Optimization and Julia Analytics SIG

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Brief intro to math. optimization

- Given a set of **options**, find the **best**

$$x^* = \arg\min \{ f(x) | \, x \in \Omega \}$$

- The set of options is the **feasible set**
- The function to measure the **best** is the **objective function**
- **Modeling** and **Solving** are the main branches

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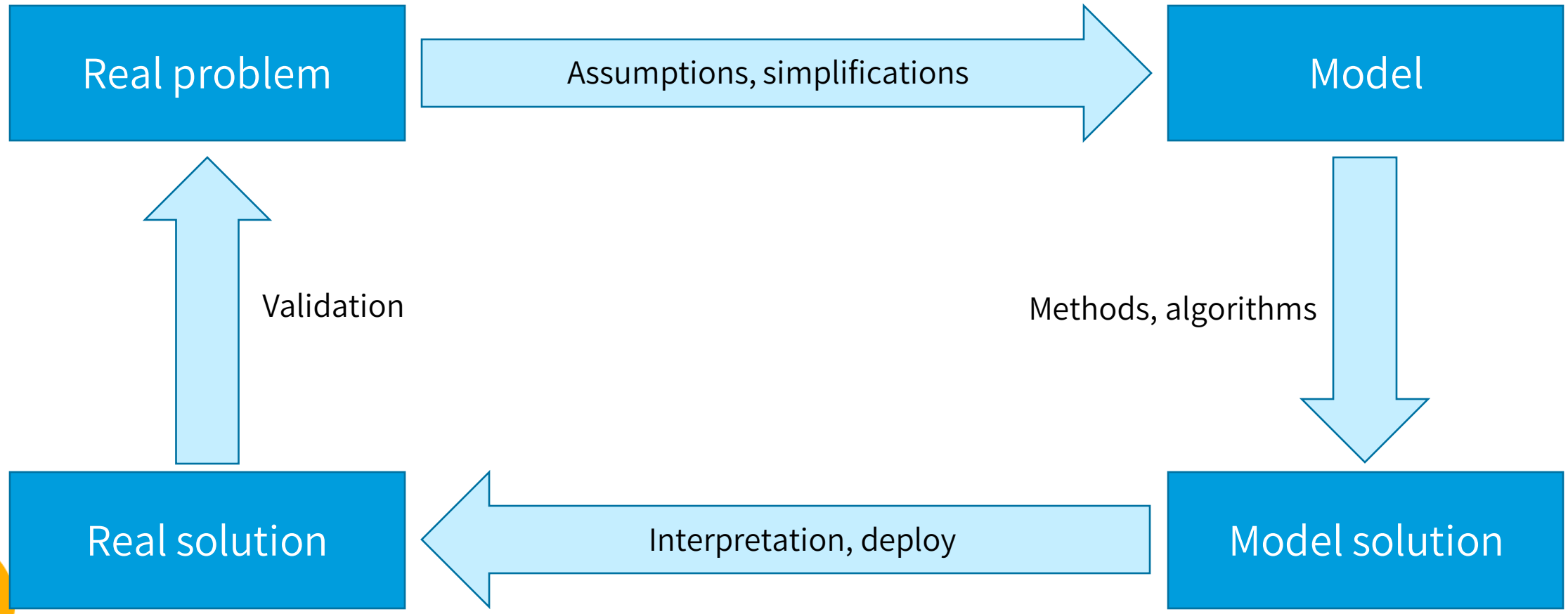
Brief intro to math. optimization



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Brief intro to math. optimization

- A **solver** is a software that solves the problem (approx)
- **Method** is the definition of the sequence
- Solver = implementation

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Brief intro to math. optimization

- I supervised and consulted on **Modeling**
  - Final term papers (Course conclusion work)
  - Software part of a paper
  - Consulting
- My research was mainly **Solver Development**
  - Theory and implementation
  - Framework for solver development

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Modeling

- Modeling = describe a problem with **math**
  - Jobs in decision support – highly requested professional
  - Research involves creating new/better models or new modeling tricks
- Modeling software = conversion from **math** to **solver input**
  - AMPL, GAMS, AIMMS ||| PuLP (python), JuMP (julia)
- Solvers (Gurobi, CPLEX, SCIP, Cbc/Clp, GLPK, Ipopt, Knitro)

