Repairing solutions

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1 Introduction

There are several approaches for including constraints into heuristics, see Chapter 12 of Gilli et al. [2011]. The notes in this vignette give some examples for simple repair mechanisms. These can be called in `DEopt`, `GAopt` and `PSopt` through the repair function; in `LSopt`/`TAopt`, they could be included in the neighbourhood function.

```
> set.seed(112233)
> options(digits = 3)
```

2 Upper and lower limits

Suppose the solution `x` is to satisfy `all(x >= lo)` and `all(x <= up)`, with `lo` and `up` being vectors of `length(x)`.

2.1 Setting values to the boundaries

One strategy is to replace elements of `x` that violate a constraint with the boundary value. Such a repair function can be implemented very concisely. An example:

```
> up <- rep(1, 4L)
> lo <- rep(0, 4L)
> x <- rnorm(4L)
> x
```

```
[1]  2.127 -0.380  0.167  1.600
```

Three of the elements of `x` actually violate the constraints.

```
> repair1a <- function(x,up,lo) pmin(up,pmax(lo,x))
> x
```

```
[1]  2.127 -0.380  0.167  1.600
```

```
> repair1a(x, up, lo)
```

```
[1] 1.000 0.000 0.167 1.000
```

We see that indeed all values greater than `1` are replaced with `1`, and those smaller than `0` become `0`. Two other possibilities that achieve the same result:

```
> repair1b <- function(x, up, lo) {
     ii <- x > up
     x[ii] <- up[ii]
     ii <- x < lo
     x[ii] <- lo[ii]
     x
```

```
}
> repair1c <- function(x, up, lo) {
     xadjU <- x - up
     xadjU <- xadjU + abs(xadjU)
     xadjL <- lo - x
     xadjL <- xadjL + abs(xadjL)
     x - (xadjU - xadjL)/2
}
```

The function `repair1b` uses comparisons to replace only the relevant elements in `x`. The function `repair1c` uses the 'trick' that

$$\mathtt{pmax}(x,y) = \frac{x+y}{2} + \left|\frac{x-y}{2}\right|,$$
$$\mathtt{pmin}(x,y) = \frac{x+y}{2} - \left|\frac{x-y}{2}\right|.$$

```
> repair1a(x, up, lo)
```

```
[1] 1.000 0.000 0.167 1.000
```

```
> repair1b(x, up, lo)
```

```
[1] 1.000 0.000 0.167 1.000
```

```
> repair1c(x, up, lo)
```

```
[1] 1.000 0.000 0.167 1.000
```

```
> trials <- 10000L
> strials <- seq_len(trials)
> system.time(for(i in strials) y1 <- repair1a(x, up, lo))
```

```
   user  system elapsed
  0.184   0.000   0.185
```

```
> system.time(for(i in strials) y2 <- repair1b(x, up, lo))
```

```
   user  system elapsed
  0.072   0.000   0.070
```

```
> system.time(for(i in strials) y3 <- repair1c(x, up, lo))
```

```
   user  system elapsed
   0.04    0.00    0.04
```

The third of these functions would also work on matrices if `up` or `lo` were scalars.

```
> X <- array(rnorm(25L), dim = c(5L, 5L))
> X
```

```
        [,1]  [,2]   [,3]    [,4]   [,5]
[1,]  0.1962 0.434 -2.155 -1.5881 -1.029
[2,]  0.2284 1.231  0.975  0.0682  1.818
[3,] -1.1492 0.580 -0.711 -0.4457 -1.315
[4,] -0.0712 0.246  0.628  1.4662  0.511
[5,] -0.5619 0.388 -0.136 -0.8412  1.337
```

```
> repair1c(X, up = 0.5, lo = -0.5)
```

```
        [,1]  [,2]   [,3]    [,4] [,5]
[1,]  0.1962 0.434 -0.500 -0.5000 -0.5
[2,]  0.2284 0.500  0.500  0.0682  0.5
[3,] -0.5000 0.500 -0.500 -0.4457 -0.5
[4,] -0.0712 0.246  0.500  0.5000  0.5
[5,] -0.5000 0.388 -0.136 -0.5000  0.5
```

The considerable speedup comes at a price, of course, since there is no checking (eg, for `NA` values) in `repair1b` and `repair1c`. We could also define new functions `pmin2` and `pmax2`.

```
> pmax2 <- function(x1, x2) ( (x1 + x2) + abs(x1 - x2) ) / 2
> pmin2 <- function(x1, x2) ( (x1 + x2) - abs(x1 - x2) ) / 2
```

A test follows.

```
> x1 <- rnorm(100L)
> x2 <- rnorm(100L)
> t1 <- system.time(for (i in strials) z1 <- pmax(x1,x2) )
> t2 <- system.time(for (i in strials) z2 <- pmax2(x1,x2))
> t1[[3L]]/t2[[3L]] ## speedup
```

```
[1] 4
```

```
> all.equal(z1, z2)
```

```
[1] TRUE
```

```
> t1 <- system.time(for (i in strials) z1 <- pmin(x1,x2) )
> t2 <- system.time(for (i in strials) z2 <- pmin2(x1,x2))
> t1[[3L]]/t2[[3L]] ## speedup
```

```
[1] 3.94
```

```
> all.equal(z1, z2)
```

```
[1] TRUE
```

One downside of this repair mechanism is that a solution may quickly become stuck at the boundaries (but of course, in some cases this is exactly what we want).

