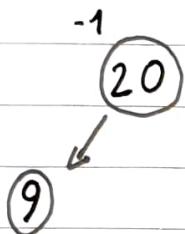


insert (20):

(20)

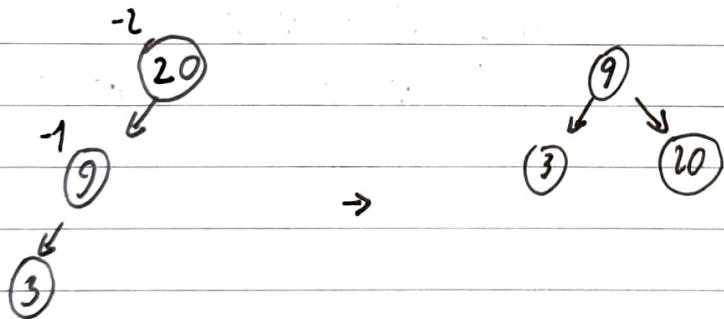
insert (9):



$9 < 20$

- to left subtree
- left is null, insert (9) to left

insert (3):

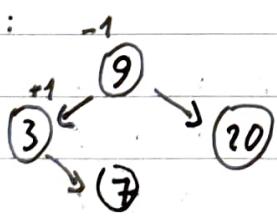


$3 < 20 \rightarrow$  to left subtree

$3 < 9 \rightarrow$  to left subtree

The tree's left is heavier  
→ 1 rotate right

insert (7):

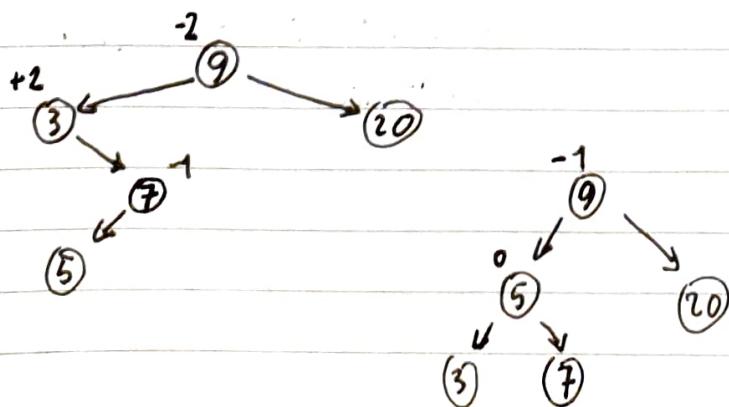


$7 < 9 \rightarrow$  to left

$7 \geq 3 \rightarrow$  to right

(3)'s right is null, insert 7

insert (5):



$5 < 9 \rightarrow$  to left

$5 \geq 3 \rightarrow$  to right

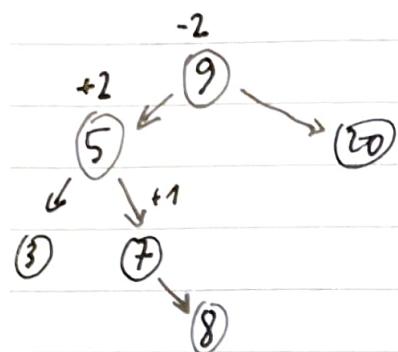
$5 < 7 \rightarrow$  to left

(7)'s left is null, insert (5)

(3) tree is ~~right~~ heavy

→ double rotate left

insert(8):



