Exercise: Fitting a Log-Normal, Low-Mass IMF

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Introduction

The functional form of the Initial Mass Function (IMF) at the low-mass end ($M < 1.0\,M_\odot$) remains a subject of debate, particularly in the context of Ultra-Faint Dwarf (UFD) galaxies and resolved stellar populations. In this exercise, you will numerically investigate how the definition of the observational lower mass limit ($M_{\rm obs}$) affects the inferred power-law slope when the underlying population actually follows a log-normal distribution.

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Part 1: Numerical Simulation

Assume the underlying stellar population follows a **Chabrier-like Log-Normal IMF** defined by the distribution:

$$\xi(m) \propto \frac{1}{m} \exp\left[-\frac{(\ln m - \ln m_c)^2}{2\sigma^2}\right] \tag{1}$$

where:

- The characteristic mass $m_c = 0.20\,M_\odot$
- The width $\sigma = 0.69$

Tasks

1. **Monte Carlo Realization:** Generate a synthetic population drawn from this IMF. Ensure your sample size is sufficient to minimize Poisson noise (e.g., $N \sim 10^5 - 10^6$ stars). You may able to do this with an analytic log-normal as well, but convention in astronomy is usually to Monte Carlo the IMF.
2. **Truncation:** Truncate the dataset to simulate an observational window. The sample should only include stars with masses $M \in [M_{\rm obs}, 0.8\,M_\odot]$.
3. **Power-Law Fitting:** Although the underlying distribution is log-normal, observers often characterize the low-mass end using a single-slope power law of the form $\xi(m) \propto m^{-\alpha}$.
   - Fit this power law to your synthetic data for a varying lower mass cutoff.
   - Vary $M_{\rm obs}$ from **0.1** $M_\odot$ **to 0.6** $M_\odot$.
   - *Note: Be explicit about your fitting method (e.g., Least Squares on binned data vs. Maximum Likelihood Estimation). MLE is strongly preferred for this regime.*
4. **Plotting:** Create a figure showing the recovered power-law slope (α) on the y-axis as a function of the lower mass limit ($M_{\rm obs}$) on the x-axis.

Part 2: Comparison with Literature

(1) The UFD Context (Geha et al. 2013)

Consult *Geha et al. (2013)* regarding the IMF of Ultra-Faint Dwarf galaxies (e.g., Hercules and Leo IV).

- Extract their derived IMF slopes and the corresponding mass limits used in their analysis.
- **Overplot** these observational data points (with error bars) onto your simulation plot from Part 1. What, if any, conclusions can you draw?

(2) Comparison with El-Badry et al. 2017

Review El-Badry et al. (2017) about the effects of fitting a true log-normal IMF with a single-slope power-law. How do your findings compare with the conclusions of this paper?