

**Advanced Laboratory**  
**Department of Physics and Astronomy**  
**Stony Brook University**

**Pre-experiment and mid-experiment evaluation sheet:**

You must meet with a staff member before the beginning of each experiment and before the lab period indicated in the middle of the experiment for an evaluation of your preparation and progress. This form should be attached as the first page of your final report. You will be penalized for each section of the report (pre-lab, mid-lab, final) that is not completed on time. You are responsible for relevant parts of the course notes, especially radiation safety.

$\gamma - \gamma$  **Correlation**

**Student name**

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**Partner name**

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**Pre-lab discussion:**

Be prepared to show that you have read the background material for this experiment by discussing the following questions with the staff:

1. How does a photomultiplier work?
2. Study the energy diagram of  $^{60}\text{Co}$  decay. What radiation is produced in addition to the two  $\gamma$ -rays?
3. What is the difference between a true and a chance coincidence?
4. What is the minimum radiation dose (in mrem) which causes rapid human death?
5. What is the typical yearly dose humans receive on the surface of the Earth?

Staff comments:

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Signature: \_\_\_\_\_ Date \_\_\_\_\_

**Mid-lab report:**

Before the 5<sup>th</sup> lab period of this experiment turn in a brief report (2 or 3 pages) showing:

1. the  $\gamma - \gamma$  correlation that you have measured (with statistical errors) and
2. an estimation of the sensitivity of your data in measuring  $a_2$  (if  $a_4$  is small).

Staff comments:

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Signature: \_\_\_\_\_ Date \_\_\_\_\_

## Angular Momentum Quantization and the $\gamma$ - $\gamma$ Angular Correlation

(Updated August 2007)

**Introduction:** When two  $\gamma$ -rays are emitted in two consecutive transitions between nuclear levels in the same nucleus, the probability of observing one  $\gamma$ -ray relative to the other depends generally on the angle  $\theta$  between the two observed rays. This is called an angular correlation. Such correlations in nuclear transitions are one of the most convincing demonstrations of the quantization of the orientation of angular momentum in space. They have also enabled us to determine the spins of the nuclear levels involved in the transitions simply from angular momentum geometry without any need for prior knowledge about nuclear dynamics.

How this comes about can be seen from the following simple consideration: Imagine a nuclear level at an excitation energy  $E_i$  has spin  $J_i = 0$  (all angular momenta given in units of  $\hbar$ ). Without any spin this level has no preferred orientation in space. This level then decays to a lower level  $E_{int}$  with spin  $J_{int} = 1$  by emission of a dipole  $\gamma$ -ray. This  $\gamma$ -ray is detected in a detector that looks at the nuclear source thereby defining an axis in space from source to detector. Quantum mechanics tells us that the spin  $S = 1$  of the  $\gamma$ -ray can only have the projections  $S_z = \pm 1$  on this axis. Since the initial angular momentum of the system was 0, the spin of the intermediate nuclear state must compensate the spin of the photon, i.e. if  $S_z = +1$  then the projection of  $J_{int}$  must be  $J_{intz} = -1$ , and equivalently for the  $S_z = -1$  case. Thus the spin of the intermediate state is now oriented in space such that its projection on the detector-source axis is  $J_{intz} = \pm 1$ . Thus the second  $\gamma$ -ray, which leaves the nucleus in its final state  $E_f$  with  $J_f = 0$ , cannot be emitted uniformly into space but only in such a way that its spin projection on the axis to the first detector is  $S_{2z} = \pm 1$ . In a semi-classical picture the probability of detecting  $\gamma$  at 90 degrees relative to the first  $\gamma$ -ray would be zero.

The theory of the angular momentum geometry can be worked out exactly and provides a clear prediction for each spin sequence  $J_i (\gamma_1) J_{int} (\gamma_2) J_f$ . From the observed correlation between  $\gamma_1$  and  $\gamma_2$  the spin sequence of the nuclear levels can be deduced.

The present experiment studies a particularly transparent case: the consecutive emission of a 1.17-MeV and a 1.33-MeV  $\gamma$ -ray from the decay of  $^{60}\text{Ni}$ , in the sequence  $4^+ (\gamma_1) 2^+ (\gamma_2) 0^+$ . The initial  $4^+$  state is populated by  $\beta$ -decay from a radioactive  $^{60}\text{Co}$  source. In this case the  $\gamma$ -rays take away 2 units of angular momentum, i.e. they have quadrupole character.

*Please study the radioactive decay scheme of  $^{60}\text{Co}$  and understand why this decay can populate such a high-spin state in  $^{60}\text{Ni}$ .*

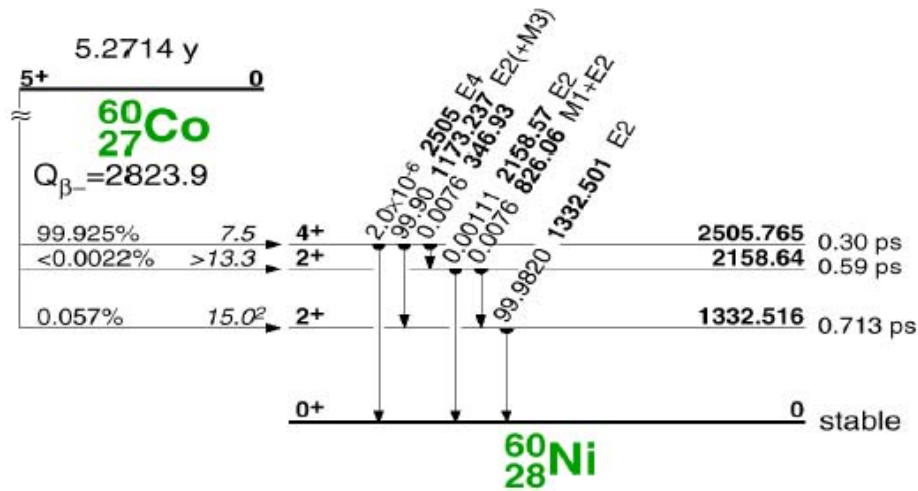


Figure 1. The energy level diagram of  $^{60}\text{Co}$  decay. Energies are given in keV.  $E1$  denotes an electric dipole transition.  $E2$  denotes an electric quadrupole transition.

The level spectrum of  $^{60}\text{Co}$  and  $^{60}\text{Ni}$  shows that the ground state of  $^{60}\text{Co}$  populates with very high probability the 4+ state at 2.505 MeV in  $^{60}\text{Ni}$ . This state then decays with 99.9% probability to the 2+ state at 1.332 MeV. This state decays within  $< 1$  ps to the ground state. Thus the two  $\gamma$ -rays (1.17 and 1.33 MeV) can be considered emitted simultaneously.

A good and complete discussion of the theory of angular correlations applied to this specific case as well as the experimental arrangement to measure it appears in reference [1].

