hw05

October 26, 2021

1 BME 6310 Homework 05

Nathan LeRoy

1.1 Problem 1

1.1.1 $w(t) = v(t)\cos(\omega t)$

Linearity:

$\mathscr{S}[av_1(t) + bv_2(t)] = [av_1(t) + bv_2(t)]\cos(\omega t)$

$\mathscr{S}[av_1(t)] + \mathscr{S}[bv_2(t)] = av_1(t)\cos(\omega t) + bv_2(t)\cos(\omega t) = [av_1(t) + bv_2(t)]\cos(\omega t)$

*These are equvalent and thus, the system **is linear**.*

Shift Invariance:

$w(t + t_0) = v(t + t_0)\cos(\omega(t + t_0))$

let: $r(t) = v(t + t_0)$

$\mathscr{S}[r(t)] = r(t)\cos(\omega(t)) = v(t + t_0)\cos(\omega t)$

*These are **not** equivalent, this the system is **not shift invariant**.*

1.1.2 $w(t) = \sin(v(t))$

Linearity:

$\mathscr{S}[av_1(t) + bv_2(t)] = \sin(av_1(t) + bv_2(t))$

$\mathscr{S}[av_1(t)] + \mathscr{S}[bv_2(t)] = \sin(av_1(t)) + \sin(bv_2(t))$

*These are **not** equivalent and thus the system is **not linear**.*

Shift invariance:

$w(t + t_0) = \sin(v(t + t_0))$

let: $r(t) = v(t + t_0)$

$\mathscr{S}[r(t)] = \sin(r(t)) = \sin(v(t + t_0))$

*These are equivalent and thus, the system is **shift invariant**.*

1.1.3 $w(t) = \frac{d}{dt}v(t)$

Linearity:

$\mathscr{S}[av_1(t) + bv_2(t)] = \frac{d}{dt}(av_1(t) + bv_2(t)) = a\frac{dv_1}{dt} + b\frac{dv_2}{dt}$

$\mathscr{S}[av_1(t)] + \mathscr{S}[bv_2(t)] = \frac{d}{dt}av_1(t) + \frac{d}{dt}bv_2(t) = a\frac{dv_1}{dt} + b\frac{dv_2}{dt}$

These are equivalent and thus the system ***is linear****.*

Shift invariance:

$w(t + t_0) = \frac{d}{dt}v(t + t_0)$

let: $r(t) = v(t + t_0)$

$\mathscr{S}[r(t)] = \frac{d}{dt}r(t) = \frac{dr}{dt} = \frac{d}{dt}v(t + t_0)$

These are equivalent and this, the system ***is shift invariant****.*

1.1.4 $w(t) = \cos(\omega t + v(t))$

Linearity:

$\mathscr{S}[av_1(t) + bv_2(t)] = \cos(\omega t + av_1(t) + bv_2(t))$

$\mathscr{S}[av_1(t)] + \mathscr{S}[bv_2(t)] = \cos(\omega t + av_1(t)) + \cos(\omega t + v_2(t))$

These are ***not equivalent*** *and thus, the system is* ***not linear****.*

Shift invariance:

$w(t + t_0) = \cos(\omega(t + t_0) + v(t + t_0))$

let: $r(t) = v(t + t_0)$

$\mathscr{S}[r(t)] = \cos(\omega t + r(t)) = \cos(\omega t + v(t + t_0))$

These are ***not equivalent*** *and thus, the system is* ***not shift invariant****.*

1.2 Problem 2

Import the systems from the `.pyc` file:

[52]:
```python
# better image quality
import matplotlib as mpl
%matplotlib inline
mpl.rcParams['figure.dpi'] = 300
```

[53]:
```python
import numpy as np
from systems_hw5 import *
import matplotlib.pyplot as plt
```

Lets create a function to display some plots for us that can generate an "eye test" for linearity:

[54]:

```python
from typing import Callable
def test_linearity(system: Callable) -> None:
    """
    Function to display plots that will test for
    a system's linearity. Given a system, this
    function will plot 3 plots:
        1.) original system
        2.) plot with homogeneity test
        3.) plot with additivity test
    """

    _, ax = plt.subplots(1,3, figsize=(20,6), sharex=False, sharey=True)

    # investigate system
    v = np.zeros(41)
    v[20] = 1
    w = system(v)
    ax[0].stem(
        w,
        linefmt='b-',
        markerfmt="bo",
        basefmt="b-",
    )
    ax[0].set_title("$v(t)$")
    ax[0].set_xlabel("Time")
    ax[0].set_ylabel("Output")

    # test homogeneity
    ax[1].stem(system(2*v),
        linefmt='b-',
        markerfmt="bo",
        basefmt="b-",
        label="$\mathscr{S}(2v(t))$"
    )
    ax[1].stem(2*system(v),
        linefmt='r-',
        markerfmt="ro",
        basefmt="r-",
        label="$2\mathscr{S}(v(t))$"
    )
    ax[1].set_title("Homogeneity")
    ax[1].set_xlabel("Time")
    ax[1].legend(loc='upper right', ncol=1)

    # test additivity
    v2 = np.zeros(41)
    v2[20] = 2
```

```python
    ax[2].stem(system(v + v2),
        linefmt='b-',
        markerfmt="bo",
        basefmt="b-",
        label="$\mathscr{S}(v_{1}(t) + v_{2}(t))$"
    )
    ax[2].stem(system(v) + system(v2),
        linefmt='r-',
        markerfmt="ro",
        basefmt="r-",
        label="$\mathscr{S}(v_{1}(t)) + \mathscr{S}(v_{2}(t))$"
    )
    ax[2].set_title("Additivity")
    ax[2].set_xlabel("Time")
    ax[2].legend(loc='upper right', ncol=1)
```

Similarly, we can define a function that will test for shift invariance of a system:

[55]:
```python
def test_shift_invariance(system: Callable) -> None:
    """
    A simple function to test for a system's
    shift invariance by generating two plots:
        1.) Original system
        2.) system shifted over
    """
    plt.rcParams["figure.figsize"] = (6,3)
    v = np.zeros(41)
    v[20] = 1
    w = system(v)

    v_shift = np.zeros(41)
    v_shift[25] = 1
    w_shift = system(v_shift)

    plt.stem(
        w,
        linefmt='b-',
        markerfmt="bo",
        basefmt="b-",
        label="$\mathscr{S}(v(t))$",
    )
    plt.stem(
        w_shift,
        linefmt='r-',
        markerfmt="ro",
        basefmt="r-",
        label="$\mathscr{S}(v(t-t_{0}))$"
```

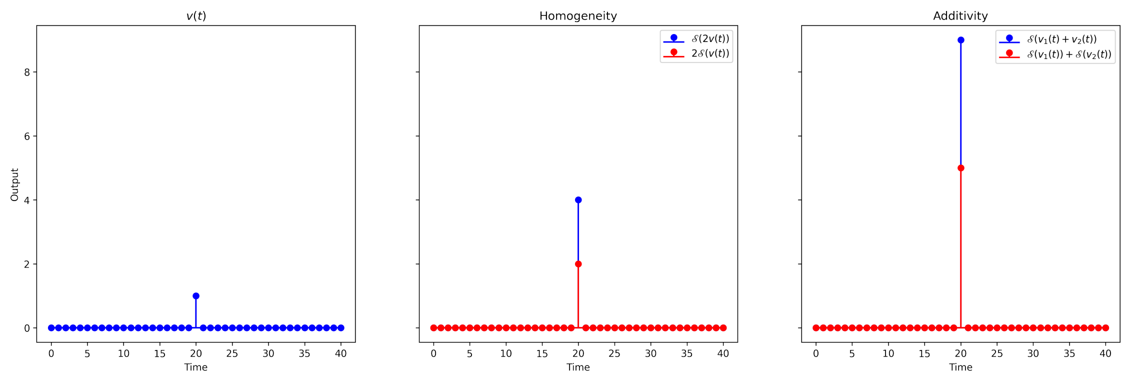
```
    )
    plt.xlabel("Time")
    plt.ylabel("Output")
    plt.legend()
```

Lets test our systems.

