|------|--|

### 13.10.4.3 starneig\_reorder\_init\_conf()

Initializes an eigenvalue reordering configuration structure with default parameters.

### **Parameters**

| out | conf | The eigenvalue reordering configuration structure. | 1 |
|-----|------|--|---|
|-----|------|--|---|

### 13.10.4.4 starneig\_eigenvectors\_init\_conf()

Initializes an eigenvectors configuration structure with default parameters.

### **Parameters**

| out | conf | The eigenvectors configuration structure. |
|-----|------|---|
|-----|------|---|

# 13.11 ScaLAPACK compatibility / BLACS matrices

Data types and functions for BLACS formatted distributed matrices.

#### **Data Structures**

struct starneig\_blacs\_descr
 BLACS descriptor. More...

#### **BLACS** contexts

· typedef int starneig\_blacs\_context\_t

BLACS context.

starneig\_blacs\_context\_t starneig\_distr\_to\_blacs\_context (starneig\_distr\_t distr)

Convers a data distribution to a BLACS context.

starneig\_distr\_t starneig\_blacs\_context\_to\_distr (starneig\_blacs\_context\_t context)

Convers a BLACS context to a data distribution.

int starneig distr is blacs compatible (starneig distr t distr)

Checks whether a data distribution is BLACS compatible.

int starneig\_distr\_is\_compatible\_with (starneig\_distr\_t distr, starneig\_blacs\_context\_t context)

Checks whether a data distribution is compatible with a given BLACS context.

### **BLACS** descriptors

typedef struct starneig\_blacs\_descr\_t

BLACS descriptor.

• void starneig\_blacs\_create\_matrix (int rows, int cols, int row\_blksz, int col\_blksz, starneig\_datatype\_t type, starneig\_blacs\_context\_t context, starneig\_blacs\_descr\_t \*descr, void \*\*local)

Creates a BLACS matrix with uninitialized matrix elements.

void starneig\_blacs\_destroy\_matrix (starneig\_blacs\_descr\_t \*descr, void \*\*local)

Destroyes a BLACS matrix.

 void starneig\_distr\_matrix\_to\_blacs\_descr (starneig\_distr\_matrix\_t matrix, starneig\_blacs\_context\_t context, starneig\_blacs\_descr\_t \*descr, void \*\*local)

Convers a distributed matrix to a BLACS descriptor and a matching local array.

starneig\_distr\_matrix\_t starneig\_blacs\_descr\_to\_distr\_matrix (starneig\_datatype\_t type, starneig\_distr\_t distr, starneig\_blacs\_descr\_t \*descr, void \*local)

Convers a BLACS descriptor and a matching local array to a distributed matrix.

int starneig\_distr\_matrix\_is\_blacs\_compatible (starneig\_distr\_matrix\_t matrix)

Checks whether a distributed matrix is BLACS compatible.

int starneig\_distr\_matrix\_is\_compatible\_with (starneig\_distr\_matrix\_t matrix, starneig\_blacs\_context\_t context)

Checks whether a distributed matrix is compatible with a given BLACS context.

### 13.11.1 Detailed Description

Data types and functions for BLACS formatted distributed matrices.

#### 13.11.2 Data Structure Documentation

#### 13.11.2.1 struct starneig\_blacs\_descr

BLACS descriptor.

#### **Data Fields**

| int                      | type    | The descriptor type.  |
|--------------------------|---------|---|
| starneig_blacs_context_t | context | The related BLACS context.  |
| int                      | m       | The number of (global) rows in the matrix.                          |
| int                      | n       | The number of (global) columns in the matrix.                       |
| int                      | sm      | The number of rows in a distribution block.                         |
| int                      | sn      | The number of columns in a distribution block.                      |
| int                      | rsrc    | The process grid row over which the first row is distributed.       |
| int                      | csrc    | The process grid column over which the first column is distributed. |
| int                      | lld     | The leading dimension of the local array.                           |

### 13.11.3 Function Documentation

### 13.11.3.1 starneig\_distr\_to\_blacs\_context()

```
\begin{tabular}{ll} starneig\_blacs\_context\_t & starneig\_distr\_to\_blacs\_context (\\ & starneig\_distr\_t & distr ) \end{tabular}
```

Convers a data distribution to a BLACS context.

#### Attention

The data distribution must describe a two-dimensional block cyclic distribution.

#### **Parameters**

| in | distr | The data distribution. |
|----|-------|------------------------|

#### Returns

The BLACS context.

### 13.11.3.2 starneig\_blacs\_context\_to\_distr()

Convers a BLACS context to a data distribution.

### **Parameters**

| in | context | The BLACS context. |
|----|---------|--------------------|

#### Returns

The data distribution.

### 13.11.3.3 starneig\_distr\_is\_blacs\_compatible()

Checks whether a data distribution is BLACS compatible.

#### **Parameters**

|    | -l' - t | The state distribution |
|----|---------|------------------------|
| ın | aistr   | The data distribution. |

#### Returns

Non-zero if the data distribution matrix is BLACS compatible.

### 13.11.3.4 starneig\_distr\_is\_compatible\_with()

Checks whether a data distribution is compatible with a given BLACS context.

#### Parameters

| in | distr   | The data distribution. |
|----|---------|------------------------|
| in | context | The BLACS context.     |

#### Returns

Non-zero if the data distribution compatible with the BLACS context.

### 13.11.3.5 starneig\_blacs\_create\_matrix()

```
int col_blksz,
starneig_datatype_t type,
starneig_blacs_context_t context,
starneig_blacs_descr_t * descr,
void ** local )
```

Creates a BLACS matrix with uninitialized matrix elements.

#### **Parameters**

| in  | rows      | The number of (global) rows in the matrix.  |
|-----|-----------|---|
| in  | cols      | The number of (global) columns in the matrix.   |
| in  | row_blksz | The number of rows in a distribution block. Can be set to -1 in which case the library    |
|     |           | decides the value.  |
| in  | col_blksz | The number of columns in a distribution block. Can be set to -1 in which case the library |
|     |           | decides the value.  |
| in  | type      | The matrix element data type.   |
| in  | context   | The BLACS context.  |
| out | descr     | The BLACS descriptor.   |
| out | local     | A pointer to the local array.   |

### 13.11.3.6 starneig\_blacs\_destroy\_matrix()

Destroyes a BLACS matrix.

### **Parameters**

| in,out | descr | The BLACS descriptor.         |
|--------|-------|-------------------------------|
| in,out | local | A pointer to the local array. |

### 13.11.3.7 starneig\_distr\_matrix\_to\_blacs\_descr()

Convers a distributed matrix to a BLACS descriptor and a matching local array.

This function creates a wrapper object. The contents of the distributed matrix may be modified by the functions that use the wrapper object.

```
starneig_distr_matrix_t dA = starneig_distr_matrix_create(...);
```

```
...
starneig_distr_t distr = starneig_distr_matrix_get_distr(A);
starneig_blacs_context_t context = starneig_distr_to_blacs_context(distr);
starneig_blacs_descr_t descr_a;
double *local_a;
starneig_distr_matrix_to_blacs_descr(
    dA, context, &descr_a, (void **)&local_a);
```

#### **Parameters**

| in  | matrix  | The distributed matrix.  |
|-----|---------|--|
| in  | context | The BLACS context. The context must have been converted from the same data distribution the distributed matrix is using or vice versa. |
| out | descr   | The BLACS descriptor.  |
| out | local   | A pointer to the local array.  |

#### 13.11.3.8 starneig\_blacs\_descr\_to\_distr\_matrix()

Convers a BLACS descriptor and a matching local array to a distributed matrix.

This function creates a wrapper object. The contents of the local array may be modified by the functions that use the wrapper object. The <a href="matrix\_destroy">starneig\_distr\_matrix\_destroy</a>() function does not de-initilize the BLACS descriptor nor free the local array.

#### **Parameters**

| in | type  | The matrix element data type.   |  |
|----|-------|---|--|
| in | distr | The data distribution. The data distribution must have been converted from the same BLACS |  |
|    |       | context the BLACS descriptor is using or vice versa.                                      |  |
| in | descr | The BLACS descriptor.   |  |
| in | local | A pointer to the local array.   |  |

#### Returns

The distributed matrix.

### 13.11.3.9 starneig\_distr\_matrix\_is\_blacs\_compatible()

Checks whether a distributed matrix is BLACS compatible.

#### **Parameters**

| in | matrix | The distributed matrix. |
|----|--------|-------------------------|
|----|--------|-------------------------|

#### Returns

Non-zero if the distributed matrix is BLACS compatible.

### 13.11.3.10 starneig\_distr\_matrix\_is\_compatible\_with()

Checks whether a distributed matrix is compatible with a given BLACS context.

#### **Parameters**

| in | matrix  | The distributed matrix. |
|----|---------|-------------------------|
| in | context | The BLACS context.      |

### Returns

Non-zero if the distributed matrix compatible with the BLACS context.

### 13.12 ScaLAPACK compatibility / BLACS helpers

Data types and helper functions for BLACS.

#### **Functions**

void starneig\_blacs\_pinfo (int \*my\_rank, int \*rank\_count)

Queries process rank information.

• int starneig\_blacs\_get (starneig\_blacs\_context\_t context, starneig\_blacs\_query\_id\_t query)

Returns BLACS context's internal defaults.

starneig\_blacs\_context\_t starneig\_blacs\_gridinit (starneig\_blacs\_context\_t system\_context, char \*order, int rows, int cols)

Initializes a BLACS process grid.

void starneig\_blacs\_gridinfo (starneig\_blacs\_context\_t context, int \*rows, int \*cols, int \*row, int \*col)

Queries BLACS process grid information.

void starneig\_blacs\_pcoord (starneig\_blacs\_context\_t context, int process, int \*row, int \*col)

Queries BLACS process grid coordinates.

void starneig\_blacs\_gridexit (starneig\_blacs\_context\_t context)

Releases process grid specific resources.

void starneig\_blacs\_exit (int cont)

Releases all contexts and related resources.

int starneig blacs numroc (int n, int nb, int iproc, int isrcproc, int nprocs)

Computes the number of matrix rows/columns owned by a given process.

• int starneig\_blacs\_descinit (struct starneig\_blacs\_descr \*descr, int m, int n, int sm, int sn, int irsrc, int icsrc, starneig\_blacs\_context\_t context, int ld)

Initializes a BLACS descriptor.

### **Query indeces**

· typedef int starneig blacs query id t

Data type for blacs\_get() function query id.

#define STARNEIG BLACS GET DEFAULT CONTEXT 0

Query id for getting the default system context.

### 13.12.1 Detailed Description

Data types and helper functions for BLACS.

### 13.12.2 Function Documentation

#### 13.12.2.1 starneig blacs pinfo()

Queries process rank information.

#### **Parameters**

|   | out | my_rank    | An unique process id (rank).                     |
|---|-----|------------|--|
| ſ | out | rank_count | The total number of processes (ranks) available. |

### 13.12.2.2 starneig\_blacs\_get()

Returns BLACS context's internal defaults.

#### **Parameters**

| in | context | The BLACS context. |
|----|---------|--------------------|
| in | query   | The query id.      |

#### Returns

The internal default value that matches the given query id.

### 13.12.2.3 starneig\_blacs\_gridinit()

Initializes a BLACS process grid.

### **Parameters**

| in | system_context | The system BLACS context to be used in creating the process grid.        |
|----|----------------|--|
| in | order          | The process mapping order. "R": Use row-major natural ordering. "C": Use |
|    |                | column-major natural ordering. ELSE: Use row-major natural ordering.     |
| in | rows           | The number of rows in the process grid.                                  |
| in | cols           | The number of columns in the process grid.                               |

### Returns

A handle to the created BLACS context.

### 13.12.2.4 starneig\_blacs\_gridinfo()

Queries BLACS process grid information.

#### **Parameters**

| in              | context | The BLACS context.                            |
|-----------------|---------|---|
| out <i>rows</i> |         | The number of rows in the process grid.       |
| out             | cols    | The number of columns in the process grid.    |
| out             | row     | The row coordinate of the calling process.    |
| out             | col     | The column coordinate of the calling process. |

### 13.12.2.5 starneig\_blacs\_pcoord()

Queries BLACS process grid coordinates.

