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Network Science Chapter 5.5

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Amsterdam, 09/05/2021

Other Preferential Models:

1. Non-linear BA
2. Attractiveness model
3. Fitness Model
4. Random walk model
5. Copy model
6. Rank model

Non-linear BA model

$$\Pi_\alpha(i \leftrightarrow j) = \frac{k_j^\alpha}{\sum_l k_l^\alpha},$$

What happens when $\alpha < 1$?
What happens when $\alpha > 1$?

When $\alpha < 1$, link probability does not grow with degree as in the BA model, the degree doesn't have a heavy tail, which means the hubs disappear.

When $\alpha > 1$, high degree nodes accumulate new links much faster than other nodes.
When $\alpha > 2$, we get a winner-takes-all effect, one node ends up connected to all other nodes, other nodes have the same, low degree.

- Strict dependency on linear preferential attachment appears unrealistic
- BA model yields a fixed pattern for degree distribution for any choice of model parameters. Real degree distributions could decay faster or more slowly
- The hubs are always the oldest nodes, new nodes cannot overcome them in degree
- It does not create many triangles, average clustering coefficient is much lower than in many real networks
- Nodes and links are only added, in real networks they can also be deleted
- Each node is attached to older nodes, therefore the network consists of only a single connected component. Many real networks have multiple components.

Attractiveness Model

$$\Pi(i \leftrightarrow j) = \frac{A + k_j}{\sum_l (A + k_l)},$$

- In BA model, single node network without links will never get neighbors
- Attractiveness model assigns constant value for intrinsic attractiveness

- Model results in heavy tailed degree distributions, slope depends on choice of constant A

$$\Pi(i \leftrightarrow j) = \frac{\eta_j k_j}{\sum_l \eta_l k_l}.$$

- In previous models, the hubs will always be the oldest nodes
- *Can you think of real world examples where this doesn't hold?*
- Nodes might have their own individual appeal that is only partly and indirectly expressed by degree.
- In the Fitness model, every node has its own individual appeal, or *fitness*.

- If fitness distribution $\rho(\eta)$ has *infinite support* (η can take arbitrarily large values), we get a winner-takes-all effect
- If fitness distirbution $\rho(\eta)$ has *finite support* (η has a finite maximum value), we end up with a heavy tailed degree distribution