1.2.1 SystemA

Linearity:

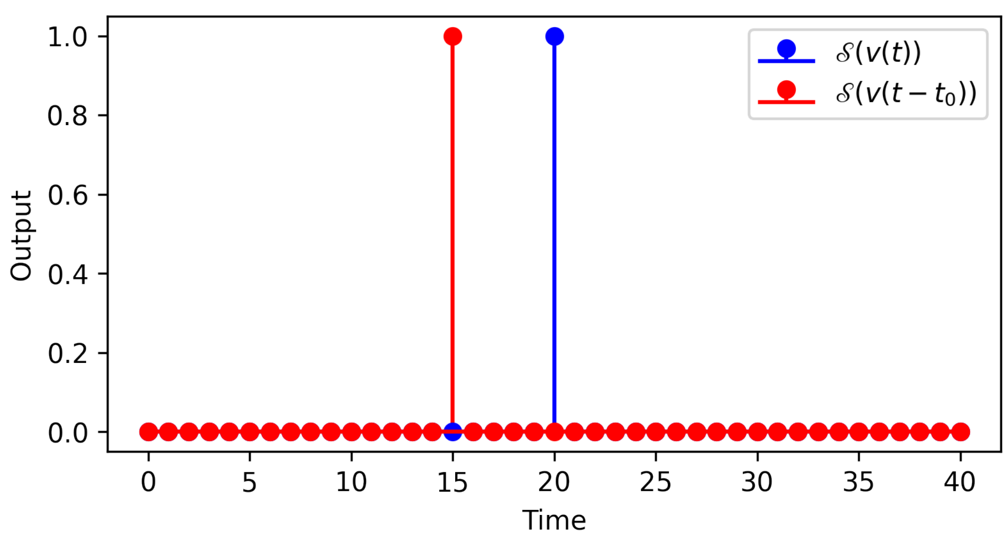
[56]:
```
test_linearity(systemA)
```



We can see that our system produces **two different** outputs based on the order in which inputs are either added or scaled. From this. We can infer that the system is **not linear**.

Shift invariance:

[57]:
```
test_shift_invariance(systemA)
```

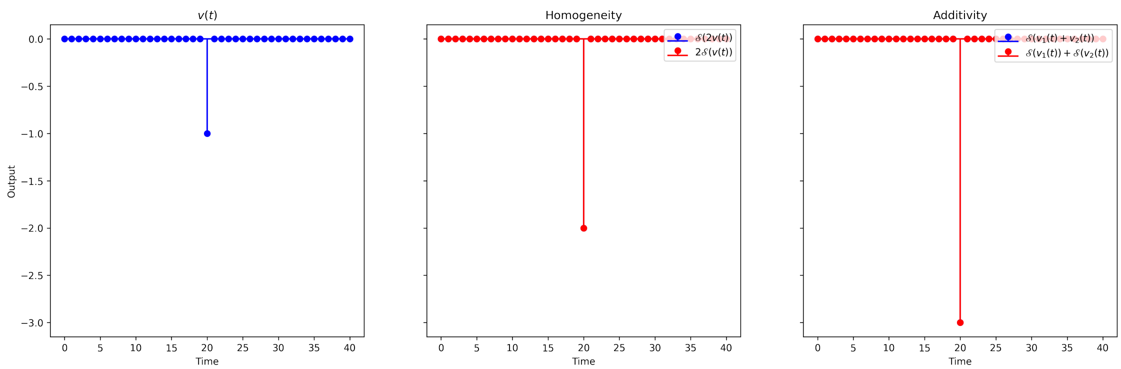


From the original and shift profile's, we can infer that the system is **shift invariant**.

1.2.2 SystemB

Linearity:

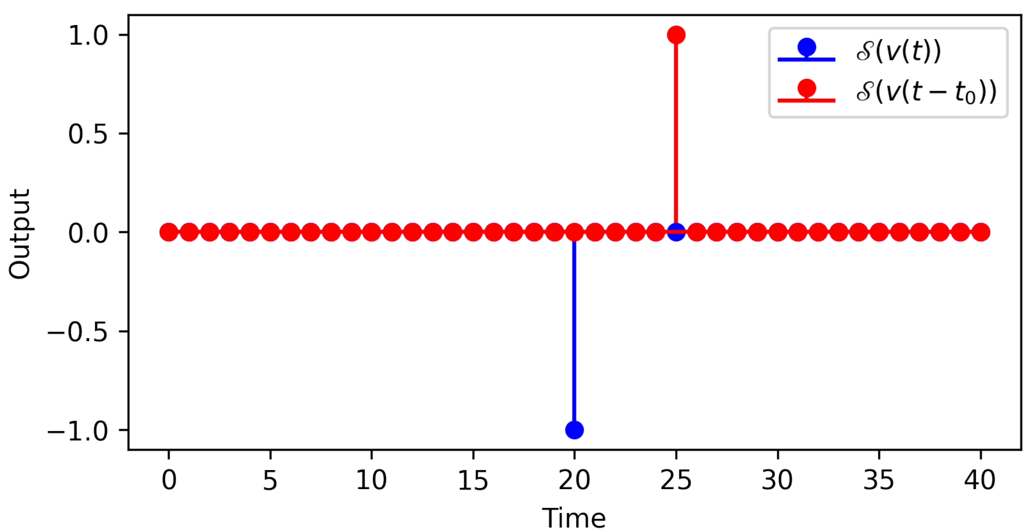
[58]:
```
test_linearity(systemB)
```



We can see that the plots perfectly overlap for but the homogeneity and additivity tests. Thus, our system can be inferred as **linear**.

Shift invariance:

[59]:
```
test_shift_invariance(systemB)
```

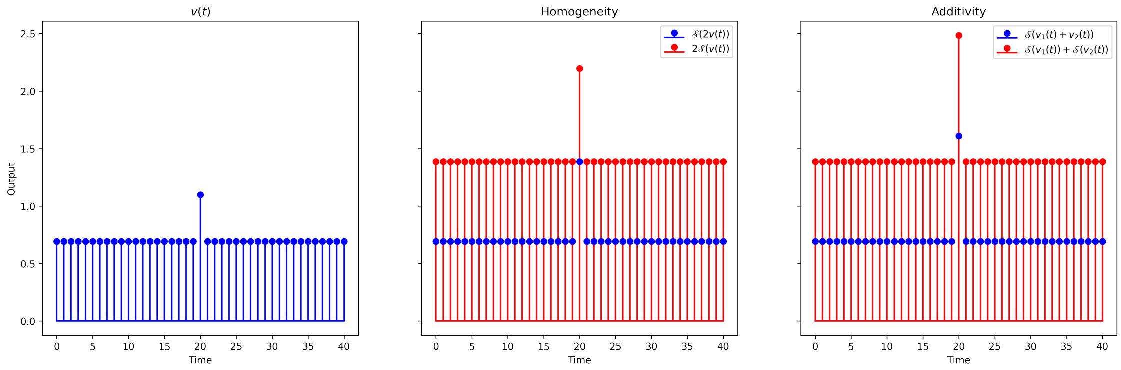


Based on the original and shifted profiles, it would appear that the system is **not shift invariant:**

1.2.3 SystemC

Linearity:

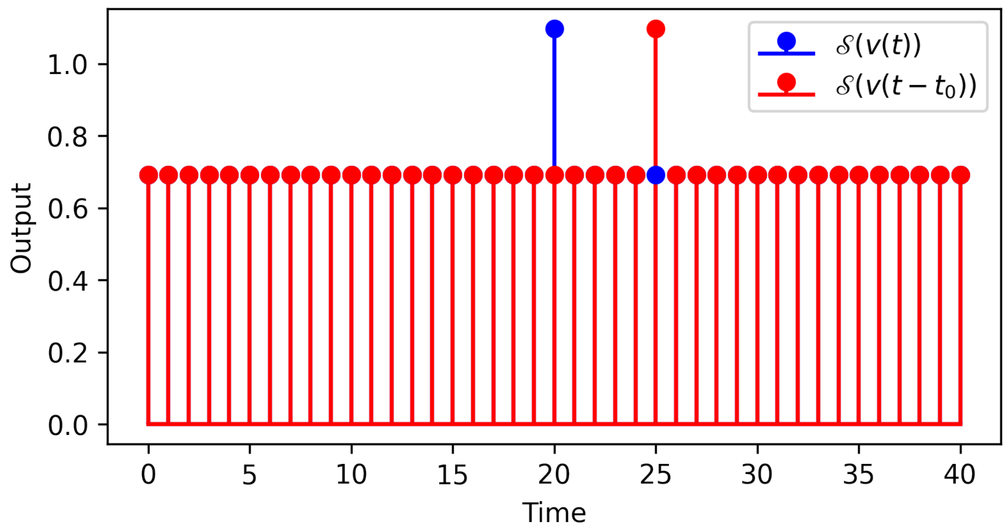
[60]:
```
test_linearity(systemC)
```



We can see that for the system both homogenity and additivity tests fail. Thus, the system can beinferred as **non linear**.