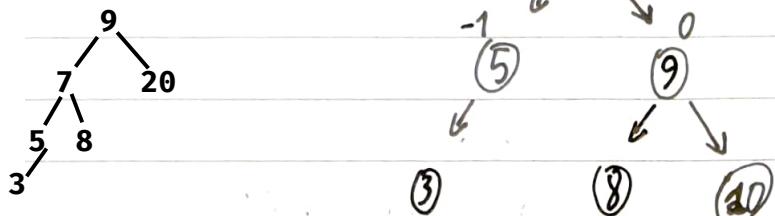
$8 < 9 \rightarrow \text{to left}$

$8 > 5 \rightarrow \text{to right}$

$8 > 7 \rightarrow \text{to right}$

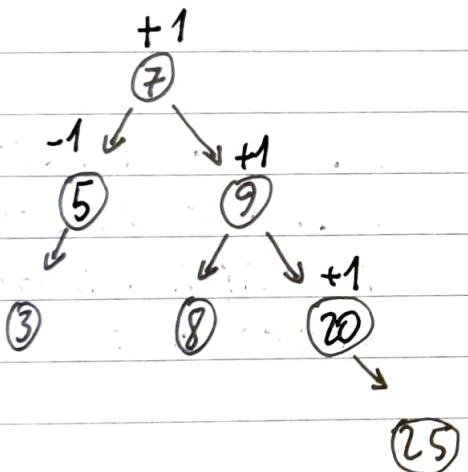
(7)'s right is null, insert 8

Tree is left heavy



Rotate Left right

insert(25)



