



$$\Delta E_{\text{in}}^* = \sqrt{\left(\frac{\Delta L^*}{k_L S_L}\right)^2 + \left(\frac{\Delta C^*}{k_C S_C}\right)^2 + \left(\frac{\Delta H^*}{k_H S_H}\right)^2} + R_T \frac{\Delta C^* \Delta H^*}{k_C S_C k_H S_H}$$

Note: The formulas below should use degrees rather than radians; the issue is significant for R_T .
The k_L , k_C , and k_H are usually unity.

$$\Delta L^* = L_2^* - L_1^*$$

$$\bar{L} = \frac{L_1^* + L_2^*}{2} \quad \bar{C} = \frac{C_1^* + C_2^*}{2}$$

$$a'_1 = \frac{a_1^* - \bar{a}^*}{\sqrt{C_1^*}}$$

$$\bar{C}^* = \frac{C_1^* + C_2^*}{2} \text{ and } \Delta C^* = C_2^* - C_1^* \text{ where } C_1^* = \sqrt{a_1^{*2} + b_1^{*2}} \quad C_2^* = \sqrt{a_2^{*2} + b_2^{*2}}$$

$$h'_1 = \begin{cases} \arctan(a'_1/b'_1) & \text{if } b'_1 > 0 \\ \arctan(a'_1/b'_1) + 180^\circ & \text{if } b'_1 < 0 \text{ and } a'_1 \geq 0 \\ \arctan(a'_1/b'_1) - 180^\circ & \text{if } b'_1 < 0 \text{ and } a'_1 < 0 \end{cases}$$

$$\Delta h = \begin{cases} h'_2 - h'_1 & |h'_1 - h'_2| \leq 180^\circ \\ h'_2 - h'_1 - 360^\circ & |h'_1 - h'_2| > 180^\circ, h'_2 > h'_1 \\ h'_2 - h'_1 + 360^\circ & |h'_1 - h'_2| > 180^\circ, h'_2 < h'_1 \end{cases}$$

$$\Delta H^* = 2\sqrt{C_1^* C_2^*} \sin(\Delta h'/2), \quad H' = \begin{cases} (h'_1 + h'_2 + 360^\circ)/2 & |h'_1 - h'_2| > 180^\circ \\ (h'_1 + h'_2)/2 & |h'_1 - h'_2| \leq 180^\circ \end{cases}$$

$$\Delta H^* = 2\sqrt{C_1^* C_2^*} \sin(\Delta h'/2), \quad H' = \begin{cases} (h'_1 + h'_2 + 360^\circ)/2 & |h'_1 - h'_2| > 180^\circ \\ (h'_1 + h'_2)/2 & |h'_1 - h'_2| \leq 180^\circ \end{cases}$$

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$$\Delta H^* = 2\sqrt{C_1^* C_2^*} \sin(\Delta h'/2), \quad H' = \begin{cases} (h'_1 + h'_2 + 360^\circ)/2 & |h'_1 - h'_2| > 180^\circ \\ (h'_1 + h'_2)/2 & |h'_1 - h'_2| \leq 180^\circ \end{cases}$$

22nd IS&T Color and Imaging Conference, November 2014, Boston MA

Outline

- **Motivation**

- Large vs. Small Differences

- **Experiments**

- Hundreds of pairs of stimuli with ΔE_{00} of 20
- Sorting of 9 pairs of differences from smallest to largest
- Web and laboratory

- **Results**

- Observers rank pairs having equal ΔE_{00} as being different

- **Similarity metric**

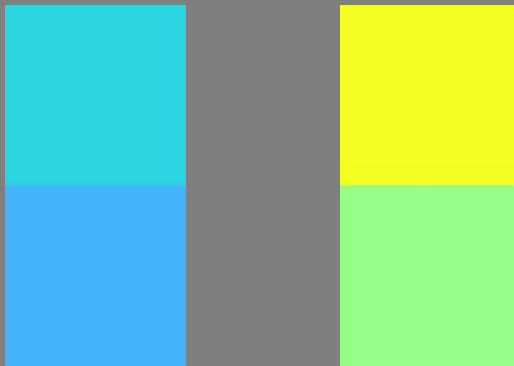
- Cosine similarity given categorical vectors
- Cosine similarity is small within a category & large across categories

- **Discussion and Speculation**



Motivation

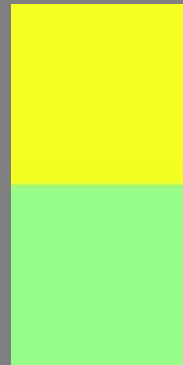
Which of these two pairs, has the larger ΔE_{00} ?



