ECE-6123 Nikita Makarov HW2

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0.0.1 ECE-GY 6123

0.0.2 Nikita Makarov, Fall 2024

0.0.3 Homework 2

Problem 1a:

[185]:
```python
from PIL import Image
import numpy as np
%matplotlib inline
from matplotlib import pyplot as plt

# Create conv2 function
def conv2(A: np.ndarray, B: np.ndarray) -> np.ndarray:
    '''
    Compute and return the same-padded 2D convolution of two matrices A and B.

    Parameters:
    A: 2D numpy array of the first matrix
    B: 2D numpy array of the second matrix

    Returns:
    C: 2D numpy array of the same-padded 2D convolution matrix
    '''

    # Calculate final shape of C and preallocate
    [ma, na] = np.shape(A)
    [mb, nb] = np.shape(B)
    mc = ma + mb - 1
    nc = na + nb - 1
    C = np.zeros([mc,nc])

    # Iterate over all elements in C
    for j in range(-1,mc):
        for k in range(-1,nc):
            for p in range(max(0, j+2-mb), min(ma, j+2), 1):
                for q in range(max(0, k+2-nb), min(na, k+2), 1):
```

```
                    C[j+1,k+1] = C[j+1,k+1] + (A[p,q] * B[j-p+1,k-q+1]) # Sum␣
↪over p and q to compute convolution

    # Subset for 'same' padding
    mi = int(abs(mc-ma)/2) # Starting index for axis 1
    ni = int(abs(nc-na)/2) # Starting index for axis 2
    return C[mi:mi+ma,ni:ni+na]
```

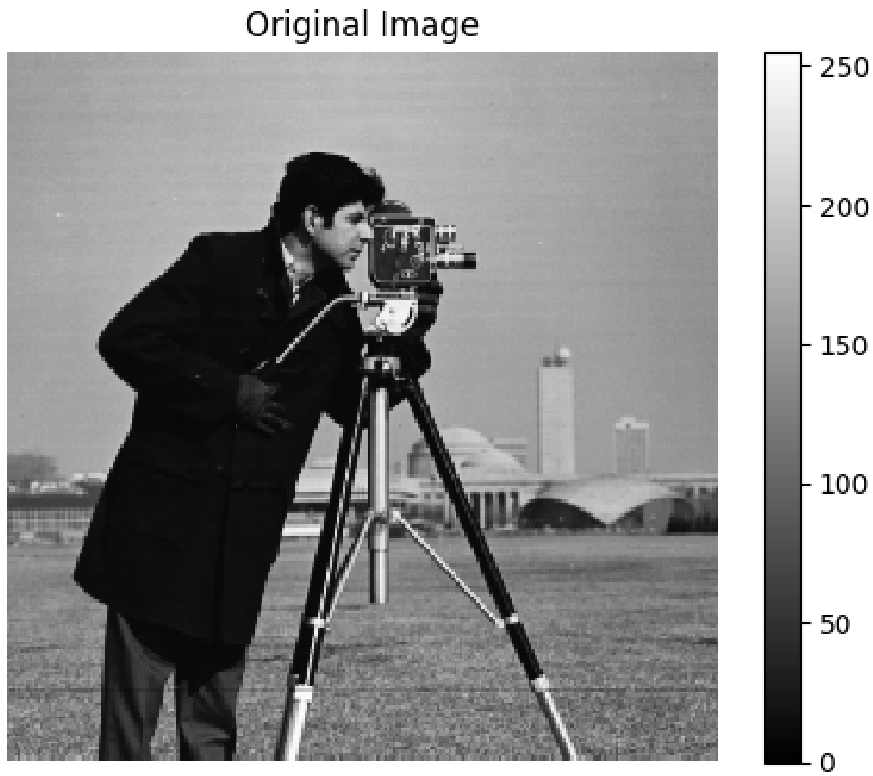
Problem 1b:

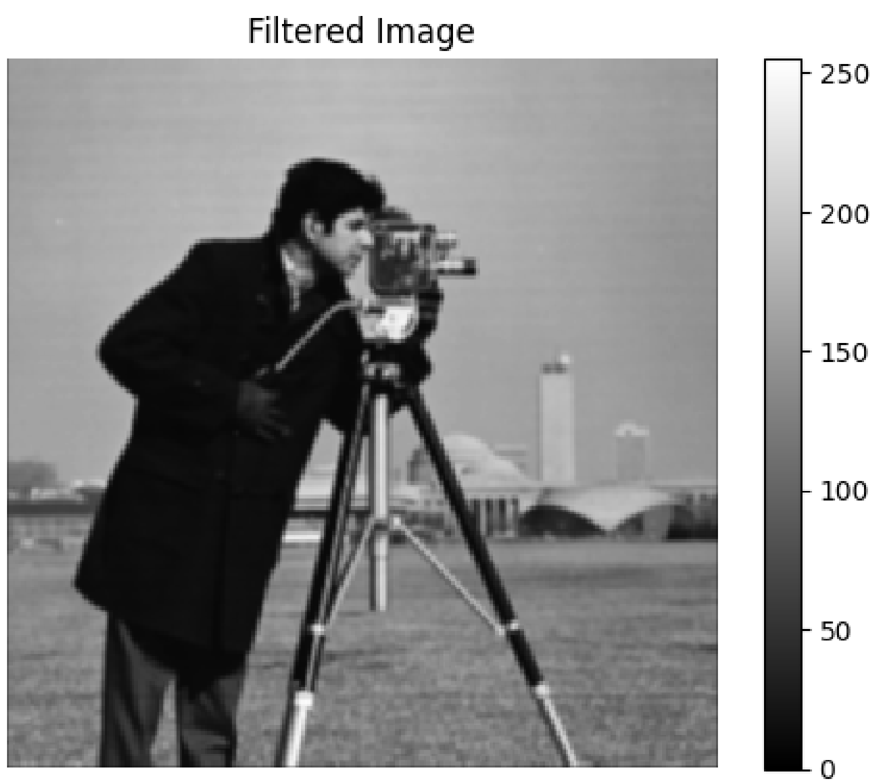
[186]:
```
# Create plot_filtering function
def plot_filtering(data: np.ndarray, filter: np.ndarray) -> None:
    '''
    Filter the given image using the given filter and plot the following:
    - Original image and filtered image
    - Log-magnitude spectrum of the original image, filter, and filtered image

    Parameters:
    data: 2D numpy array of the image data in grayscale
    filter: 2D numpy array of the filter matrix
    '''

    # Compute filtered image
    data_filter = conv2(data, filter)
    N = np.shape(data)[0]

    # Create figure for original image
    plt.figure()
    plt.imshow(data, interpolation='none', cmap='gray', vmin=0, vmax=255)
    plt.title('Original Image')
    plt.axis('off')
    plt.colorbar()

    # Create figure for filtered image
    plt.figure()
    plt.imshow(data_filter, interpolation='none', cmap='gray', vmin=0, vmax=255)
    plt.title('Filtered Image')
    plt.axis('off')
    plt.colorbar()

    # Create spectrum of original image
    H = np.abs(np.fft.fft2(data,[N,N]))
    H = np.fft.fftshift(np.log(H+1))
    plt.figure()
    plt.imshow(H, interpolation='none', cmap='gray')
    plt.title('Log-magnitude spectrum of original image')
    plt.axis('off')
```