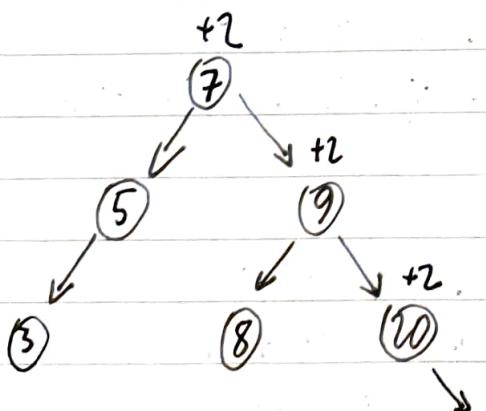
$25 > 7 \rightarrow \text{right}$

$25 > 9 \rightarrow \text{right}$

$25 > 20 \rightarrow \text{right}$

(20)'s right is null, insert 25

insert 30



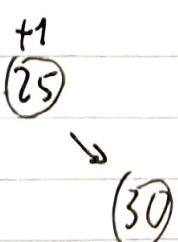
$30 > 7 \rightarrow \text{right}$

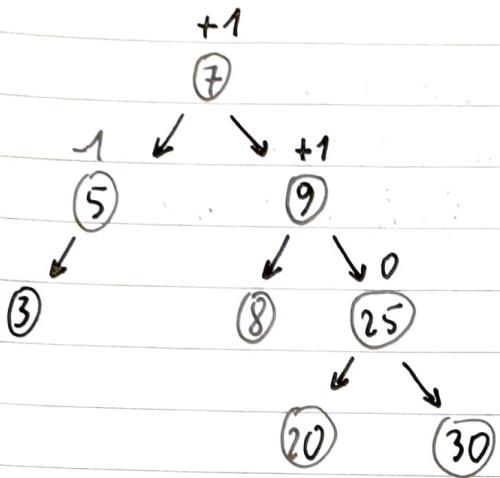
$30 > 9 \rightarrow \text{right}$

$30 > 10 \rightarrow \text{right}$

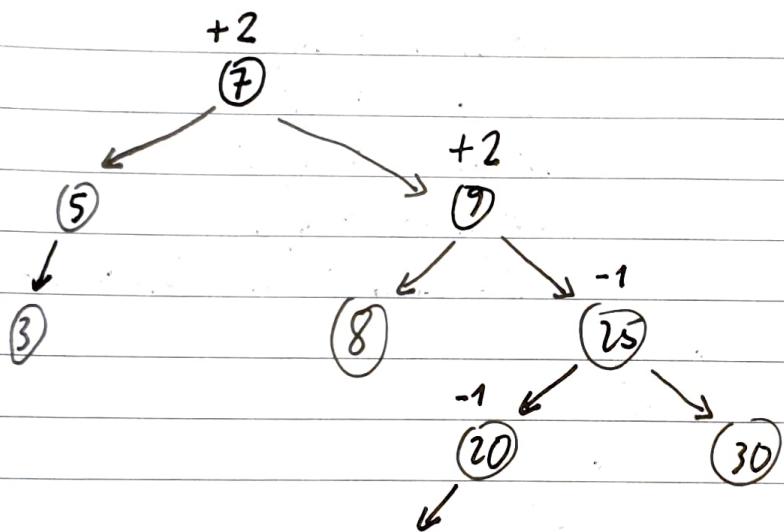
$30 > 25 \rightarrow \text{right}$

(20)'s tree is right heavy  
→ 1 rotate left

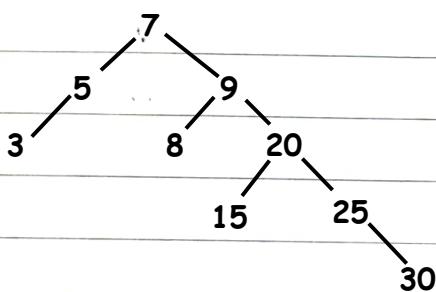
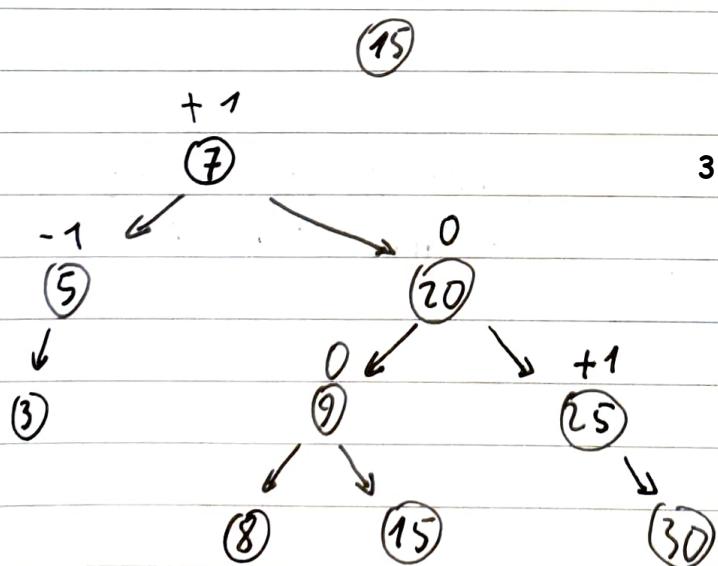




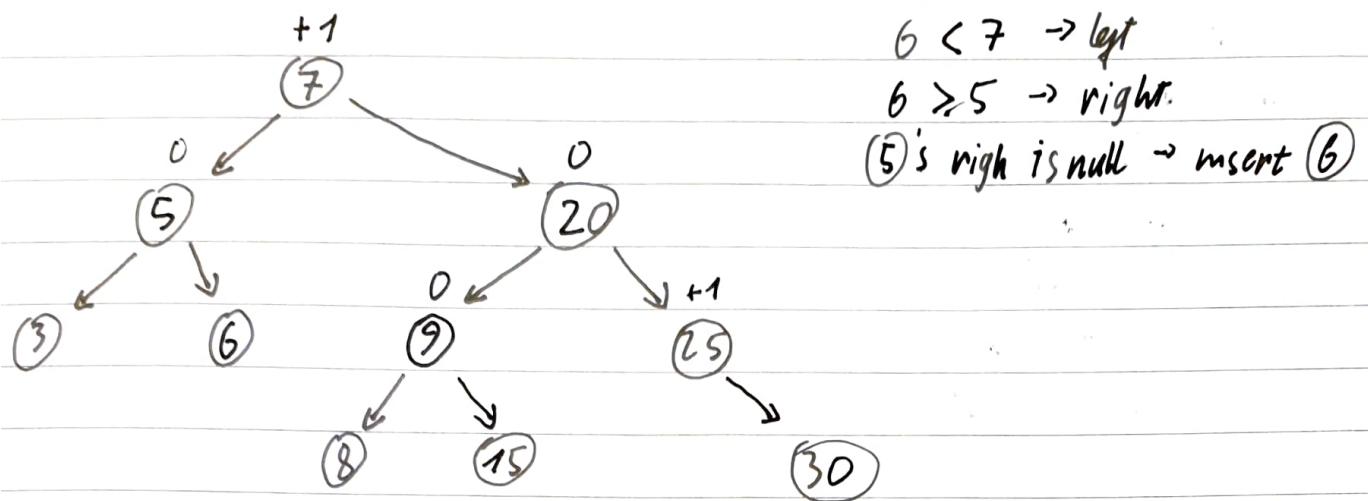
insert (15):



