ABE 55700

Homework 3 | Drying

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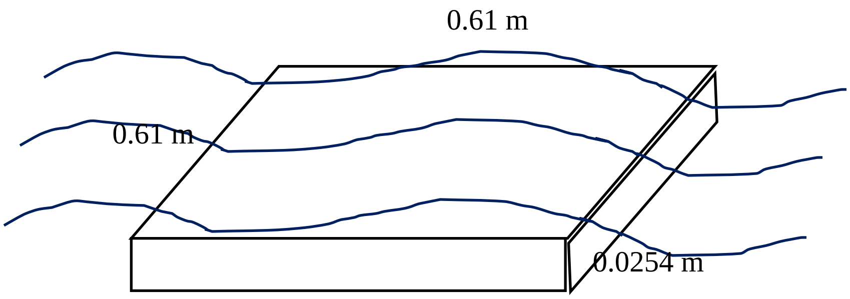
Problem 9.6-3

A food material is being dried in a pan:



We are told the following properties of our system:

V_{air} = 3.05 m/a = **10,980 m/hr**

The food material contains **11.34 kg dry solid**

The free moisture content, X_1, is initially at **0.35 kg H_2O/kg dry solid**

T_{air} = **60 °C** and the wet bulb temperature T_w = **29.4 °C**

The final free moisture content X_2 = **0.22 kg H_2O/kg dry solid**

We are also told that the drying rate was found to be in the constant rate period.

We can estimate the constant drying rate with the following formula:

$$R_c = \frac{h}{\lambda_w}(T_{air} - T_w)(3600)$$

where λ_w is the latent heat of vaporization of water (2,260 kJ/kg), and h is the convective heat transfer coefficient. In addition, we can then subsequently estimate the time of drying by the following equation:

$$t = \frac{L_s}{AR_c}(X_1 - X_2)$$

The convective heat transfer coefficient, h, can be estimated with the following equation:

$$h = 0.0204G^{0.8}, where\ G\ is\ the\ gas\ mass\ velocity$$

$$h = 0.0204\big((10{,}980 m/hr)(1.06\ kg/m^3)\big)^{0.8} = \mathbf{36.51 W/m^2K}$$

Thus,

$$R_c = \frac{36.51}{2.260e6}(60°C - 29.4°C)(3600) = 1.78\ kg\ H_2O/h - m^2$$

Thus, the time required to dry is,

$$t\ = \frac{11.34kgdry}{0.61^2 m^2\left(1.78\frac{kgH_2O}{h - m^2}\right)}\left(0.35\frac{kgH_2O}{kgdry} - 0.22\frac{kgH_2O}{kgdry}\right)\ = \boxed{2.23\ hrs}$$

In the constant rate drying regime, we can assume that the thickness is linearly proportional to the drying time:

$$\frac{t_1}{L_1} = \frac{t_2}{L_2}\ \ or\ \ \frac{L_2 t_1}{L_1} = t_2$$

Thus,

$$t_2 = \frac{44.5mm}{25.4mm}(2.23hr) = \boxed{3.90hrs}$$

Problem 9.9-3

We are given data on the relative moisture content over time. Specifically, the ratio of the current moisture content to the critical moisture content, $\frac{X}{X_c}$. The data is given in the following table:

X/Xc	t (hours)
1.0	0
0.65	0.25
0.32	7.0
0.17	11.4
0.10	14.0
0.06	16.0

We know that the data follow the diffusion equation:

$$t = \frac{4\mathrm{x}_1^2}{\pi^2 \mathrm{D_L}} ln\left(\frac{8X_c}{\pi^2 X}\right)$$

where x_1 is the half thickness, D_L is the diffusion coefficient, X_c is the critical moisture content, and X is the current moisture content at time t. We can plot the natural logarithm of $\frac{8X_c}{\pi^2 X}$ against the time and the slope will then be equal to $\frac{4x_1^2}{\pi^2 D_L}$. Using the known thickness and other constants we can then get a value for the diffusion coefficient.