Motivation

Using ΔE_{00} they are equal

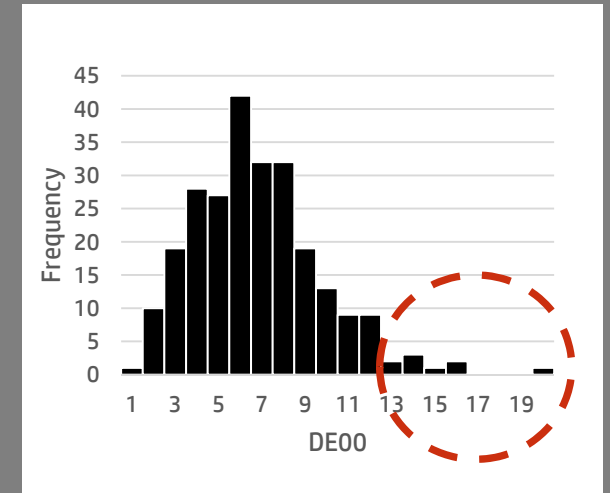
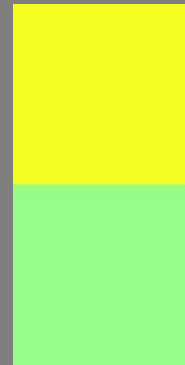
Both are $20 \Delta E_{00}$



Motivation

But isn't ΔE_{00} only for small differences ? OK, so how to measure maximum error?

Both are $20 \Delta E_{00}$



Large vs. Small Color Differences

We'll fill this in by the end of the talk

	Small Differences	Large Differences
Application	Just Noticeable Difference	
Central Question	Do 2 colors match?	
Metrics*	ΔE^*_{ab} , ΔE_{94} , ΔE_{00}	
Underlying Metric	Euclidean distance, with weighting schemes	
Input	CIELAB coordinates, weights	
Output	“Geometric” distance, where a JND is approximately < 1	

* Small color differences metrics not recommended for large differences or greater than ~5.



Experiments

Sorting of Non-Repeating Random Vectors

- **Stimuli**

- Non-repeating random vectors
- Similar to a farthest-point sampling of vector endpoints
- All vectors within 0.0001 of $20 \Delta E_{00}$,

*Not regular or centroid plus
offsets sampling*

- **Task**

- Sorting of multiple color differences
- Observer sorts 9 color differences from smallest to largest
- 21 blocks of 9 random color difference pairs

*Not forced choice paired
comparison of 2 pairs*

- **Web-Based Experiment**

- 285 participants; perform sorting of 1 block of differences; unknown displays

- **Laboratory Experiment**

- 12 participants; sort all 21 blocks of differences; sRGB mode HP DreamColor Z27x Display



Non-Repeating Random Vectors

(Insert comment about Jim King here)

```
NonRepeatingRandomVectors(n, dist, t1, t2)
i = 0
while (i < n)
    test_ave = F
    test_ends = F
    start = RandomVector()
    end = start
    while ((Distance(start, end) - dist) < epsilon)
        end = RandomStep()

    ave = Average(start, end)
    for (i = 0 to number_vectors)
        if ((ave - ave[i]) < t1) test_ave = T

        d11 = start - start[i]
        d22 = end - end[i]
        d12 = strt - end[i]
        d21 = end - start[i]
        min_diff = Min(d11, d22, d12, d21)
        if (min_diff < t2) test_ends = T

    if ((test_ave & test_ends) = T)
        AddVector(start, end)
        ++I
```