### **Parameters**

| in  | context | The BLACS context.                    |
|-----|---------|---------------------------------------|
| in  | process | The process id (rank).                |
| out | row     | The row coordinate of the process.    |
| out | col     | The column coordinate of the process. |

### 13.12.2.6 starneig\_blacs\_gridexit()

Releases process grid specific resources.

### **Parameters**

| in context The BLACS contex | t. |
|-----------------------------|----|
|-----------------------------|----|

### 13.12.2.7 starneig\_blacs\_exit()

```
void starneig_blacs_exit (
          int cont )
```

Releases all contexts and related resources.

#### **Parameters**

| in | cont | The continue flag. |
|----|------|--------------------|
|----|------|--------------------|

### 13.12.2.8 starneig\_blacs\_numroc()

```
int starneig_blacs_numroc (
    int n,
    int nb,
    int iproc,
    int isrcproc,
    int nprocs )
```

Computes the number of matrix rows/columns owned by a given process.

### **Parameters**

| in | n        | The number of rows/columns in the distributed matrix.   |
|----|----------|---|
| in | nb       | The block size.   |
| in | iproc    | The coordinate of the process whose local array row or column is to be determined.              |
| in | isrcproc | The coordinate of the process that possesses the first row or column of the distributed matrix. |
| in | nprocs   | The total number processes over which the matrix is distributed.                                |

### Returns

The number of rows/columns owned by the process.

### 13.12.2.9 starneig\_blacs\_descinit()

```
int irsrc,
int icsrc,
starneig_blacs_context_t context,
int Id )
```

### Initializes a BLACS descriptor.

### **Parameters**

| out | descr   | The matrix descriptor.  |  |
|-----|---------|---|--|
| in  | m       | The number of rows in the matrix.                                   |  |
| in  | n       | The number of columns in the matrix.                                |  |
| in  | sm      | The number of rows in a distributed block.                          |  |
| in  | sn      | The number of columns in a distributed block.                       |  |
| in  | irsrc   | The process grid row over which the first row is distributed.       |  |
| in  | icsrc   | The process grid column over which the first column is distributed. |  |
| in  | context | The BLACS context.  |  |
| in  | ld      | The local array leading dimension.                                  |  |

### Returns

Zero if the initialization was successful, non-zero otherwise.

# Chapter 14

# **File Documentation**

## 14.1 blacs helpers.h File Reference

This file contains various BLACS helper functions.

```
#include <starneig/configuration.h>
#include <starneig/blacs_matrix.h>
```

#### **Functions**

void starneig\_blacs\_pinfo (int \*my\_rank, int \*rank\_count)

Queries process rank information.

• int starneig\_blacs\_get (starneig\_blacs\_context\_t context, starneig\_blacs\_query\_id\_t query)

Returns BLACS context's internal defaults.

starneig\_blacs\_context\_t starneig\_blacs\_gridinit (starneig\_blacs\_context\_t system\_context, char \*order, int rows, int cols)

Initializes a BLACS process grid.

- void starneig\_blacs\_gridinfo (starneig\_blacs\_context\_t context, int \*rows, int \*cols, int \*row, int \*col)

  \*\*Queries BLACS process grid information.
- void starneig\_blacs\_pcoord (starneig\_blacs\_context\_t context, int process, int \*row, int \*col)

Queries BLACS process grid coordinates.

void starneig\_blacs\_gridexit (starneig\_blacs\_context\_t context)

Releases process grid specific resources.

· void starneig blacs exit (int cont)

Releases all contexts and related resources.

• int starneig\_blacs\_numroc (int n, int nb, int iproc, int isrcproc, int nprocs)

Computes the number of matrix rows/columns owned by a given process.

• int starneig\_blacs\_descinit (struct starneig\_blacs\_descr \*descr, int m, int n, int sm, int sn, int irsrc, int icsrc, starneig\_blacs\_context\_t context, int ld)

Initializes a BLACS descriptor.

### **Query indeces**

• #define STARNEIG BLACS GET DEFAULT CONTEXT 0

Query id for getting the default system context.

• typedef int starneig\_blacs\_query\_id\_t

Data type for blacs\_get() function query id.

130 File Documentation

### 14.1.1 Detailed Description

This file contains various BLACS helper functions.

**Author** 

Mirko Myllykoski ( mirkom@cs.umu.se), Umeå University

### 14.2 blacs\_matrix.h File Reference

This file contains data types and functions for BLACS formatted distributed matrices.

```
#include <starneig/configuration.h>
#include <starneig/distr_matrix.h>
```

#### **Data Structures**

struct starneig\_blacs\_descr
 BLACS descriptor. More...

### **BLACS** contexts

typedef int starneig\_blacs\_context\_t

BLACS context.

• starneig\_blacs\_context\_t starneig\_distr\_to\_blacs\_context (starneig\_distr\_t distr)

Convers a data distribution to a BLACS context.

starneig\_distr\_t starneig\_blacs\_context\_to\_distr (starneig\_blacs\_context\_t context)

Convers a BLACS context to a data distribution.

int starneig\_distr\_is\_blacs\_compatible (starneig\_distr\_t distr)

Checks whether a data distribution is BLACS compatible.

int starneig\_distr\_is\_compatible\_with (starneig\_distr\_t distr, starneig\_blacs\_context\_t context)

Checks whether a data distribution is compatible with a given BLACS context.

### **BLACS** descriptors

typedef struct starneig\_blacs\_descr starneig\_blacs\_descr\_t

BLACS descriptor.

• void starneig\_blacs\_create\_matrix (int rows, int cols, int row\_blksz, int col\_blksz, starneig\_datatype\_t type, starneig\_blacs\_context\_t context, starneig\_blacs\_descr\_t \*descr, void \*\*local)

Creates a BLACS matrix with uninitialized matrix elements.

void starneig\_blacs\_destroy\_matrix (starneig\_blacs\_descr\_t \*descr, void \*\*local)

Destroyes a BLACS matrix.

 void starneig\_distr\_matrix\_to\_blacs\_descr (starneig\_distr\_matrix\_t matrix, starneig\_blacs\_context\_t context, starneig\_blacs\_descr\_t \*descr, void \*\*local)

Convers a distributed matrix to a BLACS descriptor and a matching local array.

• starneig\_distr\_matrix\_t starneig\_blacs\_descr\_to\_distr\_matrix (starneig\_datatype\_t type, starneig\_distr\_t distr, starneig\_blacs\_descr\_t \*descr, void \*local)

Convers a BLACS descriptor and a matching local array to a distributed matrix.

• int starneig\_distr\_matrix\_is\_blacs\_compatible (starneig\_distr\_matrix\_t matrix)

Checks whether a distributed matrix is BLACS compatible.

int starneig\_distr\_matrix\_is\_compatible\_with (starneig\_distr\_matrix\_t matrix, starneig\_blacs\_context\_t context)

Checks whether a distributed matrix is compatible with a given BLACS context.

### 14.2.1 Detailed Description

This file contains data types and functions for BLACS formatted distributed matrices.

**Author** 

Mirko Myllykoski ( mirkom@cs.umu.se), Umeå University

# 14.3 configuration.h File Reference

This file contains StarNEig library configuration.

#### **Macros**

#define STARNEIG\_VERSION\_MAJOR

StarNEig major version.

• #define STARNEIG\_VERSION\_MINOR

StarNEig minor version.

• #define STARNEIG\_VERSION\_PATCH

StarNEig patch level.

• #define STARNEIG\_ENABLE\_MPI

MPI support enabled.

• #define STARNEIG\_ENABLE\_CUDA

CUDA support enabled.

• #define STARNEIG\_ENABLE\_BLACS

BLACS support enabled.

• #define STARNEIG SEP DM HESSENBERG

Distributed memory Hessenberg reduction enabled.

#define STARNEIG\_GEP\_DM\_HESSENBERGTRIANGULAR

Distributed memory Hessenberg-triangular reduction enabled.

• #define STARNEIG\_SEP\_DM\_REDUCE

Full distributed memory reduction enabled.

• #define STARNEIG\_GEP\_DM\_REDUCE

Full distributed memory generalized reduction enabled.

### 14.3.1 Detailed Description

This file contains StarNEig library configuration.

Author

Mirko Myllykoski ( mirkom@cs.umu.se), Umeå University

132 File Documentation

# 14.4 distr\_helpers.h File Reference

This file contains generic distributed memory interface functions.

```
#include <starneig/configuration.h>
#include <stddef.h>
#include <mpi.h>
```

#### **Functions**

#### **MPI** communicator

```
    void starneig_mpi_set_comm (MPI_Comm comm)
    Sets a MPI communicator for the library.
```

MPI\_Comm starneig\_mpi\_get\_comm ()
 Returns the library MPI communicator.

#### **Broadcast**

void starneig\_mpi\_broadcast (int root, size\_t size, void \*buffer)
 Broadcast a buffer.

### 14.4.1 Detailed Description

This file contains generic distributed memory interface functions.

Author

```
Mirko Myllykoski ( mirkom@cs.umu.se), Umeå University
```

# 14.5 distr\_matrix.h File Reference

This file contains data types and functions for distributed matrices.

```
#include <starneig/configuration.h>
#include <stddef.h>
```

### **Data Structures**

· struct starneig\_distr\_block

Distributed block. More...

#### **Functions**

#### **Query functions**

 void starneig\_distr\_matrix\_get\_blocks (starneig\_distr\_matrix\_t matrix, struct starneig\_distr\_block \*\*blocks, int \*num\_blocks)

Returns the locally owned distributed blocks.

starneig\_distr\_t starneig\_distr\_matrix\_get\_distr (starneig\_distr\_matrix\_t matrix)

Returns the distribution that is associated with a distributed matrix.

starneig datatype t starneig distr matrix get datatype (starneig distr matrix t matrix)

Returns the matrix element data type.

• size\_t starneig\_distr\_matrix\_get\_elemsize (starneig\_distr\_matrix\_t matrix)

Returns the matrix element size.

int starneig\_distr\_matrix\_get\_rows (starneig\_distr\_matrix\_t matrix)

Returns the number of (global) rows.

• int starneig\_distr\_matrix\_get\_cols (starneig\_distr\_matrix\_t matrix)

Returns the number of (global) columns.

int starneig\_distr\_matrix\_get\_row\_blksz (starneig\_distr\_matrix\_t matrix)

Returns the number of rows in a distribution block.

int starneig\_distr\_matrix\_get\_col\_blksz (starneig\_distr\_matrix\_t matrix)

Returns the number of columns in a distribution block.

#### **Data distributions**

 enum starneig\_distr\_order\_t { STARNEIG\_ORDER\_DEFAULT, STARNEIG\_ORDER\_ROW\_MAJOR, STARNEIG\_ORDER\_COL\_MAJOR}

Process mapping order.

typedef struct starneig\_distr \* starneig\_distr\_t

Data distribution.

starneig distr t starneig distr init ()

Creates a default data distribution.

• starneig\_distr\_t starneig\_distr\_init\_mesh (int rows, int cols, starneig\_distr\_order\_t order)

Creates a two-dimensional block cyclic data distribution.

starneig\_distr\_t starneig\_distr\_init\_func (int(\*func)(int row, int col, void \*arg), void \*arg, size\_t arg\_size)

Creates a distribution using a data distribution function.

starneig\_distr\_t starneig\_distr\_duplicate (starneig\_distr\_t distr)

Duplicates a data distribution.

void starneig\_distr\_destroy (starneig\_distr\_t distr)

Destroys a data distribution.

#### **Distributed matrices**

enum starneig\_datatype\_t { STARNEIG\_REAL\_DOUBLE }

Distributed matrix element data type.

typedef struct starneig\_distr\_matrix \* starneig\_distr\_matrix\_t

Distributed matrix.

• starneig\_distr\_matrix\_t starneig\_distr\_matrix\_create (int rows, int cols, int row\_blksz, int col\_blksz, starneig\_datatype\_t type, starneig\_distr\_t distr)

Creates a distributed matrix with uninitialized matrix elements.

• starneig\_distr\_matrix\_t starneig\_distr\_matrix\_create\_local (int rows, int cols, starneig\_datatype\_t type, int owner, double \*A, int IdA)

Creates a single-owner distributed matrix from a local matrix.

void starneig\_distr\_matrix\_destroy (starneig\_distr\_matrix\_t matrix)

134 File Documentation

Destroys a distributed matrix.