In addition to the study of angular correlations this experiment introduces you to the technique of coincidence measurements at the nanosecond level. This technique has wide application. In the present application it assures that you are measuring the correlation between two serial  $\gamma$ -rays rather than transitions from two different nuclei.

Coincidences can be observed in two ways:

1. In the first way, two pulses that pile up on top of each other trigger an output, the coincidence signal. This technology is used, *e.g.* in the logic of the muon decay experiment.
2. The second technique is to measure the time difference between two events that are correlated in time. If the events occur simultaneously, then an artificial time delay between the two can be introduced. Truly coincident events will produce always the same time delay. This is the technique that is used in the present experiment.

## Experimental Setup:

The experiment uses two 1 3/4 in x 2 in NaI(Tl) scintillation counters to detect the  $\gamma$ -rays. These are described by Melissinos [1] on p.194 (p. 333 of Melissinos and Napolitano). The crystals are bonded to their photomultipliers, which are connected to preamplifiers and a high-voltage power supply. These units should never be turned off. *Please do not disconnect the high-voltage and 20 V power cables!*

A beautiful fast scope with two measuring channels is available to observe the pulses and their coincidence with each other in real time. *Please familiarize yourself with the operation of this scope before you begin to check the electronics!*

The electronics is quite sophisticated with automatic control of the measuring cycle allowing you to take good data. In addition the angular movement of one detector is driven by a computer-controlled motor so that measurements at different angles can be made overnight.

The electronic setup is shown below. The signal processing in both channels is identical. Each signal goes first through a DL (delay line) amplifier. It provides a signal with a polarity cross-over. This cross-over point in time (which is independent of the pulse height) is later used to create the timing signal for the detected  $\gamma$ -ray. This signal then enters a timing single channel analyzer (TSCA). This unit has two functions: It creates a standard-height negative fast-logic timing pulse when the pulse changes polarity. This signal can be delayed by adjustment of a fine potentiometer with a range of 1  $\mu$ s. The unit also allows you to set a window on the pulse height. One potentiometer (E) sets the lower threshold of the window, the second one ( $\Delta E$ ) either sets the width of the window ("differential") or the upper level ("integral"). A pulse that falls within the window produces a positive slow-logic (+several V) output pulse. This enables you to set a window on the energies of the two  $\gamma$ -transitions in  $^{60}\text{Ni}$ .

*Always set the window so that both  $\gamma$ -ray energies fall into the window. This will double your counting rate without penalty since the geometry is symmetric between the two  $\gamma$ -rays and the correlation depends only on the relative angle between the detectors!*

As shown in Fig. 2 the setup contains two identical chains: one for the first and one for the second  $\gamma$ -ray. Each SCA unit delivers a timing pulse to the Time-to-Amplitude Converter (TAC). The TAC is an electronic stop watch whose linear output provides a (positive) signal whose height is proportional to the elapsed time between start and stop. The first timing signal starts the converter; the second, stopping signal, must of course come after the start signal. This is achieved by using the delay potentiometer on the SCA box. The linear signal is fed into a multi-channel analyzer board (contained inside a computer) which digitizes the pulses and sorts them by pulse height. The pulse height distribution is displayed on the computer screen. Coincident events observed between detector 1 and detector 2 will produce a peak in the distribution, at a pulse height determined by the artificial delay.

*Make sure that the timing range set for the TAC is larger than the delay time between start and stop signals!*

The use of the TAC is very important because it allows you to observe the entire range of times elapsed between the detection of the two  $\gamma$ -rays. Once you have determined the pulse height of the “prompt” peak in the coincidence (time) spectrum, you can set an electronic window around that peak height using the single channel analyzer contained in the TAC unit. The (positive) output pulse from this SCA indicates then the detection of a prompt coincidence between the two  $\gamma$ -rays. This output is sent into a counter and produces the primary data set, i.e. the number of coincidences for a fixed time as a function of angle  $\theta$  between the two counters.

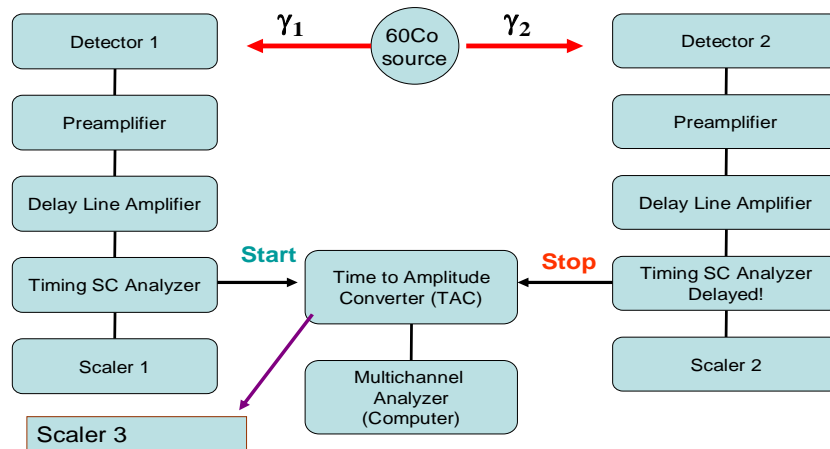


Figure 2. Block diagram of the angular correlation measurement. Each block represents an electronic module. Channels A, B, and C are counted by scalers 1, 2, and 3 respectively.

The angle between the two counters is changed mechanically by a stepping motor that is controlled by a computer. The same computer controls 3 counters that count the outputs of the scalers 1, 2 and 3 (see diagram above). It turns them on and off and stores the recorded numbers in the computer. The computer control is done through the distribution box (DB) shown in Fig. 3. The linear inputs from the two detector channels and the TAC linear output plug into the top three inputs, and the associated logic pulses from the three SCA's go into the bottom three plugs. The linear and gate outputs plug into the appropriate inputs of the MCA card in the computer. The logic inputs also feed into three scalers. Their counts are shown on a display and are also stored in the computer. The distribution box is controlled by a computer program.

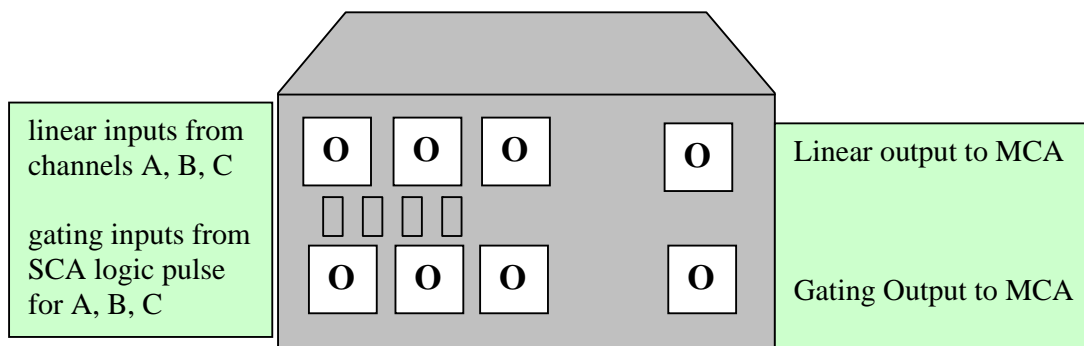


Figure 3. Distribution box which switches between channels A, B, and C as inputs to the MCA under computer control.

