Homework 01

Problem 13.2-1

<u>Given:</u>

C_1 = 2.0e-2 kg mol A/m^3 K' = 0.75

C_2 = 0.3e-2 kg mol A/m^3 D_{AB} = 3.5e-11 m^2/s

K_{c1} = 3.5e-5 m/s k_{c2} = 2.1e-5 m/s

L = 1.59e-5 m

<u>Find:</u>

R_1, R_2, R_m, R_{tot}, $R_{\%1}$, $R_{\%2}$, Membrane Area

<u>Solution:</u>

a.)

$$R_1 = \frac{1}{k_{c1}} = \frac{1}{3.5e-5\ m/s} = 28{,}571.4\ s/m$$

$$R_2 = \frac{1}{k_{c2}} = \frac{1}{2.1e-5\ m/s} = 47{,}619\ s/m$$

$$p_m = \frac{D_{AB}K'}{L} = \frac{(3.5e-11 m^2/s)(0.75)}{1.59e-5m} = 2.0e-6$$

$$R_m = \frac{1}{p_m} = \frac{1}{2.0e-6\ m/s} = 605{,}714\ s/m$$

$$R_{tot} = \Sigma R_i = R_1 + R_2 + R_m = 28{,}571.4\ s/m + 47{,}619\ s/m + 605{,}714\ s/m = 681{,}904\ s/m$$

$$\%Resistance of Films = \frac{R_1 + R_2}{R_{tot}} x100 = \frac{28{,}571.4 + 47{,}619}{681{,}904} x100 = 11.17\ \%$$

b.)

$$N_{ss} = \frac{c_1 - c_2}{R_{tot}} = \frac{2.0e-2\ kgmol/s-m^3 - 0.3e-2\ kgmol/s-m^3}{681{,}904\ s/m} = 2.49e-8 \text{ kgmol A/s-m}^2$$

Area = 0.01 kgmol/h x 1 hr/60 min x 1 min/60 s x 1/2.49e-8 kgmol A/s-m^2 = 111.557 m^2

Problem 12.2-2

<u>Given:</u>

L = 0.029 mm

C_1 = 1.0e-4 g mol/cm^3 = 100 g mol/m^3

C_2 = 5.0e-7 g mol/cm^3 = 0.5 g mol/m^3

$K_{c1} = k_{c2}$ = 5.24e-5 m/s

N_A = 8.11e-4 gmol NaCl/s-m^2

<u>Find:</u>

P_m, $D_{AB}K'$, % resistance to diffusion in the liquid films

<u>Solution:</u>

$N_A = \frac{c_1 - c_2}{\frac{1}{k_{c1}} + \frac{1}{p_m} + \frac{1}{k_{c2}}}$ which can be rearranged to give the following:

$$1/p_m = \frac{c_1 - c_2}{N_A} - \frac{1}{k_{c1}} - \frac{1}{k_{c2}} = \frac{100 gmol/m^3 - 0.5 gmol/m^3}{8.11e-4\ gmolNaCl/s-m^2} - \frac{1}{5.24e-5\ m/s} - \frac{1}{5.24e-5\ m/s}$$
$$= 84{,}520.1\ s/m$$

Thus, p_m = 1.2e-5 m/s

$D_{AB}K' = p_m L = (1.2e-5)(0.029e-3m)$ = 3.48e-10 m^2/s

$$\%Resistance = \frac{R_1 + R_2}{R_1 + R_2 + R_m} x100 = \frac{\frac{1}{k_{c1}} + \frac{1}{k_{c2}}}{\frac{1}{k_{c1}} + \frac{1}{p_m} + \frac{1}{k_{c2}}}$$
$$= \frac{\frac{1}{5.24e-5\ m/s} + \frac{1}{5.24e-5\ m/s}}{\frac{1}{5.24e-5\ m/s} + \frac{1}{5.24e-5\ m/s} + \frac{1}{1.2e-5\ m/s}} = 31.41\ \%$$

Problem 13.10-1

<u>Given:</u>

C_1 = 3500 mg NaCl/L (ρ=999.5 kg/m^3)

$A_w = 3.50 \times 10^{-4}$ kg solvent/s-m^2-atm

$A_s = 1.00 \times 10^{-7}$ m/s

ΔP = 35.50 atm, 17.20 atm, 27.20 atm, 37.20 atm

<u>Find:</u>

N_{NaCl}, solute rejection R, c_2

<u>Solution:</u>

$N_w = A_w(\Delta P - \Delta\pi), \textit{and we can get the osmotic pressure differences from table } 13.9-1$