• void starneig\_distr\_matrix\_copy (starneig\_distr\_matrix\_t source, starneig\_distr\_matrix\_t dest)

Copies the contents of a distributed matrix to a second distributed matrix.

void starneig\_distr\_matrix\_copy\_region (int sr, int sc, int dr, int dc, int rows, int cols, starneig\_distr\_matrix\_t source, starneig\_distr\_matrix\_t dest)

Copies region of a distributed matrix to a second distributed matrix.

### 14.5.1 Detailed Description

This file contains data types and functions for distributed matrices.

**Author** 

```
Mirko Myllykoski (mirkom@cs.umu.se), Umeå University Lars Karlsson (larsk@cs.umu.se), Umeå University
```

### 14.6 error.h File Reference

This file contains the library error codes.

```
#include <starneig/configuration.h>
```

#### **Macros**

• #define STARNEIG SUCCESS 0

Success.

#define STARNEIG\_GENERIC\_ERROR 1

Reneric error.

• #define STARNEIG NOT INITIALIZED 2

Not initialized.

• #define STARNEIG\_INVALID\_CONFIGURATION 3

Invalid configuration.

• #define STARNEIG\_INVALID\_ARGUMENTS 4

Invalid argument.

#define STARNEIG\_INVALID\_DISTR\_MATRIX 5

Invalid distributed matrix.

• #define STARNEIG DID NOT CONVERGE 6

Did not converge.

#define STARNEIG\_PARTIAL\_REORDERING 7

Partial reordering.

• #define STARNEIG\_CLOSE\_EIGENVALUES 8

Close eigenvalues.

### **Typedefs**

· typedef int starneig\_error\_t

Interface function return value data type.

### 14.6.1 Detailed Description

This file contains the library error codes.

**Author** 

Mirko Myllykoski ( mirkom@cs.umu.se), Umeå University

# 14.7 expert.h File Reference

This file contains configuration structures and functions for the expert interface functions.

#include <starneig/configuration.h>

### **Data Structures**

· struct starneig\_hessenberg\_conf

Hessenberg reduction configuration structure. More...

· struct starneig\_schur\_conf

Schur reduction configuration structure. More...

· struct starneig reorder conf

Eigenvalue reordering configuration structure. More...

· struct starneig\_eigenvectors\_conf

Eigenvector computation configuration structure. More...

### **Hessenberg reduction**

#define STARNEIG\_HESSENBERG\_DEFAULT\_TILE\_SIZE -1

Default tile size.

• #define STARNEIG\_HESSENBERG\_DEFAULT\_PANEL\_WIDTH -1

Default panel width.

• void starneig\_hessenberg\_init\_conf (struct starneig\_hessenberg\_conf \*conf)

Initializes a Hessenberg reduction configuration structure with default parameters.

#### **Schur reduction**

• #define STARNEIG\_SCHUR\_DEFAULT\_INTERATION\_LIMIT -1

Default iteration limit.

• #define STARNEIG\_SCHUR\_DEFAULT\_TILE\_SIZE -1

Default tile size.

• #define STARNEIG\_SCHUR\_DEFAULT\_SMALL\_LIMIT -1

Default sequential QR limit.

#define STARNEIG\_SCHUR\_DEFAULT\_AED\_WINDOW\_SIZE -1

Default AED window size.

#define STARNEIG SCHUR DEFAULT AED NIBBLE -1

Default nibble value.

• #define STARNEIG\_SCHUR\_DEFAULT\_AED\_PARALLEL\_SOFT\_LIMIT -1

136 File Documentation

Default soft sequential AED limit.

#define STARNEIG\_SCHUR\_DEFAULT\_AED\_PARALLEL\_HARD\_LIMIT -1

Default hard sequential AED limit.

#define STARNEIG\_SCHUR\_DEFAULT\_SHIFT\_ORIGIN -1

Default shift origin.

• #define STARNEIG\_SCHUR\_DEFAULT\_SHIFT\_COUNT -1

Default shift count.

#define STARNEIG\_SCHUR\_DEFAULT\_WINDOW\_SIZE -1

Default bulge chasing window size.

#define STARNEIG\_SCHUR\_ROUNDED\_WINDOW\_SIZE -2

Rounded bulge chasing window.

• #define STARNEIG SCHUR DEFAULT SHIFTS PER WINDOW -1

Default number of shifts per bulge chasing window.

• #define STARNEIG\_SCHUR\_DEFAULT\_UPDATE\_WIDTH -1

Default left-hand side update width.

#define STARNEIG SCHUR DEFAULT UPDATE HEIGHT -1

Default right-hand side update height.

#define STARNEIG SCHUR DEFAULT THRESHOLD -1

Default deflation threshold.

#define STARNEIG\_SCHUR\_NORM\_STABLE\_THRESHOLD -2

Norm stable deflation threshold.

• #define STARNEIG SCHUR LAPACK THRESHOLD -3

LAPACK-style deflation threshold.

void starneig\_schur\_init\_conf (struct starneig\_schur\_conf \*conf)

Initializes a Schur reduction configuration structure with default parameters.

### Eigenvalue reordering

• #define STARNEIG REORDER DEFAULT UPDATE WIDTH -1

Default left-hand side update task width.

• #define STARNEIG REORDER DEFAULT UPDATE HEIGHT-1

Default right-hand side update task height.

• #define STARNEIG\_REORDER\_DEFAULT\_TILE\_SIZE -1

Default tile size.

• #define STARNEIG REORDER DEFAULT VALUES PER CHAIN -1

Default number of selected eigenvalues per window.

• #define STARNEIG\_REORDER\_DEFAULT\_WINDOW\_SIZE -1

Default default window size.

#define STARNEIG\_REORDER\_ROUNDED\_WINDOW\_SIZE -2

Default rounded window size.

#define STARNEIG\_REORDER\_DEFAULT\_SMALL\_WINDOW\_SIZE -1

Default small window size.

#define STARNEIG\_REORDER\_DEFAULT\_SMALL\_WINDOW\_THRESHOLD -1

Default small window threshold.

enum starneig\_reorder\_plan\_t { STARNEIG\_REORDER\_DEFAULT\_PLAN = 1, STARNEIG\_REORDER\_ONE\_PART\_PLAN = 2, STARNEIG\_REORDER\_MULTI\_PART\_PLAN = 3 }

Reordering plan enumerator.

```
    enum starneig_reorder_blueprint_t {
        STARNEIG_REORDER_DEFAULT_BLUEPRINT = 1, STARNEIG_REORDER_DUMMY_INSERT_A = 2,
        STARNEIG_REORDER_DUMMY_INSERT_B = 3, STARNEIG_REORDER_CHAIN_INSERT_A = 4,
        STARNEIG_REORDER_CHAIN_INSERT_B = 5, STARNEIG_REORDER_CHAIN_INSERT_C = 6,
        STARNEIG_REORDER_CHAIN_INSERT_D = 7, STARNEIG_REORDER_CHAIN_INSERT_E = 8,
        STARNEIG_REORDER_CHAIN_INSERT_F = 9 }
```

Task insertion blueprint.

void starneig\_reorder\_init\_conf (struct starneig\_reorder\_conf \*conf)

Initializes an eigenvalue reordering configuration structure with default parameters.

#### **Eigenvectors**

```
    #define STARNEIG_EIGENVECTORS_DEFAULT_TILE_SIZE -1
```

Default tile size.

• void starneig eigenvectors init conf (struct starneig eigenvectors conf \*conf)

Initializes an eigenvectors configuration structure with default parameters.

#### 14.7.1 Detailed Description

This file contains configuration structures and functions for the expert interface functions.

**Author** 

```
Mirko Myllykoski ( mirkom@cs.umu.se), Umeå University
Angelika Schwarz ( angies@cs.umu.se), Umeå University
```

# 14.8 gep\_dm.h File Reference

This file contains distributed memory interface functions for generalized eigenvalue problems.

```
#include <starneig/configuration.h>
#include <starneig/error.h>
#include <starneig/expert.h>
#include <starneig/distr_matrix.h>
```

#### **Functions**

#### **Computational functions**

starneig\_error\_t starneig\_GEP\_DM\_HessenbergTriangular (starneig\_distr\_matrix\_t A, starneig\_distr\_matrix\_t B, starneig\_distr\_matrix\_t Q, starneig\_distr\_matrix\_t Z)

Computes a Hessenberg-triangular decomposition of a general matrix pencil.

• starneig\_error\_t starneig\_GEP\_DM\_Schur (starneig\_distr\_matrix\_t H, starneig\_distr\_matrix\_t R, starneig\_distr\_matrix\_t Q, starneig\_distr\_matrix\_t Z, double real[], double imag[], double beta[])

Computes a generalized Schur decomposition given a Hessenberg-triangular decomposition.

• starneig\_error\_t starneig\_GEP\_DM\_ReorderSchur (int selected[], starneig\_distr\_matrix\_t S, starneig\_distr\_matrix\_t T, starneig\_distr\_matrix\_t Q, starneig\_distr\_matrix\_t Z, double real[], double imag[], double beta[])

Reorders selected generalized eigenvalues to the top left corner of a generalized Schur decomposition.

• starneig\_error\_t starneig\_GEP\_DM\_Reduce (starneig\_distr\_matrix\_t A, starneig\_distr\_matrix\_t B, starneig\_distr\_matrix\_t Q, starneig\_distr\_matrix\_t Z, double real[], double imag[], double beta[], int(\*predicate)(double real, double imag, double beta, void \*arg), void \*arg, int selected[], int \*num\_\circ\sigma selected)

Computes a (reordered) generalized Schur decomposition given a general matrix pencil.

starneig\_error\_t starneig\_GEP\_DM\_Eigenvectors (int selected[], starneig\_distr\_matrix\_t S, starneig\_distr\_matrix\_t T, starneig\_distr\_matrix\_t X)

Computes a generalized eigenvector for each selected generalized eigenvalue.

#### **Helper functions**

• starneig\_error\_t starneig\_GEP\_DM\_Select (starneig\_distr\_matrix\_t S, starneig\_distr\_matrix\_t T, int(\*predicate)(double real, double imag, double beta, void \*arg), void \*arg, int selected[], int \*num\_\circ\sigma selected)

Generates a selection array for a Schur-triangular matrix pencil using a user-supplied predicate function.

#### **Expert computational functions**

starneig\_error\_t starneig\_GEP\_DM\_Schur\_expert (struct starneig\_schur\_conf \*conf, starneig\_distr\_matrix\_t
 H, starneig\_distr\_matrix\_t
 R, starneig\_distr\_matrix\_t
 Q, starneig\_distr\_matrix\_t
 Z, double real[], double imag[], double beta[])

Computes a generalized Schur decomposition given a Hessenberg-triangular decomposition.

• starneig\_error\_t starneig\_GEP\_DM\_ReorderSchur\_expert (struct starneig\_reorder\_conf \*conf, int selected[], starneig\_distr\_matrix\_t S, starneig\_distr\_matrix\_t T, starneig\_distr\_matrix\_t Q, starneig\_distr\_matrix\_t Z, double real[], double imag[], double beta[])

Reorders selected generalized eigenvalues to the top left corner of a generalized Schur decomposition.

starneig\_error\_t starneig\_GEP\_DM\_Eigenvectors\_expert (struct starneig\_eigenvectors\_conf \*conf, int selected[], starneig\_distr\_matrix\_t S, starneig\_distr\_matrix\_t T, starneig\_distr\_matrix\_t Z, starneig\_distr\_matrix\_t X)

Computes a generalized eigenvector for each selected generalized eigenvalue.

#### 14.8.1 Detailed Description

This file contains distributed memory interface functions for generalized eigenvalue problems.

**Author** 

```
Mirko Myllykoski (mirkom@cs.umu.se), Umeå University Lars Karlsson (larsk@cs.umu.se), Umeå University
```

# 14.9 gep\_sm.h File Reference

This file contains shared memory interface functions for generalized eigenvalue problems.

```
#include <starneig/configuration.h>
#include <starneig/error.h>
#include <starneig/expert.h>
```

14.10 node.h File Reference 139

#### **Functions**

#### **Computational functions**

starneig\_error\_t starneig\_GEP\_SM\_HessenbergTriangular (int n, double A[], int IdA, double B[], int IdB, double Q[], int IdQ, double Z[], int IdZ)

Computes a Hessenberg-triangular decomposition of a general matrix pencil.

