Corrected 23 July 2012; see below

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Supplementary Materials for

Diversity of Interaction Types and Ecological Community Stability

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This PDF file includes:

Supplementary Text
Figs. S1 to S8

Correction: The broken axis labels and symbols in fig. S1 have been corrected and the labels missing in figs. S2 and S3 have been inserted.

Supplementary Text

Consider a randomly connected N-species community network, the population dynamics of which is described as:

$$\frac{dX_i}{dt} = X_i\left(r_i - s_i X_i + \sum_{j=1,\, j\neq i}^{N} a_{ij} X_j\right), \tag{1}$$

where X_i is the abundance of species i, r_i is the intrinsic rate of change in species i, s_i is density-dependent self regulation, and a_{ij} is the interaction coefficient between species i and species j. Two randomly chosen species are interacting with probability C (connectance). At the equilibrium, it holds that

$$r_i = -s_i X_i^* + \sum_{j=1,\, j\neq i} a_{ij} X_j^* \tag{2}$$

Community matrix, M, is the linearization of equation 1 at an equilibrium point with elements:

$$M_{ij} = \left.\frac{\partial\left(dX_i/dt\right)}{\partial X_j}\right|_{X^*}. \tag{3}$$

The system is locally stable if all eigenvalues of community matrix, M, have negative real parts. For randomly connected community with $M_{ii} = -d\left(= -s_i X_i^*\right)$, $E\left(M_{ij}\right) = 0$ and $CN >> 1$, the stability condition is given by:

$$\sqrt{N \cdot Var\left(M_{ij}\right)}\left\{1 + \frac{E\left(M_{ij} M_{ji}\right)}{Var\left(M_{ij}\right)}\right\} < d \tag{4}$$

(*23*). In applying this stability condition to our model, we assumed a random network with sufficient complexity ($CN >> 1$) so that we can approximate that each species has

$(N-1)C\,p_M$ mutualists, $(N-1)C(1-p_M)/2$ predator species and $(N-1)C(1-p_M)/2$ prey species. Parameters and species abundance are set constant ($e_{ij} = e$, $g_{ij} = g$, $X_i^* = X^*$ and $s_i = s$). We further assumed that it holds that:

$$f_M e = f_A(1-g)\,, \tag{5}$$

so that $E\left(M_{ij}\right) = 0$. The diagonal elements are given by:

$$M_{ii} = -s\,X^*. \tag{6}$$

Given that p_M is not too close to one or zero, the off-diagonal elements are:

$$M_{ij} = \frac{X^* g f_A}{\left[\text{prey sp. number per sp.}\right]} = \frac{X^* g f_A}{(N\text{-}1)C(1-p_M)/2}\,, \tag{7a}$$

$$M_{ij} = \frac{-X^* f_A}{\left[\text{prey sp. number per sp.}\right]} = \frac{-X^* f_A}{(N\text{-}1)C(1-p_M)/2}\,, \tag{7b}$$

$$M_{ij} = \frac{X^* e f_M}{\left[\text{mutualist sp. number per sp.}\right]} = \frac{X^* e f_M}{(N\text{-}1)C\,p_M} \tag{7c}$$

for antagonistic consumer i and resource j, antagonistic resource i and consumer j, and mutualists i and j, occurring with probabilities, $C(1 - p_M)/2$, $C(1 - p_M)/2$ and Cp_M, respectively. Thus, $Var\left(M_{ij}\right)$ and $E\left(M_{ij}M_{ji}\right)$ are calculated as:

$$Var\left(M_{ij}\right) = E\left(M_{ij}^2\right) - E\left(M_{ij}\right)^2 = \frac{\left\{(1-g)X^* f_A\right\}^2}{p_M C(N-1)^2} + \frac{2\left(1+g^2\right)\left(X^* f_A\right)^2}{(1-p_M)C(N-1)^2} \tag{8a}$$

and

$$E\left(M_{ij}M_{ji}\right)=\frac{\left\{\left(1-g\right)X^{*}f_{A}\right\}^{2}}{p_{M}C\left(N-1\right)^{2}}-\frac{4g\left(X^{*}f_{A}\right)^{2}}{\left(1-p_{M}\right)C\left(N-1\right)^{2}} \quad , \tag{8b}$$

respectively. Substituting Eqns. 6, 8a and 8b to Eqn. 4, we have the stability condition for hybrid communities as:

$$\frac{2f_{A}\left(1-g\right)^{2}\sqrt{N}X^{*}}{\left(N-1\right)\sqrt{Cp_{M}\left(1-p_{M}\right)\left\{\left(1+g\right)^{2}p_{M}+\left(1-g\right)^{2}\right\}}}<sX^{*} \tag{9}$$