$\pi_1 = 2.24\ atm$ from linear interpolation

$\pi_2 = 0.08\ atm$

$\Delta\pi = 2.16\ atm$

From this data we can calculate the values of water flux across the membrane at each pressure differential:

ΔP, atm	**N_w, kg H_2O water/s-m^2**
17.2	0.0053
27.2	0.0088
35.5	0.017
37.2	0.012

We can then subsequently calculate R from this value.

$$R = \frac{B(\Delta P - \Delta\pi)}{1 + B(\Delta P - \Delta\pi)}, \qquad where\ B = \frac{A_w}{A_s C_{ws}} = \frac{A_w}{A_s \rho_w}$$

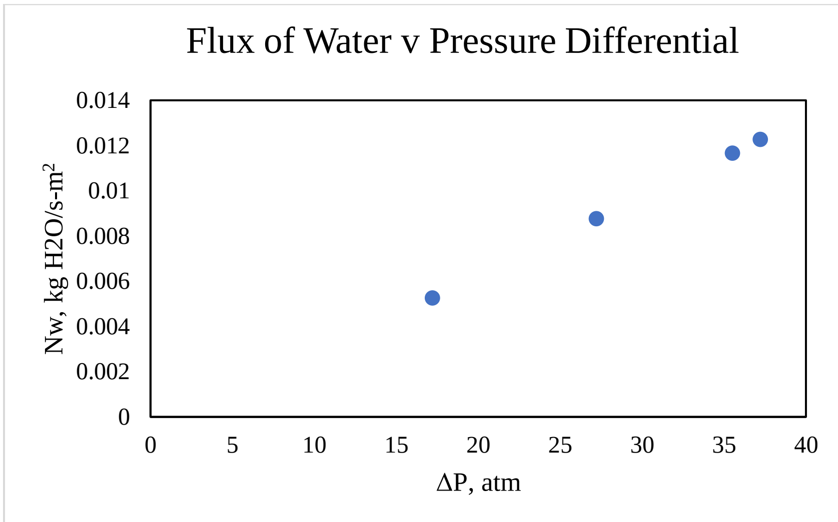
ΔP, atm	**R**
17.2	0.95
27.2	0.97
35.5	0.98
37.2	0.98

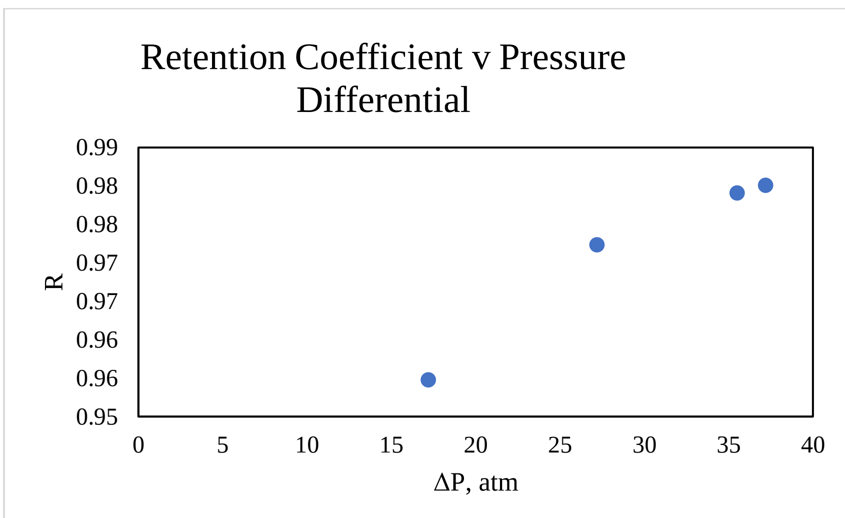
Finally, we can calculate the second concentration, c_2

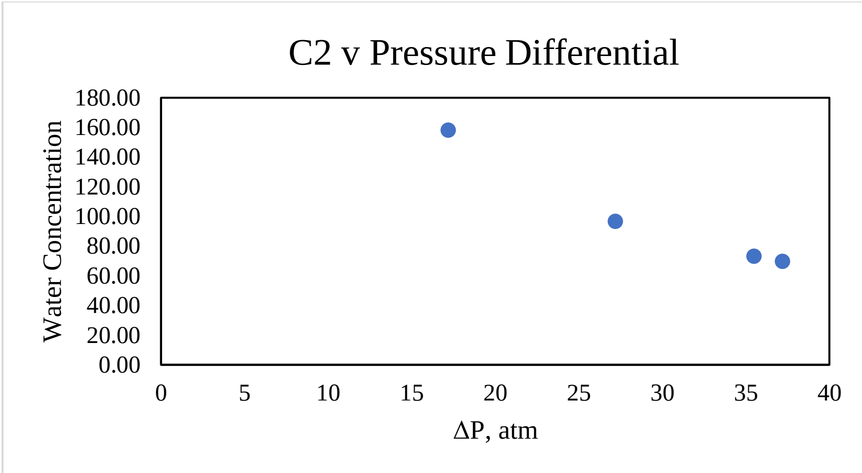
$$R = \frac{c_1 - c_2}{c_1}$$

ΔP, atm	C_2, mg NaCl/L
17.2	158.23
27.2	96.79
35.5	73.20
37.2	69.72

We can plot these data on a graph v the pressure differential:






C2 v Pressure Differential
Water Concentration
180.00
160.00
140.00
120.00
100.00
80.00
60.00
40.00
20.00
0.00
0
5
10
15
20
25
30
35
40
ΔP, atm

Problem 13.10-3

Given:

A_W = 3.50e-4 kg solvent/s-m^2

A_s = 2.00e-7 m/s

ΔP 35.50 atm

C_f = 3500 mg NaCl/L

θ = 0.1

q_2 = 100 gal/hr

Find:

Area, c_1, c_2

Solution:

We can find area with the flowrate given q_2. We can relate the water flowrate through the membrane to the following equation:

$$N_w = A_w A(\Delta P - \Delta\pi)$$

This can be rearranged to produce the following equation:

$$A = \frac{N_w}{A_w(\Delta P - \Delta\pi)}$$

N_w must be converted to kg/s before calculating area.

$$\text{Area} = \frac{0.1028 kg/s}{3.5e\text{-}4 kg/s\text{-}m^2\text{-}atm \text{ x } 35.5 atm} = 8.27\ m^2$$

We can calculate q_f using the "cut" parameter, and then use a mass balance to calculate q_1.

$$\theta = \frac{q_2}{q_f}$$

$$q_f = q_1 + q_2$$

Plugging in and calculating, we get:

$$q_f = 1.027\ kg/s$$

$$q_1 = 0.925\ kg/s$$

Now, we can plug these values into the system of equations to solve and get out concentrations:

$$c_f = (1-\theta)c_1 + \theta c_2$$

$$0 = c_2 + c_1(R-1)$$

And R can be calculated with:

$$R = \frac{B\Delta P}{1 + B\Delta P}, \quad where$$

$$B = \frac{A_w}{A_s c_{w2}}$$

This gives us the following system:

$$3.5 = 0.90c_1 + 0.1c_2$$

$$0 = -0.02c_1 + c_2$$

Solving this set of equations gives us the answers:

$C_1 = 3.88\ kg/m^3$

$C_2 = 0.08\ kg/m^3$

Problem 13.11-1

Known:

0.9 wt % protein.

ΔP = 5 psi = 0.34 atm

A_w = 1.37e-2 kg/s-m^2-atm

Find:

N_w

Solution:

We can use the following equation to calculate flux:

$N_w = A_w(\Delta P - \Delta\pi)$

Now, we must calculate the osmotic pressure from the wt % data given.

We can also assume negligible osmotic pressure differential.

$N_w = (1.37e - 2\ \ kg/s - m^2 - atm)$(0.34 atm) = 0.0047 kg/s-m^2 = 9.88 gal/day-ft^2

Problem 13.11-2

Known:

M_s = 800 kg

C_1 = 0.05 wt %

C_2 = 1.1 wt %

Area = 9.9 m^2

A_w = 2.50e-2 kg/s-m^2-atm

ΔP = 0.50 atm

Find:

N_w, time to filter

Solution:

We can calculate the flux with the following equation:

$$N_w = A_w(\Delta P - \Delta\pi)$$

With ultrafiltration processes, the osmotic pressure differential can be considered negligible.

Thus, $N_w = (2.5e - 2\ kg/s - m^2 - atm)(0.50\ atm) = 0.0125\ kg/s - m^2$

We also have the definitions for the mass percent's given. We can also redefine the second mass fraction to be in terms of the first with the parameter, r, defining amount of water recovered.

$$X_1 = \frac{m_{pro}}{800kg}$$

$$X_2 = \frac{m_{pro}}{m_{pro} + m_{H_2O} \ - r}$$

We can combine the two equations to calculate r. With the flux known, we can then calculate the time needed to filter the solution:

$$r = \frac{800(X_2 - X_1)}{(X_2)} = \frac{800kg\ (0.011\ -\ 0.0005)}{0.0005}\ =\ 16{,}800\ kg$$

Thus,

$$time = 16{,}800kg\ x\ 1/0.0125\ kg/s - m^2\ x\ 1/9.90m^2\ =\ 135758\ s\ =\ 1.57\ days$$