Shift invariance:

[61]:
```
test_shift_invariance(systemC)
```

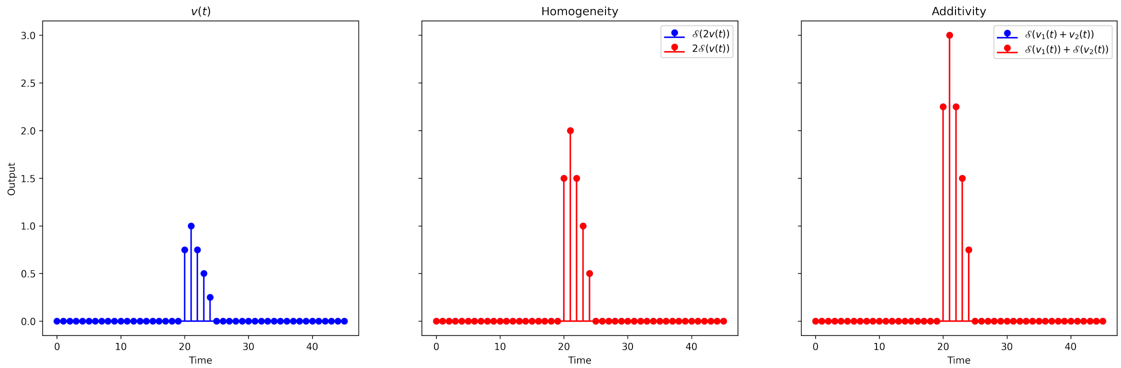


Based on the original and shifted profiles, we can infer that the system is **shift invariant**.

1.2.4 SystemD

Linearity:

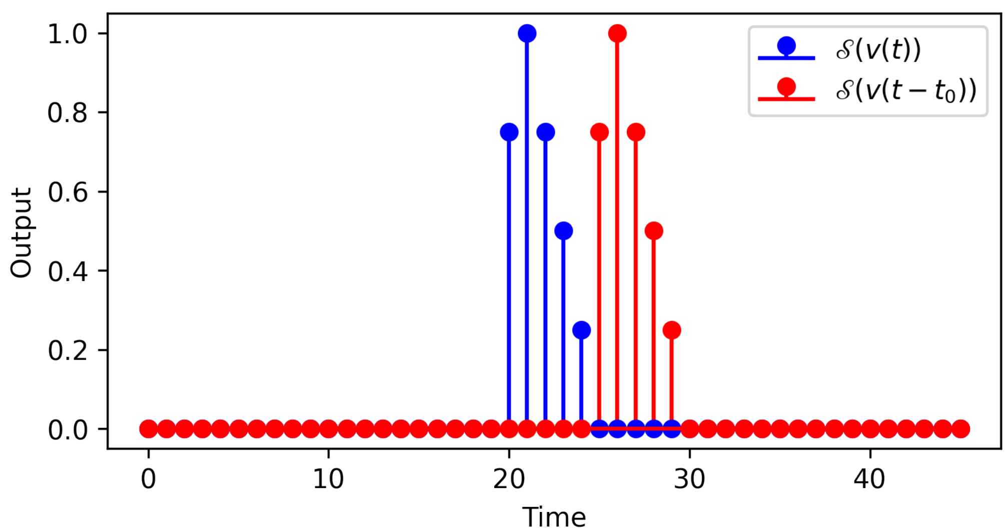
[62]:
```
test_linearity(systemD)
```



From the plots, we can see the system satisfies both the homogeneiety and addivity requirements. Our system **is linear**.

Shift invariance:

[63]:
```
test_shift_invariance(systemD)
```



Again, by looking at the original and shifted plots, we can see that the system is **shift invariant**.

1.3 Problem 3

We can assume that any function can be broken up into **even and odd** parts like so:

$f(x) = E(x) + O(x)$

where,

$E(x) = \dfrac{f(x) + f(-x)}{2}$ and $O(x) = \dfrac{f(x) - f(-x)}{2}$

1.3.1 a.) $g(x) = (x^3 + x^2)\cos(\pi x)$

$g_e(x) = \dfrac{g(x) + g(-x)}{2} = \dfrac{(x^2 - x^3)\cos(\pi x) + (x^2 - x^3 cos(-\pi x)}{2}$

We know that $\cos(-x) = \cos(x)$. As such the top equation can be drastically simplified:

$g_e(x) = \dfrac{x^2\cos(\pi x) - x^3\cos(\pi x) + x^2\cos(\pi x) + x^3\cos(\pi x)}{2}$

which simplifies to:

$g_e(x) = x^2\cos(\pi x)^{**}$.