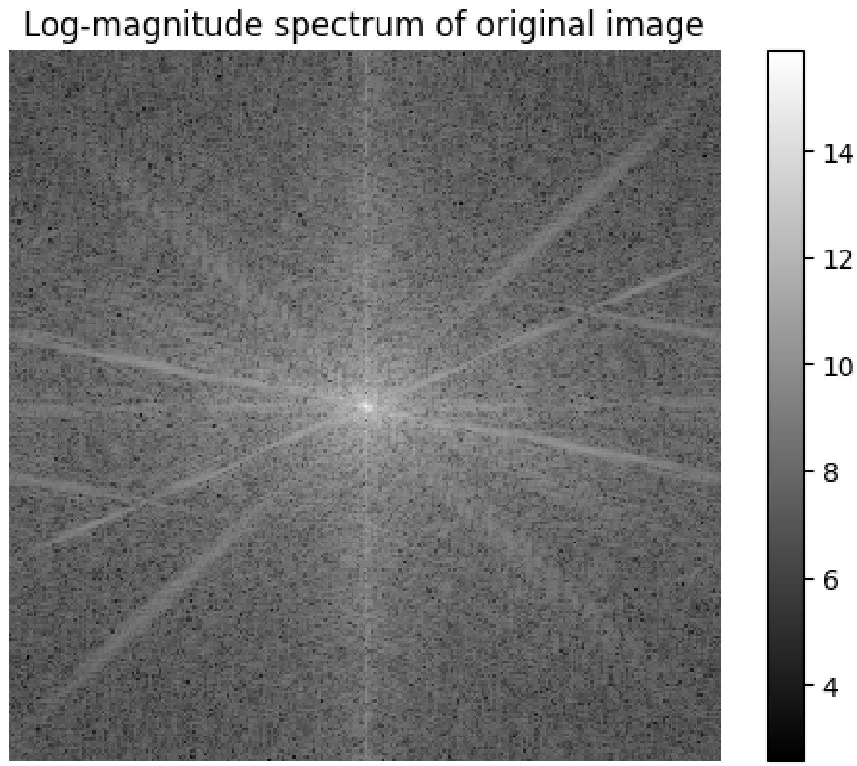
```
    plt.colorbar()

    # Create spectrum of filter
    H = np.abs(np.fft.fft2(filter,[N,N]))
    H = np.fft.fftshift(np.log(H+1))
    plt.figure()
    plt.imshow(H, interpolation='none', cmap='gray')
    plt.title('Log-magnitude spectrum of filter')
    plt.axis('off')
    plt.colorbar()

    # Create spectrum of filtered image
    H = np.abs(np.fft.fft2(data_filter,[N,N]))
    H = np.fft.fftshift(np.log(H+1))
    plt.figure()
    plt.imshow(H, interpolation='none', cmap='gray')
    plt.title('Log-magnitude spectrum of filtered image')
    plt.axis('off')
    plt.colorbar()
```

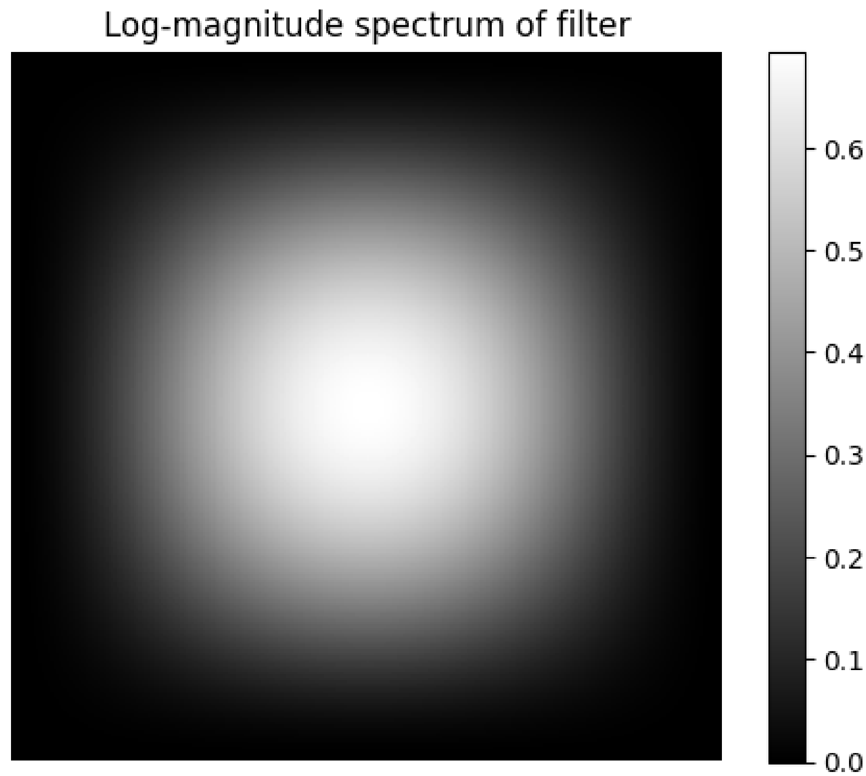
Problem 1c:

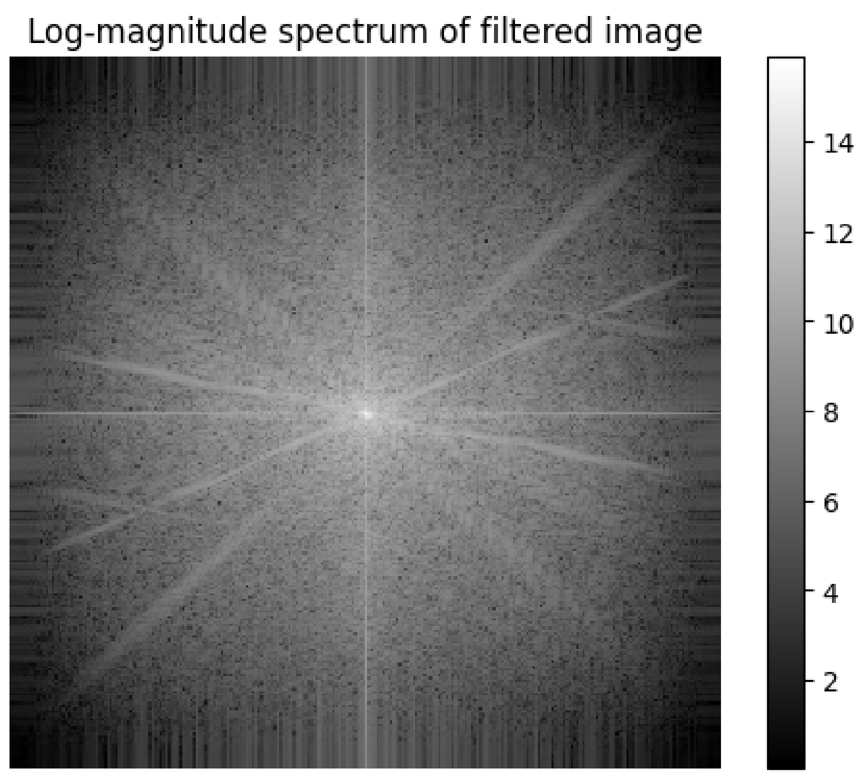
[187]:
```
# Load sample grayscale image
img = Image.open('cameraman.tif')
img.load()
image_data = np.asarray(img) # Image is in grayscale with 8 bit depth

# Apply H1 filter to image
H1 = 1/16 * np.asarray([[1,2,1],[2,4,2],[1,2,1]]) # H1 filter matrix
plot_filtering(image_data,H1)
```