The slope was found to be 0.0697 hr^{-1}. Thus, we can predict the diffusion coefficient to be equal to:

$$\frac{(0.0697)4x_1^2}{\pi^2} = 1.311e-6$$

Problem 9.7-2

We are given data on the weight of a material over time:

Time (hr)	Weight (kg)
0	4.944
0.4	4.885
0.8	4.808
1.4	4.699
2.2	4.554
3.0	4.404
4.2	4.241
5.0	4.150
7.0	4.019
9.1	3.978
12.0	3.955

We also know that the dry sample weight is 3.765 kg, while the equilibrium weight of our product is 3.955. This lets us calculate the equilibrium moisture content or bound water content, **0.190 kg.** Thus we can calculate the total free moisture in our sample at the beginning:

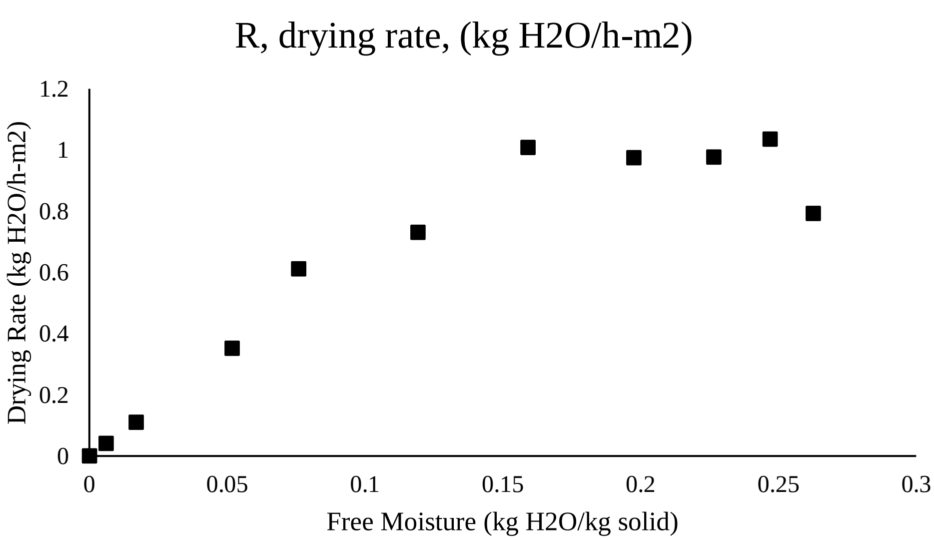
$$X_{free,o} = \frac{4.944kg - 3.765kg - 0.190kg}{3.765\ kg\ dry\ solid} = \boldsymbol{0.263\ kg\ H_2O}$$

This process is repeated for each time point to get the moisture content. We can then estimate the rate of drying between each point with the following equation:

$$R = -\frac{L_s}{A}\frac{\Delta X}{\Delta t}$$

We can then use a numerical integration technique to estimate the time to dry product between two moisture content levels. These calculations were done in excel and the sheet with the final time is attached. The final calculated to be:

4.63 hrs



Problem 9.8-1

This problem is similar to problem 9.6-3, however we need to consider that radiation is being applied to the surface of our material as well. Thus, this time around, the total energy transfer to our product can be written as:

$$q_{tot} = q_c + q_r$$

Where,

$$q_c = h_c(T - T_s)A, and q_r = h_r(T - T_s)A$$

One can estimate the radiation coefficient by the following equation,

$$\epsilon(5.676)\frac{\left(\frac{T_R}{100}\right)^4 - \left(\frac{T_s}{100}\right)^4}{T_R - T_s}$$

Thus, we can now create the following relation:

$$R_c = \frac{q}{A\lambda_s} = \frac{h_c(T - T_s) + h_R(T_R - T_s\}}{\lambda_s} = k_y M_B(H_s - H)$$

Rearranging and substituting in the specific heat capacity, we achieve an equation which related the temperature of the surface, the air, and the radiation temperature to the coefficients of heat transfer for convection and for conduction with the air humidity and saturation humidity:

$$\frac{(H_s - H)\lambda_s}{c_s} = (T - T_s) + \frac{h_R}{h_c}(T_R - T_s)$$

We know that h_R is a function of T_s and thus we can use a solving algorithm (excel) to solve for the surface temperature of our material. This was done in excel (See attached calculations)