Non-Repeating Random Vectors

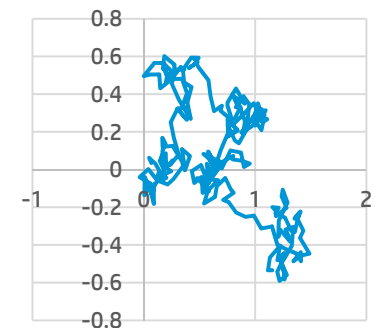
```
NonRepeatingRandomVectors(n, dist, t1, t2)
i = 0
while (i < n)
    test_ave = F
    test_ends = F
    start = RandomVector()
    end = start
    while ((Distance(start, end) - dist) < epsilon)
        end = RandomStep()

    ave = Average(start, end)
    for (i = 0 to number_vectors)
        if ((ave - ave[i]) < t1) test_ave = T

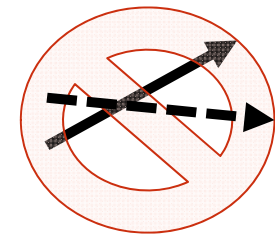
        d11 = start - start[i]
        d22 = end - end[i]
        d12 = start - end[i]
        d21 = end - start[i]
        min_diff = Min(d11, d22, d12, d21)
        if (min_diff < t2) test_ends = T

    if ((test_ave & test_ends) = T)
        AddVector(start, end)
        ++i
```