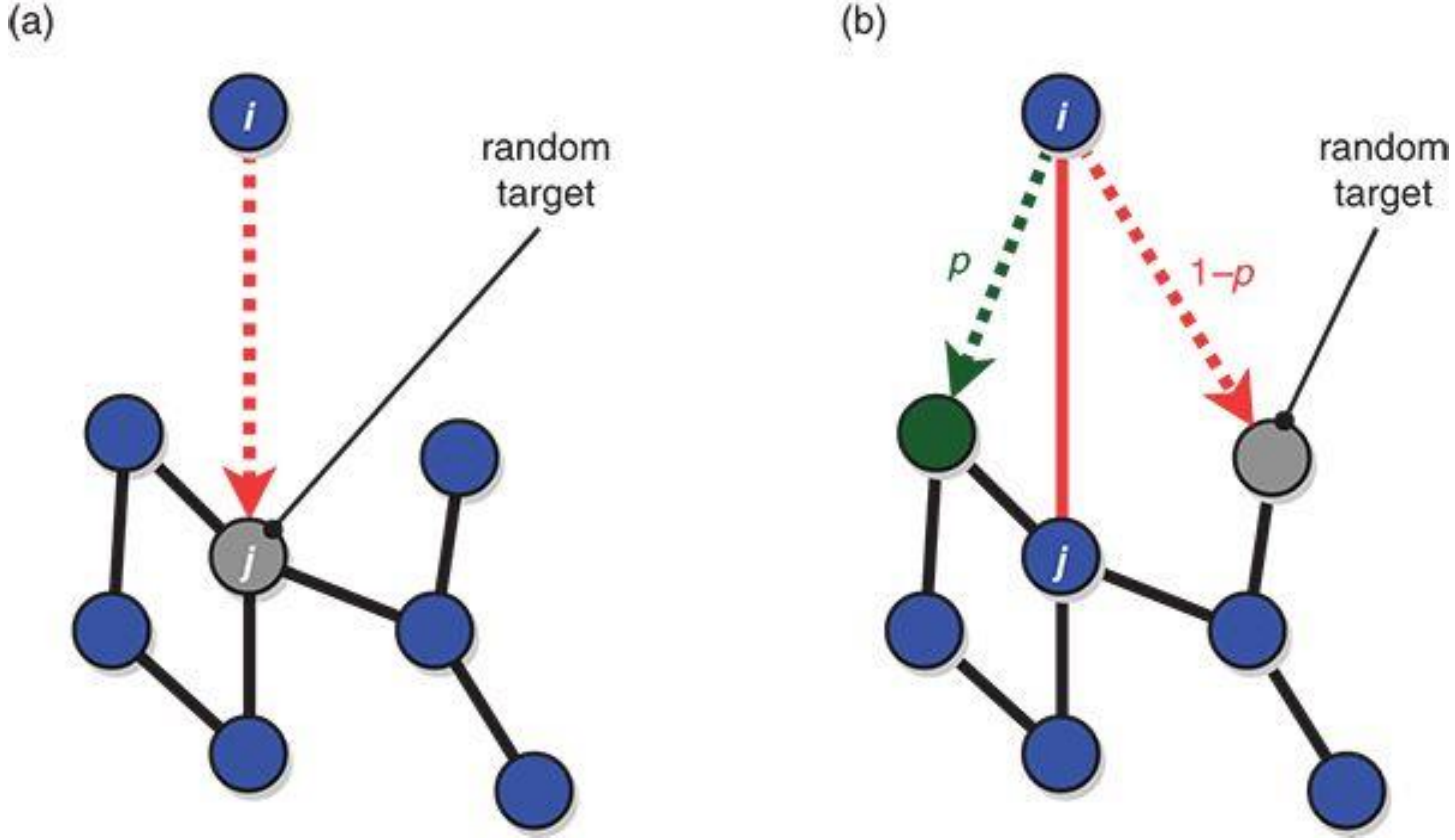
- BA model has very low clustering coefficients
- Random walk model provides mechanism that favors *triadic closures*
- In addition to creating random connections, we also connect to a new neighbor's neigbors

The random walk model can start from any small network. Each iteration of the algorithm consists of the following steps:

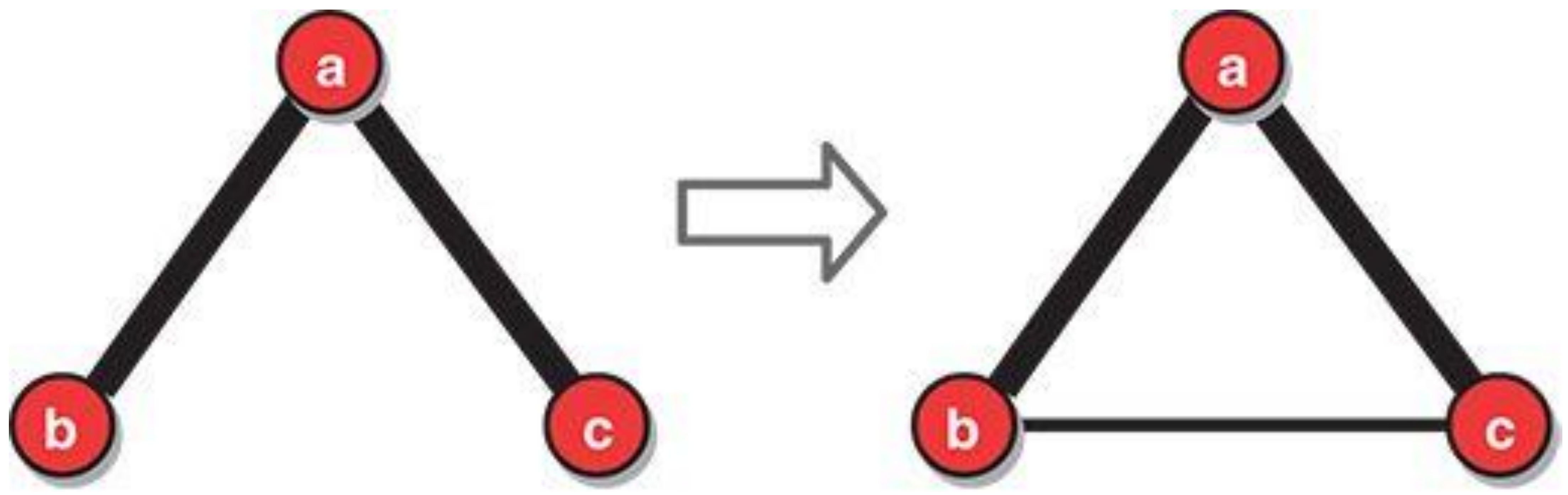
1. A new node *i* is added to the network, with new links attached to it.
2. The first link is wired to an old node j, chosen at random.
3. Each other link is attached to a randomly selected neighbor of *j*, with probability *p*, or to another randomly selected node, with probability *1-p*

The parameter p is the probability of triadic closure, because by setting a link between *i* and a neighbor of *j*, say *l*, we close the triangle *(i, j, l)*. If *p=0*, there is no triadic closure and new nodes choose their neighbors entirely at random. When *p=1* all links except the first one are wired to neighbors of the initially selected old node, thus closing triangles.