Similarly, we can calculate the odd part of $g(x)$:

$g_o(x) = \dfrac{g(x) - g(-x)}{2} = \dfrac{(x^3 + x^2)\cos(\pi x) - (x^2 - x^3)\cos(-\pi x)}{2}$

We know that $\cos(-x) = \cos(x)$. As such the top equation can be drastically simplified:

$g_o(x) = \dfrac{x^3\cos(\pi x) + x^2\cos(\pi x) - x^2\cos(\pi x) + x^3\cos(\pi x)}{2}$

$g_e(x) = \dfrac{x^3\cos(\pi x) + x^3\cos(\pi x)}{2}$

$g_e(x) = x^3\cos(\pi x)$

1.3.2 b.) Piecewise

From the graph we can see that our function is the following piecewise function:

$$f(x) = \begin{cases} -x-1 & -1 \le x \le 0 \\ x & 0 \le x \le 1 \end{cases}$$

We can test each section of the piecewise function for both even an odd parts using the following criteria:

1.3.3 Even functions:

$f(x) = f(-x)$

For the first section...

$f(x) = -x - 1$

$f(-x) = x - 1$

These are not the same – they are not even.

For the second section...

$f(x) = x$

$f(-x) = -x$

Neither of these sections are even. The function has **no even parts**.

1.3.4 Odd functions:

$f(-x) = -f(x)$

For the first section:

$f(-x) = x - 1$

$-f(x) = -(-x - 1) = x + 1$

For the second section:

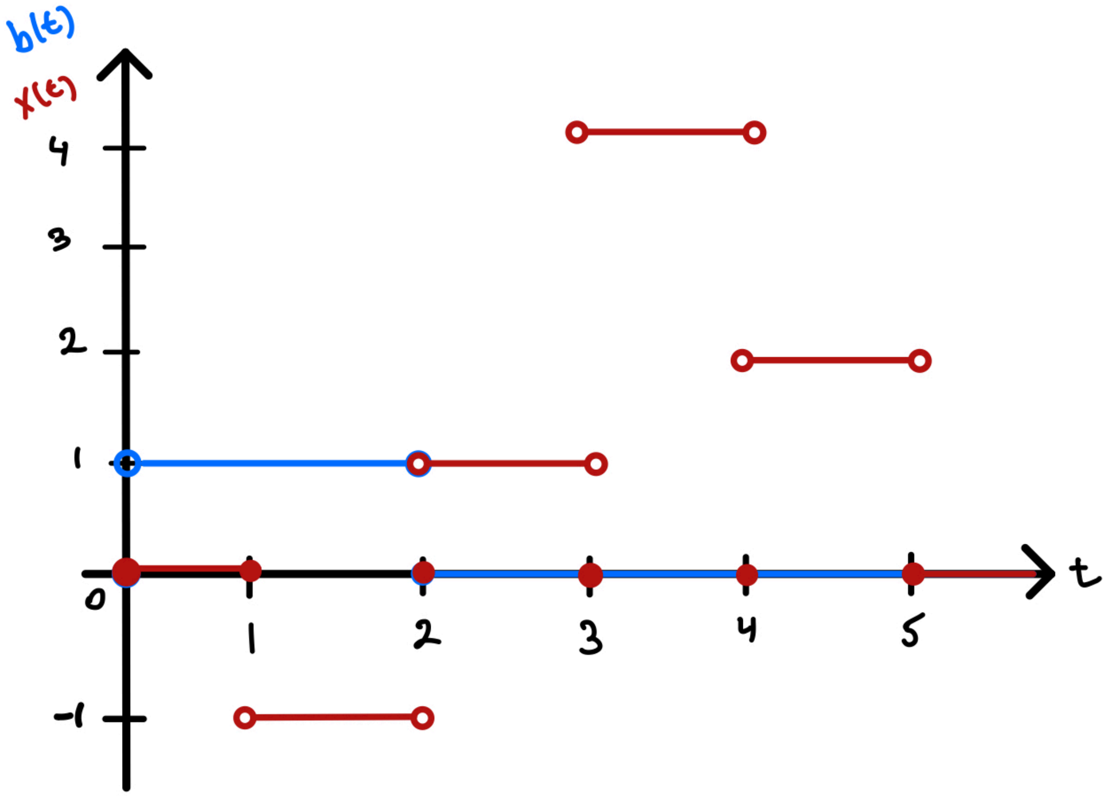
$f(-x) = -x$

$-f(x) = -x$

These are identical. The first section is not. Thus, the function is **odd only on the interval** $[0, 1]$.

