Optimization Homework

Problem 9.1:

We have costs for steam, maintenance, and effects. Our total cost is the following equation:

$$Total\ Cost\ =C_s+C_m+C_e$$

Annual steam costs can be calculated with the following equation:

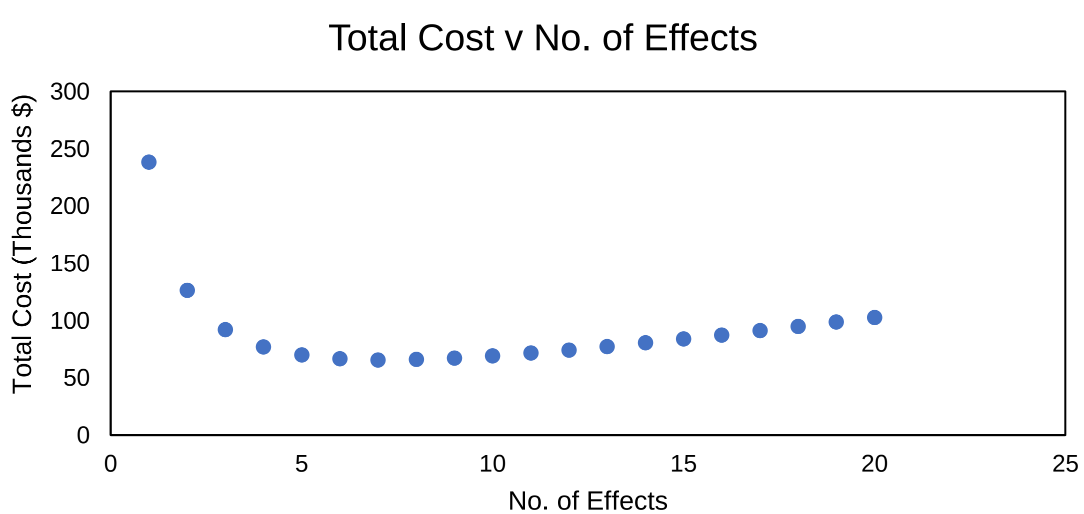
$$Cost\ of\ steam\ =\$0.0033/kg\ steam\ x\ R(kg\ steam/kg\ H2O)\ x\ 200{,}000kg\ H2O/day\ x\ 300\ days/year$$

The conversion of steam to evaporated water, R, is quantified by the following equation:

$1/R\ =0.85(n)$ where n is the number of effects.

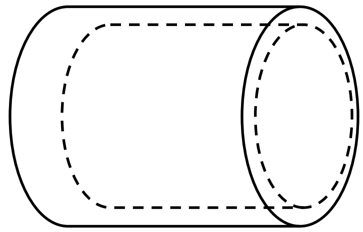
Using the given equations, we can create a table that shows the total cost v number of effects. In addition, it can be graphed, and we can optimize the number of effects:

no. effects	Cost of Effects	kg H20/kg Steam	Annual Steam Costs	Maintenance	Fixed-Depr.	Total Costs
1	18,000	0.85	232941.18	900	2700	238341.18
2	33,000	1.7	116470.59	1650	4950	126370.59
3	48,000	2.55	77647.06	2400	7200	92047.06
4	63,000	3.4	58235.29	3150	9450	77135.29
5	78,000	4.25	46588.24	3900	11700	69988.24
6	93,000	5.1	38823.53	4650	13950	66723.53
7	108,000	5.95	33277.31	5400	16200	65677.31
8	123,000	6.8	29117.65	6150	18450	66017.65
9	138,000	7.65	25882.35	6900	20700	67282.35
10	153,000	8.5	23294.12	7650	22950	69194.12
11	168,000	9.35	21176.47	8400	25200	71576.47



Problem 9.2:

We have a pipe which is losing heat through a layer of insulation:



We can quantify the steady state heat loss with the following equation:

$$q = \frac{T_{in} - T_\infty}{\frac{1}{2\pi h r_1 L} + \frac{ln(r_2/r_1)}{2\pi k L}}$$

We can correlate the heat loss with a cost through a simple parameter to get the cost of heat loss C_h.

