

1.B

1. $-v$ is the additive inverse of v , so $v + (-v) = 0$. Add the additive inverse of $-v$ to both side, we get $v + (-v) + (-(-v)) = -(-v)$. So $v + 0 = -(-v)$. Because of additive identity, we get $v = -(-v)$.
2. $av = 0 = av + (-av)$ (additive inverse). Add av to both sides we get $av + av = av$, i.e., $2av = av$. If $v \neq 0$ then $2a = a$, so $a = 0$. Else $v = 0$. So $a = 0$ or $v = 0$.
3. Suppose the answer is not unique, set them to x and x' , then $3x = w - v$ and $3x' = x - v$. So $3x = 3x'$, then $3x - 3x' = 0$, then $3(x - x') = 0$ (distributive properties). According to q.2, $x - x' = 0$, so $x = x'$.
4. Additive identity

1.C

- 1(a)
Additive identity: $0 = (0, 0, 0)$, $0 + 2 \cdot 0 + 3 \cdot 0 = 0$, so $0 \in U$.
Closed under addition: For $u, w \in U$, let $u = (x_1^u, x_2^u, x_3^u)$, $w = (x_1^w, x_2^w, x_3^w)$, then $u + w = (x_1^u + x_1^w, x_2^u + x_2^w, x_3^u + x_3^w)$.
 $(x_1^u + x_1^w) + 2(x_2^u + x_2^w) + 3(x_3^u + x_3^w) = (x_1^u + 2x_2^u + 3x_3^u) + (x_1^w + 2x_2^w + 3x_3^w) = 0 + 0 = 0$, so $u + w \in U$.
Closed under scalar multiplication: Let $u = (x_1, x_2, x_3)$, then $au = (ax_1, ax_2, ax_3)$.
 $ax_1 + ax_2 + ax_3 = a(x_1 + x_2 + x_3) = a \cdot 0 = 0$.
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Additive identity: $0 \in U_1$ and $0 \in U_2$, so $0 \in U_1 \cap U_2$.
Closed under addition: $u, w \in U_1 \cap U_2$, then $u, w \in U_1$ and $u, w \in U_2$, then $u + w \in U_1$ and $u + w \in U_2$, so $u + w \in U_1 \cap U_2$.
Closed under scalar multiplication: $au \in U_1$ and $au \in U_2$, so $au \in U_1 \cap U_2$.
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Yes. $U + W = \{u + w : u \in U, w \in W\}$, $W + U = \{w + u : w \in W, u \in U\}$. So $U + W = W + U$.
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Yes, $\{0\}$ is additive identity. Only $\{0\}$ has additive inverse.