1.4 Problem 5

Here is the sketched $b(t)$ and $x(t)$:



We can break down $x(t)$ into a linear combination of $b(t)$ that takes on the form:

$x(t) = a_1 b(t - s_1) + a_2 b(t - s_2) + a_3 b(t - s_3)$

We can combine the following 3 transformations of $b(t)$ to acheive this:

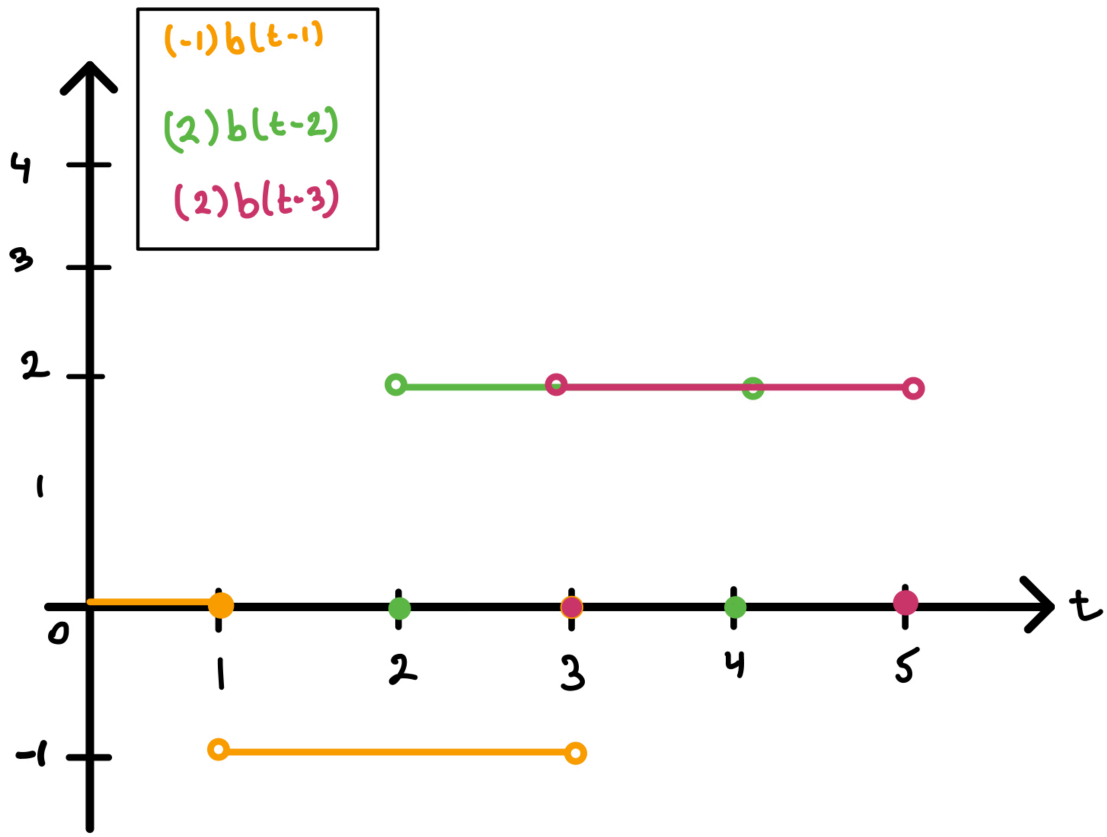
$(-1)b(t - 1)$,

$2b(t - 2)$,

$2b(t - 3)$

$x(t) = (-1)b(t - 1) + 2b(t - 2) + 2b(t - 3)$

We can sketch these 3 components here:


(-1)b(t-1)
(2)b(t-2)
(2)b(t-3)
4
3
2
1
0
-1
1
2
3
4
5
t