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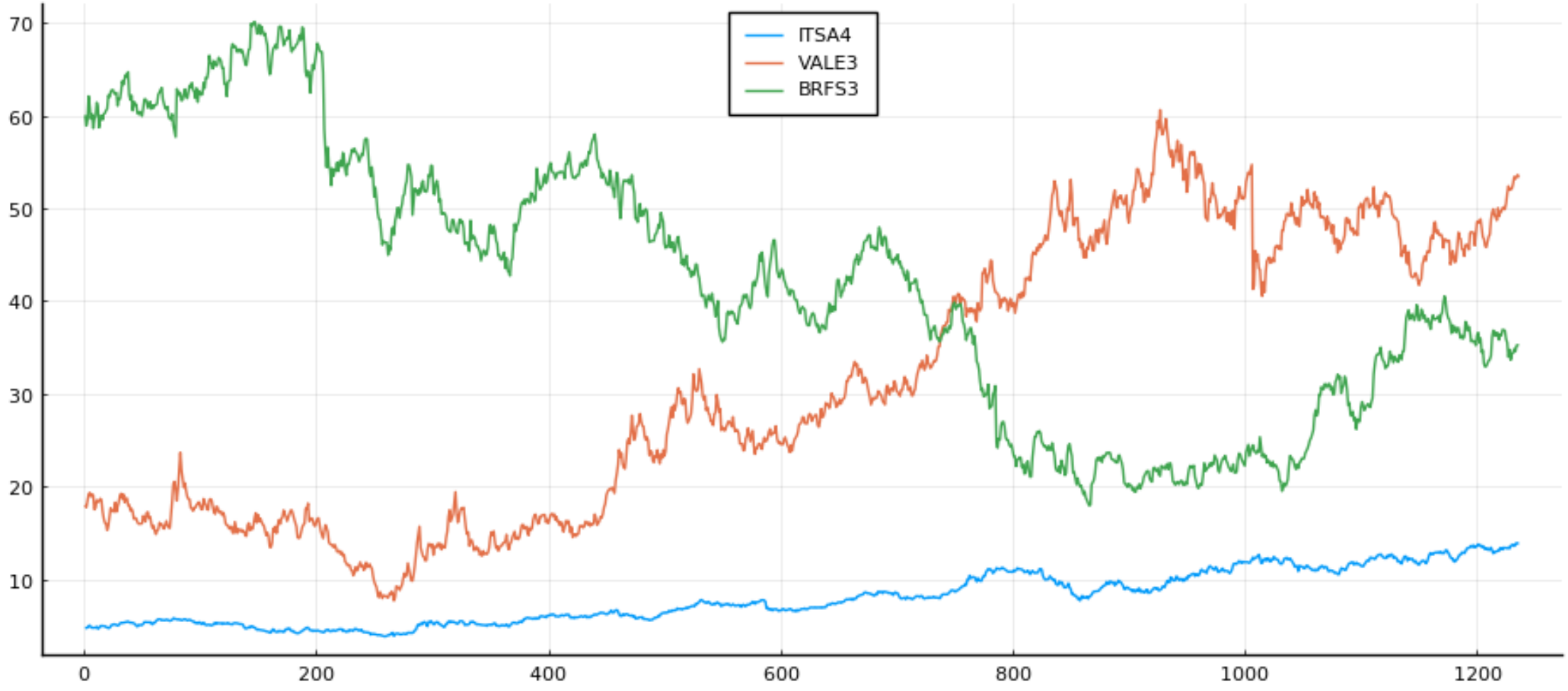
EX.: Portfolio Management (Fillipe Pierin, Aline Xavier)



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Ex.: Portfolio Management (Fillipe Pierin, Aline Xavier)