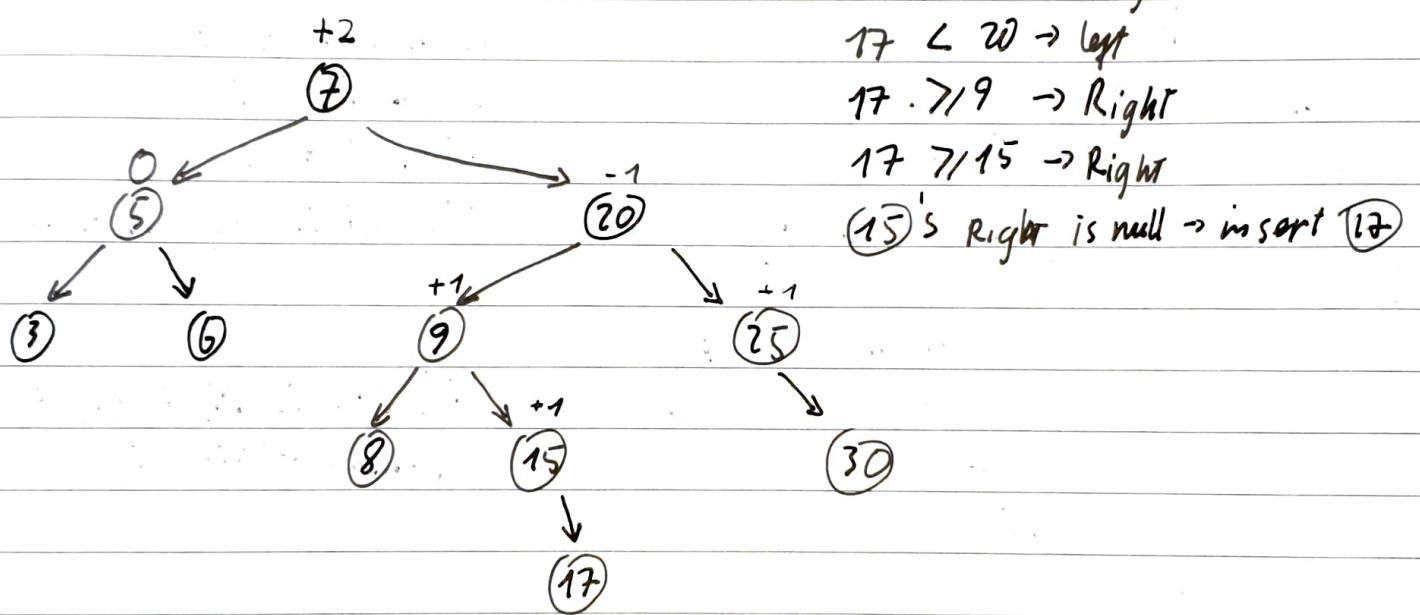
$15 > 7 \rightarrow$  right.  
 $15 > 9 \rightarrow$  right.  
 $15 < 25 \rightarrow$  left.  
 $15 < 20 \rightarrow$  left.  
 $(20)$ 's left is null, insert 15  
 $(9)$ 's tree is right weight  
 Rotate left right.



