

# Permissions

# Values

- Change permissions, write top value (left child + right child)
- 1|2 means  $\frac{1}{2}$  permission for both

## Upsweep - Initial

|   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 |
| 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 |
| 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 |
| 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 |

|   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|
|   |   |   |   |   |   |   |   |
|   |   |   |   |   |   |   |   |
|   |   |   |   |   |   |   |   |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |

## Upsweep - First row

|     |     |     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|-----|-----|
| 1   | 2   | 3   | 4   | 1   | 2   | 3   | 4   |
| 1   | 2   | 3   | 4   | 1   | 2   | 3   | 4   |
| 1   | 2   | 3   | 4   | 1   | 2   | 3   | 4   |
| 1 1 | 1 1 | 1 2 | 1 2 | 2 3 | 2 3 | 2 4 | 2 4 |

|   |   |    |    |   |   |   |   |
|---|---|----|----|---|---|---|---|
|   |   |    |    |   |   |   |   |
|   |   |    |    |   |   |   |   |
| 3 | 7 | 11 | 15 |   |   |   |   |
| 1 | 2 | 3  | 4  | 5 | 6 | 7 | 8 |

## Upsweep - Second row

|     |     |     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|-----|-----|
| 1   | 2   | 3   | 4   | 1   | 2   | 3   | 4   |
| 1   | 2   | 3   | 4   | 1   | 2   | 3   | 4   |
| 1 1 | 1 1 | 1 2 | 1 2 | 2 3 | 2 3 | 2 4 | 2 4 |
| 1 1 | 1 1 | 1 2 | 1 2 | 2 3 | 2 3 | 2 4 | 2 4 |

|    |    |    |    |   |   |   |   |
|----|----|----|----|---|---|---|---|
|    |    |    |    |   |   |   |   |
| 10 | 26 | ~  | ~  |   |   |   |   |
| 3  | 7  | 11 | 15 |   |   |   |   |
| 1  | 2  | 3  | 4  | 5 | 6 | 7 | 8 |

## Upsweep - Third row

|     |     |     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|-----|-----|
| 1   | 2   | 3   | 4   | 1   | 2   | 3   | 4   |
| 1 1 | 1 1 | 1 2 | 1 2 | 2 3 | 2 3 | 2 4 | 2 4 |
| 1 1 | 1 1 | 1 2 | 1 2 | 2 3 | 2 3 | 2 4 | 2 4 |
| 1 1 | 1 1 | 1 2 | 1 2 | 2 3 | 2 3 | 2 4 | 2 4 |

|    |    |    |    |   |   |   |   |
|----|----|----|----|---|---|---|---|
| 36 | ~  | ~  | ~  |   |   |   |   |
| 10 | 26 | ~  | ~  |   |   |   |   |
| 3  | 7  | 11 | 15 |   |   |   |   |
| 1  | 2  | 3  | 4  | 5 | 6 | 7 | 8 |

## Permissions

- Write right child (top+left), write left child (top), change permissions
- \old value of left child is required for the loop invariant (it is overwritten).

## Values

### Downsweep - Preparation

|     |     |     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|-----|-----|
| 1   | 2   | 3   | 4   | 1   | 2   | 3   | 4   |
| 1 1 | 1 1 | 1 2 | 1 2 | 2 3 | 2 3 | 2 4 | 2 4 |
| 1 1 | 1 1 | 1 2 | 1 2 | 2 3 | 2 3 | 2 4 | 2 4 |
| 1 1 | 1 1 | 1 2 | 1 2 | 2 3 | 2 3 | 2 4 | 2 4 |

|    |    |    |    |   |   |   |   |
|----|----|----|----|---|---|---|---|
| 0  | 0  | 0  | 0  |   |   |   |   |
| 10 | 26 | ~  | ~  |   |   |   |   |
| 3  | 7  | 11 | 15 |   |   |   |   |
| 1  | 2  | 3  | 4  | 5 | 6 | 7 | 8 |

### Downsweep - First row

|     |     |     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|-----|-----|
| 1   | 2   | 3   | 4   | 1   | 2   | 3   | 4   |
| 1   | 2   | 3   | 4   | 1   | 2   | 3   | 4   |
| 1 1 | 1 1 | 1 2 | 1 2 | 2 3 | 2 3 | 2 4 | 2 4 |
| 1 1 | 1 1 | 1 2 | 1 2 | 2 3 | 2 3 | 2 4 | 2 4 |

|   |    |    |    |   |   |   |   |
|---|----|----|----|---|---|---|---|
| 0 | 0  | 0  | 0  |   |   |   |   |
| 0 | 10 | ~  | ~  | ~ | ~ | ~ | ~ |
| 3 | 7  | 11 | 15 |   |   |   |   |
| 1 | 2  | 3  | 4  | 5 | 6 | 7 | 8 |

### Downsweep - Second row

|     |     |     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|-----|-----|
| 1   | 2   | 3   | 4   | 1   | 2   | 3   | 4   |
| 1   | 2   | 3   | 4   | 1   | 2   | 3   | 4   |
| 1   | 2   | 3   | 4   | 1   | 2   | 3   | 4   |
| 1 1 | 1 1 | 1 2 | 1 2 | 2 3 | 2 3 | 2 4 | 2 4 |

|   |    |    |    |   |   |   |   |
|---|----|----|----|---|---|---|---|
| 0 | 0  | 0  | 0  |   |   |   |   |
| 0 | 10 | ~  | ~  | ~ | ~ | ~ | ~ |
| 0 | 3  | 10 | 21 | ~ | ~ | ~ | ~ |
| 1 | 2  | 3  | 4  | 5 | 6 | 7 | 8 |

### Downsweep - Third row

|   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 |
| 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 |
| 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 |
| 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 |

|   |    |    |    |    |    |    |    |
|---|----|----|----|----|----|----|----|
| 0 | 0  | 0  | 0  |    |    |    |    |
| 0 | 10 | ~  | ~  | ~  | ~  | ~  | ~  |
| 0 | 3  | 10 | 21 | ~  | ~  | ~  | ~  |
| 0 | 1  | 3  | 6  | 10 | 15 | 21 | 28 |