Random walk

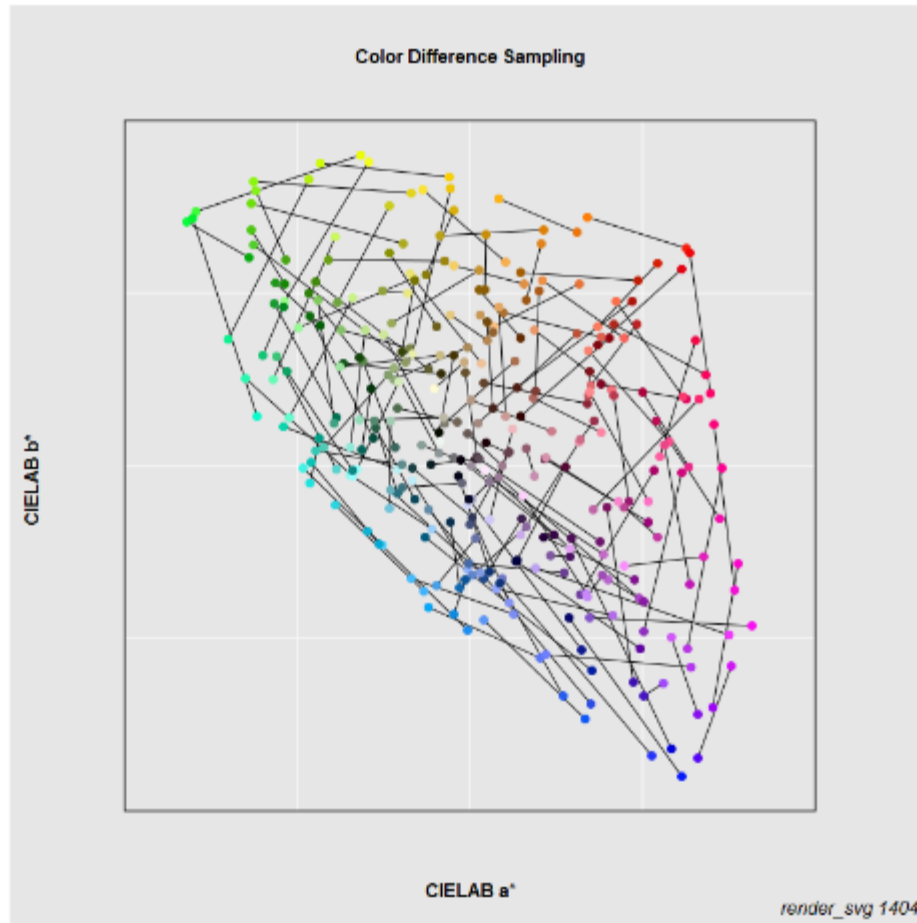


Don't duplicate vectors



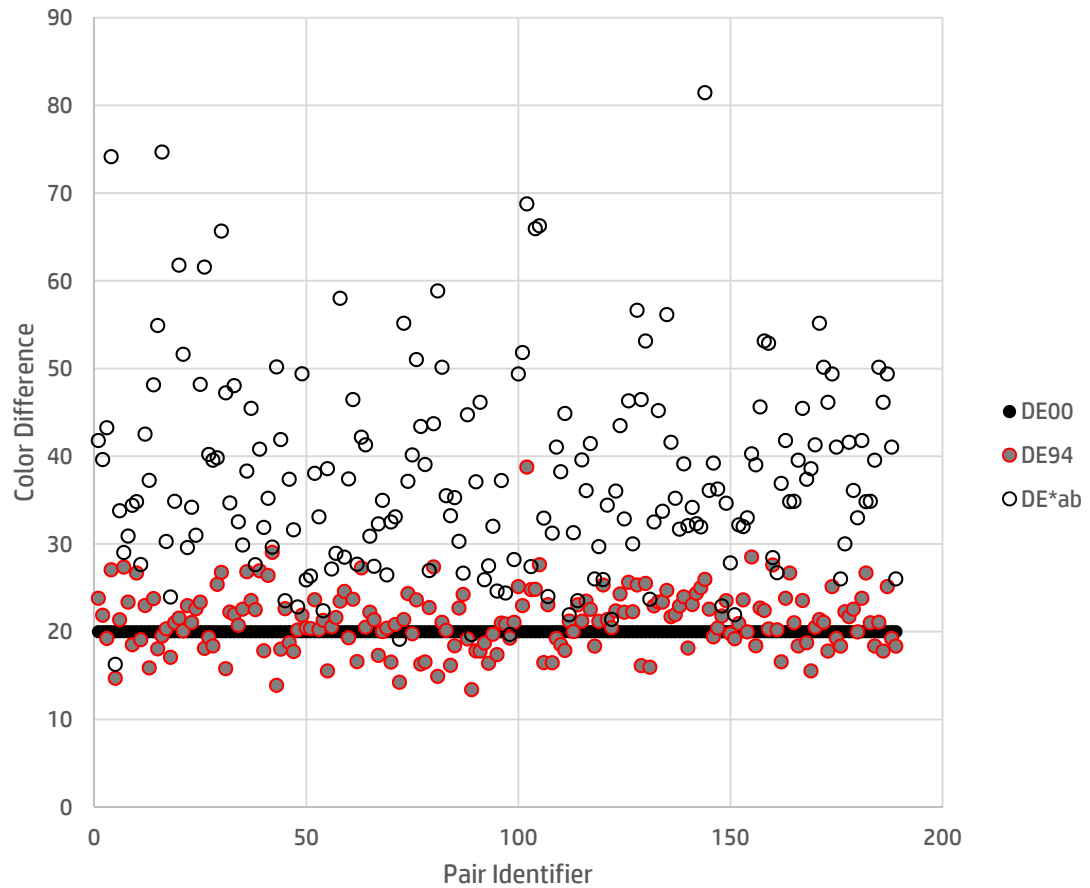
Non-Repeating Random Vectors

21 blocks rendered as RGB patches and as a CIELAB a^* versus b^* plot



Non-Repeating Random Vectors

20 ΔE_{00} and the corresponding ΔE_{94} and ΔE^*_{ab} color differences



Color Difference Sorting










HTML5 drag-and-drop interface with color patches rendered to 90x90 pixels in size

Color Difference Sorting Experiment

Please drag and drop the color differences, shown as two neighboring patches, from the top to bottom row.

As you move the colors please sort them from smallest color difference, to the left, to the largest color difference, to the right.

Note that you can not drag two items on top of each other. When you are satisfied with your sorting press the "SUBMIT" button. Thank you.

Drag Top to Bottom									
Sorted Differences									
	Smallest Difference								Largest Difference

Submit

NOTE: This experiment works with best with browsers that support the HTML5 drag-and-drop [functionality](#). [1404]