## Preparatory Steps, Angular Correlation from $^{22}\text{Na}$

The first aim is to become familiar with the electronics and the coincidence technique. For this purpose we use a  $^{22}\text{Na}$  source.  $^{22}\text{Na}$  is a positron emitter. Once emitted the positrons combine with electrons in the source and annihilate into two 511-keV  $\gamma$ -rays. Because of momentum conservation these come out back-to-back. Thus if Detector 1 catches the first  $\gamma$ -ray, the second one is sure to go into detector 2 if they are oriented at 180 degrees relative angle. This produces a very high coincidence rate, which we use to set up the system.

Place the two detectors symmetrically at a distance of 10 cm from the pivot point, facing each other, and place a  $^{22}\text{Na}$  source at the pivot. *Observe the outputs from the linear amplifiers in the two inputs of the oscilloscope.* These should be relatively slow positive signals. Observe the pulse shapes and make certain that the pulses are not saturated. The two channels may have different gains and thus different output pulse heights. It is not essential that they be matched, but they should be of similar levels. *Observe that the pulses have a bipolar shape.* Look at the cross over point and note that the pulses will all cross at the same time, with very good timing resolution, regardless of their pulse height. This sharp crossing is produced by the delay line amplifier. The timing unit in the SCA produces a logic timing pulse at this crossing point, which is thus very well defined.

*The next steps will require you to use the versatile trigger capability of the scope. Familiarize yourself with the controls that let you choose the trigger channel, the trigger pulse polarity and the capability to look at a second input triggered by the first. This allows seeing directly the time relationship between two pulses.*

Now feed the signal from the amplifier through a splitter into the SAC. Lower the lower threshold to zero and open the window all the way. You should observe the timing output on the scope by triggering one channel with the linear input pulse and observing the fast (small, fast and negative) output pulse from the SAC in the second channel. *Verify that the output is produced at the time of the cross-over!*

Next, reverse the role of the two pulses entering the scope. Trigger with the SCA output and look at the linear signal from the amplifier. Since the trigger pulse is generated after the positive lobe of the linear pulse has elapsed, you should insert a delay (see the delay box provided in each channel) between the amplifier and the scope. *Now slowly raise the threshold of the single channel and observe how the low amplitudes on the linear signal are cut out.* This is because the trigger pulse is generated only if the linear signal exceeds that threshold. Similarly the high pulse heights are cut out when you close the window (set SAC unit to “differential”). Look at the linear signals on the scope and close the window around the 511-keV energy peak. Then perform the same steps on the second channel. You have now selected two gating pulses that signify that a 511-keV  $\gamma$ -ray has been observed in each channel.

Now feed the timing signal from channel 1 into the start of the TAC and the second channel into the stop. *Be sure to delay the second channel sufficiently and check the range of the TAC!* Looking at the linear TAC output on the scope (a large positive pulse) you can observe the relative timing of the two channels. Real coincidence events will have all the same pulse height, while chance coincidences of unrelated decays will

produce a statistical background. Repeat the previous procedure for setting a window around a peak now with the single channel provided on the TAC unit. When properly set the gate output (positive logic pulse) from the TAC will signify coincident events with 511 keV energies in each channel. You should observe the peak related to actual physical coincidences (the “prompt” peak).

Feed the gate output of the TAC into scaler 3. *Now use the computer* (see the Appendix) *to move the movable detector and measure the coincidence counting rate as a function of angle  $\theta$ .* In this preliminary measurement you just want to see the angular resolution of your system to a delta function (back to back) correlation. Under ideal conditions the angular correlation should be a sharp peak at 0 degrees. But the motion of electrons in the material and the finite opening angle of the detectors introduce an angular spread. You should measure the correlation until the counting rate is down by at least a factor of 10 below the peak rate. Plot the data using the EXCEL data analysis program on a lab computer (or your own) and compare the half-width of the correlation with an estimate from the detector geometry. Attempt to measure the angular correlation at 90° and compare to the correlation at 0°.

### Angular Correlation of $^{60}\text{Co}$

You are now ready to study the consecutive  $\gamma$ -rays from a  $^{60}\text{Ni}$  nucleus and measure their angular correlation.

*Replace the  $^{22}\text{Na}$  source with all the  $^{60}\text{Co}$  sources available in the laboratory.* Since we are interested in  $\gamma$ -rays that emerge from the same nucleus, it does not matter from which specific source they emerge from, and by adding them all up we gain counting rate. The  $^{60}\text{Ni}$   $\gamma$ -rays have higher energies than those from the  $^{22}\text{Na}$  source. Thus you should start by looking at the linear outputs again and make sure that the pulses are not saturated. Adjust the pulse heights (by adjusting the amplifier gains) so that the 1.33-MeV  $\gamma$ -rays have about 8 V amplitude on the scope.

For observing the spectra and the adjustment of the SCA windows we will proceed with a bit more sophistication than before. We will use the distribution box (DB) to look at the  $\gamma$ -ray pulse height spectrum in each channel with the multi-channel analyzer in the computer. For this purpose connect the linear output after the delay amplifier into the linear input A of the DB and connect the SCA positive output into the gating input of channel A (see Fig 3). Open the SCA windows wide (by lowering the thresholds and raising the widths). Look at the linear and the logic outputs of the DB with the two scope channels. Adjust the linear channel delay until the two signals overlap in time. Then feed the linear output of the DB into the linear input of the MCA card and the gate output into the gate input of the MCA card. Now run init.exe from the desktop of the computer to initialize the system. Then run switch\_A.exe to select channel A in the DB. Click on the “Maestro” icon on the desktop and you will see an MCA display of channel A. Click go and observe the pulse height spectrum. You should see clearly the two peaks corresponding to the 1.17-MeV and the 1.33-MeV  $\gamma$ -rays. *Understand the pulse height spectrum in terms of the detection processes in the NaI crystals!*

Remember: the data are being taken in coincidence with the SCA gate output. So you can now raise the threshold and lower the window width (just like before) and observe that the pulse heights below the threshold and above the window are cut out. Set both

pots on the SCA for channel A such that the window encloses (contains) *both* peaks! Then repeat all the same steps for channel B by plugging the equivalent cables into linear and gate channels B at the DB, and running switch\_B.exe. Finally start and stop the TAC with the fast outputs from the SCA's and do all the same steps again for the linear and gating output of the TAC (by plugging these signals into the C inputs on the DB). With the TAC window wide open run switch\_C.exe and observe the timing distribution in the MCA. Measure the full width at half maximum of this distribution in *ns*.