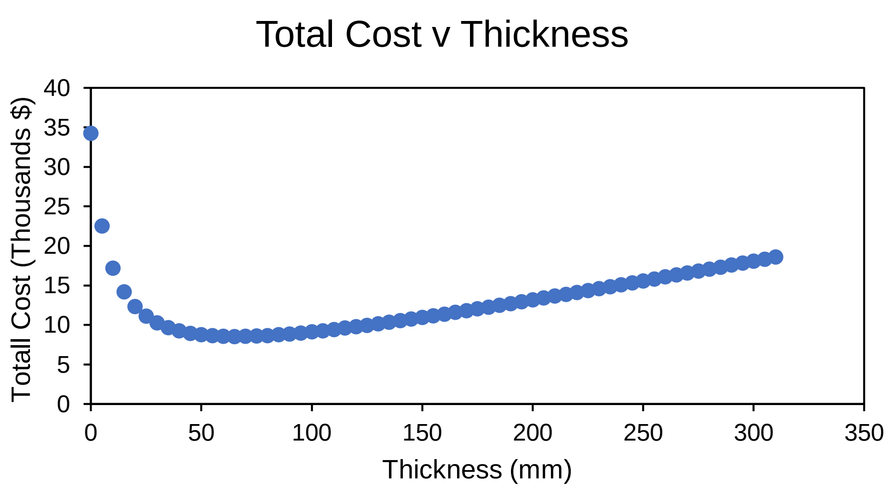
We then know the cost of insulation. It is given by the following equation:

$$C_i = 180 \text{ x } 300 \text{ x } l_t$$

The total cost equation is:

$$Total\ Cost = \frac{34220}{1 + 14.9625 ln(lt + 0.1365/0.1365)} + 180\ x\ 300\ x\ lt$$

We can sum up the total cost and plot them to find the optimal insulation thickness:



The optimal thickness was found to be 65mm.

Problem 9.4:

This is very similar to problem 9.2 in a sense that we are trying to balance the cost of buying insulation v the costs saved from lack of heat loss.

$$q = \frac{T_{in} - T_\infty}{\frac{1}{2\pi h r_1 L} + \frac{ln(^{r_2}/_{r_1})}{2\pi k L}}$$

Say we can correlate the thickness of the of the insulation to the cost by the following equation:

$$C_i = \beta L l_t$$

In addition, say we can correlate the cost due to heat loss with a parameter α:

$$C_h = \alpha q$$

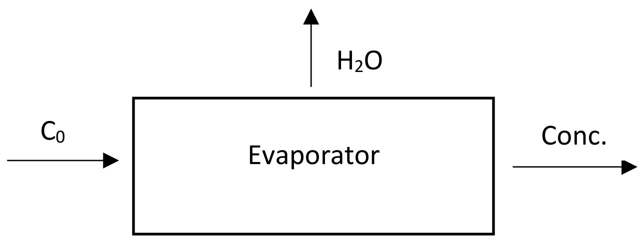
The economic cost of the insulation is the point where the two costs are equal. $C_i = C_h$

$$\alpha q = \beta L l_t$$

We can substitute in q and solve for l_t as such:

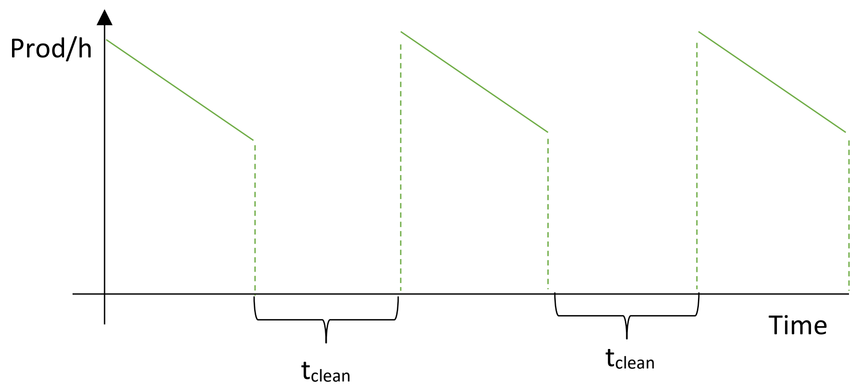
$$l_t = \sqrt{\frac{kA\beta}{\alpha}(T_{in} - T_\infty)}$$

Problem 9.5:



It is known that the evaporator decreases in efficiency over time. We can estimate that it is a linear decrease with a slope of 625/12 kg/h/h.