T_s was found to be: $30.32°C$

The drying rate can now be estimated using the following equation:

$$R_c = \frac{h}{\lambda_w}(T_{air} - T_s)(3600)$$

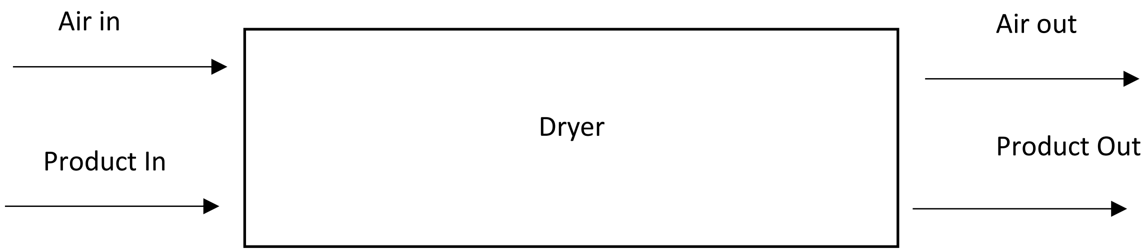
$$R_c = \frac{35.84}{2424e6}(338.6 - 303.32)(3600)$$

$$R_c = 1.88\frac{kgH_2O}{h - m^2}$$

(See attached excel document for calculation set up)

Problem 9.10-5

The system is described as such:



The material comes in wet and is to be dried to specific specifications. We are given specifications about the air coming in and out of our system and must calculate the time that the material must spend in the dryer. The exact specifications are already given in the attached excel sheet, and there is *no* reason to repeat them in this document.

We know that the drying rate is equal to:

$$R = k_y M_B (H_w - H) = \frac{h}{\lambda_w}(T_{air} - T_w)$$

And, we also know that the time required for drying of the material can be calculated using the following equation from Geankopolis:

$$t = \frac{G}{L_s}\left(\frac{L_s}{A}\right)\frac{1}{k_y M_B} ln\left(\frac{H_w - H_c}{H_w - H_1}\right)$$

The equation can be modified to include the log-mean humidity difference,

$$t = \frac{G}{L_s}\left(\frac{L_s}{A}\right)\frac{1}{k_y M_B} ln\left(\frac{H_1 - H_c}{\Delta H_{lm}}\right)$$

Where,

$\Delta H_{lm} = \frac{H_1 - H_c}{ln\left(\frac{H_w - H_c}{H_w - H_1}\right)}$ and $H_c = H_2 + \frac{L_s}{G}(X_c - X_2)$

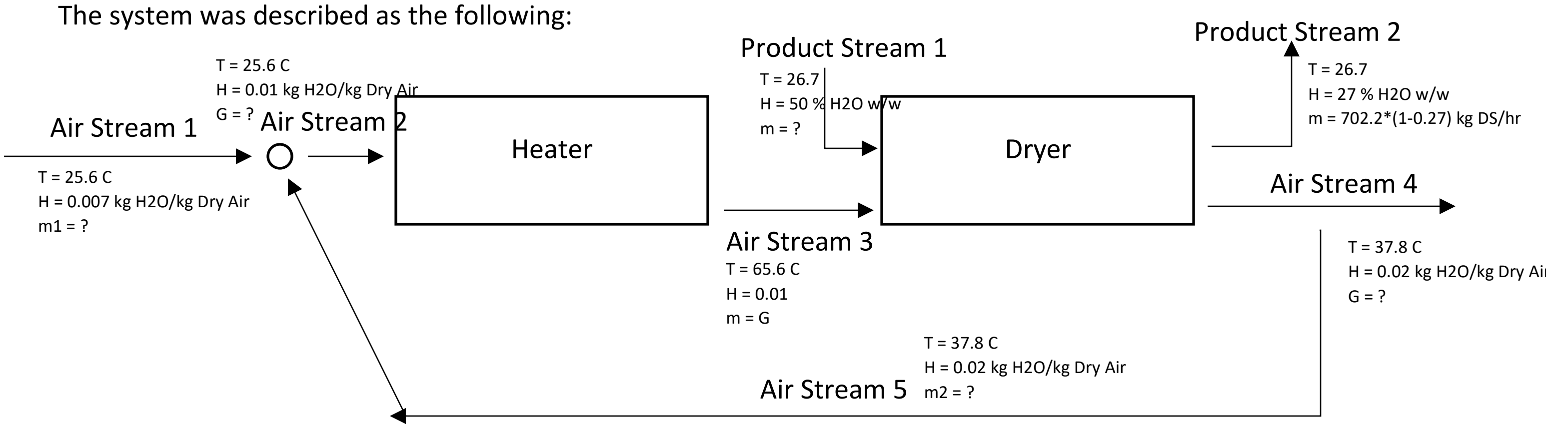
However, the first humidity is not given. But, we can obtain it from a simple mass balance

$$GH_2 + L_s X_1 = GH_1 + L_s H_2$$

The exact calculations were all done in an excel sheet, which is attached. The exact drying time was found to be 4.22 hrs.