The horizontal (timing scale) in the MCA is not calibrated in time units; it is just given in channels. But you can easily calibrate it by use of a delay cable (which you can measure with the scope) or the calibrated delay pot on the SCA. Observe the movement of the prompt peak as you change the delay in by a measured amount. Determine the time resolution of the electronics from the full width at half maximum of the peak, in *ns*. Although coincidence resolution in the *ps* range can routinely be produced, a *ns* resolution is adequate as long as the number of chance coincidences, *i.e.* the probability that two unrelated pulses fall into the same time window, is small. Look at the spectrum to convince yourself that this is the case.

What is your expected ratio of prompt to random coincidences? Then close the window around the prompt peak and measure this ratio directly using an appropriate delay. Finally plug all SCA outputs into the gate inputs of the DB. You are now ready to measure the  $\gamma$ - $\gamma$  angular correlation.

#### Definition of the relative angle:

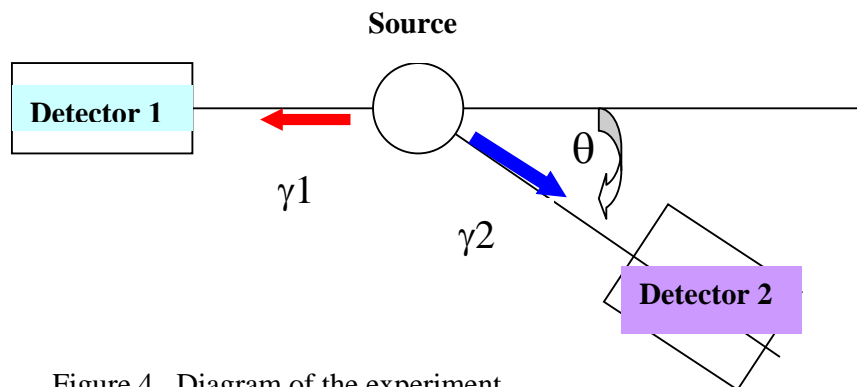


Figure 4. Diagram of the experiment.

Note that the angle  $\theta$  that is used in the following formulae describing the correlation is not the angle between the detectors, but the angle by which the second (movable) detector has been moved away from the  $180^\circ$  position relative to the stationary detector.

#### Count rate estimate:

Before you start an actual run you should determine how long you have to run at each angle to obtain meaningful counting statistics. Your aim is to verify the  $4^+ \rightarrow 2^+ \rightarrow 0^+$  transition sequence. Look into the analysis section and calculate from the theory (see Table 1) the size of the yield ratio (coincidence rates)  $Y(0^\circ)/Y(90^\circ)$  at  $0^\circ$  and  $90^\circ$ . The counting error on that ratio should be less than  $\sim 2\%$ . Nuclear processes obey the laws of statistics and thus a count of  $N$  has an uncertainty (1 standard deviation) of



$\delta N = \pm\sqrt{N}$  . Calculate how this error propagates into the ratio. Take a short run at  $0^\circ$  to get an idea of the count rate and then determine how long you have to run at each angle to obtain a meaningful measurement. Then set up the computer program that controls the experiment accordingly. *Note: You can always add the counts from individual measurements at the same angle. It is dangerous to measure for many hours at one angle because, if anything goes wrong, you have nothing to show. It is better to have runs of ~ an hour at a number of angles and then repeat the measurement at the same angle several times if necessary.*

### Description of Angular Correlations

Since the  $\gamma$ - $\gamma$  angular correlation results from spin and angular momentum involved in the nuclear transitions, and since Legendre Polynomials are eigenfunctions of angular momentum, the correlation is usually expressed in terms of Legendre Polynomials  $P_\ell(\cos \theta)$  (note that they represent a complete orthogonal set), i.e.

$$Y(\theta) = \sum_{\ell} P_{\ell} = A_0 + A_1 P_1(\cos \theta) + A_2 P_2(\cos \theta) + A_3 P_3(\cos \theta) + A_4 P_4(\cos \theta) + \dots$$

This sum can go on to larger numbers of  $\ell$  , but for the present case, where the  $\gamma$ -transitions carry away at most  $L = 2$ ,  $\ell = 4$  is the largest Polynomial needed (i.e.  $\ell \leq 2L$ ).

Legendre Polynomials are even and odd functions of  $x = \cos(\theta)$ . The first 6 polynomials are given below.

$P_0(x)$	=	1
$P_1(x)$	=	$x$
$P_2(x)$	=	$\frac{1}{2}(3x^2 - 1)$
$P_3(x)$	=	$\frac{1}{2}(5x^3 - 3x)$
$P_4(x)$	=	$\frac{1}{8}(35x^4 - 30x^2 + 3)$
$P_5(x)$	=	$\frac{1}{8}(63x^5 - 70x^3 + 15x)$
$P_6(x)$	=	$\frac{1}{16}(231x^6 - 315x^4 + 105x^2 - 5)$

Nuclear levels have specific and pure parities, and parity is conserved by the  $\gamma$ -transitions. This means that the correlation is left-right symmetric around  $\theta = 90^\circ$  and thus all polynomials with odd powers of  $x$ , i.e. with  $\ell = \text{odd}$ , must vanish. This intrinsic symmetry can, of course, be violated by any asymmetry in the experimental set-up, such as an alignment error or an error in the angle  $\theta$ .

For the current case the angular correlation is then reduced to

$$Y(\theta) = A_0 + A_2 P_2(x) + A_4 P_4(x) = A_0(1 + a_2 P_2(x) + a_4 P_4(x))$$

$A_0$  just provides an overall normalization and can be factored out.

**The real physics is then contained in the coefficients  $a_2$  and  $a_4$ .** The experiment must determine these two coefficients with sufficient accuracy to differentiate between different spin possibilities. In order to check that the experimental left-right asymmetry is small it is useful to add a  $P_1$  term. The observed  $a_1$  coefficient should be zero or small. The plot below shows the angular variations of the various  $P_\ell$  functions. In order to trace out  $P_4$  measurements should be made in  $15^\circ$  steps. In order to check for  $P_1$ , measurements must be made at angles  $>90^\circ$ .