• starneig\_error\_t starneig\_GEP\_SM\_Schur (int n, double H[], int ldH, double R[], int ldR, double Q[], int ldQ, double Z[], int ldZ, double real[], double imag[], double beta[])

Computes a generalized Schur decomposition given a Hessenberg-triangular decomposition.

• starneig\_error\_t starneig\_GEP\_SM\_ReorderSchur (int n, int selected[], double S[], int ldS, double T[], int ldT, double Q[], int ldQ, double Z[], int ldZ, double real[], double imag[], double beta[])

Reorders selected generalized eigenvalues to the top left corner of a generalized Schur decomposition.

• starneig\_error\_t starneig\_GEP\_SM\_Reduce (int n, double A[], int IdA, double B[], int IdB, double Q[], int IdQ, double Z[], int IdZ, double real[], double imag[], double beta[], int(\*predicate)(double real, double imag, double beta, void \*arg), void \*arg, int selected[], int \*num selected)

Computes a (reordered) generalized Schur decomposition given a general matrix pencil.

• starneig\_error\_t starneig\_GEP\_SM\_Eigenvectors (int n, int selected[], double S[], int ldS, double T[], int ldT, double Z[], int ldZ, double X[], int ldX)

Computes a generalized eigenvector for each selected generalized eigenvalue.

#### **Helper functions**

• starneig\_error\_t starneig\_GEP\_SM\_Select (int n, double S[], int ldS, double T[], int ldT, int(\*predicate)(double real, double imag, double beta, void \*arg), void \*arg, int selected[], int \*num\_selected)

Generates a selection array for a Schur-triangular matrix pencil using a user-supplied predicate function.

#### **Expert computational functions**

- starneig\_error\_t starneig\_GEP\_SM\_Schur\_expert (struct starneig\_schur\_conf \*conf, int n, double H[], int ldH, double R[], int ldQ, double Z[], int ldZ, double real[], double imag[], double beta[]) Computes a generalized Schur decomposition given a Hessenberg-triangular decomposition.
- starneig\_error\_t starneig\_GEP\_SM\_ReorderSchur\_expert (struct starneig\_reorder\_conf \*conf, int n, int selected[], double S[], int ldS, double T[], int ldT, double Q[], int ldQ, double Z[], int ldZ, double real[], double imag[], double beta[])

Reorders selected eigenvalues to the top left corner of a generalized Schur decomposition.

• starneig\_error\_t starneig\_GEP\_SM\_Eigenvectors\_expert (struct starneig\_eigenvectors\_conf \*conf, int n, int selected[], double S[], int ldS, double T[], int ldT, double Z[], int ldZ, double X[], int ldX)

Computes a generalized eigenvector for each selected generalized eigenvalue.

#### 14.9.1 Detailed Description

This file contains shared memory interface functions for generalized eigenvalue problems.

**Author** 

```
Mirko Myllykoski ( mirkom@cs.umu.se), Umeå University Lars Karlsson ( larsk@cs.umu.se), Umeå University
```

#### 14.10 node.h File Reference

This file contains interface to configure the intra-node execution environment.

```
#include <starneig/configuration.h>
#include <stddef.h>
#include <mpi.h>
```

#### **Functions**

• void starneig\_node\_init (int cores, int gpus, starneig\_flag\_t flags)

Initializes the intra-node execution environment.

int starneig\_node\_initialized ()

Checks whether the intra-node execution environment is initialized.

int starneig node get cores ()

Returns the number of cores (threads) per MPI rank.

void starneig\_node\_set\_cores (int cores)

Changes the number of CPUs cores (threads) to use per MPI rank.

• int starneig\_node\_get\_gpus ()

Returns the number of GPUs per MPI rank.

· void starneig node set gpus (int gpus)

Changes the number of GPUs to use per MPI rank.

• void starneig\_node\_finalize ()

Deallocates resources associated with the intra-node configuration.

#### Pinned host memory

• void starneig\_node\_enable\_pinning ()

Enable CUDA host memory pinning.

· void starneig node disable pinning ()

Disables CUDA host memory pinning.

#### Library initialization flags

• #define STARNEIG\_USE\_ALL -1

Use all resources.

• #define STARNEIG\_DEFAULT 0x0

Default mode.

#define STARNEIG\_HINT\_SM 0x0

Shared memory mode.

#define STARNEIG\_HINT\_DM 0x1

Distributed memory mode.

#define STARNEIG\_FXT\_DISABLE 0x2

No FxT traces mode.

#define STARNEIG\_AWAKE\_WORKERS 0x4

Awake worker mode.

#define STARNEIG\_AWAKE\_MPI\_WORKER 0x8

Awake MPI worker mode.

• #define STARNEIG\_FAST\_DM (STARNEIG\_HINT\_DM | STARNEIG\_AWAKE\_WORKERS | STARNEIG\_AWAKE\_MPI\_WOR

Fast distributed memory mode.

• #define STARNEIG\_NO\_VERBOSE 0x10

No verbose mode.

#define STARNEIG NO MESSAGES (STARNEIG NO VERBOSE | 0x20)

No messages mode.

• typedef unsigned starneig\_flag\_t

Library initialization flag data type.

#### 14.10.1 Detailed Description

This file contains interface to configure the intra-node execution environment.

**Author** 

```
Mirko Myllykoski ( mirkom@cs.umu.se), Umeå University Lars Karlsson ( larsk@cs.umu.se), Umeå University
```

# 14.11 sep dm.h File Reference

This file contains distributed memory interface functions for standard eigenvalue problems.

```
#include <starneig/configuration.h>
#include <starneig/error.h>
#include <starneig/expert.h>
#include <starneig/distr_matrix.h>
```

#### **Functions**

#### Computational functions

- starneig\_error\_t starneig\_SEP\_DM\_Hessenberg (starneig\_distr\_matrix\_t A, starneig\_distr\_matrix\_t Q)
   Computes a Hessenberg decomposition of a general matrix.
- starneig\_error\_t starneig\_SEP\_DM\_Schur (starneig\_distr\_matrix\_t H, starneig\_distr\_matrix\_t Q, double real[], double imag[])

Computes a Schur decomposition given a Hessenberg decomposition.

• starneig\_error\_t starneig\_SEP\_DM\_ReorderSchur (int selected[], starneig\_distr\_matrix\_t S, starneig\_distr\_matrix\_t Q, double real[], double imag[])

Reorders selected eigenvalues to the top left corner of a Schur decomposition.

• starneig\_error\_t starneig\_SEP\_DM\_Reduce (starneig\_distr\_matrix\_t A, starneig\_distr\_matrix\_t Q, double real[], double imag[], int(\*predicate)(double real, double imag, void \*arg), void \*arg, int selected[], int \*num\_selected)

Computes a (reordered) Schur decomposition of a general matrix.

starneig\_error\_t starneig\_SEP\_DM\_Eigenvectors (int selected[], starneig\_distr\_matrix\_t S, starneig\_distr\_matrix\_t Q, starneig\_distr\_matrix\_t X)

Computes an eigenvector for each selected eigenvalue.

#### **Helper functions**

• starneig\_error\_t starneig\_SEP\_DM\_Select (starneig\_distr\_matrix\_t S, int(\*predicate)(double real, double imag, void \*arg), void \*arg, int selected[], int \*num\_selected)

Generates a selection array for a Schur matrix using a user-supplied predicate function.

#### **Expert computational functions**

• starneig\_error\_t starneig\_SEP\_DM\_Hessenberg\_expert (struct starneig\_hessenberg\_conf \*conf, int begin, int end, starneig\_distr\_matrix\_t A, starneig\_distr\_matrix\_t Q)

Computes a Hessenberg decomposition of a general matrix.

starneig\_error\_t starneig\_SEP\_DM\_Schur\_expert (struct starneig\_schur\_conf \*conf, starneig\_distr\_matrix\_t
 H, starneig\_distr\_matrix\_t Q, double real[], double imag[])

Computes a Schur decomposition given a Hessenberg decomposition.

starneig\_error\_t starneig\_SEP\_DM\_ReorderSchur\_expert (struct starneig\_reorder\_conf \*conf, int selected[], starneig\_distr\_matrix\_t S, starneig\_distr\_matrix\_t Q, double real[], double imag[])

Reorders selected eigenvalues to the top left corner of a Schur decomposition.

starneig\_error\_t starneig\_SEP\_DM\_Eigenvectors\_expert (struct starneig\_eigenvectors\_conf \*conf, int selected[], starneig\_distr\_matrix\_t S, starneig\_distr\_matrix\_t Q, starneig\_distr\_matrix\_t X)

Computes an eigenvector for each selected eigenvalue.

#### 14.11.1 Detailed Description

This file contains distributed memory interface functions for standard eigenvalue problems.

**Author** 

```
Mirko Myllykoski ( mirkom@cs.umu.se), Umeå University Lars Karlsson ( larsk@cs.umu.se), Umeå University
```

# 14.12 sep\_sm.h File Reference

This file contains shared memory interface functions for standard eigenvalue problems.

```
#include <starneig/configuration.h>
#include <starneig/error.h>
#include <starneig/expert.h>
```

#### **Functions**

#### Computational functions

- starneig\_error\_t starneig\_SEP\_SM\_Hessenberg (int n, double A[], int ldA, double Q[], int ldQ)
   Computes a Hessenberg decomposition of a general matrix.
- starneig\_error\_t starneig\_SEP\_SM\_Schur (int n, double H[], int ldH, double Q[], int ldQ, double real[], double imag[])

Computes a Schur decomposition given a Hessenberg decomposition.

• starneig\_error\_t starneig\_SEP\_SM\_ReorderSchur (int n, int selected[], double S[], int ldS, double Q[], int ldQ, double real[], double imag[])

Reorders selected eigenvalues to the top left corner of a Schur decomposition.

Computes a (reordered) Schur decomposition of a general matrix.

• starneig\_error\_t starneig\_SEP\_SM\_Eigenvectors (int n, int selected[], double S[], int ldS, double Q[], int ldQ, double X[], int ldX)

Computes an eigenvector for each selected eigenvalue.

#### **Helper functions**

starneig\_error\_t starneig\_SEP\_SM\_Select (int n, double S[], int ldS, int(\*predicate)(double real, double imag, void \*arg), void \*arg, int selected[], int \*num\_selected)

Generates a selection array for a Schur matrix using a user-supplied predicate function.

#### **Expert computational functions**

• starneig\_error\_t starneig\_SEP\_SM\_Hessenberg\_expert (struct starneig\_hessenberg\_conf \*conf, int n, int begin, int end, double A[], int IdA, double Q[], int IdQ)

Computes a Hessenberg decomposition of a general matrix.

• starneig\_error\_t starneig\_SEP\_SM\_Schur\_expert (struct starneig\_schur\_conf \*conf, int n, double H[], int IdH, double Q[], int IdQ, double real[], double imag[])

Computes a Schur decomposition given a Hessenberg decomposition.

• starneig\_error\_t starneig\_SEP\_SM\_ReorderSchur\_expert (struct starneig\_reorder\_conf \*conf, int n, int selected[], double S[], int ldS, double Q[], int ldQ, double real[], double imag[])

Reorders selected eigenvalues to the top left corner of a Schur decomposition.

starneig\_error\_t starneig\_SEP\_SM\_Eigenvectors\_expert (struct starneig\_eigenvectors\_conf \*conf, int n, int selected[], double S[], int ldS, double Q[], int ldQ, double X[], int ldX)

Computes an eigenvector for each selected eigenvalue.

# 14.12.1 Detailed Description

This file contains shared memory interface functions for standard eigenvalue problems.

Author

```
Mirko Myllykoski ( mirkom@cs.umu.se), Umeå University Lars Karlsson ( larsk@cs.umu.se), Umeå University
```

# 14.13 starneig.h File Reference

This file includes most StarNEig header files.

```
#include <starneig/configuration.h>
#include <starneig/node.h>
#include <starneig/gep_sm.h>
#include <starneig/sep_sm.h>
#include <starneig/distr_helpers.h>
#include <starneig/gep_dm.h>
#include <starneig/sep_dm.h>
#include <starneig/blacs_helpers.h>
#include <starneig/blacs_matrix.h>
```

### 14.13.1 Detailed Description

This file includes most StarNEig header files.