Original Image
250
200
150
100
50
0


Filtered Image
250
200
150
100
50
0

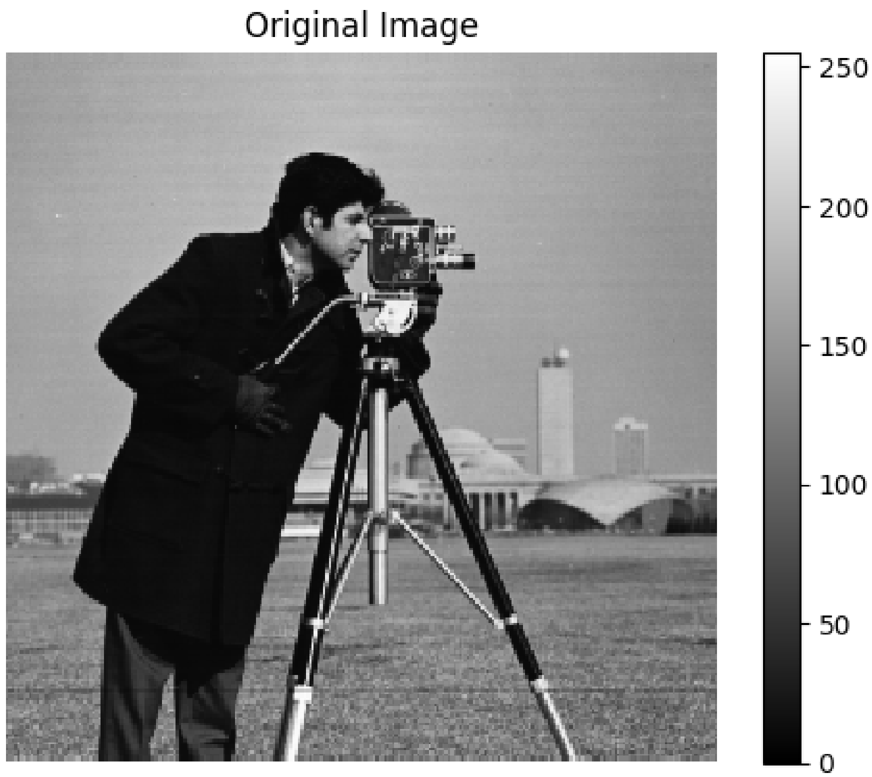

Log-magnitude spectrum of original image
14
12
10
8
6
4

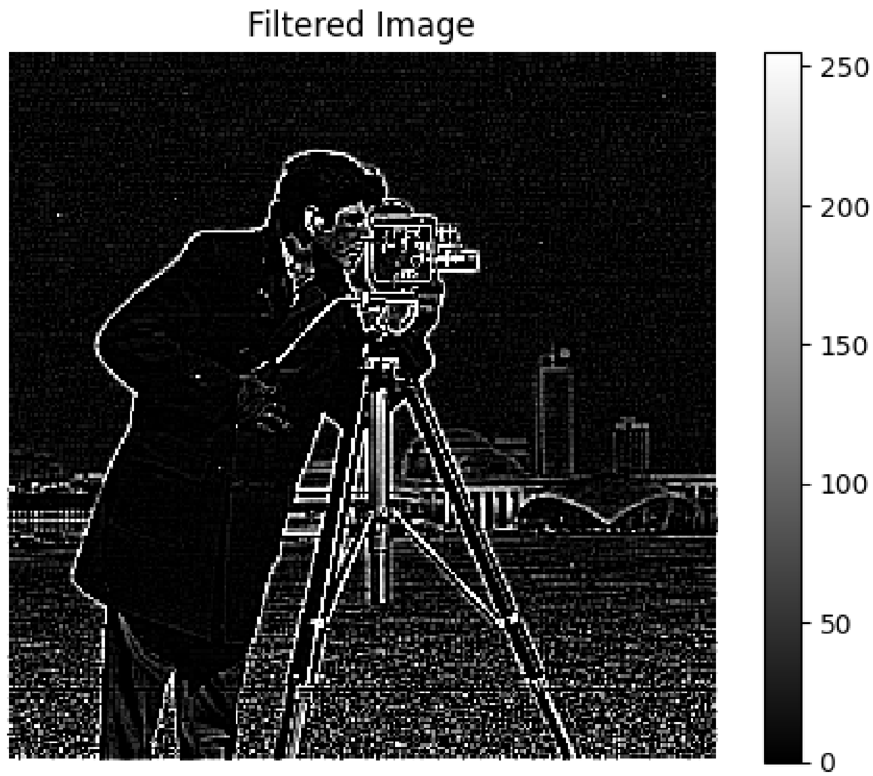

Log-magnitude spectrum of filter
0.6
0.5
0.4
0.3
0.2
0.1
0.0

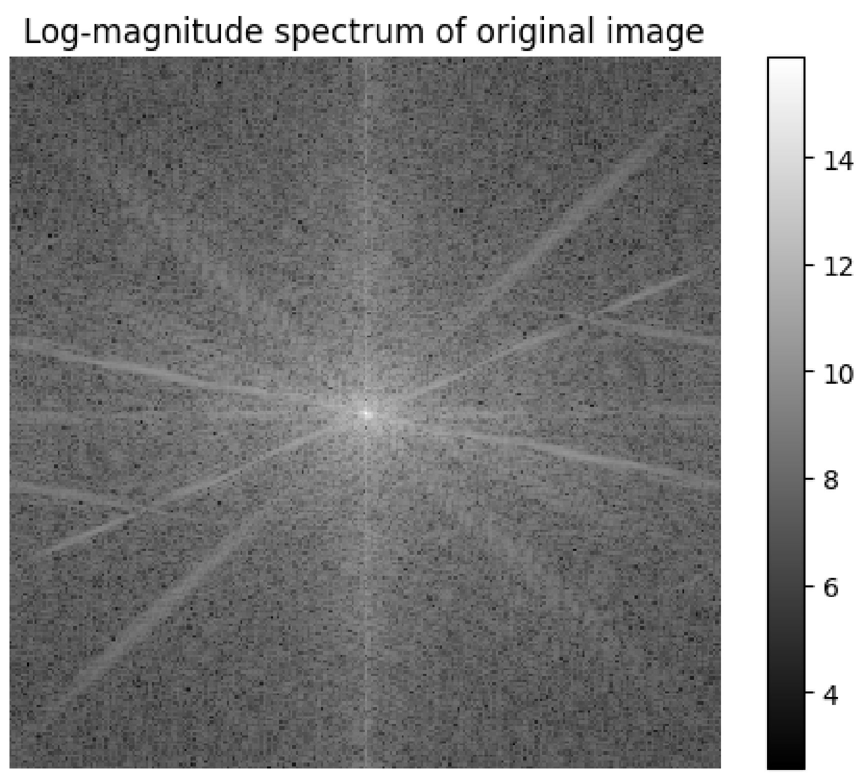

Log-magnitude spectrum of filtered image
14
12
10
8
6
4
2

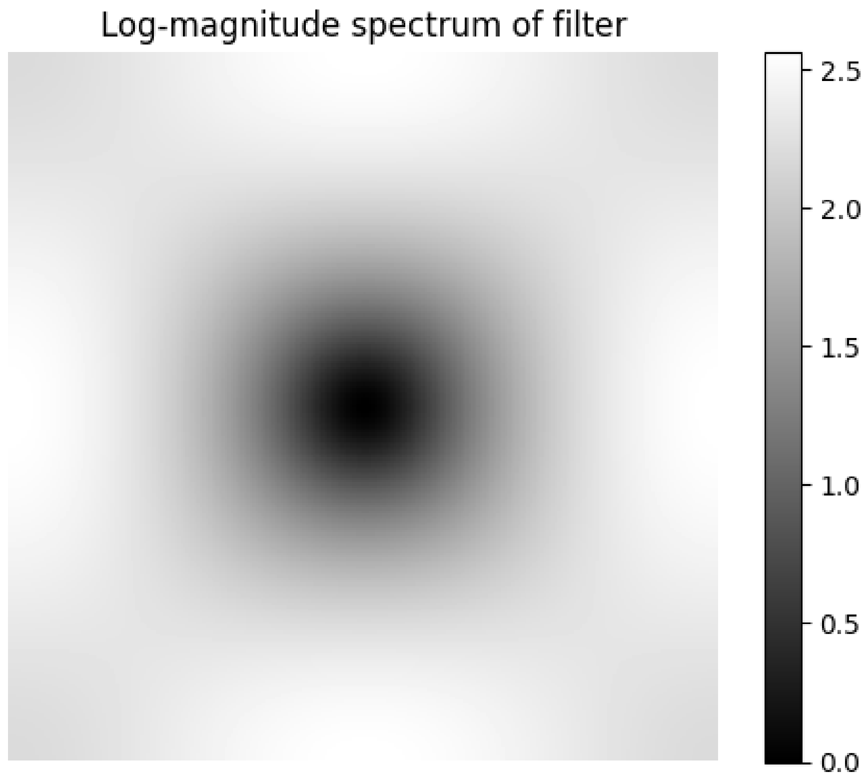
This filter is a low pass filter, since its coefficients are all positive and they sum to 1. These filters are used for noise removal or image smoothing. The frequency response of the filter is in the shape of a Gaussian, meaning it is high for the lower frequencies in the center and low for the higher frequencies at the edges, which agrees with the purpose of the filter. The filtered image does have smoother edges and looks like a noise removal filter has been applied to it. Convolution in the spatial domain is multiplication in the frequency domain, and the frequency response of the filtered image does look like the frequency response of the original image multiplied by the frequency response of the filter.