Results: *Block 118*

- **Initial visualizations**

- Each set of 9 sub-plots is for 1 block

A **X-axis is sorted rank**

- Left: smallest difference
- Right: largest difference

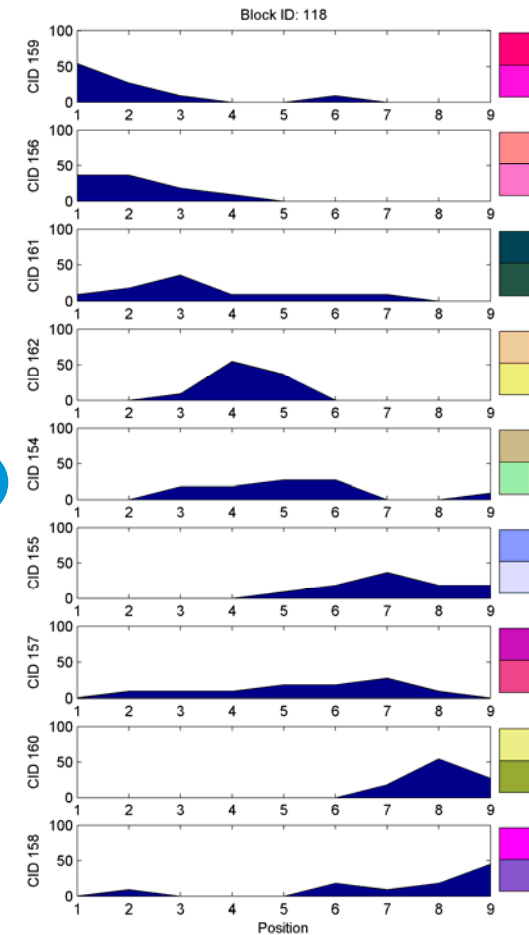
B **Y-axis is relative frequency**

- Larger: observers consistently used this rank
- Zero: observers did not use this rank