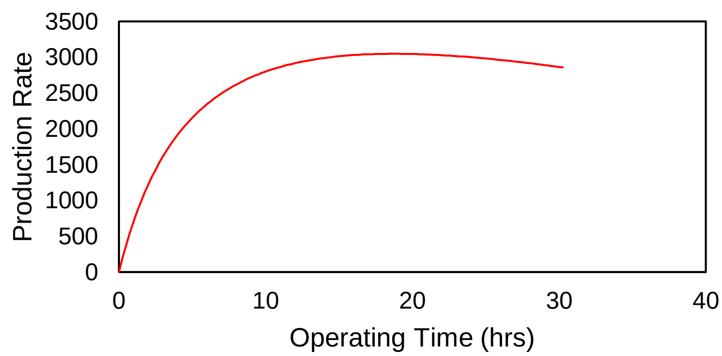
The total time to run a cycle = t_{op} + t_{clean}. We are trying to optimize the operating time, while the cleaning time is a fixed 6 hours. A graph of the production of concentrate over time will look something like this:



We can calculate the total product processed by integrating over the rate equation:

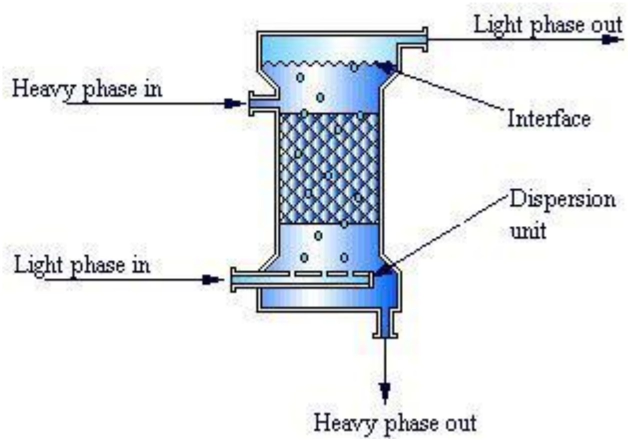
$$Product\ Processed\ = \int_0^{t_{op}} (rate)dt\ = \int_0^{t_{op}} (5000\ - \frac{625}{12}t)dt$$

Including the cleaning time, we can get the total production rate, and this can then be optimized against the operating time.



The max rate of production is found to be 30.25 hours.

Problem 9.6:

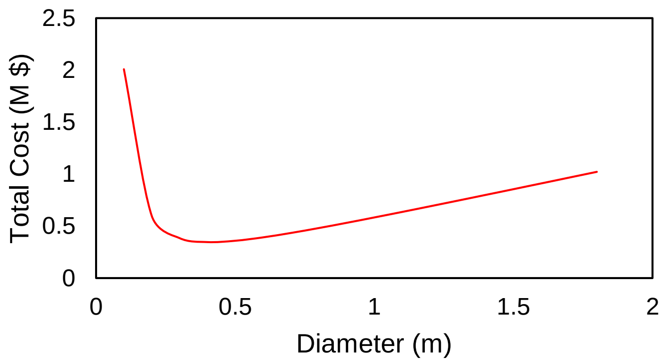


The plate column solvent extractor is operated 24 hr a day, 300 days/year. It is fed at 40 m³/day. The total allowable velocity per cross sectional area is 12.2 m³ of solvent and feed. We can quantify the cost of the equipment based on the following:

$$C = 8800F_{st}^2 - 51{,}000F_{st} + 110{,}000\ \$/yr$$

We also need to consider the cost of the solvent recovery: $ 1.41/m³.

This cost can then be optimized, as we need more solvent to extract more product, but it costs more to recover the solvent as well. A graph of this optimized cost can be shown below. The optimization occurs with the size of the system we create since we can push in more solvent if our cross-sectional area increases. And the velocity needs to be kept constant. So we can optimize the diameter:



The optimized diameter was found to be: 0.4m

Problem 9.9:

