Exercise: Fitting a Log-Normal, Low-Mass IMF

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Introduction

The functional form of the Initial Mass Function (IMF) at the low-mass end ($M < 1.0\,M_\odot$) remains a subject of debate, particularly in the context of Ultra-Faint Dwarf (UFD) galaxies and resolved stellar populations. In this exercise, you will numerically investigate how the definition of the observational lower mass limit ($M_{\rm obs}$) affects the inferred power-law slope when the underlying population actually follows a log-normal distribution.

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Part 1: Numerical Simulation

Assume the underlying stellar population follows a **Chabrier-like Log-Normal IMF** defined by the distribution:

$$\xi(m) \propto \frac{1}{m} \exp\left[-\frac{(\log_{10} m - \log_{10} m_c)^2}{2\sigma^2}\right] \tag{1}$$

where:

- The characteristic mass $m_c = 0.20\,M_\odot$
- The width $\sigma = 0.69$

Tasks

1. **Monte Carlo Realization:** Generate a synthetic population drawn from this IMF. Ensure your sample size is sufficient to minimize Poisson noise (e.g., $N \sim 10^5 - 10^6$ stars). You may able to do this with an analytic log-normal as well, but convention in astronomy is usually to Monte Carlo the IMF.
2. **Truncation:** Truncate the dataset to simulate an observational window. The sample should only include stars with masses $M \in [M_{\rm obs}, 0.8\,M_\odot]$.
3. **Power-Law Fitting:** Although the underlying distribution is log-normal, observers often characterize the low-mass end using a single-slope power law of the form $\xi(m) \propto m^{-\alpha}$.
   - Fit this power law to your synthetic data for a varying lower mass cutoff.
   - Vary $M_{\rm obs}$ from **0.1** $M_\odot$ **to 0.6** $M_\odot$.
   - *Note: Be explicit about your fitting method (e.g., Least Squares on binned data vs. Maximum Likelihood Estimation). MLE is strongly preferred for this regime.*
4. **Plotting:** Create a figure showing the recovered power-law slope (α) on the y-axis as a function of the lower mass limit ($M_{\rm obs}$) on the x-axis.

Part 2: Comparison with Literature

(1) The UFD Context (Geha et al. 2013)

Consult *Geha et al. (2013)* regarding the IMF of Ultra-Faint Dwarf galaxies (e.g., Hercules and Leo IV).

- Extract their derived IMF slopes and the corresponding mass limits used in their analysis.
- **Overplot** these observational data points (with error bars) onto your simulation plot from Part 1. What, if any, conclusions can you draw?

(2) Comparison with El-Badry et al. 2017

Review El-Badry et al. (2017) about the effects of fitting a true log-normal IMF with a single-slope power-law. How do your findings compare with the conclusions of this paper?

Part 3: Design an observation

(1) JWST Observations of Hercules

Ask you AI assistant to design a JWST observing program to robustly constrain a low-mass, log-normal IMF measurement in the Hercules dwarf galaxy.

(2) Validation

Try validating the above proposed observing plan from (1) using the JWST Exposure time calculator (ETC) for the NIRCam. Does the proposed observing plan seem reasonable to you? You may try and see if your AI assistant can teach you how to use the ETC.