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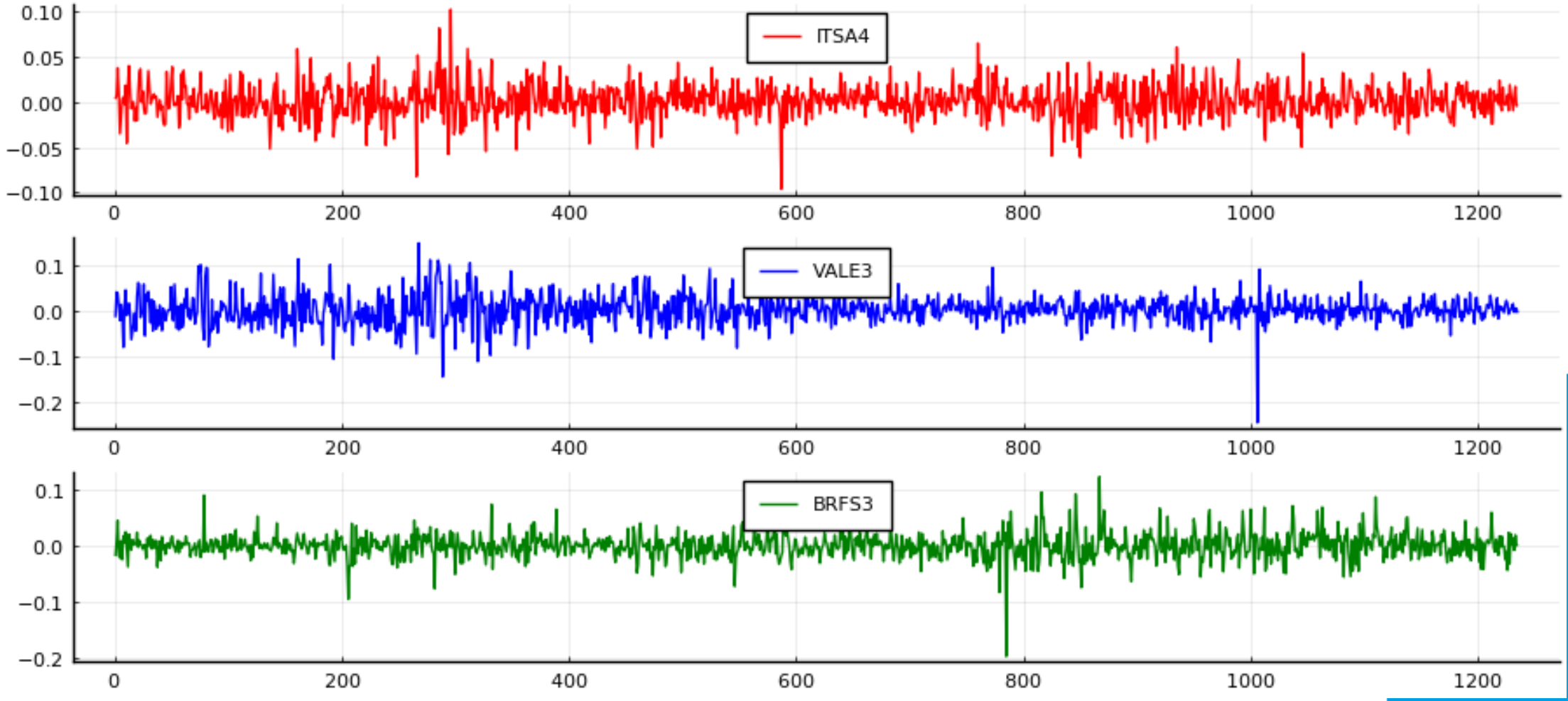

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Markowitz Mean-Variance model

$$x_i = \%\ \text{invested in asset}\ i$$

$$\min \text{risk}(x) \text{ and } \max \text{return}(x)$$

$$\text{return}(x) = x^T \mu$$

$$\text{risk}(x) = x^T S x$$

μ

0.0010
0.0014
-0.0002

S

0.0003	0.0002	0.0001
0.0002	0.0010	0.0001
0.0001	0.0001	0.0005

Ex.: Portfolio Management (Fillipe Pierin, Aline Xavier)

$$
\begin{aligned}
\min_{x} \quad & x^T S x \\
\text{suj. a} \quad & \sum_i x_i = 1 \\
& \mu^T x \geq \mu_{\min} \\
& x \geq 0
\end{aligned}
$$