[188]:
```
# Apply H2 filter to image
H2 = np.asarray([[-1,-1,-1],[-1,8,-1],[-1,-1,-1]]) # H2 filter matrix
plot_filtering(image_data,H2)
```

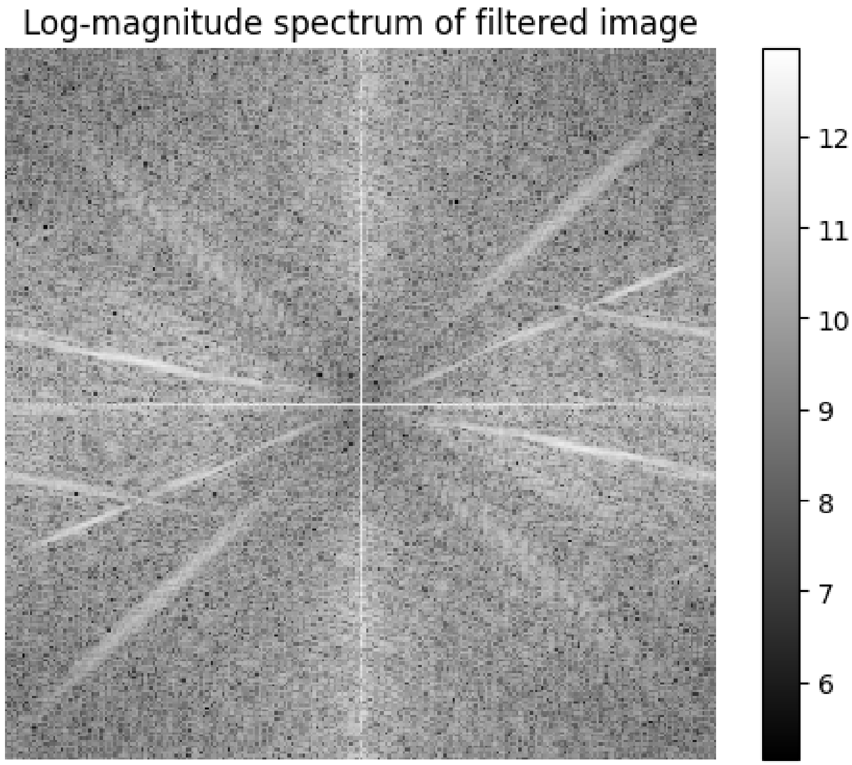



Filtered Image
250
200
150
100
50
0


Log-magnitude spectrum of original image
14
12
10
8
6
4

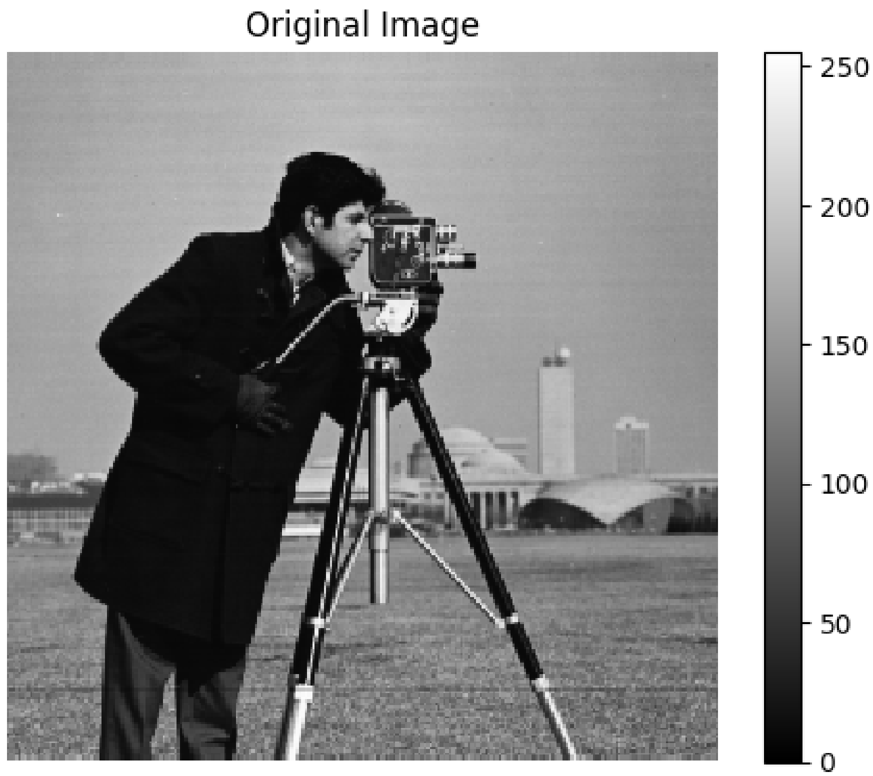


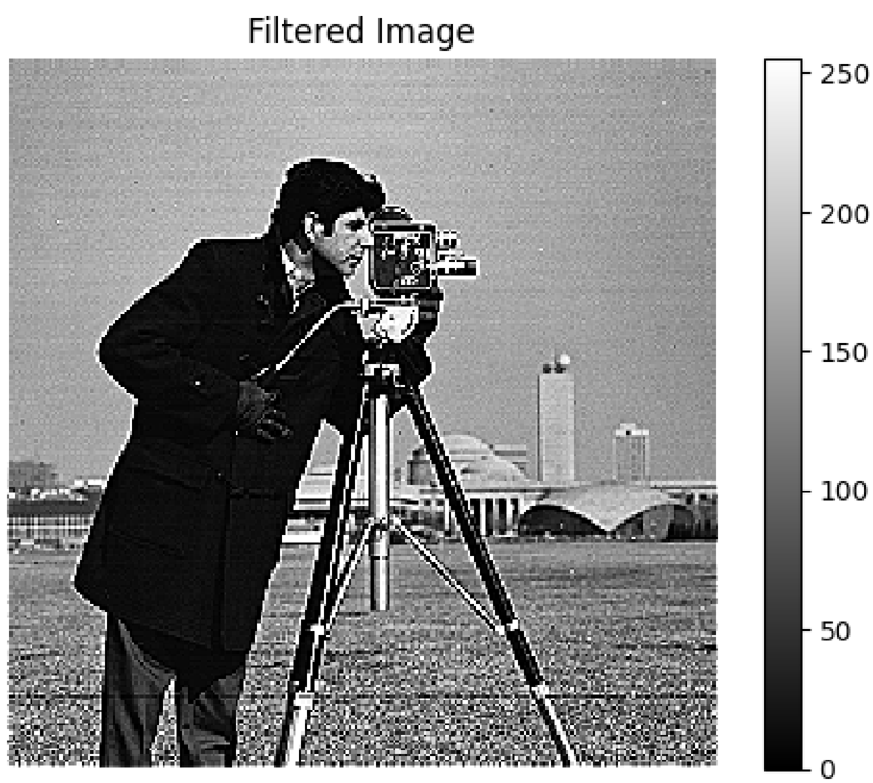
Log-magnitude spectrum of filter
2.5
2.0
1.5
1.0
0.5
0.0