C **Approximate color rendering**

- Shown to the right of each sub-plot

- **Approximate sorting top to bottom**



Results: *Block 118*

A Smallest difference

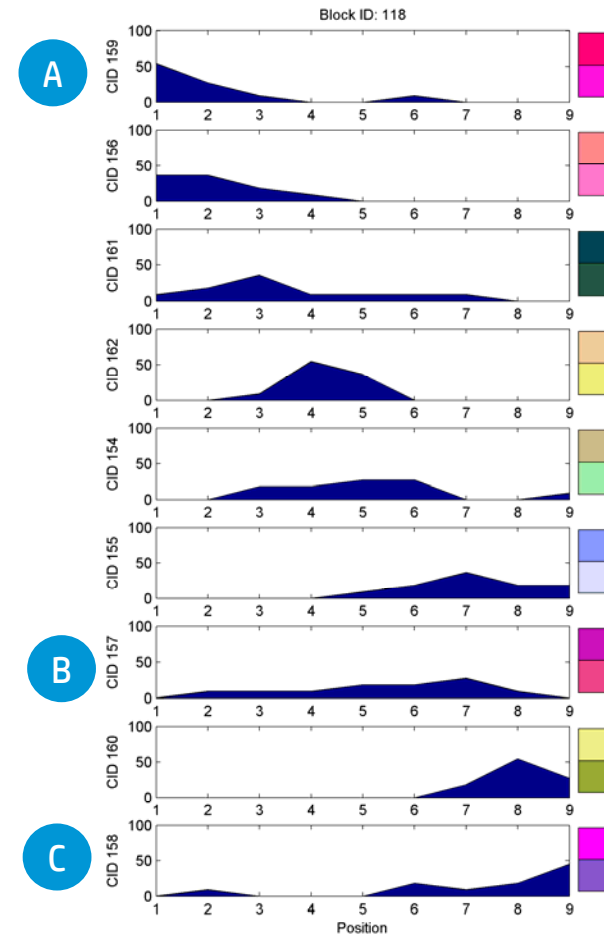
- Relatively narrower distribution

B Intermediate differences

- Relatively wider distributions

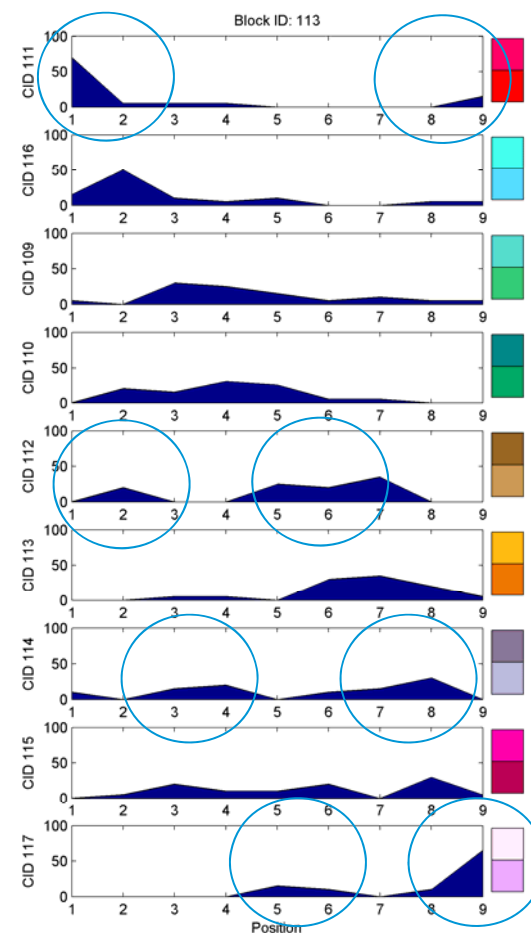
C Largest difference

- Relatively narrower distribution
- Qualitatively, at crossing of naming boundaries?
- **For ideal set of 9 equal differences**
 - Flat histograms for each pair
 - Not seen in the experimental data...



Results: *Block 113*

- Similar results seen across other blocks
- Similar results for web-based & laboratory
- But also multi-modal distributions?
 - Complicates even basic analysis
 - Can occur for any ranking, shown circled to right
 - Almost as if there were multiple criterion for sorting....



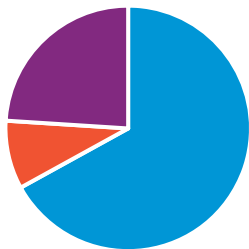
Results: *Block 103*

- **Name boundary crossing**

- Tends to result in a mode with a larger rank or sorted color difference
- Shown circled

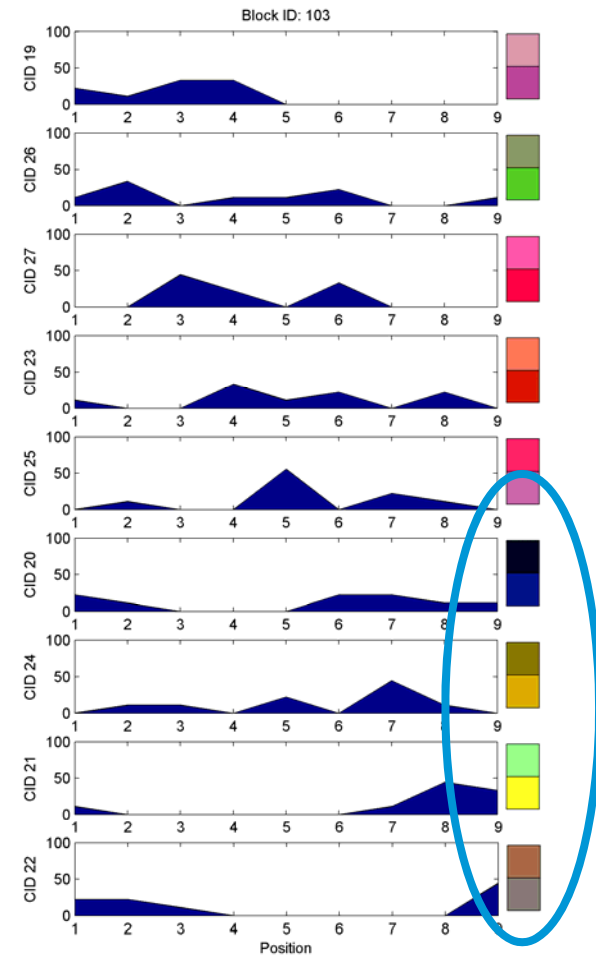
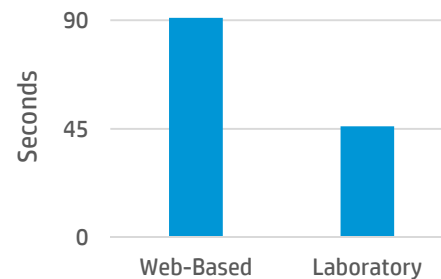
- **Other interesting results**

