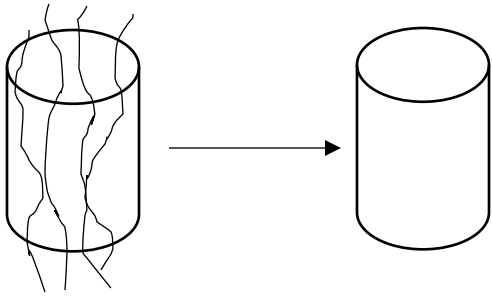


Problem 10.1

Given $C^* = 7.3$ mg/L and dissolved oxygen (DO) v time data... estimate $K_L a$.

Time (min)	DO (mg/L)
-1	3.3
0	3.3
1	2.4
2	1.3
.	.
.	.
.	.

At unsteady-state we know that the accumulation of oxygen in the system is a function of the oxygen transfer rate and the oxygen uptake rate.

$$\frac{dC_L}{dt} = OTR - OUR$$

$$\frac{dC_L}{dt} = k_L a (C^* - C_L) - X \cdot q_{O_2}$$

We can estimate oxygen uptake rate with data as seen in Fig. 1:

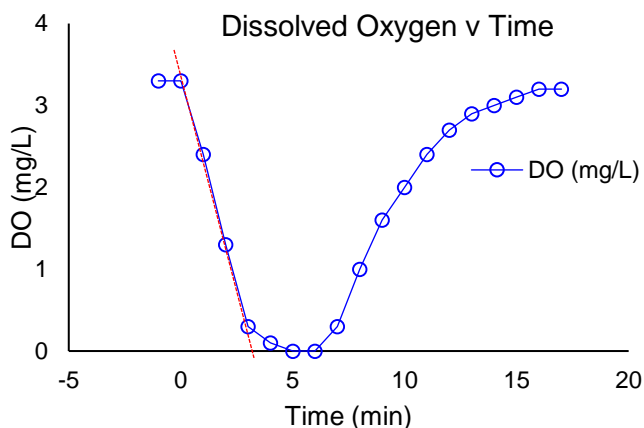


Figure 1. The plot of DO v time. The red dotted line estimates the oxygen uptake rate (OUR).

The slope of the red line represents the decay of O_2 in the system, or the oxygen uptake rate.

Then, $\frac{dC_L}{dt} + OUR = k_L a (C^* - C_L)$ is our new equation.

We can plot the slope of the curve plus the OUR against the C^* minus C_L value to get a linear plot. The slope of the subsequent linear plot is our estimated $k_L a$.

$\frac{dC_L}{dt}$ can be estimated by: $\frac{C_{n+1} - C_n}{t_{n+1} - t_n}$. The subsequent

Linear regression in excel was used to compute the slope of the OUR data points. It was also used to compute the slope of subsequent linearized equation to estimate the $k_L a$. (See excel sheets). Figure 2 shows the linearized data plot:

Linearized Data

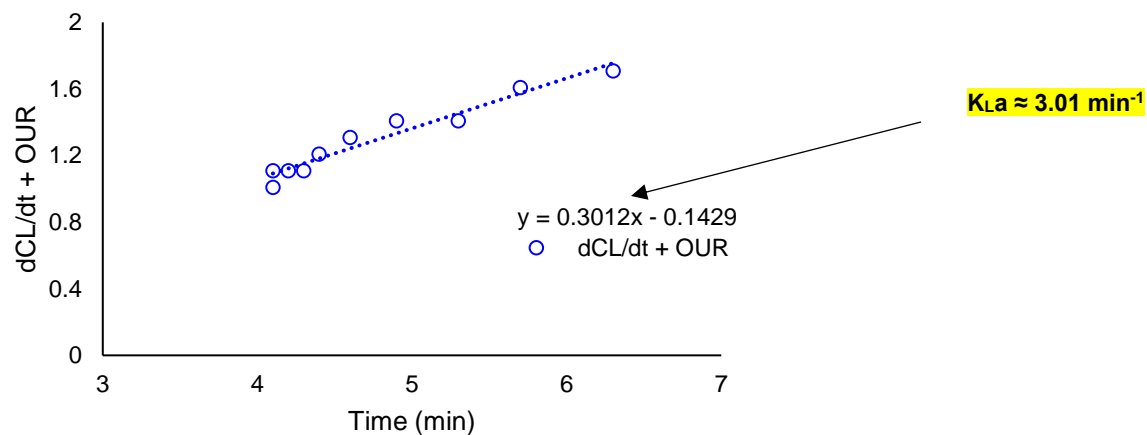


Figure 2. Estimating the k_{La} of our system requires the linearization of our equation and plotting of the proper data transformations. Linear regression allows us to estimate the specific k_{La} .

a.)