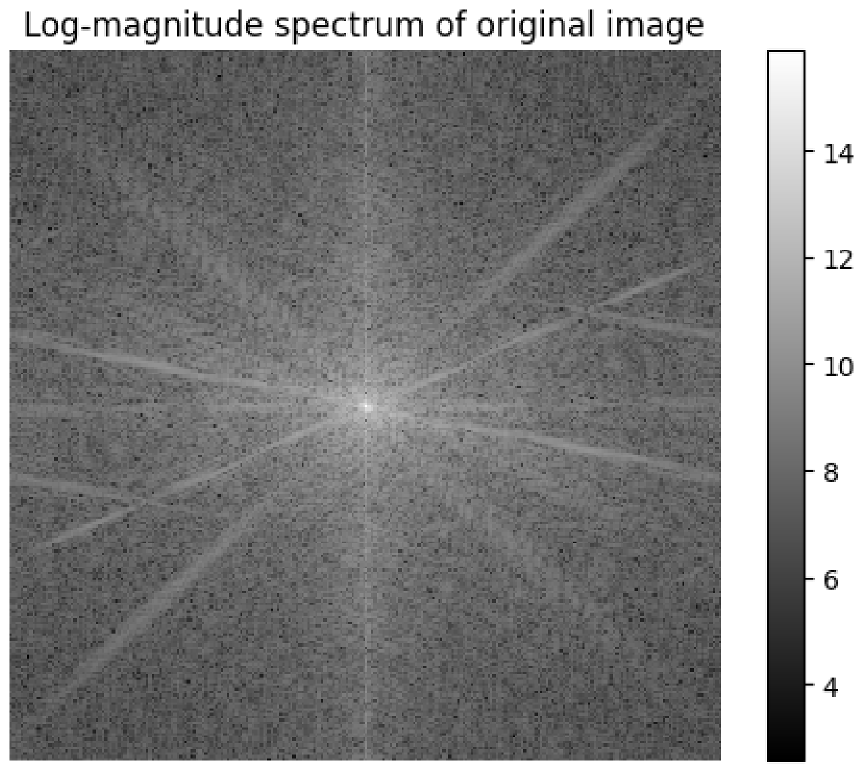


This filter is a high pass filter, since its coefficients sum to 1. These filters are used for edge detection. The shape of the frequency response of the filter is zero in the middle for the lower frequencies, rising up to maximum around the edges for the higher frequencies, which agrees with the purpose of the filter. The filtered image is primarily showing the edges of the original image. Again, the frequency response of the filtered image does look like the frequency response of the original image multiplied by the frequency response of the filter.

[189]:
```
# Apply H3 filter to image
H3 = np.asarray([[0,-1,0],[-1,5,-1],[0,-1,0]]) # H3 filter matrix
plot_filtering(image_data,H3)
```

