A First Course in Network Science: 3.3-3.6

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Content

So far…

- Degree measures number of links on a node
- Betweenness measures how often a node is traversed following shortest paths
- In large networks statistical tools are necessary to analyze global features
- Distributions of centrality measures are often heterogeneous in real networks. Degree distribution often has heavy tail
- Nodes with a large degree are called hubs

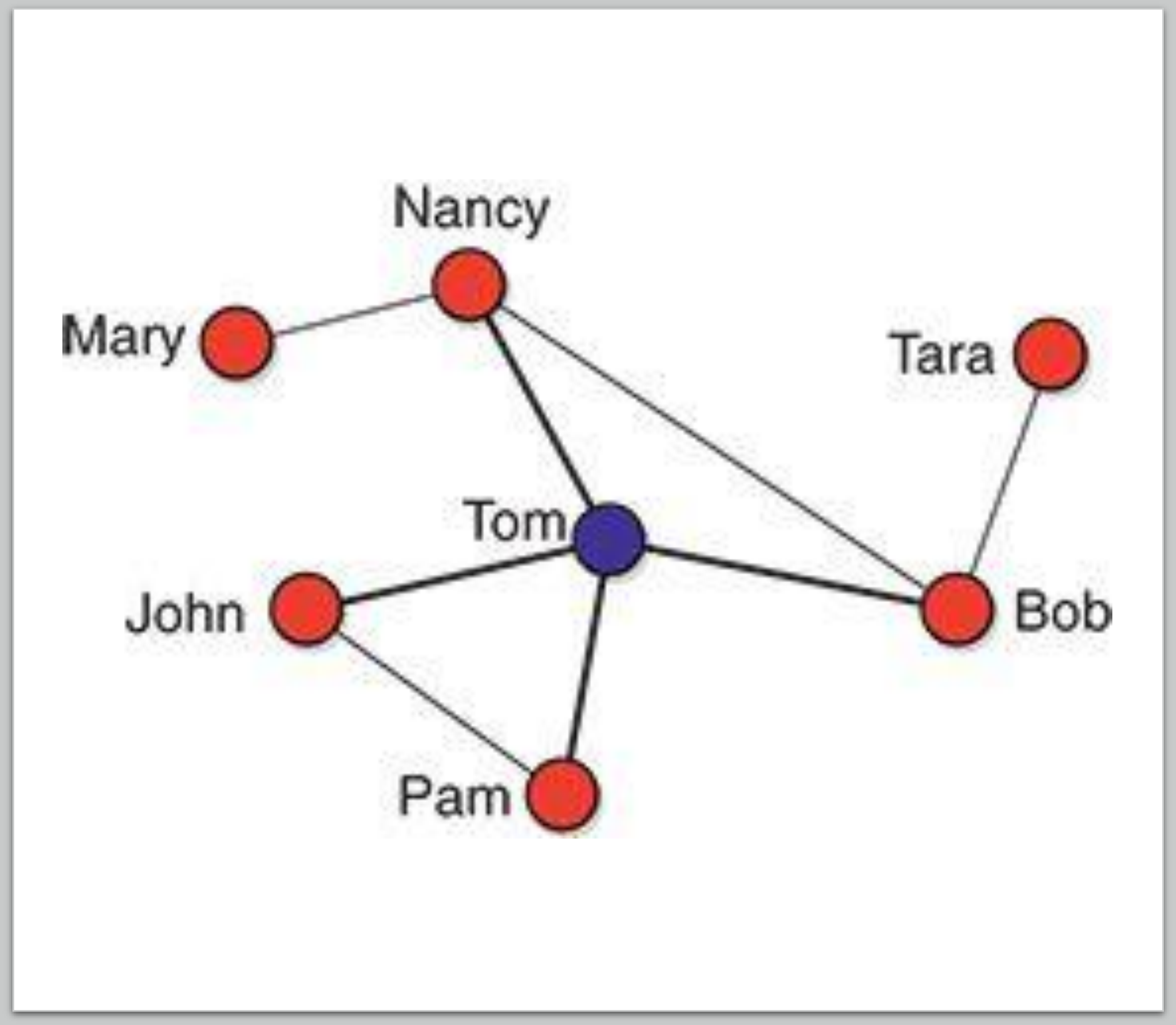
3.3: The Friendship Paradox

In social networks, most people have fewer friends than their friends have, on average