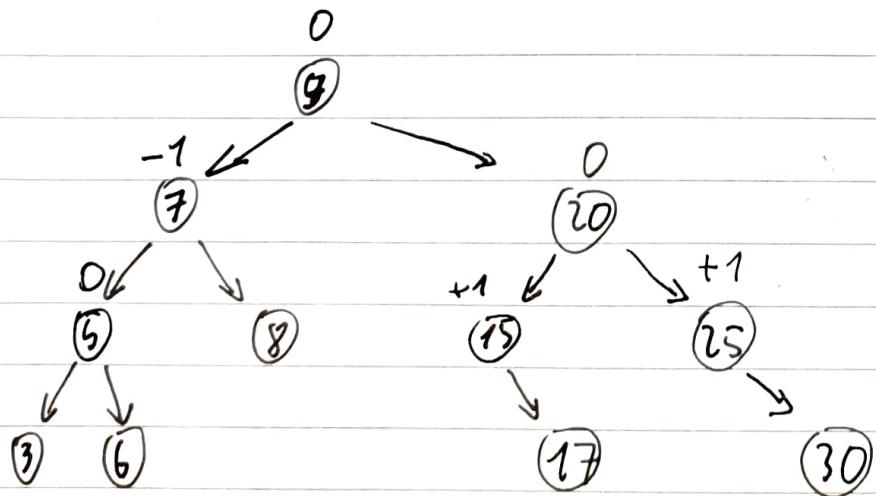
insert (6) :



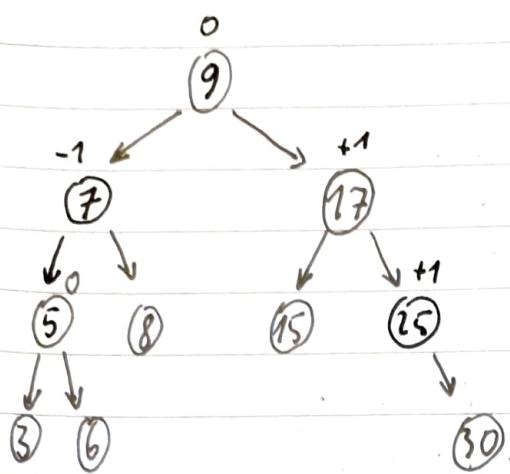
insert (17) :



The tree is right heavy  
→ double rotate left



Delete (20)



$20 > 9 \rightarrow \text{right}$

$20 = 20$

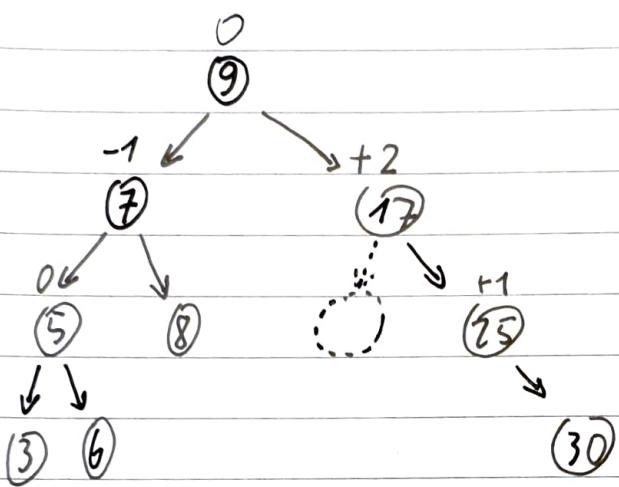
node (20) has (15) at left and (25) at right

node (20) has 2 children

$\rightarrow$  the largest in the left sub tree  
is (17)

copy (17) to (20), remove  
the old (17)