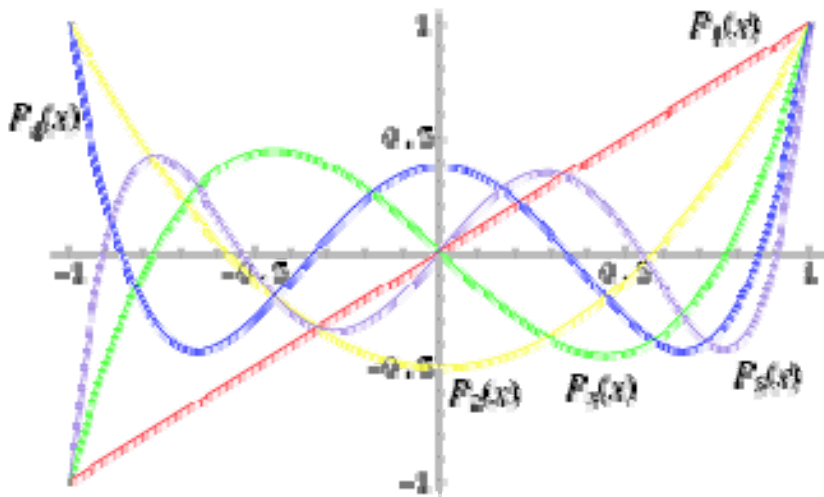


Figure 5. The Legendre polynomials are plotted as a function of  $x = \cos \theta$ .

*You should be able to get 10 data points.* The ratio of the yield at  $90^\circ$  to that at  $0^\circ$  is called the anisotropy,  $Y(90^\circ)/Y(0^\circ)$ . It can be smaller or larger than 1. You should calculate the value for this anisotropy for the expected spin sequence in  $^{60}\text{Ni}$ :  $4 \rightarrow 2 \rightarrow 0$ , using the table below. Start your measurements by observing this anisotropy to see that your results are reasonable.

#### **Background subtraction:**

The background at each angle comes from the counting of accidental coincidences from  $\gamma$ -rays that originate in unrelated decays from two different nuclei. They have a statistical, isotropic distribution. The set-up with a SCA window over the prompt peak in the time spectrum makes it easy to determine this background. You simply delay one detector pulse against the other *without* shifting the window. This way your timing acceptance remains the same, but the true coincidences are shifted outside of the window. You should determine how accurately you have to measure this background. If background counts are 10% of the true coincidences then a 10% measurement of the chance coincidences results in a 1% error on the true coincidences. Since the background is (should be) isotropic you may not have to measure it at all angles.

#### **Data Analysis:**

Your result is a set of measurements  $Y(\theta)$  of the coincidence yield (corrected for chance coincidences) for a fixed time as a function of angle  $\theta$  (see Figure 4 for definition of

this angle). Use the Excel plotting routine on your (or a lab) computer to display your data with their errors as a function of angle  $\theta$  or of  $x = \cos\theta$ .

Then make a least-square fit to the data points using the polynomial expansion including the  $P_1$  term. The MS Excel program is very nicely suited for least squares fits, using the “solver” routine. You find it at the task bar under “Tools”. The program will determine the *best* values for the four fit parameters  $A_0$ ,  $a_1$ ,  $a_2$  and  $a_4$  by minimizing  $\chi^2$ . To do this select “min” in the solver dialog box.

$$\chi^2 = \sum_i \frac{\{Data(\theta_i) - Y(\theta_i)\}^2}{\{Error(\theta_i)\}^2} \rightarrow \text{Minimum}$$

The “Error” here is the one-standard-deviation uncertainty. The first thing to decide is whether a fit is good or bad. Of course you should display your fit against the data in a plot. But  $\chi^2$  also gives a numerical indication of the quality of the fit. If  $\chi^2$  were zero you certainly would have an excellent fit. But in view of the errors on each data point a fit that goes exactly through all the points is very unlikely. Alternatively, if  $\chi^2$  is large the fit exceeds the errors and is obviously bad. What is a reasonable fit? One might think that a reasonable fit is one where the deviation at each point is of the order of one standard deviation and each term in the sum is  $\sim 1$ . Thus with 10 data points, a  $\chi^2 = 10$  would seem reasonable. However this is incorrect. If your curve had 4 data points for a function with 4 variables, the fit would necessarily come out *exact*, i.e.  $\chi^2 = 0$ . Only the over-determination of the fit with  $N=10 - 4 = 6$  free parameters allows you to judge the believability of the fit. Thus one would expect (in our case) a reasonable  $\chi^2(\text{min}) \sim 6$ . In fact  $\chi^2$  is often normalized to the number of free parameters, and then  $\chi^2(\text{min})/N \equiv 1$ .

After you determine the best (most reasonable) values for your function parameters you will need to determine the uncertainty on these values. The  $\chi^2$ -procedure allows you to do that again using “solver”. The 1-standard deviation uncertainty on each parameter is determined by varying *this one parameter* until  $\chi^2 = \chi^2(\text{min}) + 1$ , while using solver to vary all the *other* parameters (keeping the one parameter in question *fixed* during each variation) to minimize  $\chi^2$ . This produces the  $1\sigma$  deviation around the best value. To very good approximation the  $\chi^2$  value increases quadratically as each parameter is varied around  $\chi^2(\text{min})$ , and the deviation is fairly symmetric around the value at  $\chi^2(\text{min})$ . Repeat this same procedure to find the uncertainties on all the parameters of your fit.

Remember that the odd term  $a_1$  should be small, ideally zero. If there is a significant odd  $\ell$  term it means that the setup geometry and/or your angle measurement is wrong. You can try to save the data by recalibrating the angle  $\theta$  values until the odd term vanishes. This will yield a different set of parameters  $a_2$  and  $a_4$ . *These two parameters contain all the nuclear physics of the spin sequence in  $^{60}\text{Ni}$ .*

The theory of angular momentum provides *precise values* for  $a_2$  and  $a_4$  for all possible spin sequences involved in the two  $\gamma$ -transitions. However, the theory assumes exact angles whereas the experimental setup smears the angle over the finite solid angle of the detectors. Fortunately the use of the expansion in Legendre polynomials allows correction for this smearing out in an analytical form.

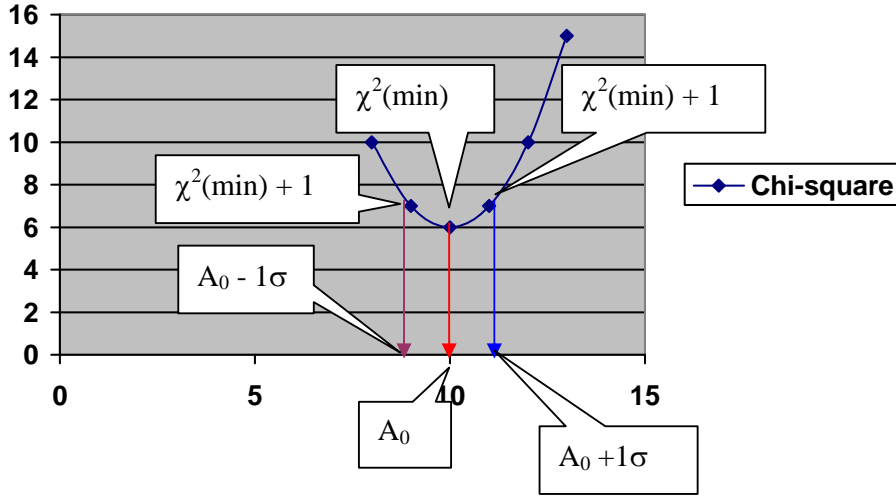


Figure 6. The  $1\sigma$  uncertainty in any parameter is found by varying the parameter in question (minimizing  $\chi^2$  using the other parameters at each step) until  $\chi^2(\min) \rightarrow \chi^2(\min) + 1$ .