Mary
Tara
John
Bob
Pam

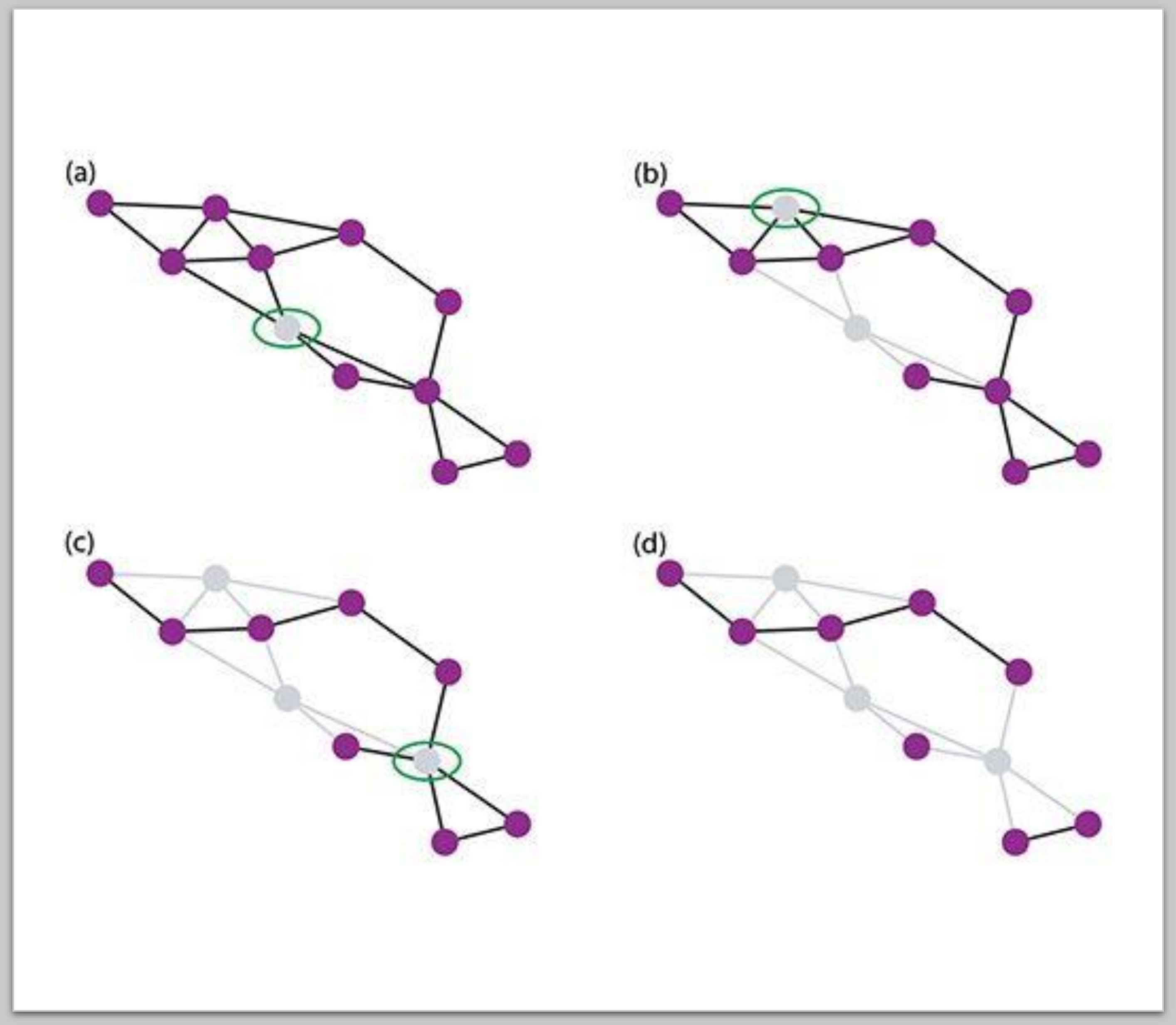
Exercise

1. What is the average degree?
2. What is the average number of friends of friends?



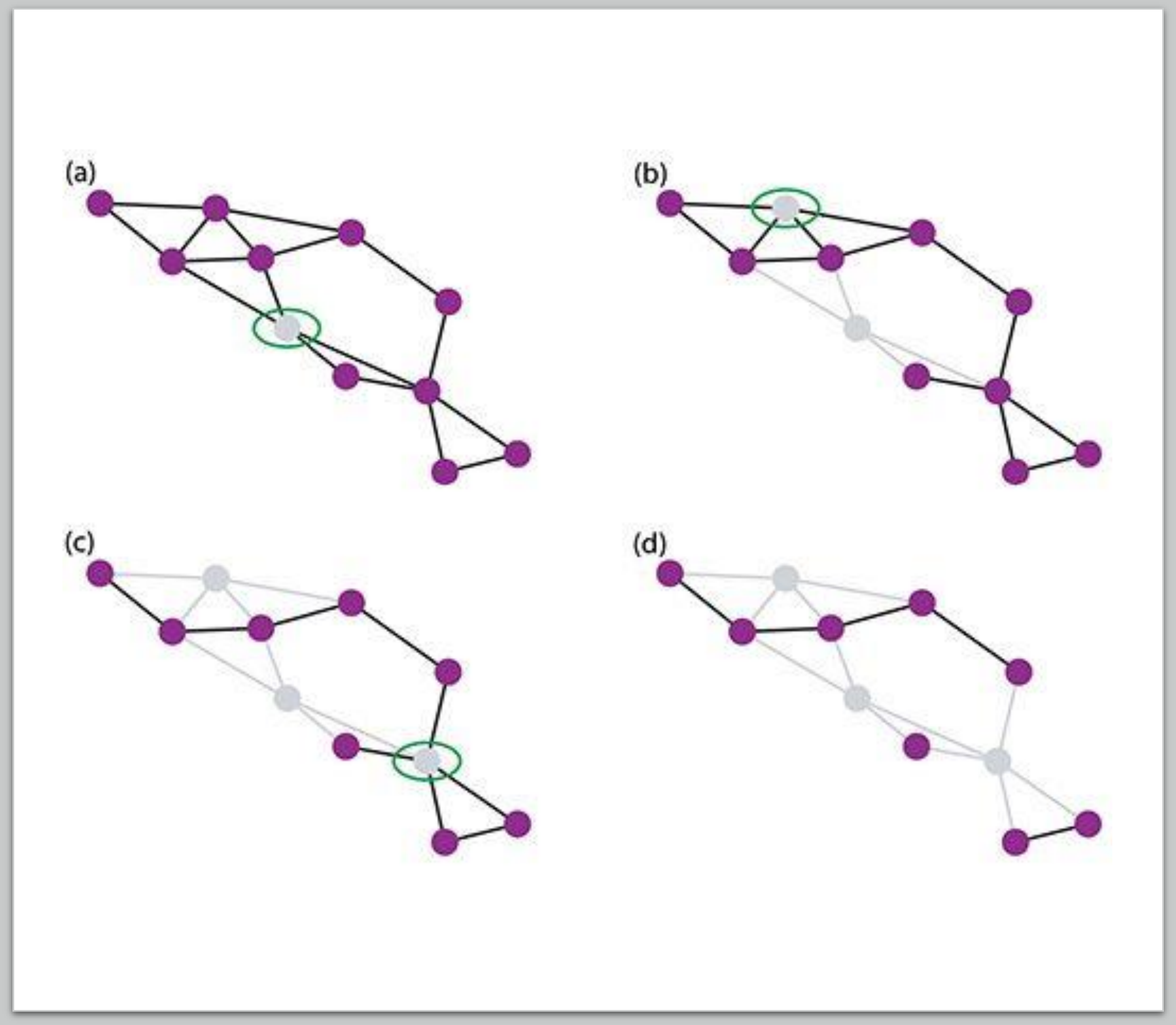
3.5 Robustness

- A system is *robust* if the failure of some of its components does not affect its function
- Robustness depends on *which* components fail
- A way to define robustness:
    - How many nodes stay connected after removal of one or several nodes?
    - What is the proportion of the giant component after removal?