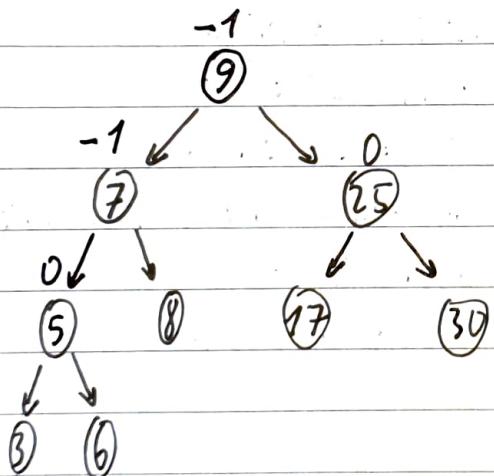
Delete (15)



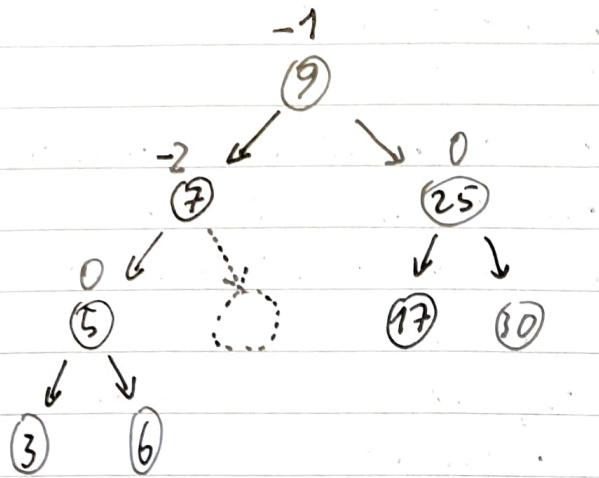
$15 > 9 \rightarrow \text{right}$

$15 < 17 \rightarrow \text{left}$ , found  $15 = 15$   
remove (15)

(17)'s tree is right heavy  
single rotate left

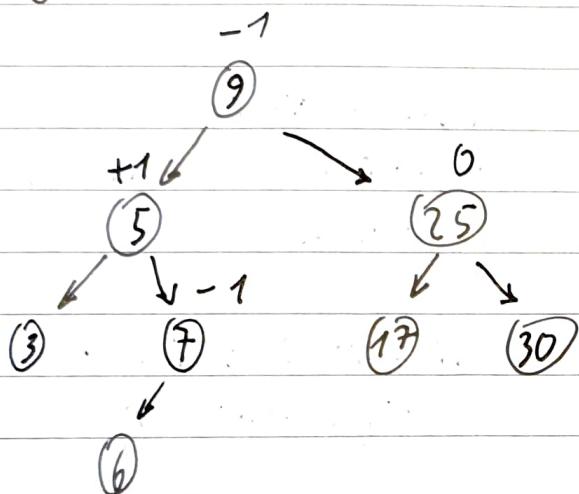


Delete (8):

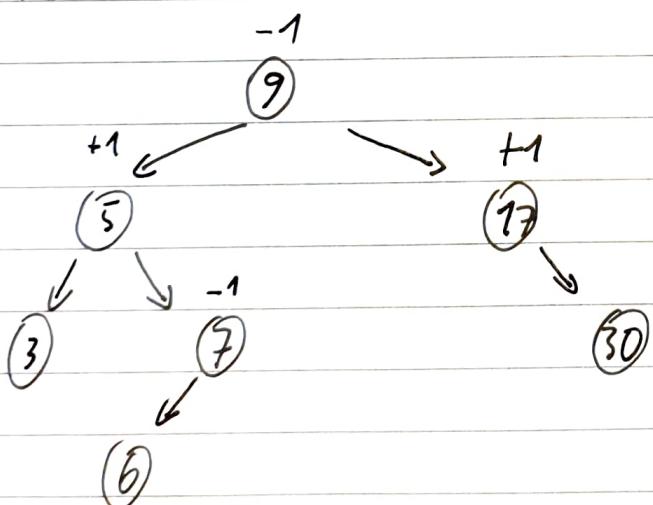


$8 < 9 \rightarrow \text{left}$   
 $8 > 7 \rightarrow \text{right}$   
found  $8 = 8$   
remove 8

