The effect of heterogeneity on hypergraph contagion models

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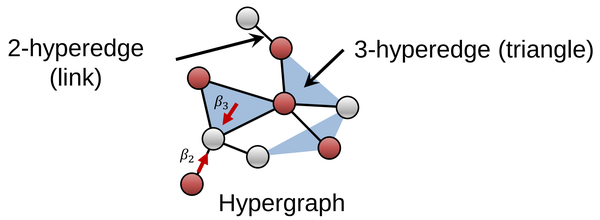
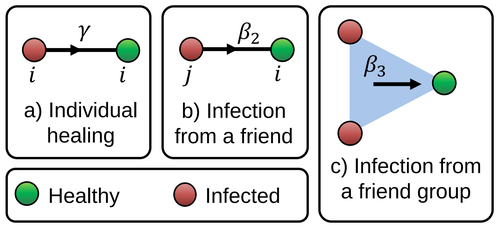
Background

GOAL: Developing more realistic models of social and viral contagion by not only including pairwise (i.e., person-to-person) interactions, but higher-order interactions as well (i.e., group interactions). We use the generalization of a network known as a hypergraph.



We used a modified SIS model to include higher order interactions. The mechanisms of contagion for a node are



The network-based SIS model is simply a) and b).

Notation

- N nodes with degrees k
- x_k is the fraction of infected nodes with degree k
- $P(k)$ is the number of nodes with degree k
- The infected population average is

$$\langle I \rangle = \sum_k \frac{P(k)x_k}{N}$$

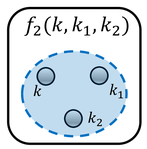
- β_2^c is the epidemic threshold, above which epidemics occur
- β_3^c is the threshold at which explosive transitions to appear

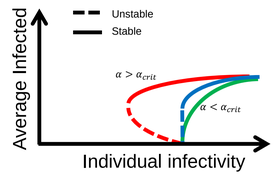
Mean-Field Theory Continued

The mean-field equation is

$$\frac{dx_k}{dt} = \underbrace{-\gamma x_k}_{\substack{\text{Rate of} \\ \text{healing}}} + \underbrace{\beta_2(1-x_k)\sum_{k_1} P(k_1) f_2(k,k_1) x_{k_1}}_{\substack{\text{Rate of infection} \\ \text{by infected neighbors}}} + \underbrace{\frac{\beta_3}{2}(1-x_k)\sum_{k_1,k_2} P(k_1)P(k_2) f_3(k,k_1,k_2) x_{k_1} x_{k_2}}_{\substack{\text{Rate of infection by infected simplices of which the node is a member}}}$$

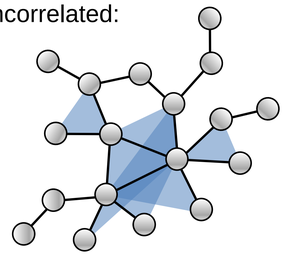
$\frac{dx_k}{dt}$: Rate of Infection of a node of degree k



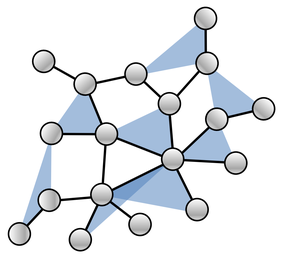


We used the mean-field equation to find the theoretical onset of bistability, where two stable equilibria can exist, which we denote β_3^c. If the system crosses over the unstable branch (also called a "critical mass"), the system explosively transitions to an epidemic equilibrium; below it, the infection dies out.

We used the undirected configuration model for the pairwise network and two ways of choosing the 3-hyperedge connections, degree-correlated and uncorrelated:

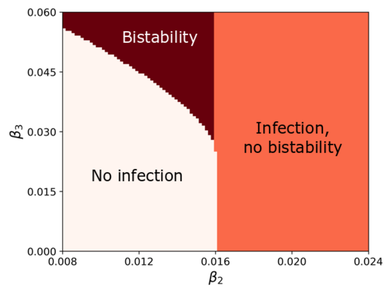


Degree-correlated

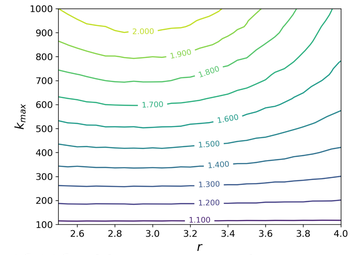


Uncorrelated

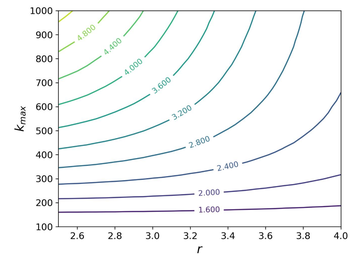
Phase diagram as a function of β_2 and β_3, found by numerically solving for the equilibria of the mean-field equation



Mean-Field Simulation

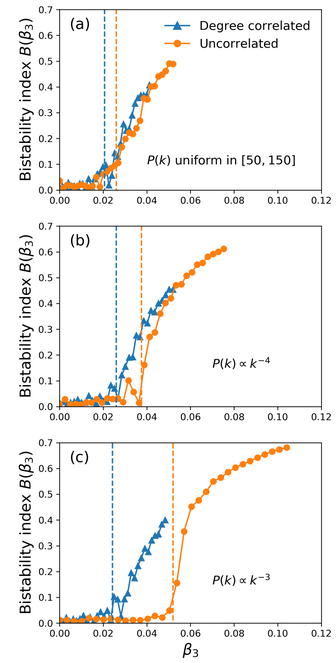


β_3^c/β_2^c with respect to the power-law exponent, r, and the maximum degree in the uncorrelated case



β_3^c/β_2^c with respect to the power-law exponent, r, and the maximum degree in the degree-correlated case

Discrete-Time Markov Chain Simulation



Measure of bistability for different values of β_3 and different networks

Conclusions

- Heterogeneous networks suppress explosive transitions
- Triangles uncorrelated with the network degree suppress explosive transitions

References

- Iacopini, I., Petri, G., Barrat, A. *et al*. Simplicial models of social contagion. *Nat Commun* 10, 2485 (2019). doi:10.1038/s41467-019-10431-6
- de Arruda, Guilherme Ferraz, Giovanni Petri, and Yamir Moreno. Social contagion models on hypergraphs. *Physical Review Research* 2.2 (2020): 023032.
- Cisneros-Velarde, Pedro, and Francesco Bullo. Multi-group SIS Epidemics with Simplicial and Higher-Order Interactions. *arXiv preprint arXiv:2005.11404* (2020).

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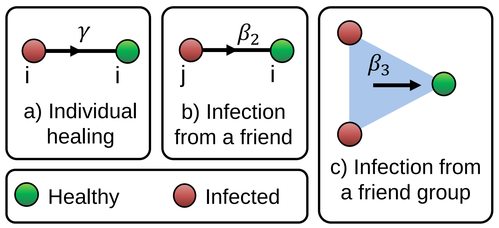
γ
i
i
a) Individual healing
β_2
j
i
b) Infection from a friend
β_3
c) Infection from a friend group
Healthy
Infected