**Author** 

```
Mirko Myllykoski ( mirkom@cs.umu.se), Umeå University
```

# **Chapter 15**

# **Example Documentation**

# 15.1 gep\_dm\_full\_chain.c

```
#include "validate.h"
#include <stdlib.h>
#include <stdio.h>
#include <time.h>
#include <mpi.h>
#include <starneig/starneig.h>
// a predicate function that selects all finate eigenvalues that have positive
static int predicate (double real, double imag, double beta, void *arg)
     if (0.0 < real && beta != 0.0)</pre>
          return 1;
     return 0;
int main(int argc, char **argv)
    const int n = 3000; // matrix dimension const int root = 0; // root rank
     // initialize MPI
     int thread_support;
     MPI_Init_thread(
          &argc, (char ***)&argv, MPI_THREAD_MULTIPLE, &thread_support);
     int world rank;
    Int world_rank;
MPI_Comm_rank(MPI_COMM_WORLD, &world_rank);
// the root node initializes the matrices locally
int ldA = 0, ldB = 0, ldQ = 0, ldZ = 0, ldC = 0, ldD = 0;
double *A = NULL, *B = NULL, *Q = NULL, *Z = NULL, *C = NULL, *D = NULL;
     if (world_rank == root) {
          srand((unsigned) time(NULL));
          // generate a full random matrix A and a copy C 1dA = ((n/8)+1)*8, 1dC = ((n/8)+1)*8;
          A = malloc(n*ldA*sizeof(double));
          C = malloc(n*ldC*sizeof(double));
          1dB = ((n/8)+1)*8, 1dD = ((n/8)+1)*8;
          B = malloc(n*ldB*sizeof(double));
          D = malloc(n*ldD*sizeof(double));
          for (int j = 0; j < n; j++)

for (int i = 0; i < n; i++)

B[j*ldB+i] = D[j*ldD+i] = 2.0*rand()/RAND_MAX - 1.0;

// generate an identity matrix Q
          1dQ = ((n/8)+1)*8;
          Q = malloc(n*ldA*sizeof(double));
          for (int j = 0; j < n; j++)
    for (int i = 0; i < n; i++)
        Q[j*ldQ+i] = i == j ? 1.0 : 0.0;</pre>
          // generate an identity matrix Z
          1dZ = ((n/8)+1)*8;
          Z = malloc(n*1dZ*sizeof(double));
          for (int j = 0; j < n; j++)
  for (int i = 0; i < n; i++)
      Z[j*ldZ+i] = i == j ? 1.0 : 0.0;</pre>
     // allocate space for the eigenvalues and the eigenvalue selection vector
     double *real = malloc(n*sizeof(double));
```

```
double *imag = malloc(n*sizeof(double));
double *beta = malloc(n*sizeof(double));
int *select = malloc(n*sizeof(int));
// Initialize the {\tt StarNEig} library using all available CPU cores and
// GPUs. The STARNEIG_FAST_DM flag indicates that the library should
// initialize itself for distributed memory computations and keep StarPU
// worker threads and StarPU-MPI communication thread awake between
// interface function calls.
starneig_node_init(STARNEIG_USE_ALL, STARNEIG_USE_ALL, STARNEIG_FAST_DM);
// create a two-dimensional block cyclic distribution with column-major
// ordering
starneig_distr_t distr = starneig_distr_init_mesh(
    -1, -1, STARNEIG_ORDER_COL_MAJOR);
// Convert the local matrix A to a distributed matrix lA that is owned by
// the root node. This is done in-place, i.e., the matrices A and lA point
// to the same data.
starneig\_distr\_matrix\_t 1A = starneig\_distr\_matrix\_create\_local(
n, n, STARNEIG_REAL_DOUBLE, root, A, ldA);

// create a distributed matrix dA using the earlier created data
// distribution and default distributed block size
starneig_distr_matrix_t dA =
    starneig_distr_matrix_create(n, n, -1, -1, STARNEIG_REAL_DOUBLE, distr);
// copy the local matrix lA to the distributed matrix dA (scatter) \,
starneig_distr_matrix_copy(lA, dA);
// scatter the matrix B
starneig_distr_matrix_t lB = starneig_distr_matrix_create_local(
    n, n, STARNEIG_REAL_DOUBLE, root, B, ldB);
starneig_distr_matrix_t dB =
   starneig_distr_matrix_create(n, n, -1, -1, STARNEIG_REAL_DOUBLE, distr);
starneig_distr_matrix_copy(lB, dB);
// scatter the matrix O
starneig_distr_matrix_t 1Q = starneig_distr_matrix_create_local(
    n, n, STARNEIG_REAL_DOUBLE, root, Q, ldQ);
starneig_distr_matrix_t dQ =
    starneig_distr_matrix_create(n, n, -1, -1, STARNEIG_REAL_DOUBLE, distr);
starneig_distr_matrix_copy(1Q, dQ);
// scatter the matrix Z
starneig_distr_matrix_t 1Z = starneig_distr_matrix_create_local(
    n, n, STARNEIG_REAL_DOUBLE, root, Z, ldZ);
starneig_distr_matrix_t dZ =
    starneig_distr_matrix_create(n, n, -1, -1, STARNEIG_REAL_DOUBLE, distr);
starneig_distr_matrix_copy(1Z, dZ);
// reduce the dense-dense matrix pair (A,B) to Hessenberg-triangular form
printf("Hessenberg-triangular reduction...\n");
starneig_GEP_DM_HessenbergTriangular(dA, dB, dQ, dZ);
// reduce the Hessenberg-triangular matrix pair (A,B) to generalized Schur
// form
printf("Schur reduction...\n");
starneig_GEP_DM_Schur(dA, dB, dQ, dZ, real, imag, beta);
// select eigenvalues that have positive a real part
int num_selected;
starneig_GEP_DM_Select(dA, dB, &predicate, NULL, select, &num_selected);
printf("Selected %d eigenvalues out of %d.\n", num_selected, n);
// reorder selected eigenvalues to the upper left corner of the generalized // Schur form (A,B)
printf("Reordering...\n");
starneig_GEP_DM_ReorderSchur(select, dA, dB, dQ, dZ, real, imag, beta);
// copy the distributed matrix dA back to the local matrix lA (gather)
starneig_distr_matrix_copy(dA, lA);
// free the distributed matrix 1A (matrix A is not freed)
starneig distr matrix destroy(1A);
// free the distributed matrix dA (all local resources are freed)
starneig_distr_matrix_destroy(dA);
// gather the matrix B
starneig_distr_matrix_copy(dB, lB);
starneig_distr_matrix_destroy(lB);
starneig_distr_matrix_destroy(dB);
// gather the matrix O
starneig_distr_matrix_copy(dQ, 1Q);
starneig_distr_matrix_destroy(lQ);
starneig_distr_matrix_destroy(dQ);
// gather the matrix Z
starneig_distr_matrix_copy(dZ, 1Z);
starneig_distr_matrix_destroy(1Z);
starneig_distr_matrix_destroy(dZ);
// free the data distribution
starneig_distr_destroy(distr);
// de-initialize the StarNEig library
starneig_node_finalize();
// de-initialize MPT
MPI Finalize();
if (world_rank == root) {
    // check residual ||
                          Q A Z^T - C ||_F / || C ||_F
    check_residual(n, ldQ, ldA, ldZ, ldC, Q, A, Z, C);
// check residual || Q B Z^T - D ||_F / || D ||_F
    check_residual(n, ldQ, ldB, ldZ, ldD, Q, B, Z, D); // check residual || Q Q^T - I ||_F / || I ||_F
```

```
check_orthogonality(n, ldQ, Q);
    // check residual || Z Z^T - I ||_F / || I ||_F
    check_orthogonality(n, ldZ, Z);
}
// cleanup
free(A);
free(C);
free(B);
free(D);
free(Q);
free(Z);
free(real);
free(imag);
free(beta);
free(select);
return 0;
}
```

# 15.2 gep\_sm\_eigenvectors.c

```
#include "validate.h"
#include <stdlib.h>
#include <stdio.h>
#include <time.h>
#include <starneig/starneig.h>
// a predicate function that selects all finite eigenvalues that have positive
static int predicate(double real, double imag, double beta, void *arg)
    if (0.0 < real && beta != 0.0)
        return 1;
   return 0;
int main()
    const int n = 3000; // matrix dimension
    srand((unsigned) time(NULL));
// generate a full random matrix A and a copy C
    int 1dA = ((n/8)+1)*8, 1dC = ((n/8)+1)*8;
    double *A = malloc(n*ldA*sizeof(double));
    double *C = malloc(n*ldC*sizeof(double));
   int 1dB = ((n/8)+1)*8, 1dD = ((n/8)+1)*8;
    double *B = malloc(n*ldB*sizeof(double));
    double *D = malloc(n*ldD*sizeof(double));
    // generate an identity matrix Q
    int 1dQ = ((n/8)+1)*8;
    double *Q = malloc(n*ldA*sizeof(double));
    for (int j = 0; j < n; j++)
  for (int i = 0; i < n; i++)
        Q[j*ldQ+i] = i == j ? 1.0 : 0.0;</pre>
    // generate an identity matrix Z
    int 1dZ = ((n/8)+1)*8;
    double *Z = malloc(n*ldZ*sizeof(double));
    for (int j = 0; j < n; j++)
    for (int i = 0; i < n; i++)</pre>
    Z[j*1dZ+i] = i == j ? 1.0 : 0.0;
double *X = NULL; int ldX = 0;
    // allocate space for the eigenvalues and the eigenvalue selection vector
    double *real = malloc(n*sizeof(double));
    double *imag = malloc(n*sizeof(double));
double *beta = malloc(n*sizeof(double));
    int *select = malloc(n*sizeof(int));
    // Initialize the StarNEig library using all available CPU cores and
    // GPUs. The STARNEIG_HINT_SM flag indicates that the library should
    // initialize itself for shared memory computations.
    starneig_node_init(STARNEIG_USE_ALL, STARNEIG_USE_ALL, STARNEIG_HINT_SM);
    // reduce the dense-dense matrix pair (A,B) to generalized Schur form
    // (skip reordering)
    printf("Reduce...\n");
    starneig_GEP_SM_Reduce(
        n, A, ldA, B, ldB, Q, ldQ, Z, ldZ, real, imag, beta,
        NULL, NULL, NULL, NULL);
    // select eigenvalues that have positive a real part and allocate space for
    // the eigenvectors
    int num_selected;
    starneig_GEP_SM_Select(
```

```
n, A, ldA, B, ldB, &predicate, NULL, select, &num_selected);
printf("Selected %d eigenvalues out of %d.\n", num_selected, n);
1dX = ((n/8)+1)*8;
X = malloc(num_selected*ldX*sizeof(double));
// compute a selected set of eigenvectors
printf("Eigenvectors...\n");
starneig_GEP_SM_Eigenvectors(n, select, A, ldA, B, ldB, Q, ldQ, X, ldX);
// de-initialize the StarNEig library
starneig_node_finalize();
// check residual || Q A Z^T - C ||_F / || C ||_F check_residual(n, ldQ, ldA, ldZ, ldC, Q, A, Z, C); // check_residual || Q B Z^T - D ||_F / || D ||_F
check_residual(n, ldQ, ldB, ldZ, ldD, Q, B, Z, D);
// check_residual || Q Q^T - I ||_F / || I ||_F
check_orthogonality(n, ldQ, Q);
// check residual || Z Z^T - I ||_F / || I ||_F
\verb|check_orthogonality(n, ldZ, Z);|\\
// cleanup
free(A);
free(C);
free(B);
free (D);
free (0);
free (7):
free(X);
free (real);
free (beta);
free(select);
return 0;
```