7's tree is left heavy  
single rotate right

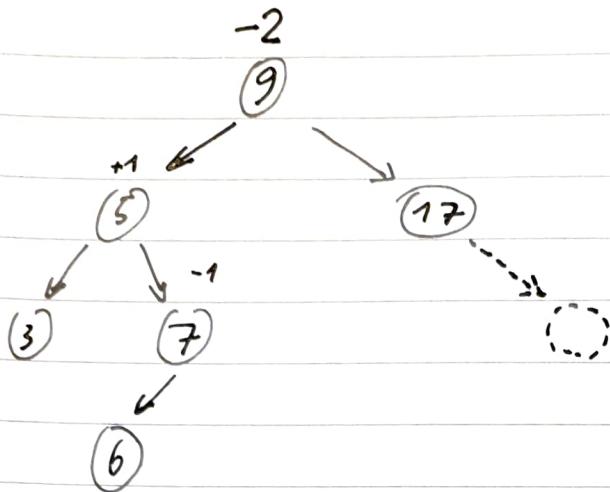


Delete (25)



$25 > 9 \rightarrow \text{right}$   
found  $25 = 25$   
 $25$  has 2 children  
~~swap with the largest on the left (17)~~  
copy 17 to 25  
remove old 17

Delete (30)



$30 > 9 \rightarrow \text{right}$

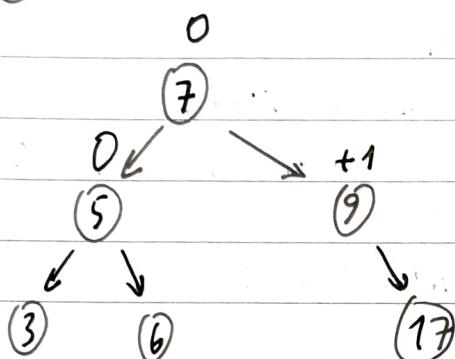
$30 > 17 \rightarrow \text{right}$

found  $30 = 30$

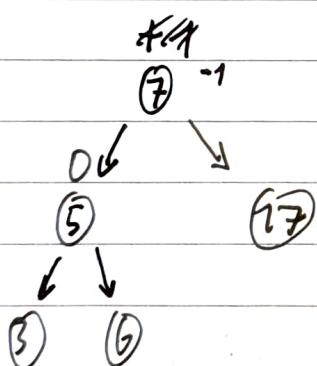
$30$  has no children

$\rightarrow$  remove 30

The tree is left heavy  
double rotate right



Delete (9)



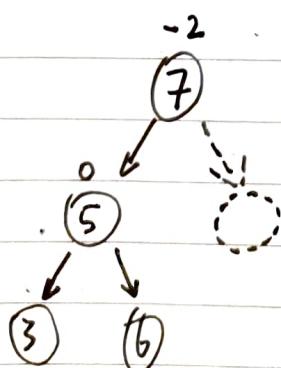
$9 > 7 \rightarrow \text{right}$

found:  $9 = 9$

$9$  has no left child

Set  $(17)$  (left right child) ~~as~~ in  $(9)$ 's position.