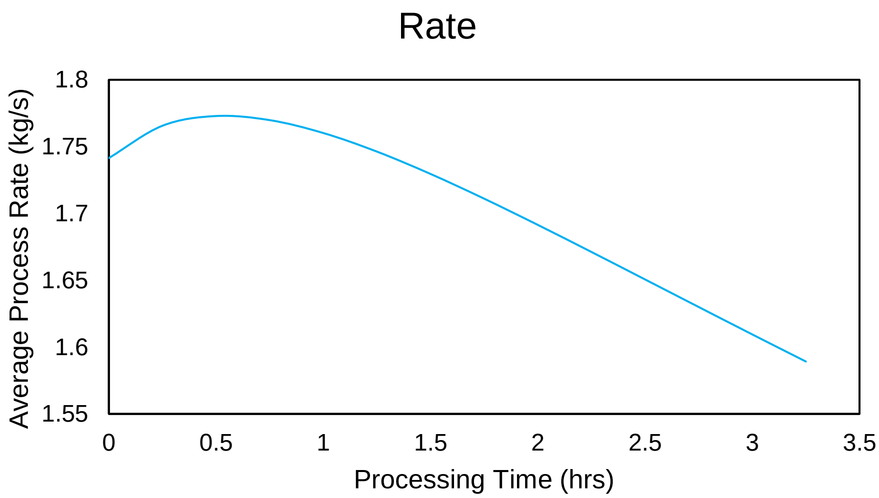
We need to optimize an evaporator process. Similarly, to problem 9.5 we want to find the average rate of production. This time, however, the decrease in efficiency is non-linear. The decrease comes with the decrease in efficiency of the heat exchanger.

The change in overall heat transfer rate is characterized by the following equation:

$$U^{-2} = 6.88e - 5\,\theta_b + 0.186$$

Where, the units of U are in (kg H_2O/s)(1/m^2K). Similar to problem 9.5, we can know that the average rate of water evaporation is the integrated rate over the process time theta$_b$. Divided by the total time in the process including cleaning.

When we optimize the process rate, we find that the optimal time of process, when cleaning takes 4 hours, is found to be about half an hour.



Problem 9.13:

Again, we are trying to balance the process time with the cost of stopping and starting to clean/regenerate. Here, the cost of catalyst regeneration is $800. We are feeding the reactor at 70 kg/day. It costs $5.50 per kg. We are operating 300 days per year and the daily costs are $300 while the annual fixed costs are $100,000. We can sell the product for $31/kg.

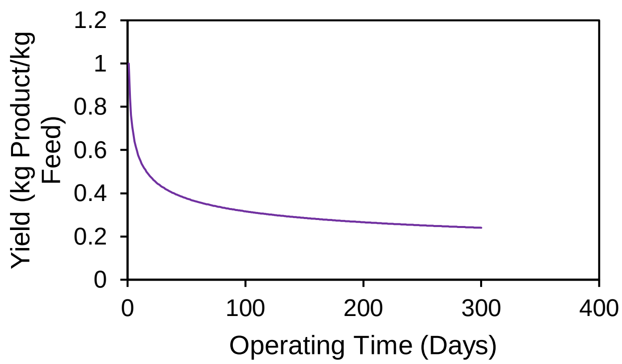
The yield from the catalyst decreases with time, characterized by the following equation:

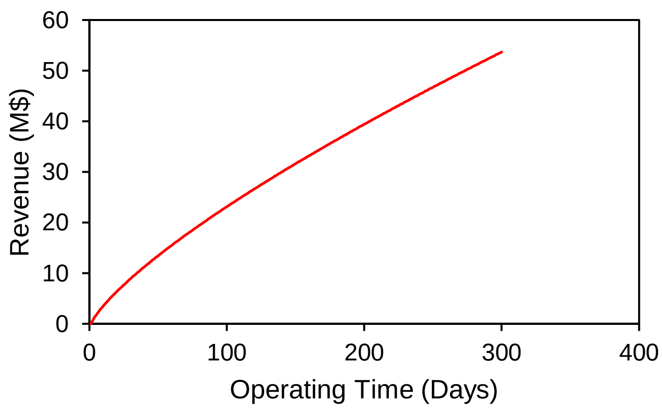
$$Y_p = \frac{0.87}{\theta_D^{0.25}}$$

Where θ_D is the process time in days. We can find the total accumulated yield over the entire process time by integrating over this equation.

$$Total\ Yield = \int_0^t \frac{0.87}{\theta} d\theta$$

We can multiply this equation by the feed rate, we can get the total yield of product per day. Multiplying by the going rate for product, we can get total revenue per day.

