

1.

i.

Since `rand()` always returns a value between 0 and 1, so the best case is `rand() = 1`

The worst case is `rand() < 0.5`; and average case would be between 0 and 1.

- a. Best case: $1(\text{initialize}) + 1(\text{increasement}) + 2(\text{increasement}) = 4$ operations
Worst case: $1(\text{initialize}) + 1(\text{increasement}) + 6(\text{increasement}) = 8$ operations
Average case: $1(\text{initialize}) + 1(\text{increasement}) + 4(\text{increasement}) = 6$ operations
- b. Best case, worst case, average case: $\Omega(1)$, $O(1)$, $\Theta(1)$
- c. $f = O(1)$
- d. $f = \Omega(1)$
- e. $f = \Theta(1)$

ii.

Best case would be $N \leq 0$, so the loop doesn't run, worst and average case would be $N > 0$ with `rand() < 0.5`

- a. Best case: $1 + 1 + 1 = 3$ operations
Worst case: $1 + 1 + (n + 1) + n + n + n + n = 5n + 3$ operations
Average case: $(5n)/2 + 3$
- b. Best case: $\Omega(1)$
Worst case: $O(n)$
Average case: $\Theta(n)$
- c. $f = O(n)$
- d. $f = \Omega(1)$

iii.

Best case would be $N \leq 0$ so the loop doesn't run

Worst case would be $N > 0$ and `unlucky == true`

- a. Best case: $1 + 1 + 1 = 3$ operations
Worst case: $1(\text{Initial "count"}) + 1(\text{initial "i"}) + n(\text{checks N}) + 1(\text{check unlucky}) + 1(\text{assign "j"}) + (n^2 + n)/2(\text{checks}) + 2(n^2 + n)/2(\text{increasement}) + (n^2 + n)/2(\text{decrement}) + n(\text{increment}) = 2n^2 + 4n + 7/2$ operations
Average case: $n^2 + 2n + 4$
- b. Best case: $\Omega(1)$
Worst case: $O(n^2)$
Average case: $\Theta(n^2)$
- c. $f = O(n^2)$
- d. $f = \Omega(1)$

iv.

Best case would be `unlucky == false` so the loop doesn't run

Worst case would be `lucky == true` and $N > 0$ so the loop can run

- a. Best case: $1(\text{Init "count"}) + 1(\text{Init "i"}) + 1(\text{check unlucky}) = 3$ operations
Worst case: $1(\text{Init "count"}) + 1(\text{Init "i"}) + 1(\text{check unlucky}) + n(\text{check "i"}) + n(\text{increment for "count"}) + n(\text{decrement for "i"}) = 3n + 3$ operations
Average case: $3n/n + 3$ operations
- b. Best case: $\Omega(1)$
Worst case: $O(n)$
Average case: $O(n)$
- c. $f = O(n)$

d. $f = \Omega(1)$

v.

The best case would be $N \leq 0$ so that the count remains 0, the second loop also inactive

The worst case would be $N > 0$ then the 2 loops can run, $\text{rand}() < 0.5$

a. Best case: 1(init "count") + 1(init "i") + 1(check "i") + 1 (init "num") + 1 (init "j") + 1 (check "j") = 6 operations

Worst case: 1(init "count") + 1(init "i") + n (check "i") + n (init "num") + n(check "num") + n("count" increment) + n("i" increment) + 1(init "num") + 1 (init "j") + n(check "j") + n("count" increment) + n("j" increment) = $8n + 4$ operations

Average case: $4n + 5$ operations

b. Best case : $\Omega(1)$

Worst case: $O(n)$

Average case: $\Theta(n)$

c. $f = O(n)$

d. $f = \Omega(1)$

vi.

the best case would be $N \leq 1$ so the loops doesn't run

the worst case would be $N > 1$

a. Best case: 1(init "i") + 1(check) = 2 operations

Worst case: 1(init "i") + $n-1$ (check) + $n-1$ (init "j") + $n(n+1)/2-2$ (check) + $n(n+1)/2-1$ (check) + $n(n+1)/2-2$ (swap) + $n(n+1)/2-2$ (increment) $n-1$ (increment) = $2x^2 + 5x - 10$ operations

Average: $(2n^2 + 5n - 8)/2$

b. Best case : $\Omega(1)$

Worst case: $O(n^2)$

Average case: $\Theta(n^2)$

c. $f = O(n^2)$

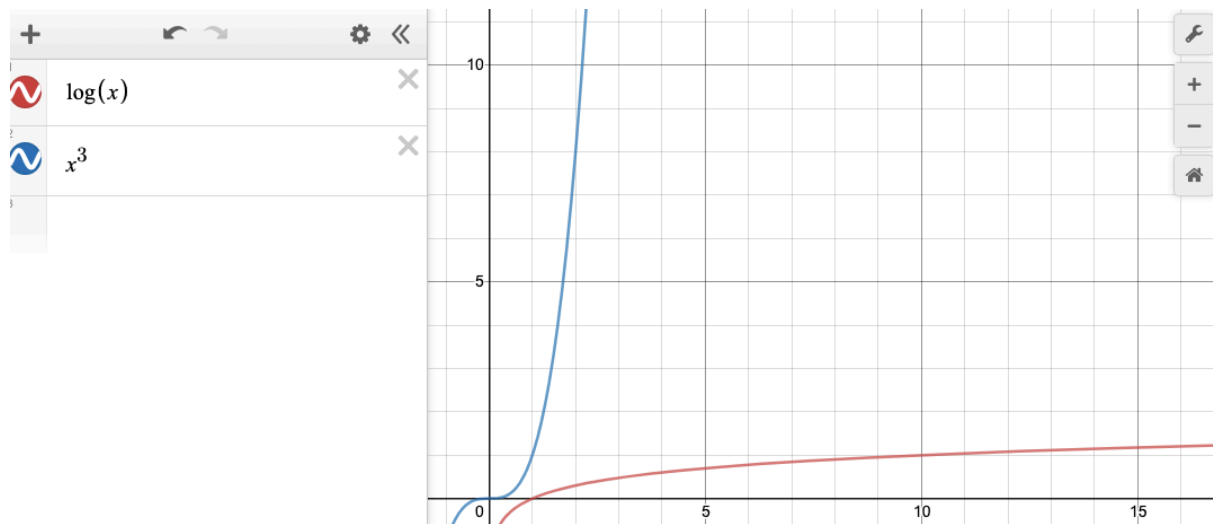
d. $f = \Omega(1)$

f. The best way to describe the performance of algorithms is using big O notation, because it represent the worst case scenarios, which are regularly occurs during normal usage.

2.

Big O describe the worst-case scenario, can be used to describe the execution time or space used by algorithms.

3. $\Theta(n^3)$ always takes longer to run than $\Theta(\log(n))$ with every $n \geq 1$. Because when $n < 1$, $\log(n) < 0$. In common usage, the size of input to algorithms is > 0 so $\Theta(\log(n))$ will be faster.



4.

True, the highest power of variable n is 2, therefore $O(n^2)$

True, the highest power of variable n is 1, therefore $O(n)$

False, as n^4 is not a lower bound to n^3

True, because n is a lower bound for it