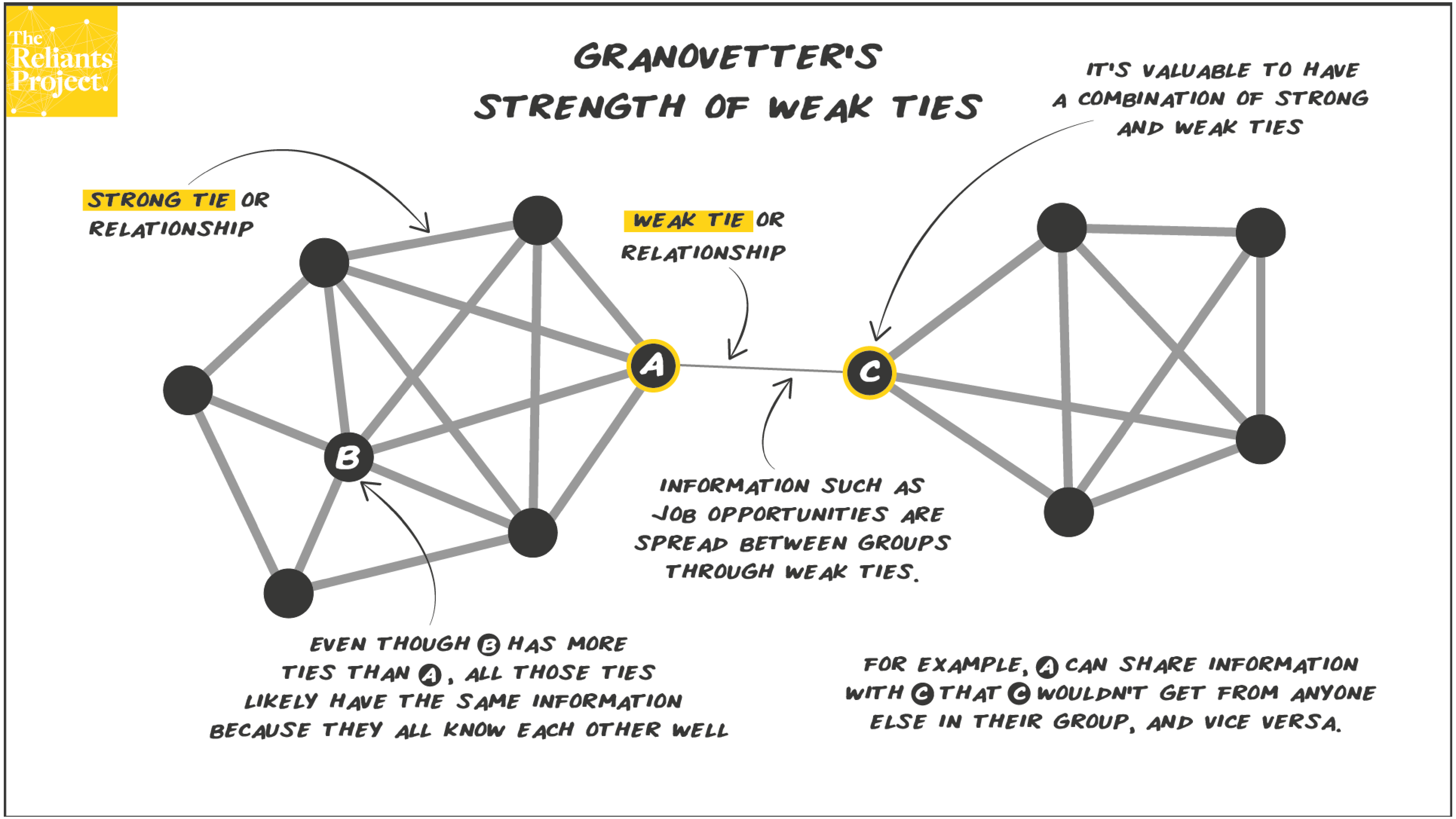
Random Walk model



- In 1973, socioligist Mark S. Granovetter published a paper entitled "The Strength of weak ties", which would become the most cited article in sociology
- Granovetter introduced principle of *strong triadic closure* for link formation in social networks
- If a has strong link with b and c, it is likely that b and c are, or will become, friends.



Triadic Closure and The strength of weak ties



- *Suppose that we wish to build a network such that there is a relevant amount of squares. Based on what you learned about triadic closure, could you suggest a mechanism that incentivises the formation of squares?*

- In social networks, triadic closure implies that an individual *copies* the contacts of someone else
- Scientists often discover new articles in the list of references of publications they read and cite them in their own papers
- While browsing online, web content creators might discover relevant web pages that provide lists of resources. They then copy the hyperlinks leading to these resources to their own newly created pages

- The *copy model* captures these scenarios.
- Similar to random walk model

- A new node gets wired to one of the following:
  - Randomly selected old node
  - One of the neigbors of the randomly selected node
- No triadic closure in copy model

- In realistic settings, it is more common to have a perception of the *relative value* rather than the absolute value.
- Rank model keeps node ranked by one of their properties, e.g. degree
- Probability of node to receive links is proportional to some invers power of their rank

The rank model can start from any small graph with nodes. A node property, such as the degree, age, or some measure of fitness is selected to rank the nodes. Each iteration of the algorithm consists of the following steps:

1. All nodes are ranked based on the property of interest. Nodes are assigned ranks *R=1,2* etc. Node *l* receives rank R_l.
2. A new node i is added to the network, with $m \leftarrow m_0$ new links attached to it.
3. Each new link from *i* is wired to an old node *j* with probability

$$\Pi(i \leftrightarrow j) = \frac{R_j^{-\alpha}}{\sum_l R_l^{-\alpha}},$$

where the exponent $\alpha > 0$ is a parameter.

Nodes may have to be re-ranked at each iteration if the ranking property depends on the links from new nodes joining the network, as happens for example when nodes are ranked by degree.

Let’s stay in touch

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