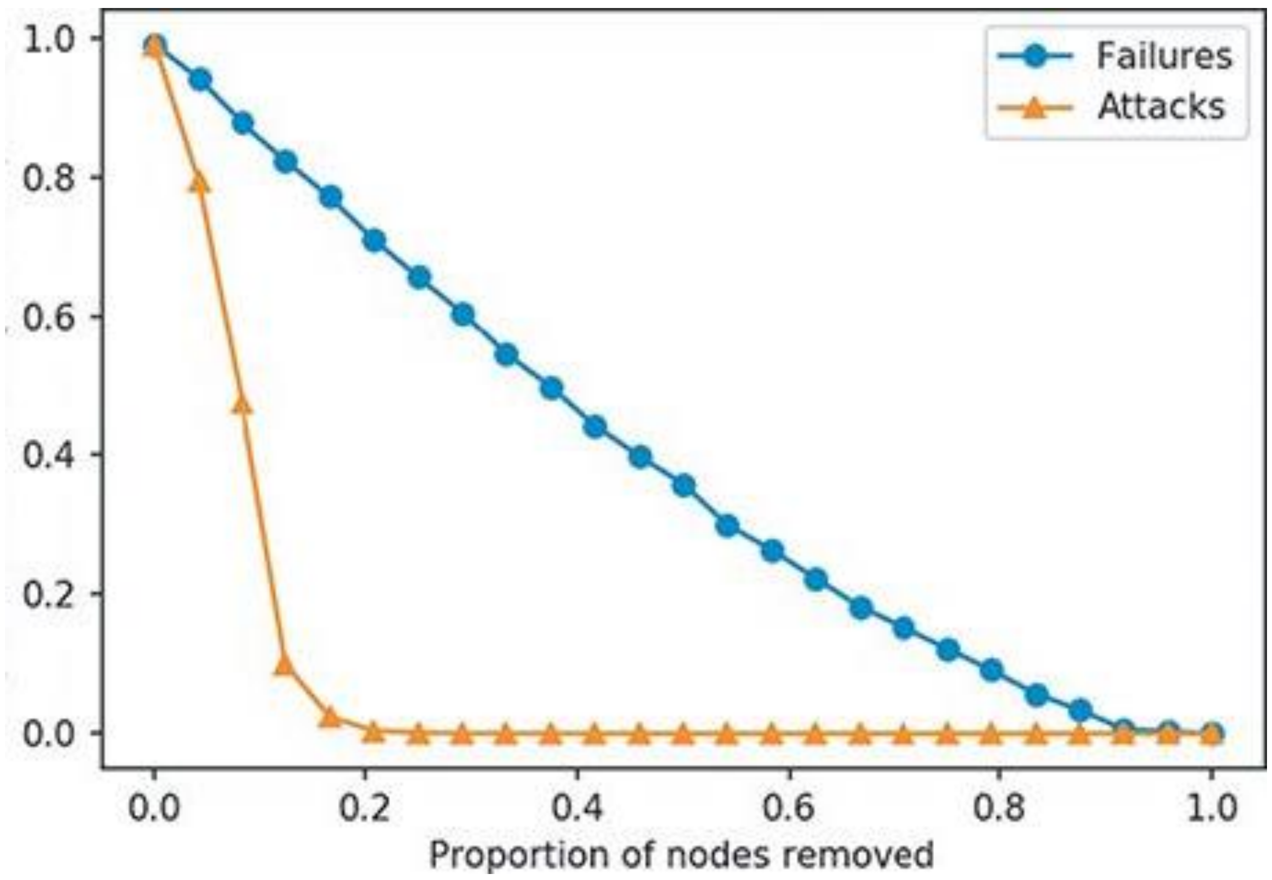
3.5 Robustness

- Exercise:
- Compute proportion of nodes in giant component in a, b, c, d



Robustness

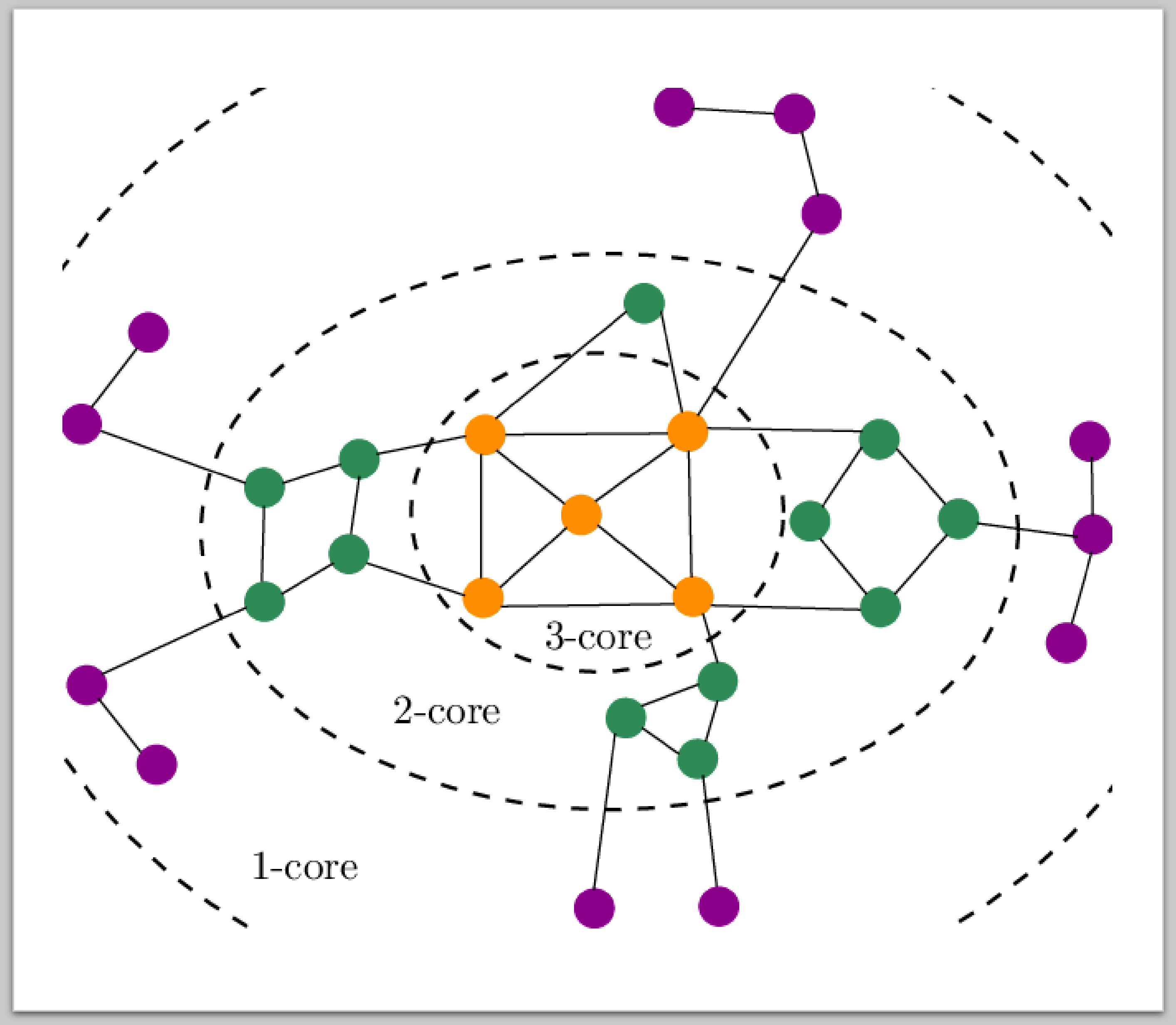
- Picture illustrates robustness of OpenFlight World network
- Failures: random nodes are removed from network
- Attacks: node removal prioritized based on degree



Proportion of giant component in Openflight World network after random failures and targeted attacks