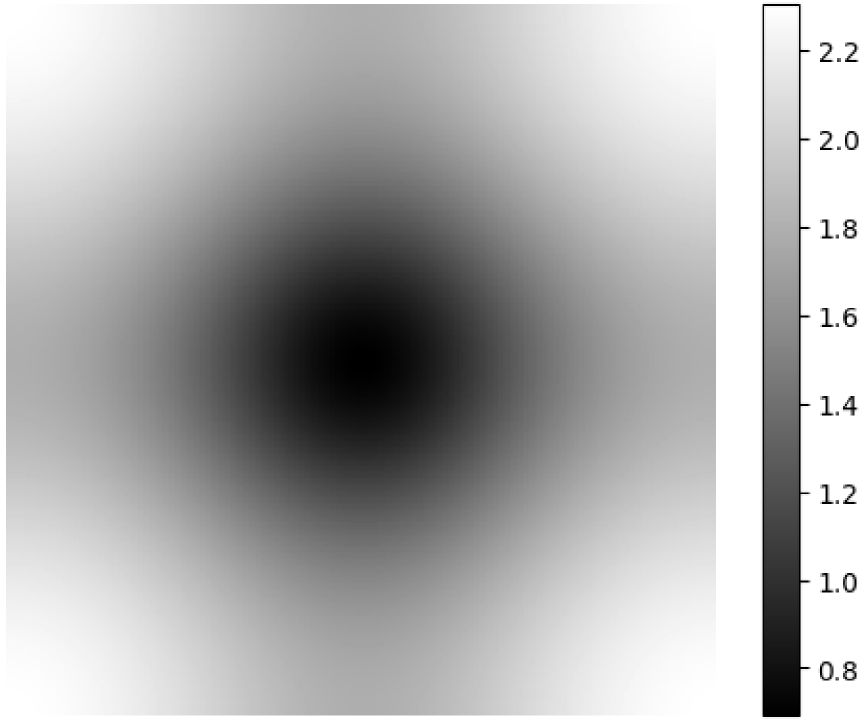

Original Image
250
200
150
100
50
0


Filtered Image
250
200
150
100
50
0

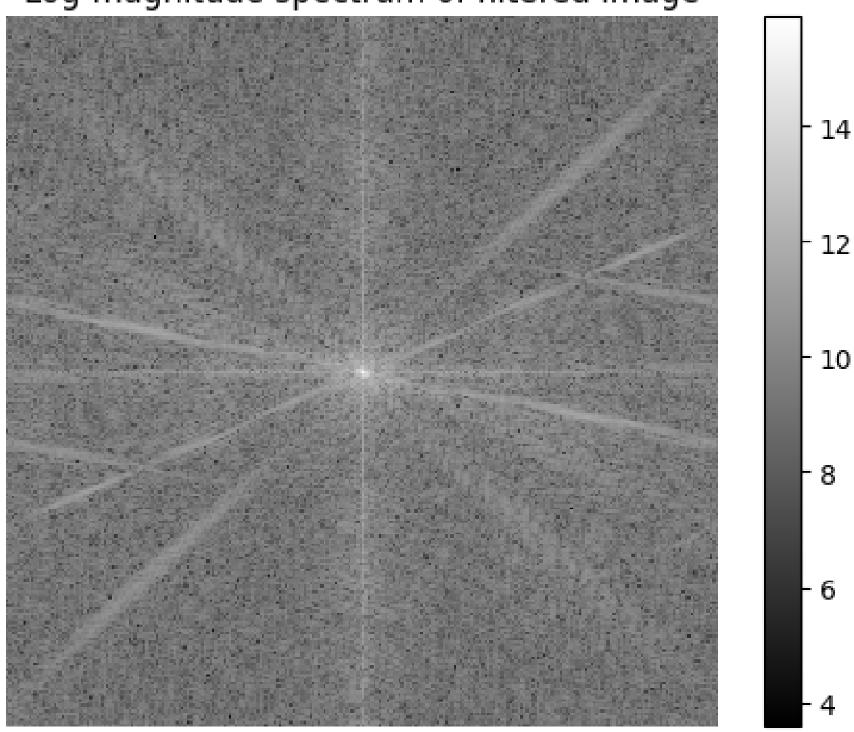


Log-magnitude spectrum of original image
14
12
10
8
6
4

Log-magnitude spectrum of filter



Log-magnitude spectrum of filtered image



This filter is a high emphasis filter, since some of its coefficients are negative and they sum to 1. These filters are used for edge sharpening. The shape of the frequency response of the filter is minimum in the middle for the lower frequencies, rising up to maximum around the corners for the higher frequencies, which agrees with the purpose of the filter. Unlike H2, this filtered image still looks like the original image but with the edges enhanced and sharpened. Again, the frequency response of the filtered image does look like the frequency response of the original image multiplied by the frequency response of the filter.

Problem 2a:

[221]:
```
# Create image noise generator function
def awgn(data: np.ndarray, sigma: float) -> np.ndarray:
    '''
    Apply IID zero-mean Gaussian distribution random noise with the given␣
 ↪standard deviation to the given image.

    Parameters:
    data: 2D numpy array of the image data, range [0,1]
    sigma: Float of the Gaussian standard distribution

    Returns:
    data: 2D numpy array of the resulting image
    '''

    # Set up random generator and create noise data
    rng = np.random.default_rng()
    noise = rng.normal(0, sigma, np.shape(data))

    # Apply noise to image
    data = data + noise

    # Re-scale image to [0,1]
    data = data + np.abs(np.min(data))
    data = data / np.max(data)

    # Return final noisy image data
    return data
```

Problem 2b:

[213]:
```
# Create gaussian filter generator function
def gaussian_filter(sigma: int) -> np.ndarray:
    '''
    Generate a 2D Gaussian filter of size MxM with the given standard deviation␣
 ↪sigma, where M = 5*sigma
```

```
    Parameters:
    sigma: Int of the standard deviation

    Returns:
    H: 2D numpy array of the Gaussian filter
    '''

    # Create initial arrays for Gaussian function
    x = np.linspace(-(5*sigma-1)/2,(5*sigma-1)/2,5*sigma) # Array of x, spaced␣
↪by 1, centered at 0
    x,y = np.meshgrid(x,x) # Create 2D arrays of x and y values
    gauss = np.exp(-(x**2+y**2)/(2*sigma**2)) # Create 2D array of gaussian␣
↪function

    # Rescale Gaussian to sum to 1
    H = gauss/(np.sum(gauss))

    # Return Gaussian filter
    return H
```

Problem 2c:

[216]:
```
# Load sample grayscale image
img = Image.open('cameraman.tif')
img.load()
img_data = np.asarray(img) # Image is in grayscale with 8 bit depth
img_orig = img_data/255 # Rescale image to [0,1]

# Create figure for original image
plt.figure()
plt.imshow(img_orig, interpolation='none', cmap='gray')
plt.title('Original Image')
plt.axis('off')
```

[216]:
```
(np.float64(-0.5), np.float64(255.5), np.float64(255.5), np.float64(-0.5))
```

Original Image

[241]:
```python
# Add first noise to original image
sigma = 0.1
img_noisy = awgn(img_orig, sigma)

# Apply Gaussian filter to noisy image
N = 1
H_gaus = gaussian_filter(N) # Create Gaussian filter
img_filtered_gaus = conv2(img_noisy, H_gaus) # Apply filter

# Apply average filter to noisy image
H_avg = np.ones([N*5,N*5]) / (25*N**2) # Create average filter
img_filtered_avg = conv2(img_noisy, H_avg) # Apply filter

# Calculate PSNR for noisy image
[m,n] = np.shape(img_orig)
MSE = np.sum((img_orig - img_noisy)**2) / (m*n) # Mean squared error
PSNR = -10*np.log10(MSE) # Peak signal to noise ratio, for image in range [0,1]

# Create figure for noisy image
plt.figure()
plt.imshow(img_noisy, interpolation='none', cmap='gray')
plt.title(f'Noisy image with Sigma = {sigma}, PSNR = {PSNR:.2f}')
```

```
plt.axis('off')

# Calculate PSNR for noisy image
[m,n] = np.shape(img_orig)
MSE = np.sum((img_orig - img_filtered_gaus)**2) / (m*n) # Mean squared error
PSNR = -10*np.log10(MSE) # Peak signal to noise ratio, for image in range [0,1]

# Create figure for gaussian filtered image
plt.figure()
plt.imshow(img_filtered_gaus, interpolation='none', cmap='gray')
plt.title(f'Filtered with Gaussian {N*5}x{N*5}, PSNR = {PSNR:.2f}')
plt.axis('off')

# Calculate PSNR for noisy image
[m,n] = np.shape(img_orig)
MSE = np.sum((img_orig - img_filtered_avg)**2) / (m*n) # Mean squared error
PSNR = -10*np.log10(MSE) # Peak signal to noise ratio, for image in range [0,1]

# Create figure for average filtered image
plt.figure()
plt.imshow(img_filtered_avg, interpolation='none', cmap='gray')
plt.title(f'Filtered with Average {N*5}x{N*5}, PSNR = {PSNR:.2f}')
plt.axis('off')
```