# 15.3 gep\_sm\_full\_chain.c

```
#include "validate.h"
#include <stdlib.h>
#include <stdio.h>
#include <time.h>
#include <starneig/starneig.h>
// a predicate function that selects all finate eigenvalues that have positive
// a real part
static int predicate(double real, double imag, double beta, void \star arg)
    if (0.0 < real && beta != 0.0)
        return 1;
    return 0;
int main()
    const int n = 3000; // matrix dimension
    srand((unsigned) time(NULL));
    // generate a full random matrix A and a copy C
    int 1dA = ((n/8)+1)*8, 1dC = ((n/8)+1)*8;
    double *A = malloc(n*ldA*sizeof(double));
    double *C = malloc(n*ldC*sizeof(double));
    for (int j = 0; j < n; j++)
for (int i = 0; i < n; i++)
            A[j*ldA+i] = C[j*ldC+i] = 2.0*rand()/RAND_MAX - 1.0;
    // generate a full random matrix B and a copy D
    int 1dB = ((n/8)+1)*8, 1dD = ((n/8)+1)*8;
    double *B = malloc(n*ldB*sizeof(double));
    double *D = malloc(n*ldD*sizeof(double));
    for (int j = 0; j < n; j++)
for (int i = 0; i < n; i++)
            B[j*ldB+i] = D[j*ldD+i] = 2.0*rand()/RAND_MAX - 1.0;
    // generate an identity matrix Q
    int 1dQ = ((n/8)+1)*8;
    double *Q = malloc(n*ldA*sizeof(double));
    for (int j = 0; j < n; j++)

for (int i = 0; i < n; i++)

Q[j*ldQ+i] = i == j ? 1.0 : 0.0;
    // generate an identity matrix {\tt Z}
    int 1dZ = ((n/8)+1)*8;
    double *Z = malloc(n*ldZ*sizeof(double));
    for (int j = 0; j < n; j++)
for (int i = 0; i < n; i++)
             Z[j*1dZ+i] = i == j ? 1.0 : 0.0;
    // allocate space for the eigenvalues and the eigenvalue selection vector
    double *real = malloc(n*sizeof(double));
double *imag = malloc(n*sizeof(double));
    double *beta = malloc(n*sizeof(double));
    int *select = malloc(n*sizeof(int));
    // Initialize the StarNEig library using all available CPU cores and
```

```
// GPUs. The STARNEIG_HINT_SM flag indicates that the library should
// initialize itself for shared memory computations and the
// STARNEIG_AWAKE_WORKERS indicates that the library should should keep
// reduce the dense-dense matrix pair (A,B) to Hessenberg-triangular form
printf("Hessenberg-triangular reduction...\n");
starneig_GEP_SM_HessenbergTriangular(n, A, ldA, B, ldB, Q, ldQ, Z, ldZ);
// reduce the Hessenberg-triangular matrix pair (A,B) to generalized Schur
// form
printf("Schur reduction...\n");
starneig_GEP_SM_Schur(n, A, 1dA, B, 1dB, Q, 1dQ, Z, 1dZ, real, imag, beta);
// select eigenvalues that have positive a real part
int num_selected;
starneig_GEP_SM_Select(
n, A, ldA, B, ldB, &predicate, NULL, select, &num_selected);
printf("Selected %d eigenvalues out of %d.\n", num_selected, n);
// reorder selected eigenvalues to the upper left corner of the generalized
// Schur form (A,B)
printf("Reordering...\n");
starneig_GEP_SM_ReorderSchur(
    n, select, A, 1dA, B, 1dB, Q, 1dQ, Z, 1dZ, real, imag, beta);
// de-initialize the StarNEig library
starneig_node_finalize();
// check residual || Q A Z^T - C ||_F / || C ||_F
check_residual(n, ldQ, ldA, ldZ, ldC, Q, A, Z, C); // check residual || Q B Z^T - D ||_F / || D ||_F
check_residual(n, ldQ, ldB, ldZ, ldD, Q, B, Z, D);
// check residual || Q Q^T - I ||_F / || I ||_F
check_orthogonality(n, ldQ, Q);
// check_residual || Z Z^T - I |
                                  I ||_F / || I ||_F
check_orthogonality(n, ldZ, Z);
// cleanup
free(A);
free(C):
free (B);
free(D);
free(Q);
free(Z);
free (real);
free (imag);
free (beta):
free (select);
return 0;
```

# 15.4 sep dm full chain.c

```
#include "validate.h"
#include <stdlib.h>
#include <stdio.h>
#include <time.h>
#include <mpi.h>
#include <starneig/starneig.h>
// a predicate function that selects all eigenvalues that have positive a real
// part
static int predicate(double real, double imag, void *arg)
    if (0.0 < real)</pre>
         return 1;
    return 0;
int main(int argc, char **argv)
    const int n = 3000; // matrix dimension
    const int root = 0; // root rank
    // initialize MPI
    int thread support;
    MPI_Init_thread(
         &argc, (char ***)&argv, MPI_THREAD_MULTIPLE, &thread_support);
    int world_rank;
    MPI_Comm_rank(MPI_COMM_WORLD, &world_rank);
    // the root node initializes the matrices locally int ldA = 0, ldQ = 0, ldC = 0; double *A = NULL, *Q = NULL, *C = NULL; if (world_rank == root) {
         srand((unsigned) time(NULL));
         // generate a full random matrix A and a copy C
         1dA = ((n/8)+1)*8; 1dC = ((n/8)+1)*8;
        A = malloc(n*ldA*sizeof(double));
        C = malloc(n*ldC*sizeof(double));
         for (int j = 0; j < n; j++)
```

```
for (int i = 0; i < n; i++)</pre>
             A[j*ldA+i] = C[j*ldC+i] = 2.0*rand()/RAND_MAX - 1.0;
    // generate an identity matrix Q
    1d0 = ((n/8)+1)*8;
    O = malloc(n*ldA*sizeof(double));
    for (int j = 0; j < n; j++)
for (int i = 0; i < n; i++)
             Q[j*ldQ+i] = i == j ? 1.0 : 0.0;
, // allocate space for the eigenvalues and the eigenvalue selection vector
double *real = malloc(n*sizeof(double));
double *imag = malloc(n*sizeof(double));
int *select = malloc(n*sizeof(int));
// Initialize the StarNEig library using all available CPU cores and
// GPUs. The STARNEIG_HINT_DM flag indicates that the library should
// initialize itself for distributed memory computations.
starneig_node_init(STARNEIG_USE_ALL, STARNEIG_USE_ALL, STARNEIG_HINT_DM);
// create a two-dimensional block cyclic distribution with row-major
// ordering
starneig_distr_t distr = starneig_distr_init_mesh(
    -1, -1, STARNEIG_ORDER_ROW_MAJOR);
// Convert the local matrix A to a distributed matrix lA that is owned by
// the root node. This is done in-place, i.e., the matrices A and lA point
// to the same data.
starneig_distr_matrix_t 1A = starneig_distr_matrix_create_local(
   n, n, STARNEIG_REAL_DOUBLE, root, A, ldA);
// create a distributed matrix dA using default data distribution and
// distributed block size
starneig_distr_matrix_t dA =
starneig_distr_matrix_create(n, n, -1, -1, STARNEIG_REAL_DOUBLE, distr);
// copy the local matrix lA to the distributed matrix dA (scatter)
starneig_distr_matrix_copy(lA, dA);
// scatter the matrix Q
starneig_distr_matrix_t 1Q = starneig_distr_matrix_create_local(
    n, n, STARNEIG_REAL_DOUBLE, root, Q, ldQ);
starneig_distr_matrix_t dQ =
    starneig_distr_matrix_create(n, n, -1, -1, STARNEIG_REAL_DOUBLE, distr);
starneig_distr_matrix_copy(1Q, dQ);
// reduce the full matrix dA to upper Hessenberg form
printf("Hessenberg reduction...\n");
starneig_SEP_DM_Hessenberg(dA, dQ);
// reduce the upper Hessenberg matrix dA to Schur form
printf("Schur reduction...\n");
starneig_SEP_DM_Schur(dA, dQ, real, imag);
// select eigenvalues that have positive a real part
int num_selected;
starneig_SEP_DM_Select(dA, &predicate, NULL, select, &num_selected);
printf("Selected %d eigenvalues out of %d.\n", num_selected, n);
printf("Reordering...\n");
starneig_SEP_DM_ReorderSchur(select, dA, dQ, real, imag);
\ensuremath{//} copy the distributed matrix dA back to the local matrix lA (gather)
starneig_distr_matrix_copy(dA, lA);
// free the distributed matrix lA (matrix A is not freed)
starneig_distr_matrix_destroy(lA);
// free the distributed matrix dA (all local resources are freed)
starneig_distr_matrix_destroy(dA);
// gather the matrix Q
starneig\_distr\_matrix\_copy(dQ, \ lQ);
{\tt starneig\_distr\_matrix\_destroy(1Q);}
starneig_distr_matrix_destroy(dQ);
// free the data distribution
starneig_distr_destroy(distr);
// de-initialize the StarNEig library
starneig_node_finalize();
// de-initialize MPI
MPI Finalize():
if (world_rank == root) {
    // check residual || Q A Q^T - C ||_F / || C ||_F
    check_residual(n, ldQ, ldA, ldQ, ldC, Q, A, Q, C);
// check residual || Q Q^T - I ||_F / || I ||_F
    check_orthogonality(n, ldQ, Q);
// cleanup
free(A);
free(C);
free(Q);
free (real);
free (imag):
free (select);
return 0;
```

# 15.5 sep sm eigenvectors.c

```
#include "validate.h"
#include <stdlib.h>
#include <stdio.h>
#include <time.h>
#include <starneig/starneig.h>
// a predicate function that selects all eigenvalues that have positive a real
static int predicate(double real, double imag, void *arg)
    if (0.0 < real)</pre>
         return 1;
    return 0;
int main()
    const int n = 3000; // matrix dimension
    srand((unsigned) time(NULL));
    // generate a full random matrix A and a copy C
    int 1dA = ((n/8)+1)*8, 1dC = ((n/8)+1)*8;
    double *A = malloc(n*ldA*sizeof(double));
double *C = malloc(n*ldC*sizeof(double));
    for (int j = 0; j < n; j++)
    for (int i = 0; i < n; i++)
        A[j*ldA+i] = C[j*ldC+i] = 2.0*rand()/RAND_MAX - 1.0;</pre>
    // generate an identity matrix Q
    int 1dQ = ((n/8)+1)*8;
    double *Q = malloc(n*ldA*sizeof(double));
for (int j = 0; j < n; j++)
    for (int i = 0; i < n; i++)
        Q[j*ldQ+i] = i == j ? 1.0 : 0.0;
double *X = NULL; int ldX = 0;</pre>
    // allocate space for the eigenvalues and the eigenvector selection vector
    double *real = malloc(n*sizeof(double));
    double *imag = malloc(n*sizeof(double));
int *select = malloc(n*sizeof(int));
    // Initialize the StarNEig library using all available CPU cores and
    // GPUs. The STARNEIG_HINT_SM flag indicates that the library should
    // initialize itself for shared memory computations.
    starneig_node_init(STARNEIG_USE_ALL, STARNEIG_USE_ALL, STARNEIG_HINT_SM);
    // reduce the full matrix matrix {\tt A} to Schur form (skip reordering)
    printf("Reduce...\n");
starneig_SEP_SM_Reduce(
         n, A, ldA, Q, ldQ, real, imag, NULL, NULL, NULL, NULL);
     // select eigenvalues that have positive a real part and allocate space for
    // the eigenvectors
    int num_selected;
    starneig_SEP_SM_Select(n, A, ldA, &predicate, NULL, select, &num_selected);
    printf("Selected %d eigenvalues out of %d.\n", num_selected, n);
    1dX = ((n/8)+1)*8;
    X = malloc(num_selected*ldX*sizeof(double));
    // compute a selected set of eigenvectors
    printf("Eigenvectors...\n");
    starneig_SEP_SM_Eigenvectors(n, select, A, ldA, Q, ldQ, X, ldX);
    // de-initialize the StarNEig library
    starneig_node_finalize();
    // check residual || Q A Q^T - C ||_F / || C ||_F
    check_residual(n, ldQ, ldA, ldQ, ldC, Q, A, Q, C);
// check residual || Q Q^T - I ||_F / || I ||_F
    check_orthogonality(n, ldQ, Q);
    // cleanup
    free(A);
    free(C);
    free(Q);
    free(X);
    free (real);
    free (imag):
    free (select);
```