Initial Sorting Rank



■ Smallest ■ Intermediate ■ Largest

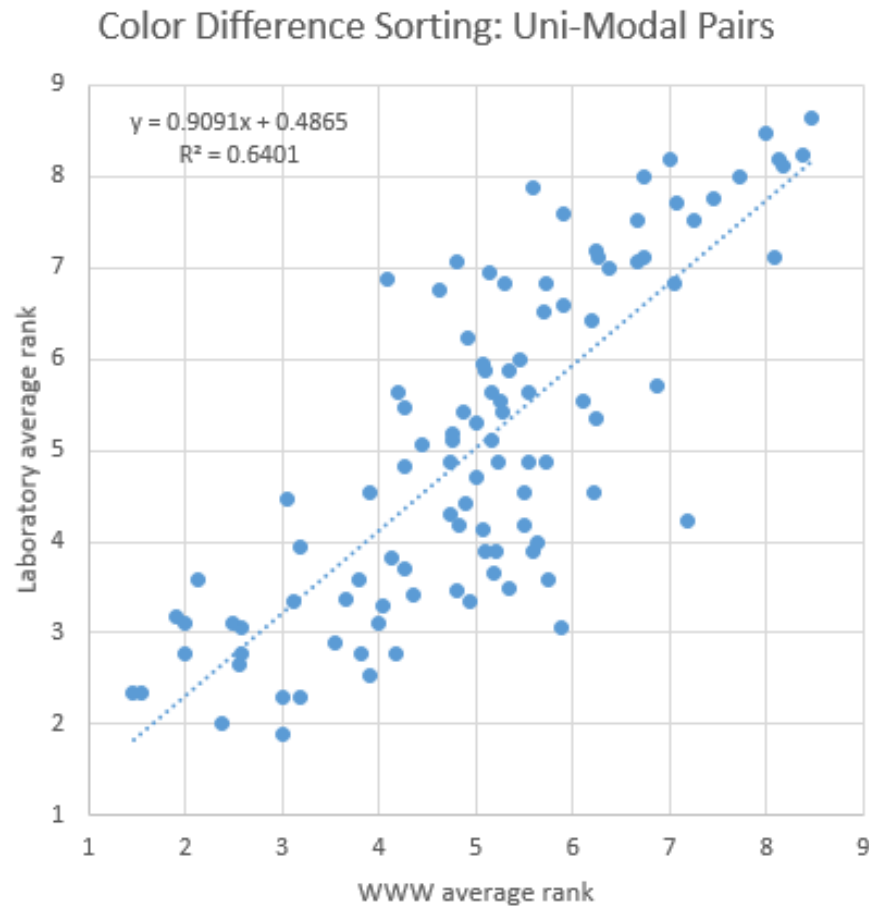
Median Time to Sort



Correlation of Web-Based and Laboratory Experiments

Limited to the approximately 2/3rds of pairs with uni-modal distributions* for both experiments

* F. Schwaiger, H. Holzmann, and S. Vollmer, "bimodalitytest: Testing for bimodality in a normal mixture", R package version 1.0, (2013)