$$\frac{dC_L}{dt} = OTR - OUR$$

$$\frac{dC_L}{dt} = k_L a(C^* - C_L) - X \cdot q_{O_2}$$

where X is the # of cells in g dry wt per L

Our process is assumed to be at steady state, that is, $X \cdot q_{O_2} = k_L a(C^* - C_L)$. We can rearrange this equation to obtain an equation for number of cells as a function of the critical oxygen level, the oxygen solubility, the $k_L a$, and the OUR, q . We then obtain...

$$X = \frac{k_L a(C^* - C_L)}{q_{O_2}}$$

We know that $C^* = 7.3 \text{ mg/L}$, $k_L a = 30 \text{ hr}^{-1}$, and $C_L = 0.2 \text{ mg/L}$. We need to do some conversion to get the OUR into units we can use. It is currently in mMol, while we need it in mg:

$$\begin{aligned} q_{O_2} &= \frac{10 \text{ mmol } O_2}{g - \text{dry wt} - h} \times \frac{1 \text{ mol}}{1000 \text{ mmol}} \times \frac{0.01 \text{ mol } O_2}{g - \text{dr wt} - h - L} \times \frac{32.02 \text{ g } O_2}{1 \text{ mol}} \times \frac{1000 \text{ mg}}{1 \text{ g}} \\ &= 320.2 \text{ mg/L} - g - \text{drywt} - h \end{aligned}$$

Now, we can plug in these values into our derived equation above for X to get a final answer for maximum number of cells per L in our system.

$$X_{max} = \frac{30 \text{ hr}^{-1}(7.3 \text{ mg/L} - 0.2 \text{ mg/L})}{320 \text{ mg/L} - g - \text{drwt} - hr} = 0.655 \text{ g - dr wt / L}$$

b.)

If we are using pure oxygen... the partial pressure of the oxygen now becomes 1 atm, and thus the solubility is now **40 mg/L**. We just need to replace the C^* in our equation and recalculate...

$$X_{max} = \frac{30 \text{ hr}^{-1}(40 \text{ mg/L} - 0.2 \text{ mg/L})}{320 \text{ mg/L} - g - \text{drwt} - hr} = 3.73 \text{ g - dr wt / L}$$

This answer intuitively makes sense as we can now supply a larger mass of cells in our processes than before. This makes sense because we are using pure oxygen in our system and we have less problems delivering a sufficient quantity to our organisms in the fermenter.

a.)

Thermal transport phenomena are a bit different than mass transport, but the general principle still applies.
Accumulation = In – Out.

We are given:

- OUR = 10 mmol/L-h *For this problem, we can assume that the*
- $T_{op} = 35\text{ C}$ *specific heat capacities of the broth and the*
- $T_{w,in} = 15\text{ C}$ *water are identical... $c_{p,w} = c_{p,b} = 1\text{ kcal/kg-K}$*
- $\Delta T > 5$

At steady state, the rate of energy transfer to the water is equal to the rate of energy transfer out of the fermenter. This can also be looked at in discrete quantities over the course of the entire process as well.

Mathematically, the total change of energy in the fermenter is equal to the total change in energy of the water, or...

$$\Delta E_{water} = \Delta E_{fermenter}$$

We can estimate the energy generation of the fermenter with the following equation:

$$\Delta E_{fermenter} \approx 0.12 q_{O_2}$$

Thus,

$$\Delta E_{fermenter} \approx 0.12 (12\text{kcal/L-h})(80,000\text{L}) \approx 96,00\text{ kcal/hr}$$

We know that the energy change in the water can be quantified using the following equation:

$$\Delta E_w = c_p \dot{m} (T_{out} - T_{in})$$

Thus,

$$96,000\text{ kcal/h} = c_p \dot{m} (T_{out} - T_{in})$$

Thus,

$$\begin{aligned} \dot{m} &= \frac{96,000\text{ kcal/h}}{c_p (T_{out} - T_{in})} \\ \dot{m} &= \frac{96,000\text{ kcal/h}}{(1\text{kcal/h})(30\text{ C} - 15\text{ C})} \\ \dot{m} &= 6,400\text{ kg/hr} = 6,400\text{ L/hr} \end{aligned}$$

b.)