[241]:
```
(np.float64(-0.5), np.float64(255.5), np.float64(255.5), np.float64(-0.5))
```

Noisy image with Sigma = 0.1, PSNR = 18.07

Filtered with Gaussian 5x5, PSNR = 18.29

The Gaussian filter does a much better job of removing some noise from the image while still maintaining the details of the original image, particularly in the edge details. The average filter might remove slightly more noise, but it makes the image look blurry and as a result it loses lots of detail in the sharp edges of the original image. The PSNR is actually lower after applying the average filter than before it, whereas the Gaussian filter increases the PSNR.

[240]:
```
# Add second noise to original image
sigma = 0.3
img_noisy = awgn(img_orig, sigma)

# Apply Gaussian filter to noisy image
N = 1
H_gaus = gaussian_filter(N) # Create Gaussian filter
img_filtered_gaus = conv2(img_noisy, H_gaus) # Apply filter

# Apply average filter to noisy image
H_avg = np.ones([N*5,N*5]) / (25*N**2) # Create average filter
img_filtered_avg = conv2(img_noisy, H_avg) # Apply filter

# Calculate PSNR for noisy image
[m,n] = np.shape(img_orig)
MSE = np.sum((img_orig - img_noisy)**2) / (m*n) # Mean squared error
```

```
PSNR = -10*np.log10(MSE) # Peak signal to noise ratio, for image in range [0,1]

# Create figure for noisy image
plt.figure()
plt.imshow(img_noisy, interpolation='none', cmap='gray')
plt.title(f'Noisy image with Sigma = {sigma}, PSNR = {PSNR:.2f}')
plt.axis('off')

# Calculate PSNR for noisy image
[m,n] = np.shape(img_orig)
MSE = np.sum((img_orig - img_filtered_gaus)**2) / (m*n) # Mean squared error
PSNR = -10*np.log10(MSE) # Peak signal to noise ratio, for image in range [0,1]

# Create figure for gaussian filtered image
plt.figure()
plt.imshow(img_filtered_gaus, interpolation='none', cmap='gray')
plt.title(f'Filtered with Gaussian {N*5}x{N*5}, PSNR = {PSNR:.2f}')
plt.axis('off')

# Calculate PSNR for noisy image
[m,n] = np.shape(img_orig)
MSE = np.sum((img_orig - img_filtered_avg)**2) / (m*n) # Mean squared error
PSNR = -10*np.log10(MSE) # Peak signal to noise ratio, for image in range [0,1]

# Create figure for average filtered image
plt.figure()
plt.imshow(img_filtered_avg, interpolation='none', cmap='gray')
plt.title(f'Filtered with Average {N*5}x{N*5}, PSNR = {PSNR:.2f}')
plt.axis('off')
```

[240]: (np.float64(-0.5), np.float64(255.5), np.float64(255.5), np.float64(-0.5))

Noisy image with Sigma = 0.3, PSNR = 14.24

Filtered with Gaussian 5x5, PSNR = 14.89

[239]:
```python
# Add second noise to original image
sigma = 0.3
img_noisy = awgn(img_orig, sigma)

# Apply Gaussian filter to noisy image
N = 2
H_gaus = gaussian_filter(N) # Create Gaussian filter
img_filtered_gaus = conv2(img_noisy, H_gaus) # Apply filter

# Apply average filter to noisy image
H_avg = np.ones([N*5,N*5]) / (25*N**2) # Create average filter
img_filtered_avg = conv2(img_noisy, H_avg) # Apply filter

# Calculate PSNR for noisy image
[m,n] = np.shape(img_orig)
MSE = np.sum((img_orig - img_noisy)**2) / (m*n) # Mean squared error
PSNR = -10*np.log10(MSE) # Peak signal to noise ratio, for image in range [0,1]

# Create figure for noisy image
```

```python
plt.figure()
plt.imshow(img_noisy, interpolation='none', cmap='gray')
plt.title(f'Noisy image with Sigma = {sigma}, PSNR = {PSNR:.2f}')
plt.axis('off')

# Calculate PSNR for noisy image
[m,n] = np.shape(img_orig)
MSE = np.sum((img_orig - img_filtered_gaus)**2) / (m*n) # Mean squared error
PSNR = -10*np.log10(MSE) # Peak signal to noise ratio, for image in range [0,1]

# Create figure for gaussian filtered image
plt.figure()
plt.imshow(img_filtered_gaus, interpolation='none', cmap='gray')
plt.title(f'Filtered with Gaussian {N*5}x{N*5}, PSNR = {PSNR:.2f}')
plt.axis('off')

# Calculate PSNR for noisy image
[m,n] = np.shape(img_orig)
MSE = np.sum((img_orig - img_filtered_avg)**2) / (m*n) # Mean squared error
PSNR = -10*np.log10(MSE) # Peak signal to noise ratio, for image in range [0,1]

# Create figure for average filtered image
plt.figure()
plt.imshow(img_filtered_avg, interpolation='none', cmap='gray')
plt.title(f'Filtered with Average {N*5}x{N*5}, PSNR = {PSNR:.2f}')
plt.axis('off')
```

[239]: (np.float64(-0.5), np.float64(255.5), np.float64(255.5), np.float64(-0.5))

Noisy image with Sigma = 0.3, PSNR = 14.17

Filtered with Gaussian 10x10, PSNR = 14.63

After increasing the noise in the image, it is clear the average filter does a better job of removing noise across the entire image. However, is makes the image appear very blurry and it loses lots of detail in the edges, which is especially apparent in the larger 10x10 filter. Overall, the larger the filter, the more noise is removed, but the blurrier the image appears and the more detail is lost. This is true for both the Gaussian and average filters. In both examples, the Gaussian filter has a higher PSNR than the noisy image and the average filter. As the filter size increases, the outer boundary increases, and the more pixels around the edge of the image go to zero.