If the *true* angular correlation is given by the form

$$Y_{true}(\theta) = \sum_{\ell} b_{\ell} P_{\ell}(\cos \theta) \quad ,$$

the experimental correlation including the finite-opening-angle correction is given by

$$Y(\theta) = \sum_{\ell=0}^4 \left[ \frac{J_{\ell}}{J_0} \right]_{\det-1} \cdot \left[ \frac{J_{\ell}}{J_0} \right]_{\det-2} \cdot a_{\ell} \cdot P_{\ell}(\cos \theta)$$

since the opening-angle correction, together with the correlation can be also expanded in Legendre polynomials.

The entire multiplier in front of  $P_{\ell}(\cos \theta)$  is the experimentally observed coefficient. It needs to be corrected by taking out the J functions for each detector. These correction functions are given on the next two pages for the  $a_2$  and  $a_4$  coefficients (computed for the 1¾ inch by 2 inch NaI crystals that are being used and as a function of  $\gamma$ -ray energy and distance between source and detector front). The normalization is such that  $a_0 = 1$ .

After these corrections have been applied you can compare the experimental data, with their errors, to the theoretical predictions of various possible spin sequences. Note: these predictions are exact and have no errors! Table 1 lists the coefficients for the sequence  $J_i \rightarrow J_{int} \rightarrow J_f$ , with the multiplicities of the first and second

$\gamma$ -ray, respectively. These are often named according to their multipole order, dipole for  $\ell = 1$  and quadrupole for  $\ell = 2$ .



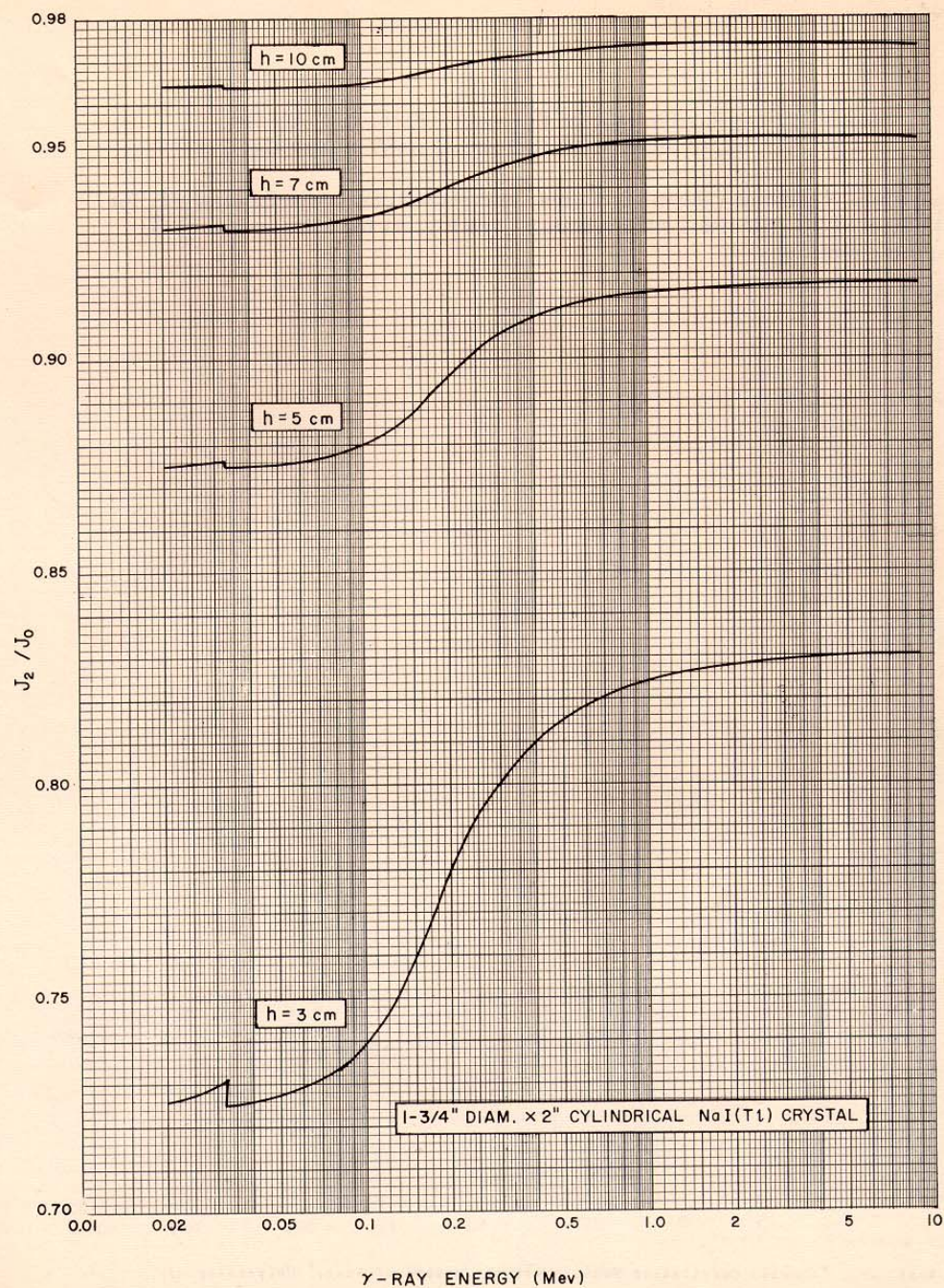


Figure 31. Angular Correlation Correction Factors:  $J_2/J_0$  for 1-3/4" diam. x 2" Cylindrical NaI(Tl) Crystals