Problem 9.10-6

The system was described as the following:



First, we need to find the mass flow rate of air through the system:

$$G = \frac{L_s(X_2 - X_1)}{(H_2 - H_1)}$$

(In excel) G was found to be, **32,301.2 kg dry air/hr**

To get the mass flow rates of the other streams we must perform a mass balance around the water and air in our system. We know that the total dry air flowing into our heater is equal to the total dry air flowing our of the heater, which was calculated to be 32,301.2 kg/hr.

In addition, the water coming into the heater is equal to the amount of water coming out of our heater. Mathematically, this can be written as such:

$$m_1 + m_2 = 32{,}301.2 kg dry air/hr$$

$$m_1(0.007) + m_2(0.02) = 32{,}301.2(0.01)$$

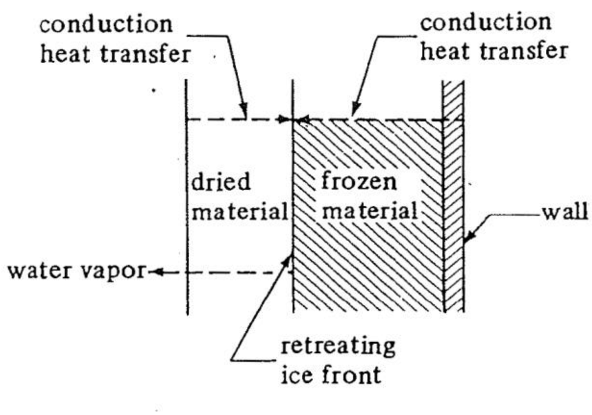
This is a system of two equations with two unknowns which is in fact solvable. m_1 was found to be 24,847.1 kg dry air per hour, while m_2 was found to be 7454.17.12 kg dry air per hour. These are the fresh air and recycled air streams respectively. We can then calculate the percent of air recycled:

$$\%Recycled = \frac{m_2}{m_1 + m_2}$$

This number was found to be 23.02%

Freeze Drying Derivation

Freeze drying with only conduction against the wall:



We know that the heat transfer through the ice material is equal to the heat transfer through the dried material which is equal to the heat of sublimation:

$$q_{dry} = q_{ice} = \Delta H_{sub} N_A$$

$$\frac{k_d}{L - \Delta L}\left(T_s - T_f\right) = \frac{k_i}{\Delta L}\left(T_w - T_f\right) = \Delta H_{sub} N_A$$

Knowing that $\Delta L = (1 - X)L$ and that $N_A = \frac{-dx}{dt}$

We can simplify our equation to become:

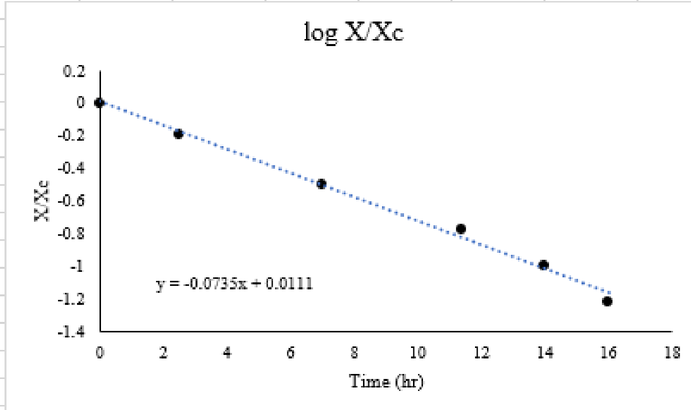
$$(T_w - T_s)\left(\frac{M_A V_s}{L \Delta H_s}\right) dt = \frac{1}{k_d k_i}(k_i L X + (1 - X) L k_d)\mathrm{dx}$$

We can integrate and then get an equation for the drying time

Appendix/Excel Sheets

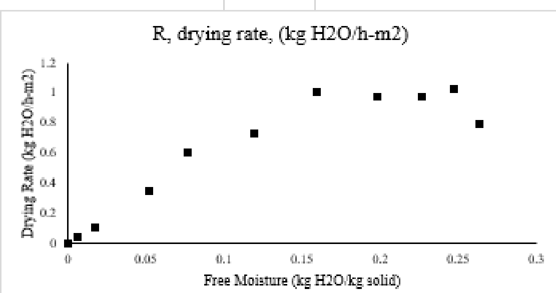
Problem 9.9-3:

	A	B	C	D	E
1	X/Xc	log X/Xc	Xc/X	DLt	Time (hrs)
2	1	0	1	#DIV/0!	0
3	0.65	-0.187086643	1.538462	9.1271E-07	2.5
4	0.32	-0.494850022	3.125	1.37232E-06	7
5	0.17	-0.769551079	5.882353	1.41613E-06	11.4
6	0.1	-1	10	1.54488E-06	14
7	0.06	-1.22184875	16.66667		16
8			Average	1.31151E-06	
9	x1	0.00505			
10					
11	Slope	0.069713713			
12	Dt	1.80136E-07			



Problem 9.7-2:

	A	B	C	D	E	F	G	H	I	J
1	Time(Hr)	Weight (k	Free Moisture Content (kg H2O/kg	R, drying rate, (kg H2O/h-m2	Rc		Drying Time			
2	0	4.944	0.262682603	0.793010753	0.9985		X	R	delt	
3	0.4	4.885	0.247011952	1.034946237			0.2	0.9985	1.0136	
4	0.8	4.808	0.226560425	0.976702509			0.15	0.9446	0.2143	
5	1.4	4.699	0.197609562	0.974462366			0.14	0.8749	0.2314	
6	2.2	4.554	0.159096946	1.008064516			0.13	0.8052	0.2514	
7	3	4.404	0.119256308	0.730286738			0.12	0.7355	0.2752	
8	4.2	4.241	0.075962815	0.61155914			0.11	0.7049	0.2872	
9	5	4.15	0.051792829	0.352150538			0.1	0.6775	0.2988	
10	7	4.019	0.016998672	0.110215054			0.09	0.6501	0.3114	
11	9	3.978	0.006108898	0.041218638			0.08	0.6226	0.3251	
12	12	3.955	0	0			0.07	0.5476	0.3697	
13							0.06	0.4402	0.4598	Total
14	Bound/Equilibrium Water (kg)	0.19					0.05	0.3397	0.5959	4.6337
15	Dry Solid Weight (kg)	3.765					0.04	0.2702	2.9971	
16	Area (m^2)	0.186								



Problem 9.8-1:

	A	B	C	D	E	F	G
1	*Parameter*	*Units*	*Value*		*LHS*	*RHS*	*Error^2*
2	*hc*	W/m^2-K	35.84262792		49.72065	49.72065	1.6E-11
3	*hr*	W/m^2-K	8.178236586				
4	*T air*	K	338.6				
5	*T R*	K	366.6				
6	*v,air*	m/s	3.05				
7	*v,air*	m/h	10980				
8	*rho,air*	kg/m^3	1.036				
9	*G*	kg/m^2-h	11375.28				
10	*λs*	kJ/kg	2424				
11	*λs*	j/kg	2424000				
12	*A*	m^2	0.093025				
13	*H*	kg-H2O/kg dry air	0.01				
14	*Hs*	kg-H2O/kg dry air	0.031				
15	*ε*	-	0.95				
16	*cs*	J/kg-K	1023.8				
17	*Ts*	K	303.3183616				
18	*Ts*	C	30.31836156				
19	*Rc*		1.878098969				

Problem 9.10-5:

	A	B	C	D	E	F	G
1	**Parameter**	**Description**	**Units**	**Value**			
2	*Ls*	*Dry product flow rate*	lbm/h	700		time for drying (hrs)	4.217646
3	*X1*	*Food Moisture Content In*	lbm H2O/lbm DS	0.4133			
4	*X2*	*Food Moisture Content Out*	lbm H2O/lbm DS	0.0374			
5	*Xc*	*Critical Moisture Content*	lbm H2O/lbm DS	0.0959			
6	*G*	*Gas Mass Flow Rate*	lbm dry air/h	13280			
7	*H1*	*Air Humidity Out*	lbm H2O/lbm dry air	0.076014006			
8	*H2*	*Air Humidity In*	lbm H2O/lbm dry air	0.0562			
9	*Hc*	*Critical Humidity of the Air*	lbm H2O/lbm dry air	0.059283584			
10	*Hw*	*Saturation Humidity*	lbm H2O/lbm dry air	0.0786			
11	*delHlm*	*Log mean Humidity Difference*	lbm H2O/lbm dry air	0.008320094			
12	*A/Ls*	*Area to mass flow ratio*	ft^2/lbmDS	0.3			
13	*kyMb*	*Empirically derived cosntants*	lbm air/h-ft^2	30.15			