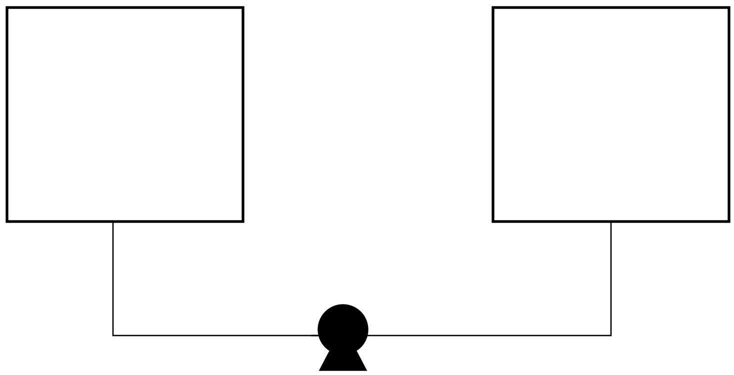




Quickly, we can see that our revenue does not seem to fall off that quickly, in fact, it stays linear for a long period of time (300 days). This is a clear indication that we need not change our catalyst that often. Because of this, it is recommended that the catalyst be changed every 2 years or so (600 days).

Extra Optimization Problem #2:

We must pump a liquid from one tank to another:



We are interested in optimizing the diameter of the piping for our pump system. First, we must calculate the pressure head required to push through 20 kg/s (0.0192 m^3/s, ρ kg/m^3).

This pressure is correlated with the friction factor in our piping system. The friction factor, a dimensionless constant, is related to the generalized Reynolds number, Re_{gen}, via the following equation:

$$f = \frac{16}{\mathrm{Re_{gen}}}$$

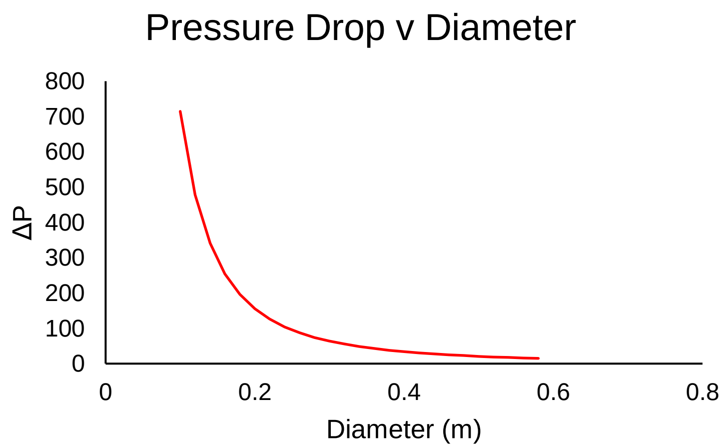
The pressure drop across the pump can be calculated as such:

$$\Delta P = \frac{2\mathrm{fv}^2}{\mathrm{DL}}$$

And the generalized Reynolds number is calculated as such:

$$\mathrm{Re_{gen}} = \frac{\mathrm{D^n v^{2\text{-}n}}}{\mathrm{K'8^{n\text{-}1}}}$$

Using this info, we can see how the pressure drop decreases as the diameter of the pipe increases:

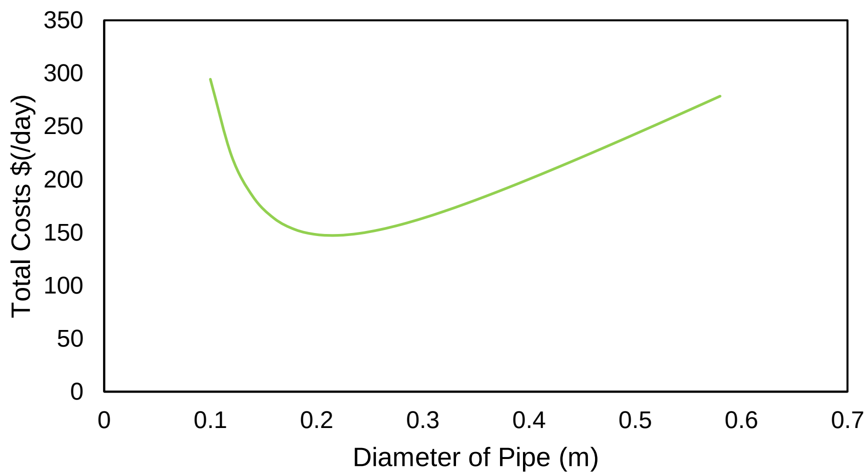


It can be seen that the pressure drop quickly drops off as the diameter increase. This is expected.

Increasing the diameter decreases the pressure drop, and thus the energy requirements of the pump. This can be seen from the following equation:

$$P = \frac{Q\Delta P}{1000\,\eta}$$

However, the cost of the piping material increases with diameter as the amount of material required will increase. The cost of the piping material is directly proportional to the surface area of the pipe. This cost can be optimized after graphing:



The optimized pipe size was found to be about 0.22 m in diameter