# 15.6 sep\_sm\_full\_chain.c

```
#include "validate.h"
#include <stdlib.h>
#include <stdio.h>
#include <time.h>
#include <starneig/starneig.h>
// a predicate function that selects all eigenvalues that have positive a real
// part
static int predicate(double real, double imag, void *arg)
{
```

```
if (0.0 < real)
          return 1;
     return 0;
int main()
     const int n = 3000; // matrix dimension
     srand((unsigned) time(NULL));
     // generate a full random matrix {\tt A} and a copy {\tt C}
     int 1dA = ((n/8)+1)*8, 1dC = ((n/8)+1)*8; double *A = malloc(n*1dA*sizeof(double)); double *C = malloc(n*1dC*sizeof(double));
     for (int j = 0; j < n; j++)
    for (int i = 0; i < n; i++)
        A[j*ldA+i] = C[j*ldC+i] = 2.0*rand()/RAND_MAX - 1.0;</pre>
     // generate an identity matrix {\tt Q}
     int ldQ = ((n/8)+1)*8;
double *Q = malloc(n*ldA*sizeof(double));
     for (int j = 0; j < n; j++)

for (int i = 0; i < n; i++)

Q[j*ldQ+i] = i == j ? 1.0 : 0.0;
     \ensuremath{//} allocate space for the eigenvalues and the eigenvalue selection vector
     double *real = malloc(n*sizeof(double));
double *imag = malloc(n*sizeof(double));
     int *select = malloc(n*sizeof(int));
     // Initialize the StarNEig library using all available CPU cores and
     // GPUs. The STARNEIG_HINT_SM flag indicates that the library should
     // initialize itself for shared memory computations.
     starneig_node_init(STARNEIG_USE_ALL, STARNEIG_USE_ALL, STARNEIG_HINT_SM);
     // reduce the full matrix {\tt Matrix}\ {\tt A} to upper Hessenberg form
     printf("Hessenberg reduction...\n");
starneig_SEP_SM_Hessenberg(n, A, 1dA, Q, 1dQ);
     // reduce the upper Hessenberg matrix A to Schur form
     printf("Schur reduction...\n");
     starneig_SEP_SM_Schur(n, A, 1dA, Q, 1dQ, real, imag);
// select eigenvalues that have positive a real part
     int num selected;
     starneig_SEP_SM_select(n, A, ldA, &predicate, NULL, select, &num_selected); printf("Selected %d eigenvalues out of %d.\n", num_selected, n);
     // reorder the selected eigenvalues to the upper left corner of the matrix A
     printf("Reordering...\n");
     starneig_SEP_SM_ReorderSchur(n, select, A, ldA, Q, ldQ, real, imag);
// de-initialize the StarNEig library
     starneig_node_finalize();
     // check residual || Q A Q^T - C ||_F / || C ||_F
     // check residual || Q Q^T - I ||_F / || I ||_F
     check\_orthogonality(n, ldQ, Q);
     // cleanup
     free(A):
     free(C);
     free(Q);
     free (real);
     free(imag);
     free (select);
     return 0;
```

# Index

| blacs_helpers.h, 129                          | starneig_SEP_DM_Hessenberg_expert, 94   |
|---|---|
| blacs_matrix.h, 130                           | starneig_SEP_DM_Reduce, 92              |
|   | starneig_SEP_DM_ReorderSchur, 92        |
| configuration.h, 131                          | starneig_SEP_DM_ReorderSchur_expert, 95 |
|   | starneig_SEP_DM_Schur, 91               |
| distr_helpers.h, 132                          | starneig_SEP_DM_Schur_expert, 95        |
| distr_matrix.h, 132                           | starneig_SEP_DM_Select, 94              |
| Distributed Memory / Distributed matrices, 76 | <b>5</b> ,                              |
| starneig_datatype_t, 79                       | Error codes, 48                         |
| starneig_distr_destroy, 81                    | STARNEIG_CLOSE_EIGENVALUES, 50          |
| starneig_distr_duplicate, 80                  | STARNEIG_DID_NOT_CONVERGE, 49           |
| starneig_distr_init, 79                       | STARNEIG_GENERIC_ERROR, 48              |
| starneig_distr_init_func, 80                  | STARNEIG_INVALID_ARGUMENTS, 49          |
| starneig_distr_init_mesh, 79                  | STARNEIG_INVALID_CONFIGURATION, 49      |
| starneig_distr_matrix_copy, 83                | STARNEIG_INVALID_DISTR_MATRIX, 49       |
| starneig_distr_matrix_copy_region, 83         | STARNEIG_NOT_INITIALIZED, 49            |
| starneig_distr_matrix_create, 81              | STARNEIG_PARTIAL_REORDERING, 49         |
| starneig_distr_matrix_create_local, 82        | STARNEIG_SUCCESS, 48                    |
| starneig_distr_matrix_destroy, 83             | error.h, 134                            |
| starneig_distr_matrix_get_blocks, 84          | Expert configuration structures, 106    |
| starneig_distr_matrix_get_col_blksz, 87       | starneig_eigenvectors_init_conf, 117    |
| starneig_distr_matrix_get_cols, 86            | starneig_hessenberg_init_conf, 116      |
| starneig_distr_matrix_get_datatype, 85        | starneig_reorder_blueprint_t, 115       |
| starneig_distr_matrix_get_distr, 84           | STARNEIG REORDER CHAIN INSERT A, 115    |
| starneig_distr_matrix_get_elemsize, 85        | STARNEIG_REORDER_CHAIN_INSERT_B, 115    |
| starneig_distr_matrix_get_row_blksz, 86       | STARNEIG_REORDER_CHAIN_INSERT_C, 115    |
| starneig_distr_matrix_get_rows, 85            | STARNEIG REORDER CHAIN INSERT D, 115    |
| starneig_distr_order_t, 77                    | STARNEIG_REORDER_CHAIN_INSERT_E, 115    |
| STARNEIG_ORDER_COL_MAJOR, 79                  | STARNEIG_REORDER_CHAIN_INSERT_F, 116    |
| STARNEIG_ORDER_DEFAULT, 79                    | STARNEIG_REORDER_DEFAULT_BLUEPRINT,     |
| STARNEIG_ORDER_ROW_MAJOR, 79                  | 115                                     |
| STARNEIG_REAL_DOUBLE, 79                      | STARNEIG_REORDER_DEFAULT_PLAN, 114      |
| Distributed Memory / Generalized EVP, 98      | STARNEIG_REORDER_DUMMY_INSERT_A, 115    |
| starneig_GEP_DM_Eigenvectors, 102             | STARNEIG_REORDER_DUMMY_INSERT_B, 115    |
| starneig_GEP_DM_Eigenvectors_expert, 105      | starneig_reorder_init_conf, 116         |
| starneig GEP DM HessenbergTriangular, 99      | STARNEIG REORDER MULTI PART PLAN, 114   |
| starneig_GEP_DM_Reduce, 101                   | STARNEIG REORDER ONE PART PLAN, 114     |
| starneig_GEP_DM_ReorderSchur, 100             | starneig_reorder_plan_t, 112            |
| starneig_GEP_DM_ReorderSchur_expert, 104      | starneig_reorder_plan_t, 112            |
| starneig_GEP_DM_Schur, 99                     | expert.h, 135                           |
| starneig_GEP_DM_Schur_expert, 103             | expertin, rec                           |
| starneig_GEP_DM_Select, 102                   | gep dm.h, 137                           |
| Distributed Memory / Helper functions, 88     | gep_sm.h, 138                           |
| starneig mpi broadcast, 89                    | 900_511111, 100                         |
| starneig_mpi_get_comm, 88                     | Intra-node execution environment, 51    |
| starneig_mpi_set_comm, 88                     | STARNEIG AWAKE MPI WORKER, 53           |
| Distributed Memory / Standard EVP, 90         | STARNEIG_AWAKE_WORKERS, 53              |
| starneig_SEP_DM_Eigenvectors, 93              | STARNEIG DEFAULT, 52                    |
| starneig_SEP_DM_Eigenvectors_expert, 96       | STARNEIG FAST DM, 53                    |
| starneig SEP DM Hessenberg, 91                | STARNEIG FXT DISABLE, 53                |

154 INDEX

|       | STARNEIG_HINT_DM, 52   | Shared Memory / Standard EVP, 57                                  |
|-------|--|---|
|       | STARNEIG_HINT_SM, 52   | starneig_SEP_SM_Eigenvectors, 61                                  |
|       | STARNEIG_NO_MESSAGES, 54                                       | starneig_SEP_SM_Eigenvectors_expert, 65                           |
|       | STARNEIG_NO_VERBOSE, 54  | starneig_SEP_SM_Hessenberg, 58                                    |
|       | starneig_node_disable_pinning, 56                              | starneig_SEP_SM_Hessenberg_expert, 62                             |
|       | starneig_node_enable_pinning, 56                               | starneig_SEP_SM_Reduce, 60  |
|       | starneig_node_get_cores, 55                                    | starneig_SEP_SM_ReorderSchur, 59                                  |
|       | starneig_node_get_gpus, 56                                     | starneig_SEP_SM_ReorderSchur_expert, 64                           |
|       | starneig_node_init, 54   | starneig_SEP_SM_Schur, 58   |
|       | starneig node initialized, 55                                  | starneig_SEP_SM_Schur_expert, 63                                  |
|       | starneig_node_set_cores, 55                                    | starneig SEP SM Select, 61  |
|       | starneig_node_set_gpus, 56                                     | starneig.h, 143   |
|       | STARNEIG_USE_ALL, 52   | STARNEIG_AWAKE_MPI_WORKER   |
|       |  | Intra-node execution environment, 53                              |
| Libra | ary configuration, 45  | STARNEIG_AWAKE_WORKERS  |
|       | STARNEIG_ENABLE_BLACS, 46                                      | Intra-node execution environment, 53                              |
|       | STARNEIG_ENABLE_CUDA, 46                                       | starneig_blacs_context_to_distr                                   |
|       | STARNEIG_ENABLE_MPI, 46  | Seel ADACK competibility / DLACS matrices 110                     |
|       | STARNEIG_GEP_DM_HESSENBERGTRIANGULA                            | Rstarneig blacs create matrix                                     |
|       | 46   | ScaLAPACK compatibility / BLACS matrices, 120                     |
|       | STARNEIG_GEP_DM_REDUCE, 47                                     | starneig blacs descinit   |
|       | STARNEIG_SEP_DM_HESSENBERG, 46                                 | ScaLAPACK compatibility / BLACS helpers, 127                      |
|       | STARNEIG_SEP_DM_REDUCE, 46                                     | starneig_blacs_descr, 118   |
|       |  | starneig_blacs_descr_to_distr_matrix                              |
| node  | e.h, 139   | ScaLAPACK compatibility / BLACS matrices, 122                     |
| _     |  | starneig_blacs_destroy_matrix                                     |
| Sca   | LAPACK compatibility / BLACS helpers, 124                      | ScaLAPACK compatibility / BLACS matrices, 121                     |
|       | starneig_blacs_descinit, 127                                   | starneig_blacs_exit   |
|       | starneig_blacs_exit, 127                                       | ScaLAPACK compatibility / BLACS helpers, 127                      |
|       | starneig_blacs_get, 125  | starneig_blacs_get  |
|       | starneig_blacs_gridexit, 126                                   | ScaLAPACK compatibility / BLACS helpers, 125                      |
|       | starneig_blacs_gridinfo, 125                                   | starneig_blacs_gridexit   |
|       | starneig_blacs_gridinit, 125                                   | ScaLAPACK compatibility / BLACS helpers, 126                      |
|       | starneig_blacs_numroc, 127                                     | starneig_blacs_gridinfo   |
|       | starneig_blacs_pcoord, 126                                     | ScaLAPACK compatibility / BLACS helpers, 125                      |
| _     | starneig_blacs_pinfo, 124                                      | starneig blacs gridinit   |
| Sca   | LAPACK compatibility / BLACS matrices, 118                     | ScaLAPACK compatibility / BLACS helpers, 125                      |
|       | starneig_blacs_context_to_distr, 119                           | starneig blacs numroc   |
|       | starneig_blacs_create_matrix, 120                              | ScaLAPACK compatibility / BLACS helpers, 127                      |
|       | starneig_blacs_descr_to_distr_matrix, 122                      | starneig_blacs_pcoord   |
|       | starneig_blacs_destroy_matrix, 121                             | ScaLAPACK compatibility / BLACS helpers, 126                      |
|       | starneig_distr_is_blacs_compatible, 120                        | starneig_blacs_pinfo  |
|       | starneig_distr_is_compatible_with, 120                         | ScaLAPACK compatibility / BLACS helpers, 124                      |
|       | starneig_distr_matrix_is_blacs_compatible, 122                 | STARNEIG_CLOSE_EIGENVALUES  |
|       | starneig_distr_matrix_is_compatible_with, 123                  | Error codes, 50   |
|       | starneig_distr_matrix_to_blacs_descr, 121                      | starneig_datatype_t   |
| 000   | starneig_distr_to_blacs_context, 119                           | Distributed Memory / Distributed matrices, 79                     |
|       | _dm.h, 141   | STARNEIG_DEFAULT  |
|       | _sm.h, 142   | Intra-node execution environment, 52                              |
| ona   | red Memory / Generalized EVP, 66                               |   |
|       | starneig_GEP_SM_Eigenvectors, 71                               | STARNEIG_DID_NOT_CONVERGE Error codes, 49                         |
|       | starneig_GEP_SM_Eigenvectors_expert, 74                        | starneig_distr_block, 77  |
|       | starneig_GEP_SM_HessenbergTriangular, 67                       | starneig_distr_destroy  |
|       | starneig_GEP_SM_Reduce, 69                                     |   |
|       | starneig_GEP_SM_ReorderSchur, 68                               | Distributed Memory / Distributed matrices, 81                     |
|       | starneig_GEP_SM_ReorderSchur_expert, 73                        | starneig_distr_duplicate  |
|       | starneig_GEP_SM_Schur, 67                                      | Distributed Memory / Distributed matrices, 80                     |
|       | starneig_GEP_SM_Schur_expert, 72<br>starneig_GEP_SM_Select, 71 | starneig_distr_init Distributed Memory / Distributed matrices, 79 |
|       | Stattleiu GEP SIVI Select. / I                                 | Distributed interriory / Distributed matrices. 79                 |