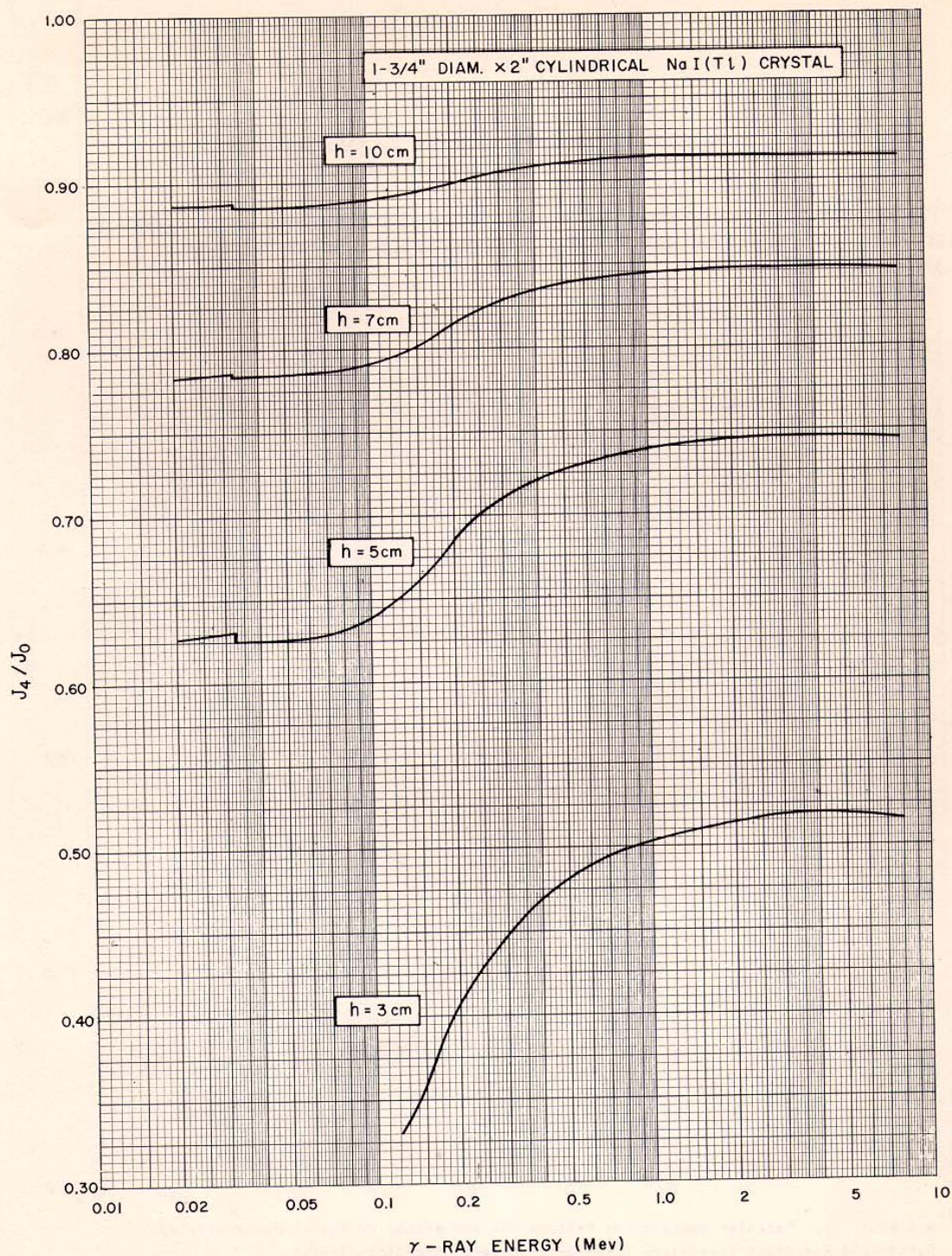


Figure 32. Angular Correlation Correction Factors:  $J_4/\bar{J}_0$  for 1-3/4" diam. 2 x 2" Cylindrical NaI(Tl) Crystals

Table 1.

	initial spin $J_i$	intermed. spin $J_{int}$	final spin $J_f$	EM Multipoles	$a_2$	$a_4$
1	0	1	0	Dipole/dipole	0.500	0
2	1	1	0	Dipole/dipole	-0.250	0
3	2	1	0	Dipole/dipole	0.050	0
4	1	1	0	Quad/dipole	-0.250	0
5	2	1	0	Quad/dipole	0.250	0
6	3	1	0	Quad/dipole	-0.071	0
7	3	2	0	Dipole/quad	-0.071	0
8	2	2	0	Dipole/quad	0.250	0
9	1	2	0	Dipole/quad	-0.250	0
10	0	2	0	Quad/quad	0.357	1.143
11	1	2	0	Quad/quad	0.179	-0.762
12	2	2	0	Quad/quad	-0.077	0.327
13	3	2	0	Quad/quad	-0.204	-0.082
14	4	2	0	Quad/quad	0.102	0.0091

You should note several items in this table.

1. Only those spin sequences are noted which end with the ground state with spin 0. This can be done because the ground state spins can often be determined from other experiments. For the even-even nucleus (even number of protons and neutrons)  $^{60}\text{Ni}$  it is very reasonable that the ground-state spin should be 0.
2. Please ascertain that all these spin sequences are geometrically possible.
3. All angular correlations that involve a dipole transition cannot have a  $P_4$ -term. It is thus important that your experiment convinces you whether you do observe such a term or not!
4. The theory also predicts the signs of the coefficients. Thus the sign of your observed coefficients is important.
5. Finally, you can see that the predicted values often differ grossly, so that the correct spin sequence can be picked out clearly. Obviously there is no error on these spins!

### Some nuclear physics:

After you determine the correct spin sequence go back and look at the level scheme of  $^{60}\text{Ni}$ . Note from the  $\gamma$ -ray branching ratios of the various levels that there are other levels that from their spin assignments could decay strongly to the ground-state, but do not. This tells you that there is something special about the 1.332-MeV and 2.506-MeV levels. Also note that their transition energies are almost the same, and that the upper level spacing is a bit less than the lower one.

*Think harmonic oscillator (HO) and describe the character of these excitations from the ground state! (You don't need any nuclear physics to figure this out!). Then, from the HO parameters, obtain an estimate of the size of the  $^{60}\text{Ni}$  nucleus.*

**References:**

1. A. Melissinos, Experiments in Modern Physics, Academic Press (1960), p. 412.  
A. Melissinos and J. Napolitano, Experiments in Modern Physics, Academic Press (2003), p. 409.
2. K. Siegbahn,  $\alpha$ - $\beta$ - $\gamma$ -Ray Spectroscopy, North Holland (1965), p. 997
3. R. Evans, The Atomic Nucleus, (1955), p.234 – 243. This is an old book but provides an instructive introduction.
4. D. R. Hamilton: Phys. Rev. 58, 122 (1940).
5. Brady and Deutsch, Phys. Rev. 78, 558 (1950).
6. H. Fraunfelder, Annual review of Nuclear Science 2, 129 (1953).



# Appendix

## Automated Data Collection for the $\gamma$ - $\gamma$ Correlation Experiment:

The executable (.exe) program files below are located in the c:\gamma\exe\ directory. They can be executed via icons on the desktop.

**init.exe** (initialize prior to scan ... run from a Maestro JOB file)

The stepper motor is reset to the zero degree position.