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```julia
function markowitz_minrisco(μ, σ; λ=0.0)
    n = length(μ)
    μmin, μmax = extrema(μ)

    model = Model(optimizer_with_attributes(Ipopt.Optimizer, "print_level" => 0))
    @variable(model, 0 ≤ x[1:n] ≤ 1)
    @objective(model, Min, dot(x, σ, x))
    @constraint(model, dot(x, μ) ≥ μmin + λ * (μmax - μmin))
    @constraint(model, sum(x) == 1)
    optimize!(model)

    return value.(x)
end
```

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Ex.: Portfolio Management (Fillipe Pierin, Aline Xavier)

$$\min_{x,y} \quad x^T S x$$

$$\text{suj. a} \quad \sum_i x_i = 1$$

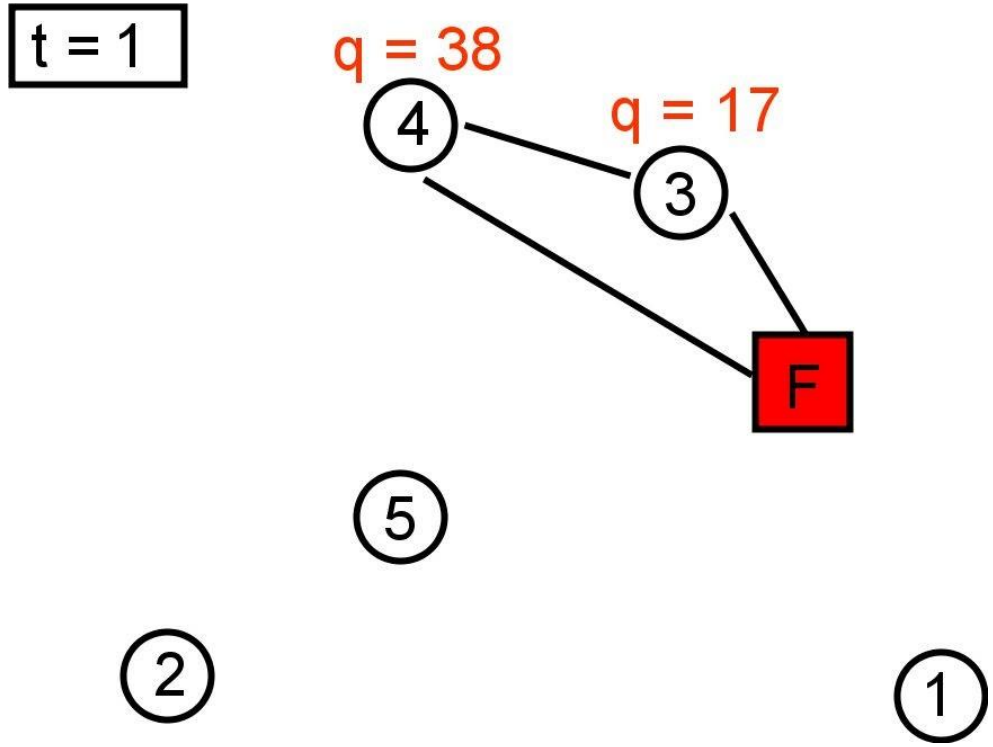
$$\mu^T x \geq \mu_{\min}$$

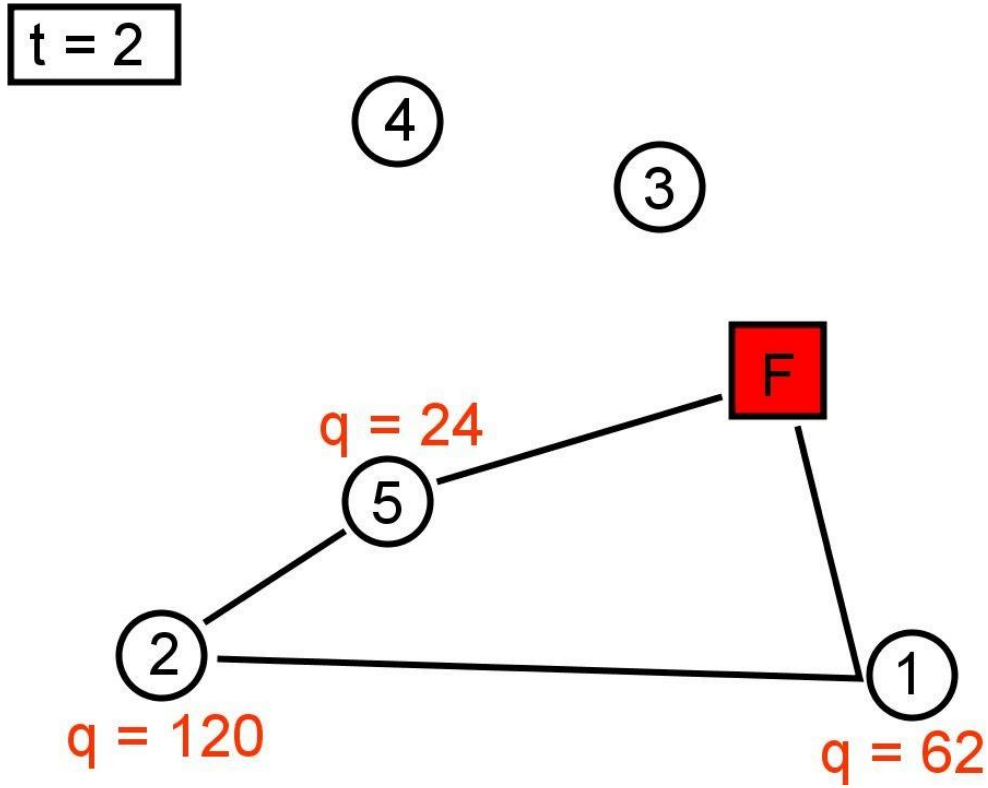
$$0.05 y_i \leq x_i \leq y_i$$

$$5 \leq \sum_i y_i \leq 8$$

$$x \geq 0, y_i \in \{0, 1\}$$

Ex.: Vehicle Routing Problem (Thaiza Rievrs)





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Ex.: Vehicle Routing Problem (Thaiza Rievrs)