Delete (17)



$17 > 7 \rightarrow \text{right}$

found  $17 = 17$

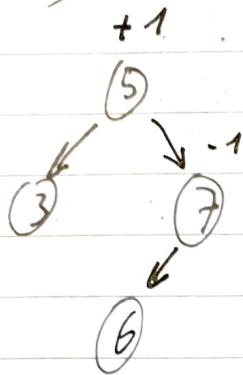
$17$  has no children

$\rightarrow$  remove 17

Tree is left heavy

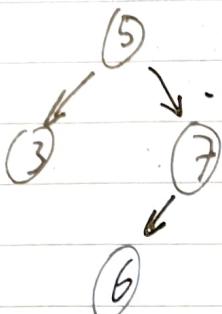
$\rightarrow$  single rotate right

Delete(5):



+1

-1



Delete(5)

+2

(3)

-1

(7)

(6)

found 5 = 5

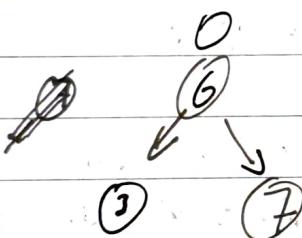
5 has 2 children

copy the largest on the left (3)  
to 5

remove old 3

Tree is right heavy

~~double~~ rotate left



(0)

6

3

7

Delete (6)

+1

(3)

7

found (6)

6 has 2 children

copy the largest on the left  
(7)

remove the old 3

Delete(3)

found 3 = 3

(7)

3 don't have left child

set the right child as parent

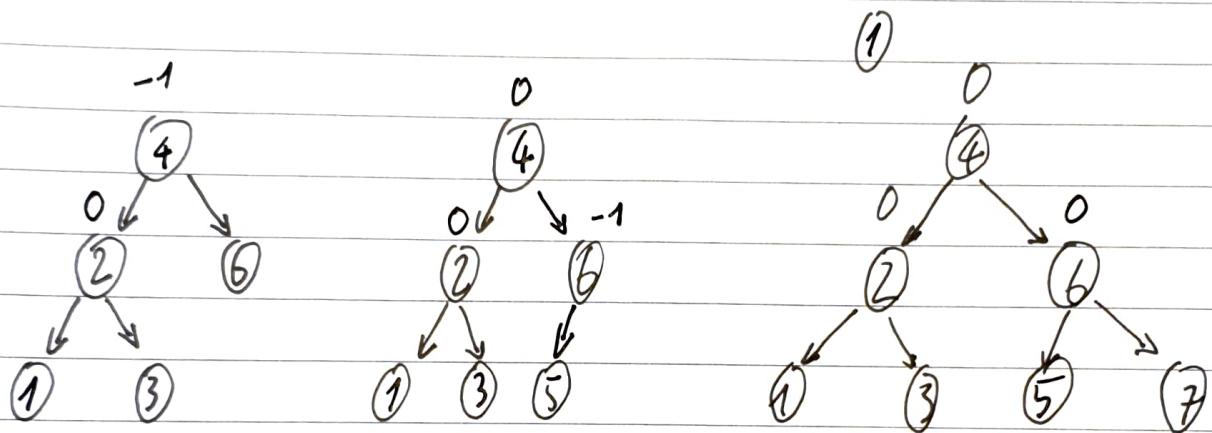
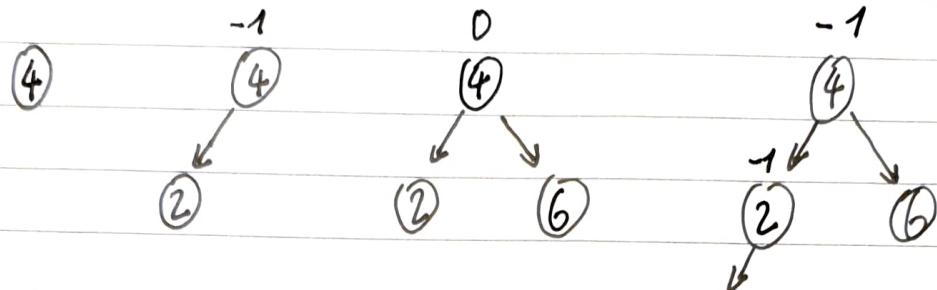
delete(7)

found 7

remove 7

1 2 3 4 5 6 7

order to insert: 4, 2, 6, 1, 3, 5, 7



no rotation!



**Perfectly Balanced...**



**...As all things should be**