3.6: Core Decomposition

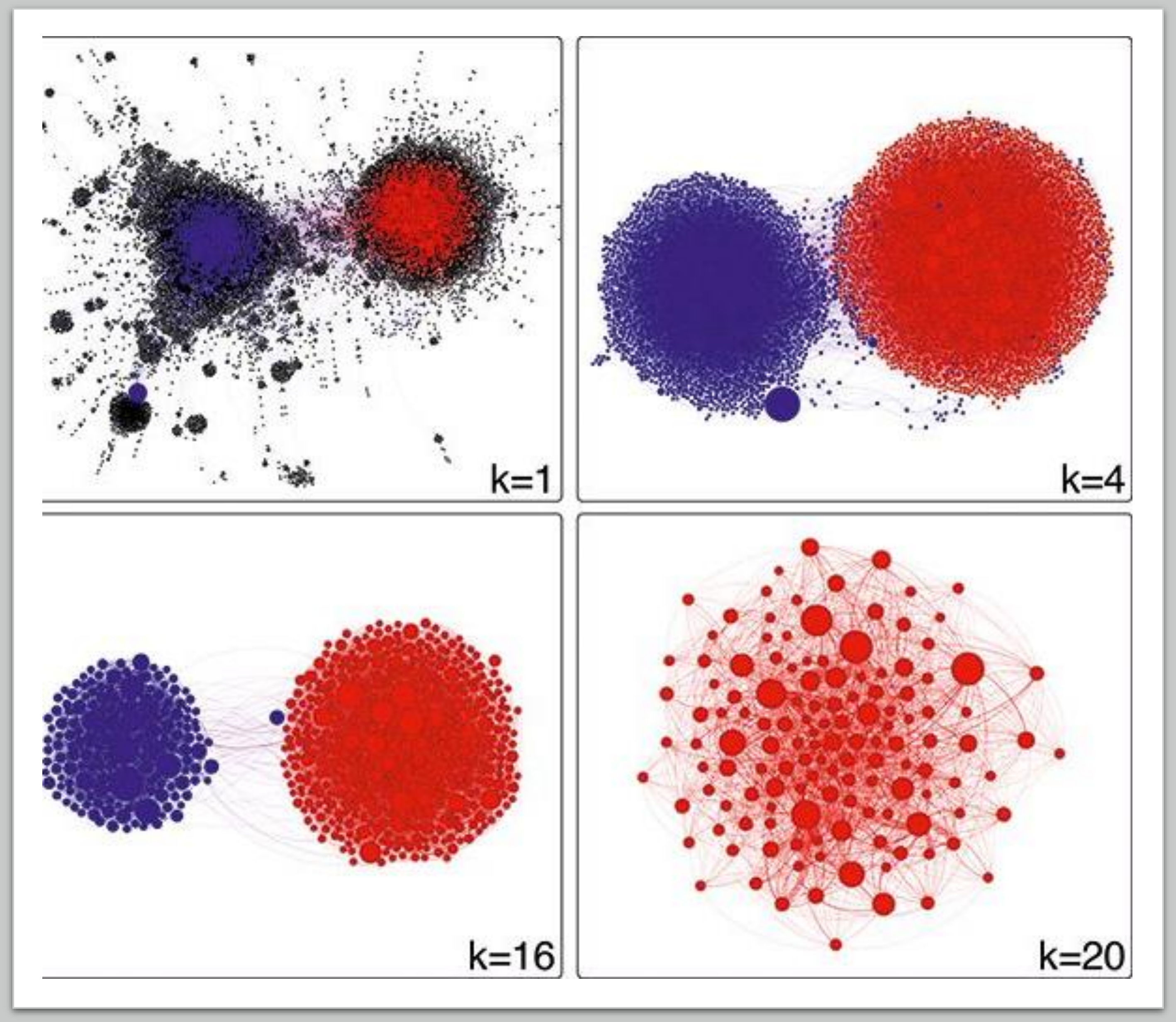
- Formally, the k-core decomposition algorithm starts by setting k=1 Then it proceeds iteratively. Each iteration corresponds to a value of k and consists of a few simple steps:
- 1. Recursively remove all nodes of degree k, until there are no more left.
- 2. The removed nodes make up the k-shell and the remaining nodes make up the -core, because they all have degree or larger.
- 3. If there are no nodes left in the core, terminate; else, increment k for the next iteration.



Source: Malliaros, Fragkiskos & Giatsidis, Christos &Papadopoulos, Apostolos &; Vazirgiannis, Michalis. (2020). The Core Decomposition of Networks: Theory, Algorithms and Applications. The VLDB Journal. 29. 10.1007/s00778-019-00587-4.

3.6: Core Decomposition

- Image visualizes the Twitter political retweet network
- As k increases, peripheral nodes are removed and core becomes smaller and denser
- The innermost core contains only red nodes, corresponding to conservative accounts.



Core decomposition in networkx

```
nx.core_number(G) # return dict with core number of each node
nx.k_shell(G,k)   # subnetwork induced by nodes in k-shell
nx.k_core(G,k)    # subnetwork induced by nodes in k-core
nx.k_core(G)      # innermost (max-degree) core subnetwork
```


The End