$$\min_{x,y,q,I} \sum_{i,t} h_i I_i^t + \sum_{i<j,k,t} c_{ij} x_{ij}^{kt}$$

$$I_0^t = I_0^{t-1} + r^t - \sum_{k,i} q_i^{kt}$$

$$I_i^t = I_i^{t-1} + \sum_k q_i^{kt} - d_i^t$$

$$\sum_k q_i^{kt} \leq C_i - I_i^{t-1}$$

$$C_i y_i^{kt} - I_i^{t-1} \leq q_i^{kt} \leq C_i y_i^{kt}$$

$$\sum_i q_i^{kt} \leq Q_k y_0^{kt}$$

$$\sum_{i<j} x_{ij}^{kt} + \sum_{i>j} x_{ij}^{kt} = 2y_i^{kt}$$

$$\sum_{i<j} x_{ij}^{kt} \leq \sum_i y_i^{kt} - y_m^{kt}$$

$$0 \leq I_i^t \leq C_i, q_i^{kt} \geq 0$$

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Solver Development Pipeline

- Format
- Information
- Derivatives
- Special characteristics

- Theory
- Numerical Linear Algebra

- Against what?
- Problem suite
- Tables
- *Performance Profiles*

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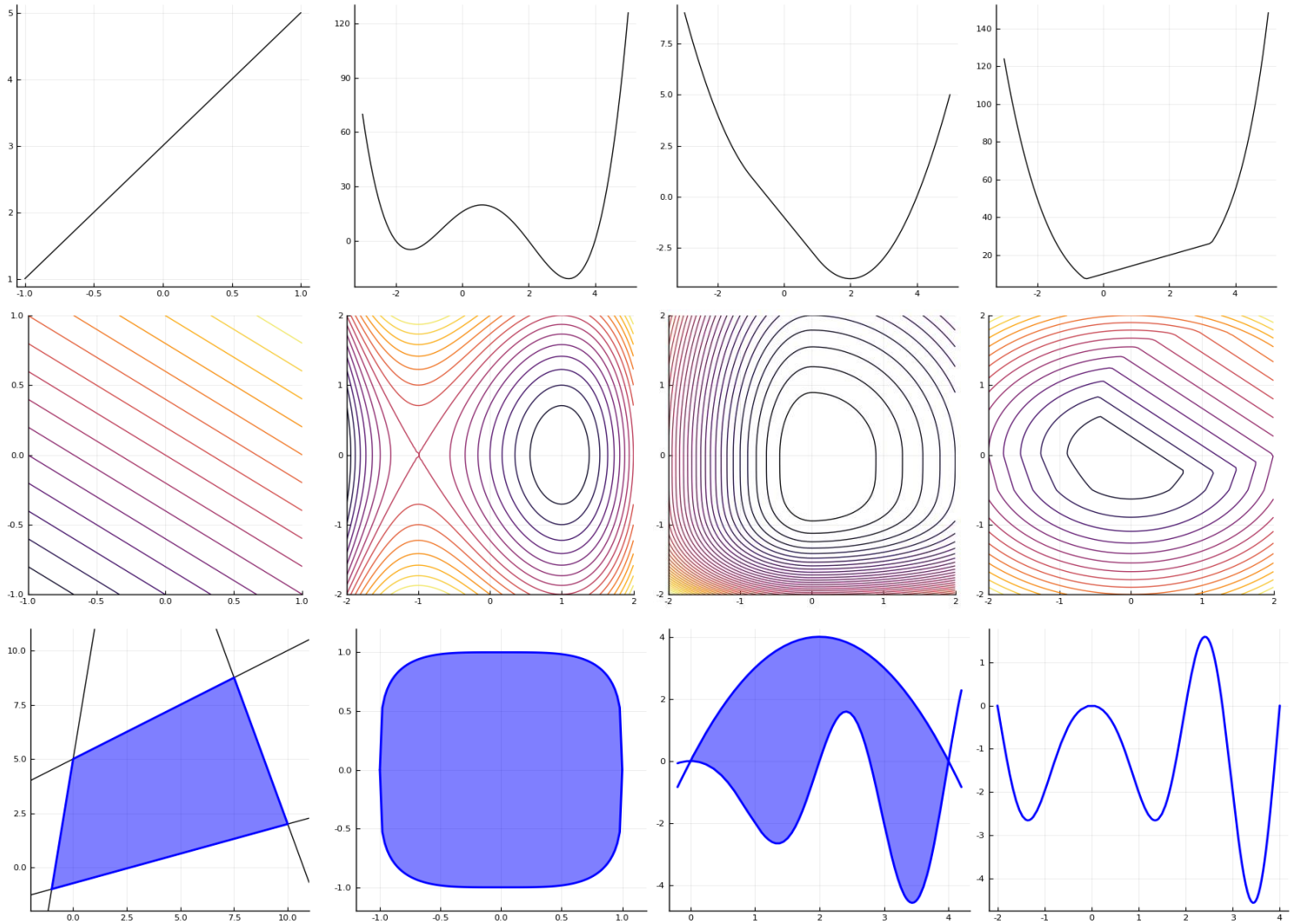
NLP = Nonlinear Programming

$$
\begin{aligned}
\min_{x \in \mathbb{R}^n} \quad & f(x) \\
\text{subject to} \quad & c_E(x) = 0 \\
& c_I(x) \leq 0 \\
& \ell \leq x \leq u
\end{aligned}
$$

where

$$
\begin{aligned}