Similarity Metric

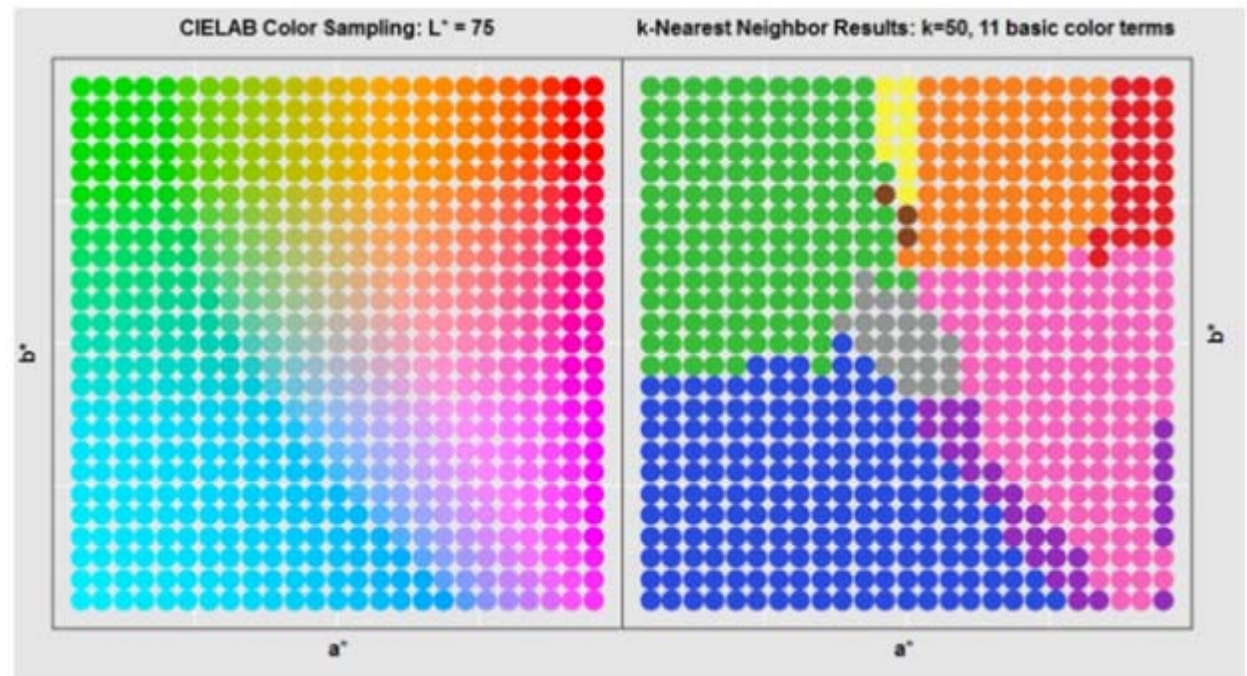
- **Given these results can a similarity metric be designed that is:**
 - Smaller within a color category
 - Larger across color categories
 - Not unlike results seen in Categorical Perception from vision science
- **Not based on weighted Euclidean distances?**
- **Similar to other similarity metrics?**
 - Value of 1 for identical
 - Value of 0 for dissimilar



Similarity Metric

Use Categorical Vectors

- **K-nearest neighbors** used to transform input CIELAB data, on left, to categorical counts, maximum shown color coded on right
- Start with basic 11 terms as the vocabulary
- Similar processing used for document classification



Similarity Metric

Use Cosine Similarity of Categorical Vectors

- **K = 100**
- **Same example from the motivation slides earlier**
- **These two have ΔE_{00} differences of 20...**

$$\Delta S = \frac{\sum A \cdot B}{\sqrt{\sum A^2 \cdot \sum B^2}}$$

	Top	Bottom	Top	Bottom
sRGB	43, 212, 224	67, 179, 253	244, 254, 32	150, 253, 137
Gray	3	1	0	0
Blue	96	98	0	0
Green	0	1	0	83
Pink	0	0	0	0
Black	1	0	0	0
Green	0	0	0	0
Orange	0	0	0	1
Purple	0	0	0	0
Red	0	0	0	0
White	0	0	0	0
Yellow	0	0	100	16
Similarity	0.99		0.18	



Large vs. Small Color Differences

We'll fill this in by the end of the talk

	Small Differences	Large Differences*
Application	Just Noticeable Difference	Consistently Describable Difference(s)
Central Question	Do 2 colors match?	When do 2 colors stop looking similar?
Metrics*	ΔE^*_{ab} , ΔE_{94} , ΔE_{00}	ΔS
Underlying Metric	Euclidean distance, with weighting schemes	Cosine similarity
Input	CIELAB coordinates, weights	Categorical or lexical vectors
Output	Geometric distance, where a JND is approximately < 1	Similarity measure, where 0 is completely dissimilar

* Large color differences also relevant to image segmentation, analysis & retrieval.



Discussion & Speculation

- **Large color differences do not necessarily have a single sorting**
 - Can be consistently sorted by multiple criterion
 - More sophisticated analysis needed to detect systematic trends in multi-modal sorting
- **Small difference metrics probably not a good idea for maximum errors**
 - At a minimum probably want to visualize
- **Cosine similarity of categorical vectors is a promising metric**
 - Initial stages of optimization but already useful in ways that differ from the ΔE 's
 - Training data and algorithms are key aspects of the metric
- **Experiment and data are public and ongoing**
 - Same data could also be used to investigate more uniform color spaces
 - Would like to *expert-source* additional analysis & related experiments