We can quantify the energy transfer rate to the water using the following equation:

$$\dot{q} = UA\Delta T, \text{ more accurately } \dot{q} = UA\Delta T_{lm}$$

Lets convert the heat transfer rate to J/s:

$$\dot{q} = \frac{96,000 \text{ kcal}}{\text{hr}} \times \frac{1 \text{ hr}}{60 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ s}} \times \frac{4184 \text{ J}}{1 \text{ kcal}} = 111,573.3 \text{ J/s}$$

We can substitute in the surface area of a cylinder for A and rearrange to obtain an equation for the length of pipe required. Note that the temperature change is also the log-mean temperature difference:

$$l = \frac{\dot{q}}{U \Delta T_{lm} \pi D}, \text{ and } \Delta T_{lm} = \frac{\Delta T_{in} - \Delta T_{out}}{\ln T_{in} - \ln T_{out}}$$

$$\Delta T_{lm} = \frac{20 - 5}{\ln(20) - \ln(5)} = 10.82$$

Thus,

$$l = \frac{111,573.3 \text{ J/s}}{(1420 \text{ J/s-m}^2\text{-C}) (10.82) \pi (0.025 \text{ m})} = 92.46 \text{ m}$$

We can assume that the concentration of oxygen in our system doesn't change much with time and that the system in our fermenter is at steady-state. Thus,

$$OTR = OUR, \text{ or } X \cdot q_{O_2} = k_L a(C^* - C_L).$$

In this case, the oxygen uptake per weight of bacteria can be mathematically characterized as the following equation:

$$q_{O_2} = \frac{q_{O_2, \max} C_L}{0.2 + C_L}$$

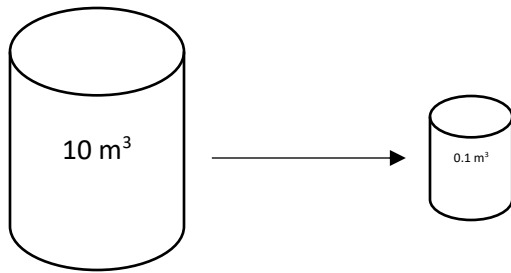
Substitute into our original equation and we obtain,

$$X \cdot \frac{q_{O_2, \max} C_L}{0.2 + C_L} = k_L a(C^* - C_L)$$

Check units... RHS = 1/hr(mg/L) = mg/L-h, LHS = (mg/g-h)(g/L) = mg/L-h. These match so our equation is valid. We know X must be in units of g/L. For this problem we are trying to find the critical oxygen level, C_L . It is impossible (or at least very difficult) to isolate C_L in this case, so we must use excel and its integrated excel to find the C_L value when X is at 20 g/L.

$$20 \text{ g/L} \left[\frac{240 \text{ mg/g} - h C_L}{0.2 \text{ mg/L} + C_L} \right] = (120 \text{ h}^{-1})(28 \text{ mg/L} - C_L)$$

A reasonable initial guess for C_L would be around 1 mg/L. When we input this and have excel solve for C_L , we get the value of **0.44 mg/L**. (See attached spreadsheet for exact details of the solver).



Scale down reactor from 10 m^3 to 0.1 m^3 . Since the tank scale-down is geometrically similar, we can calculate the scale down factor as such.

$$\frac{V_1}{V_2} = \frac{10}{0.1} = 100 = \alpha^3$$

Thus, our scale down factor, α , 4.64.

The large tank has dimensions: $D_t = 2\text{m}$, $D_i = 0.5\text{m}$, $N = 100 \text{ RPM}$. Because of the geometric similarity, we know that the following relationship must apply:

$$\frac{D_{t_0}}{D_{t_s}} = \frac{h_0}{h_s} = \frac{D_{i_0}}{D_{i_s}} = 4.64$$

a.)

Thus,

$$D_{t_s} = \frac{D_{t_0}}{4.64} = \frac{2\text{m}}{4.64} = 0.43\text{m}$$

$$D_{i_s} = \frac{D_{i_0}}{4.64} = \frac{0.5\text{m}}{4.64} = 0.108\text{m}$$

What is h_0 ? We can obtain this from the volume of the original tank:

$$V_0 = h_0 \pi \frac{D_{t_0}^2}{4}, \text{ which can be rearranged to, } h_0 = \frac{4V_0}{\pi D_{t_0}^2}$$

Thus,

$$h_0 = \frac{4(10\text{m}^3)}{\pi(2\text{m}^2)} = 3.185\text{m}$$

So,

$$h_s = \frac{h_0}{4.64} = \frac{3.185\text{m}}{4.64} = 0.686\text{m}$$

b.)

(i) For constant tip speed, the equation $N \cdot D$ must be equal to each other:

$$N_0 D_{i_0} = N_s D_{i_s} \text{ or } N_s = \frac{N_0 D_{i_0}}{D_{i_s}} = \frac{(100 \text{ RPM})(0.5\text{m})}{(0.108\text{m})} = 462.96 \text{ RPM}$$

(ii) For constant Re , the equation $N \cdot D_t^2$ must be the same for each tank:

$$N_0 D_{t_0}^2 = N_s D_{t_s}^2 = \frac{N_s D_{t_s}^2}{D_{t_s}^2} = \frac{(100 \text{ RPM})2\text{m}^2}{0.43\text{m}^2} = 2163.33 \text{ RPM}$$

[illegible]