**c:\gamma\gamma\gamma.dat** determines *dwelt time*, *angle*, and *maximum angle*

**c:\gamma\gamma\gamma.ini** is read to set system parameters

**c:\gamma\data\scaler.dat** file is created (overwritten)

**scan.exe** (perform a scan ... run from a Maestro JOB file)

Read parameters from **c:\gamma\gamma\gamma.dat**

The scalers are reset, started (*dwelt time*), stopped

The scaler data is appended to **c:\gamma\data\scaler.dat**

The movable detector is moved to the next position (*angle*)

The preceding three steps are repeated until the last position is reached

The movable detector is reset to the zero degree position

**switch\_A.exe** (select Detector A, Coincidence A)

The MCA input is connected to DETECTOR A

The MCA gate input is connected to COINCIDENCE A

**switch\_B.exe** (select Detector B, Coincidence B)

The MCA input is connected to DETECTOR B

The MCA gate input is connected to COINCIDENCE B

**switch\_C.exe** (select Detector C, Coincidence C)

The MCA input is connected to DETECTOR C

The MCA gate input is connected to COINCIDENCE C

**switch\_OFF.exe** (de-select MCA)

The MCA input is disconnected

The MCA gate input is connected GND (zero volts)

**switch\_NEXT.exe** (switch to next detector ... run from a Maestro JOB file)

Read file **c:\gamma\data\scaler.dat** to determine the last detector selection

The MCA input is switched to the "NEXT" detector

If the last selection was A ... switch to B

If the last selection was B ... switch to C

If the last selection was C ... switch to A

## **Basic Procedures:**

1. Edit the file C:\gamma\gamma.dat to enter the desired dwell time, position angle, and the maximum angle. The dwell time is the time (in seconds) that the movable detector will sit at a given position. The scalers are enabled during the dwell time. The movable detector starts at zero degrees and will move to the next position (determined by the position angle) after the specified dwell time. When the movable detector reaches the physical limit, or if the next position is greater than the maximum angle, the movable detector will be reset to the zero angle position.
2. Edit the file C:\gamma\gamma.job to specify the number of “loops” (look for the “loop n” statement in the gamma.job file, where n specifies the number of loops. Each loop runs the C:\gamma\scan.exe program which steps the movable detector from position to position starting at zero degrees. At each position, the scalers will be enabled for the dwell time. At the completion of the dwell time the scaler data is stored in the C:\gamma\data\scaler.dat file. During the “scan”, Maestro acquires a spectrum from the selected input (A, B, or C). The first loop selects input A. Each successive loop selects the next input (i.e. B, then C, then A etc.). At the completion of each loop, the acquired spectrum is stored as C:\gamma\data\spec???.chn” where ??? is the loop number. Therefore, if four loops are specified (loop 4), at the end of the process, the following files will be saved to the C:\gamma\data\ directory:

spec000.chn (spectrum with detector ‘A’ as the input)  
spec001.chn (spectrum with detector ‘B’ as the input)  
spec002.chn (spectrum with detector ‘C’ as the input)  
spec003.chn (spectrum with detector ‘A’ as the input)  
scaler.dat (text file with scaler data from each position for each loop)

3. Copy the data files to a new location. Any subsequent scans will overwrite files in the C:\gamma\data\ directory.

Note: Due to the real time system resources required by the Maestro program, the movable detector motion may seem jerky or erratic. This motion is normal and should not interfere with proper operation.

Note: Every time you reload Maestro you must set the ADC gate to coincidence. The sequence is Acquire → MCB properties → ADC gate → coincidence

Sample gamma.dat file:

This data file contains integer data constants used to specify the dwell time (seconds) at each data taking position, the angle (degrees) between each position and the maximum angle (last position).

A valid entry must have a positive numeric value (>0) as the first entry on a line (excluding tabs and spaces). Only one entry per line is recognized. The order is fixed (i.e. the first entry is the dwell time, second entry is the position angle and the final entry specifies the maximum angle for the last position).

If the maximum angle is > the mechanical stop, the mechanical stop will be used as the maximum angle.

Edit this file to meet experimental needs. Each data line entry MUST be terminated with a carriage return.

1800	dwell time in seconds
15	position angle in degrees
105	maximum angle

Sample gamma.ini file:

\*\*\* DO NOT EDIT THIS FILE \*\*\*

This data file contains integer data constants used to specify several control parameters such as stepper motor speed, stepper motor calibration and the maximum angle to process etc.

Any changes to this file will effect the operation of the low level c code.

\*\*\* DO NOT EDIT THIS FILE \*\*\*

34	steps/degree
12	steps/degree delta
1000	stepper motor pulse width
6000	stepper motor repitition rate
450	scaler readout delay1
50	scaler readout delay2
140	max limit
700	travel time/deg
5	live time offset

Sample scaler.dat file:

Sun Nov 07 04:56:42 AM

Det.	Angle (sec.)	Scaler #1	Scaler #2	Scaler #3	Time
A	0	0	0	300794	10.060
A	15	0	0	300707	10.050
A	30	61896	38430	16528	10.100
A	45	60874	71898	22367	10.100
A	60	0	300875	0	10.050
A	75	0	300845	0	10.060
A	90	0	302308	0	10.110
A	105	0	300655	0	10.050
B	0	0	300758	0	10.050
B	15	0	299980	0	10.050
B	30	0	301508	0	10.060
B	45	0	300125	0	10.050
B	60	0	300872	0	10.050
B	75	0	302377	0	10.100
B	90	0	301232	0	10.050
B	105	0	300707	0	10.050
C	0	0	299969	0	10.060
C	15	0	302346	0	10.100
C	30	0	303174	0	10.100
C	45	0	301406	0	10.050
C	60	0	300792	0	10.050
C	75	0	302663	0	10.100
C	90	0	300933	0	10.050
C	105	0	302570	0	10.100
A	0	0	300439	0	10.050
A	15	0	302536	0	10.100
A	30	0	300691	0	10.050
A	45	0	299727	0	10.060
A	60	0	300982	0	10.050
A	75	0	302398	0	10.100
A	90	0	302406	0	10.100
A	105	0	301106	0	10.050

Note: The “Time” column in the scaler.dat file is the dwell time as measured by the PC’s real time clock.  
Each “loop” of the gamma.job script is represented by a given detector sequencing through each angle position (i.e. the last eight lines of the sample file above, represent the final (4<sup>th</sup>) “loop” of the gamma.job script).

Sample gamma.job file:

```
run_minimized "c:\gamma\exe\init.exe"
wait "c:\gamma\exe\init.exe"
set_detector 1
set_preset_clear
call "c:\gamma\coinc.job"
run_minimized "c:\gamma\exe\switch_A.exe"
wait 1
loop 4
clear
start
run "c:\gamma\exe\scan.exe"
wait "c:\gamma\exe\scan.exe"
stop
run_minimized "c:\gamma\exe\switch_NEXT.exe"
wait 1
fill_buffer
set_detector 0
describe_sample "Sweep ??? spectrum"
save "c:\gamma\data\spec???.chn"
set_detector 1
end_loop
```