INDEX 155

| starneig_distr_init_func                      | Error codes, 48                           |
|---|---|
| Distributed Memory / Distributed matrices, 80 | starneig_GEP_DM_Eigenvectors              |
| starneig_distr_init_mesh                      | Distributed Memory / Generalized EVP, 102 |
| Distributed Memory / Distributed matrices, 79 | starneig_GEP_DM_Eigenvectors_expert       |
| starneig_distr_is_blacs_compatible            | Distributed Memory / Generalized EVP, 105 |
| ScaLAPACK compatibility / BLACS matrices, 120 | STARNEIG_GEP_DM_HESSENBERGTRIANGULAR      |
|   |   |
| starneig_distr_is_compatible_with             | Library configuration, 46                 |
| ScaLAPACK compatibility / BLACS matrices, 120 | starneig_GEP_DM_HessenbergTriangular      |
| starneig_distr_matrix_copy                    | Distributed Memory / Generalized EVP, 99  |
| Distributed Memory / Distributed matrices, 83 | STARNEIG_GEP_DM_REDUCE                    |
| starneig_distr_matrix_copy_region             | Library configuration, 47                 |
| Distributed Memory / Distributed matrices, 83 | starneig_GEP_DM_Reduce                    |
| starneig_distr_matrix_create                  | Distributed Memory / Generalized EVP, 101 |
| Distributed Memory / Distributed matrices, 81 | starneig_GEP_DM_ReorderSchur              |
| starneig_distr_matrix_create_local            | Distributed Memory / Generalized EVP, 100 |
| Distributed Memory / Distributed matrices, 82 | starneig_GEP_DM_ReorderSchur_expert       |
| starneig_distr_matrix_destroy                 | Distributed Memory / Generalized EVP, 104 |
| Distributed Memory / Distributed matrices, 83 | starneig GEP DM Schur                     |
| starneig_distr_matrix_get_blocks              | Distributed Memory / Generalized EVP, 99  |
| Distributed Memory / Distributed matrices, 84 | starneig_GEP_DM_Schur_expert              |
|   | <del>-</del> ·                            |
| starneig_distr_matrix_get_col_blksz           | Distributed Memory / Generalized EVP, 103 |
| Distributed Memory / Distributed matrices, 87 | starneig_GEP_DM_Select                    |
| starneig_distr_matrix_get_cols                | Distributed Memory / Generalized EVP, 102 |
| Distributed Memory / Distributed matrices, 86 | starneig_GEP_SM_Eigenvectors              |
| starneig_distr_matrix_get_datatype            | Shared Memory / Generalized EVP, 71       |
| Distributed Memory / Distributed matrices, 85 | starneig_GEP_SM_Eigenvectors_expert       |
| starneig_distr_matrix_get_distr               | Shared Memory / Generalized EVP, 74       |
| Distributed Memory / Distributed matrices, 84 | starneig_GEP_SM_HessenbergTriangular      |
| starneig_distr_matrix_get_elemsize            | Shared Memory / Generalized EVP, 67       |
| Distributed Memory / Distributed matrices, 85 | starneig_GEP_SM_Reduce                    |
| starneig_distr_matrix_get_row_blksz           | Shared Memory / Generalized EVP, 69       |
| Distributed Memory / Distributed matrices, 86 | starneig_GEP_SM_ReorderSchur              |
| starneig_distr_matrix_get_rows                | Shared Memory / Generalized EVP, 68       |
| Distributed Memory / Distributed matrices, 85 | starneig_GEP_SM_ReorderSchur_expert       |
|   | · · ·                                     |
| starneig_distr_matrix_is_blacs_compatible     | Shared Memory / Generalized EVP, 73       |
| ScaLAPACK compatibility / BLACS matrices, 122 | starneig_GEP_SM_Schur                     |
| starneig_distr_matrix_is_compatible_with      | Shared Memory / Generalized EVP, 67       |
| ScaLAPACK compatibility / BLACS matrices, 123 | starneig_GEP_SM_Schur_expert              |
| starneig_distr_matrix_to_blacs_descr          | Shared Memory / Generalized EVP, 72       |
| ScaLAPACK compatibility / BLACS matrices, 121 | starneig_GEP_SM_Select                    |
| starneig_distr_order_t                        | Shared Memory / Generalized EVP, 71       |
| Distributed Memory / Distributed matrices, 77 | starneig_hessenberg_conf, 108             |
| starneig_distr_to_blacs_context               | starneig_hessenberg_init_conf             |
| ScaLAPACK compatibility / BLACS matrices, 119 | Expert configuration structures, 116      |
| starneig_eigenvectors_conf, 112               | STARNEIG HINT DM                          |
| starneig eigenvectors init conf               | Intra-node execution environment, 52      |
| Expert configuration structures, 117          | STARNEIG HINT SM                          |
| STARNEIG_ENABLE_BLACS                         | Intra-node execution environment, 52      |
|   |   |
| Library configuration, 46                     | STARNEIG_INVALID_ARGUMENTS                |
| STARNEIG_ENABLE_CUDA                          | Error codes, 49                           |
| Library configuration, 46                     | STARNEIG_INVALID_CONFIGURATION            |
| STARNEIG_ENABLE_MPI                           | Error codes, 49                           |
| Library configuration, 46                     | STARNEIG_INVALID_DISTR_MATRIX             |
| STARNEIG_FAST_DM                              | Error codes, 49                           |
| Intra-node execution environment, 53          | starneig_mpi_broadcast                    |
| STARNEIG_FXT_DISABLE                          | Distributed Memory / Helper functions, 89 |
| Intra-node execution environment, 53          | starneig_mpi_get_comm                     |
| STARNEIG GENERIC ERROR                        | Distributed Memory / Helper functions, 88 |

156 INDEX

| starneig_mpi_set_comm  | Expert configuration structures, 116  |
|--|---------------------------------------|
| Distributed Memory / Helper functions, 88  | STARNEIG REORDER MULTI PART PLAN      |
| STARNEIG_NO_MESSAGES   | Expert configuration structures, 114  |
| Intra-node execution environment, 54   | STARNEIG_REORDER_ONE_PART_PLAN        |
| STARNEIG_NO_VERBOSE  | Expert configuration structures, 114  |
|  | starneig reorder plan t               |
| Intra-node execution environment, 54   | <del>-</del>                          |
| starneig_node_disable_pinning  | Expert configuration structures, 112  |
| Intra-node execution environment, 56   | starneig_schur_conf, 108              |
| starneig_node_enable_pinning   | starneig_schur_init_conf              |
| Intra-node execution environment, 56   | Expert configuration structures, 116  |
| starneig_node_get_cores  | starneig_SEP_DM_Eigenvectors          |
| Intra-node execution environment, 55   | Distributed Memory / Standard EVP, 93 |
| starneig_node_get_gpus   | starneig_SEP_DM_Eigenvectors_expert   |
| Intra-node execution environment, 56   | Distributed Memory / Standard EVP, 96 |
| starneig_node_init   | STARNEIG_SEP_DM_HESSENBERG            |
| Intra-node execution environment, 54   | Library configuration, 46             |
| starneig_node_initialized  | starneig_SEP_DM_Hessenberg            |
| Intra-node execution environment, 55   | Distributed Memory / Standard EVP, 91 |
| starneig_node_set_cores  | starneig_SEP_DM_Hessenberg_expert     |
| Intra-node execution environment, 55   | Distributed Memory / Standard EVP, 94 |
|  | STARNEIG SEP DM REDUCE                |
| starneig_node_set_gpus   | Library configuration, 46             |
| Intra-node execution environment, 56   | starneig_SEP_DM_Reduce                |
| STARNEIG_NOT_INITIALIZED   | Distributed Memory / Standard EVP, 92 |
| Error codes, 49  |                                       |
| STARNEIG_ORDER_COL_MAJOR   | starneig_SEP_DM_ReorderSchur          |
| Distributed Memory / Distributed matrices, 79  | Distributed Memory / Standard EVP, 92 |
| STARNEIG_ORDER_DEFAULT   | starneig_SEP_DM_ReorderSchur_expert   |
| Distributed Memory / Distributed matrices, 79  | Distributed Memory / Standard EVP, 95 |
| STARNEIG_ORDER_ROW_MAJOR   | starneig_SEP_DM_Schur                 |
| Distributed Memory / Distributed matrices, 79  | Distributed Memory / Standard EVP, 91 |
| STARNEIG_PARTIAL_REORDERING  | starneig_SEP_DM_Schur_expert          |
| Error codes, 49  | Distributed Memory / Standard EVP, 95 |
| STARNEIG REAL DOUBLE   | starneig_SEP_DM_Select                |
| Distributed Memory / Distributed matrices, 79  | Distributed Memory / Standard EVP, 94 |
| The state of the s | starneig_SEP_SM_Eigenvectors          |
| starneig_reorder_blueprint_t   | Shared Memory / Standard EVP, 61      |
| Expert configuration structures, 115   | starneig_SEP_SM_Eigenvectors_expert   |
| STARNEIG_REORDER_CHAIN_INSERT_A  | Shared Memory / Standard EVP, 65      |
| Expert configuration structures, 115   | starneig_SEP_SM_Hessenberg            |
| STARNEIG_REORDER_CHAIN_INSERT_B  | Shared Memory / Standard EVP, 58      |
| Expert configuration structures, 115   | starneig_SEP_SM_Hessenberg_expert     |
| STARNEIG_REORDER_CHAIN_INSERT_C  | Shared Memory / Standard EVP, 62      |
| Expert configuration structures, 115   | starneig SEP SM Reduce                |
| STARNEIG_REORDER_CHAIN_INSERT_D  | Shared Memory / Standard EVP, 60      |
| Expert configuration structures, 115   |                                       |
| STARNEIG REORDER CHAIN INSERT E  | starneig_SEP_SM_ReorderSchur          |
| Expert configuration structures, 115   | Shared Memory / Standard EVP, 59      |
| STARNEIG REORDER CHAIN INSERT F  | starneig_SEP_SM_ReorderSchur_expert   |
| Expert configuration structures, 116   | Shared Memory / Standard EVP, 64      |
| starneig_reorder_conf, 110   | starneig_SEP_SM_Schur                 |
| STARNEIG_REORDER_DEFAULT_BLUEPRINT   | Shared Memory / Standard EVP, 58      |
| Expert configuration structures, 115   | starneig_SEP_SM_Schur_expert          |
| •  | Shared Memory / Standard EVP, 63      |
| STARNEIG_REORDER_DEFAULT_PLAN  | starneig_SEP_SM_Select                |
| Expert configuration structures, 114   | Shared Memory / Standard EVP, 61      |
| STARNEIG_REORDER_DUMMY_INSERT_A  | STARNEIG_SUCCESS                      |
| Expert configuration structures, 115   | Error codes, 48                       |
| STARNEIG_REORDER_DUMMY_INSERT_B  | STARNEIG_USE_ALL                      |
| Expert configuration structures, 115   | Intra-node execution environment, 52  |
| starneig_reorder_init_conf   | -, -                                  |