2.2 Reflecting values into the feasible range

The function `repair2` reflects a value that is too large or too small around the boundary. It restricts the change in a variable `x[i]` to the range `up[i] - lo[i]`.

```
> repair2 <- function(x,up,lo) {
     done <- TRUE
     e <- sum(x - up  + abs(x - up) + lo - x  + abs(lo - x))
     if (e > 1e-12) done <- FALSE
     r <- up - lo
     while (!done) {
         adjU <- x - up
         adjU <- adjU + abs(adjU)
         adjU <- adjU + r - abs(adjU - r)
```

```
        adjL <- lo - x
        adjL <- adjL + abs(adjL)
        adjL <- adjL + r - abs(adjL - r)

        x <- x - (adjU - adjL)/2
        e <- sum(x - up  + abs(x - up) + lo - x  + abs(lo - x))
        if (e < 1e-12) done <- TRUE
    }
    x
}
> x
```

```
[1]  2.127 -0.380  0.167  1.600
```

```
> repair2(x, up, lo)
```

```
[1] 0.873 0.380 0.167 0.600
```

```
> system.time(for (i in strials) y4 <- repair2(x,up,lo))
```

```
   user  system elapsed
  0.272   0.000   0.273
```

2.3 Adjusting a cardinality limit

Let x be a logical vector.

```
> T <- 20L
> x <- logical(T)
> x[runif(T) < 0.4] <- TRUE
> x
```

```
 [1] FALSE  TRUE  TRUE FALSE  TRUE  TRUE FALSE FALSE FALSE FALSE FALSE FALSE
[13]  TRUE  TRUE FALSE FALSE  TRUE FALSE FALSE  TRUE
```

Suppose we want to impose a minimum and maximum cardinality, `kmin` and `kmax`.

```
> kmax <- 5L
> kmin <- 3L
```

We could use an approach like the following (for the definition of `resample`, see `?sample`):

```
> resample <- function(x, ...) x[sample.int(length(x), ...)]
> repairK <- function(x, kmax, kmin) {
     sx <- sum(x)
     if (sx > kmax) {
         i <- resample(which(x), sx - kmax)
         x[i] <- FALSE
     } else if (sx < kmin) {
         i <- resample(which(!x), kmin - sx)
         x[i] <- TRUE
     }
     x
 }
```

```
> printK <- function(x)
     cat(paste(ifelse(x, "o", "."), collapse = ""),
         "-- cardinality", sum(x), "\n")
```

For kmax:

```
> for (i in 1:10) {
     if (i==1L) printK(x)
     x1 <- repairK(x, kmax, kmin)
     printK(x1)
 }
```

```
.oo.oo......oo..o..o -- cardinality 8
.oo.o.......oo...... -- cardinality 5
.o...o......oo.....o -- cardinality 5
.o..o.......o...o..o -- cardinality 5
..o.oo......o......o -- cardinality 5
.o...o......oo..o... -- cardinality 5
....oo......oo..o... -- cardinality 5
.o..oo.......o.....o -- cardinality 5
.oo..o......oo...... -- cardinality 5
.oo..o..........o..o -- cardinality 5
.oo..o......o...o... -- cardinality 5
```

For kmin:

```
> x <- logical(T); x[10L] <- TRUE
> for (i in 1:10) {
     if (i==1L) printK(x)
     x1 <- repairK(x, kmax, kmin)
     printK(x1)
 }
```

```
.........o.......... -- cardinality 1
.........oo..o...... -- cardinality 3
o........o.....o.... -- cardinality 3
...o.....o..o....... -- cardinality 3
.........o..o...o... -- cardinality 3
....o....oo......... -- cardinality 3
....o...oo.......... -- cardinality 3
.......o.o.........o -- cardinality 3
.......o.o..o....... -- cardinality 3
.....o...o...o...... -- cardinality 3
....o....o...o...... -- cardinality 3
```

References

Manfred Gilli, Dietmar Maringer, and Enrico Schumann. *Numerical Methods and Optimization in Finance*. Elsevier, 2011.