f &: \mathbb{R}^n \to \mathbb{R} \\
c_E &: \mathbb{R}^n \to \mathbb{R}^{m_E} \\
c_I &: \mathbb{R}^n \to \mathbb{R}^{m_I}
\end{aligned}
$$



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What is research in NLP

- Convergence (Does it? When? How fast? Complexity?)
- Different structures (e.g., Riemannian manifolds)
- Specialization of methods for given problems

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What was my research in NLP

- New methods (+modifications/extensions)
  - Some theory
  - At least prototyping and benchmarking
- Better implementations of existing methods
  - Benchmark is necessary

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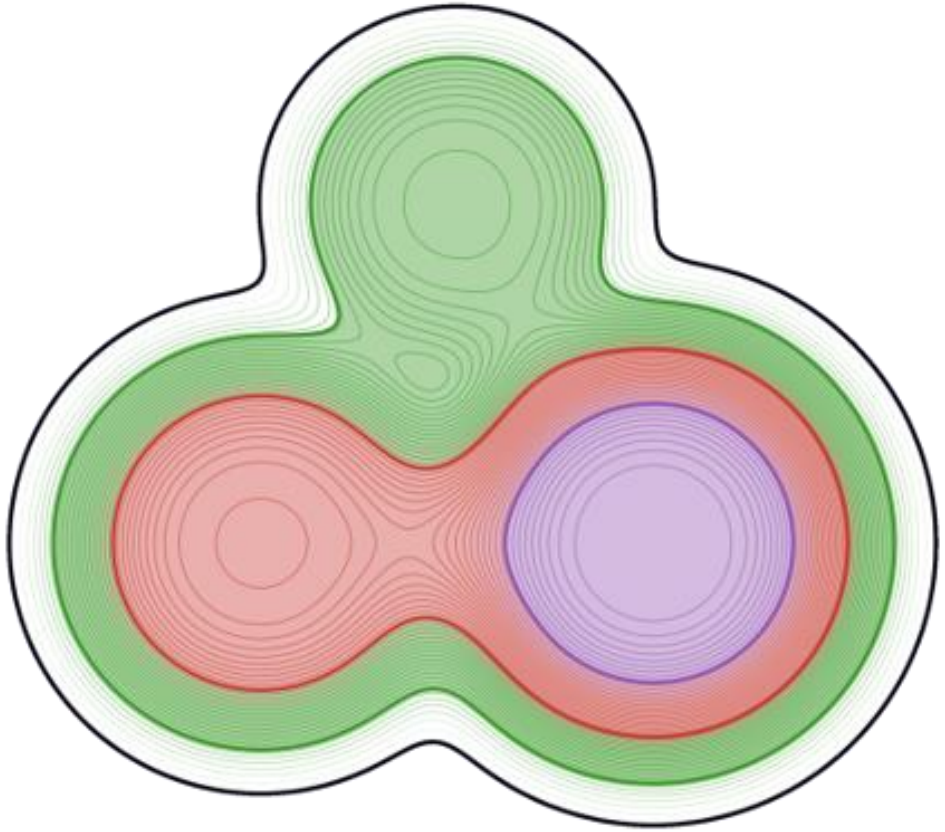
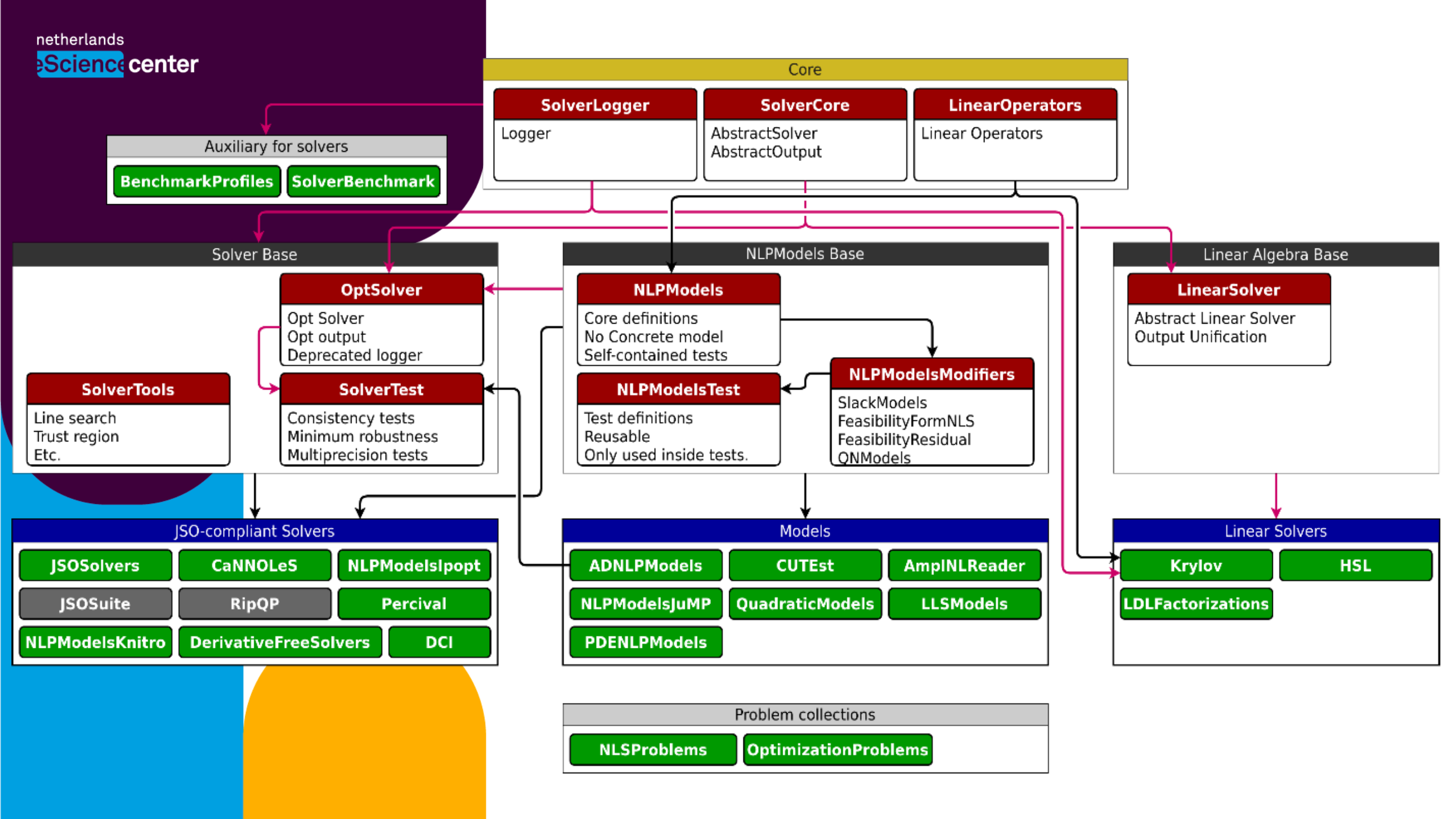
What was my research in NLP

- **Julia Language**
  - Free software
  - Fortran/C interface
  - Speed
  - Numerical LinearAlgebra
- **JuliaSmoothOptimizers**
  - Prototype easier, follow good practices, scale well

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Core
SolverLogger
Logger
SolverCore
AbstractSolver
AbstractOutput
LinearOperators
Linear Operators
Auxiliary for solvers
BenchmarkProfiles
SolverBenchmark
Solver Base
OptSolver
Opt Solver
Opt output
Deprecated logger
SolverTools
Line search
Trust region
Etc.
SolverTest
Consistency tests
Minimum robustness
Multiprecision tests
NLPModels Base
NLPModels
Core definitions
No Concrete model
Self-contained tests
NLPModelsTest
Test definitions
Reusable
Only used inside tests.
NLPModelsModifiers
SlackModels
FeasibilityFormNLS
FeasibilityResidual
QNModels
Linear Algebra Base
LinearSolver
Abstract Linear Solver
Output Unification
JSO-compliant Solvers
JSOSolvers
CaNNOLeS
NLPModelsIpopt
JSOSuite
RipQP
Percival
NLPModelsKnitro
DerivativeFreeSolvers
DCI
Models
ADNLPModels
CUTEst
AmplNLReader
NLPModelsJuMP
QuadraticModels
LLSModels
PDENLPModels
Linear Solvers
Krylov
HSL
LDLFactorizations
Problem collections
NLSProblems
OptimizationProblems

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Streamlined Workflow

- **Prototype** fast
  - JSO-compliant
  - Test with manual problems (easy to write)
  - Access known problem sets
  - Compare against solvers
    - Native Julia
    - Classic wrapped solvers

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Streamlined Workflow

- **Improve** the code
  - Linear algebra
    - *Matrix-free* tools
    - Factorizations
    - Solvers for linear systems and least squares problems
  - Auxiliary tools
    - Subsolvers, loggers, etc.

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Streamlined Workflow

- **Publish**
  - Results available as DataFrames
  - Easily convert to Markdown tables or LaTeX tables
  - Create *Performance Profiles*
  - Package is ready for use

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JSO-compliant solver example

- Percival (https://github.com/JuliaSmoothOptimizers/Percival.jl)
- Egmara Antunes' Master's dissertation

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Thank you!


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