For $CN >> 1$, thus assuming $N-1\approx N$, it follows that:

$$\frac{2f_{A}\left(1-g\right)^{2}}{\sqrt{CNp_{M}\left(1-p_{M}\right)\left\{\left(1+g\right)^{2}p_{M}+\left(1-g\right)^{2}\right\}}}<s \quad , \tag{10}$$

suggesting that increasing connectance (C) or species number (N) is stabilizing. Noting that the left hand side of Ineq. 10 is a continuous convex function of p_M for $0 < p_M < 1$ and goes to infinity as p_M approach zero or one for $0 < p_M < 1$, it follows that, when the system can be stable for $0 < p_M < 1$, there are positive constants, p_M^L and p_M^U, such that the system is stable if p_M is in the range, $0 < p_M^L \le p_M \le p_M^U < 1$.

The analytically derived stability condition (Ineq. 10) is supported by simulations, where p_M and C were systematically varied with the other parameters set to $(N, X^*, s, e, g, f_A, f_M) = (200, 1.0, 0.1, 0.5, 0.5, 1.0, 1.0)$ and we obtained the frequency of stable systems across 1000 sample communities (Fig. S1). With this setting, a clear transition between unstable and stable systems was observed as predicted by the stability condition (Ineq. 10).

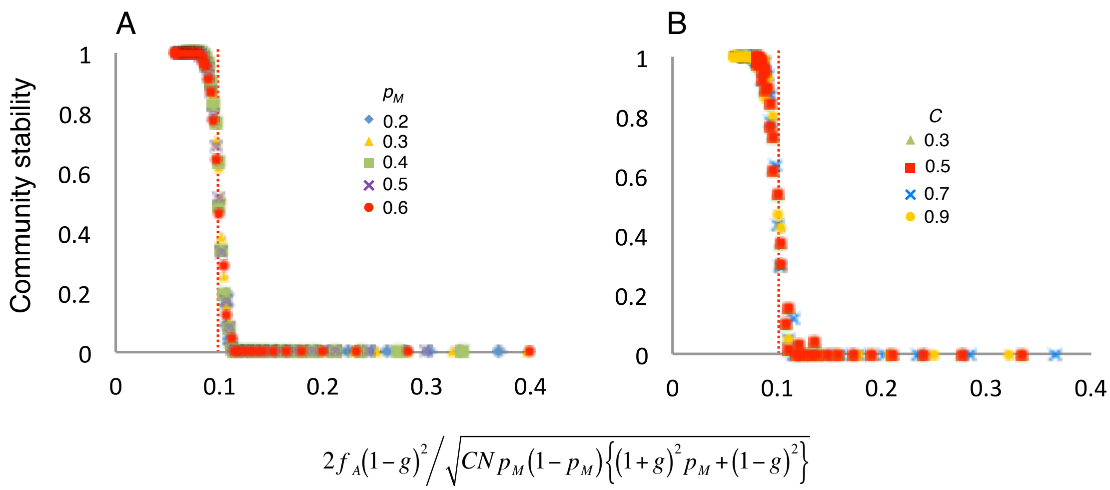

Fig. S1. Community stability evaluated from simulations. Relationship between $2f_A(1-g)^2 \Big/ \sqrt{CNp_M(1-p_M)\left\{(1+g)^2 p_M + (1-g)^2\right\}}$ (LHS of Ineq. 10) and stability were obtained. **A.** Connectance (C) or **B.** proportion of mutualistic link (p_M) was varied with the other parameters being fixed. The red vertical lines indicate the critical value ($s = 0.1$) at which the transition between stable and unstable systems is predicted to occur by the analytical analysis. Other parameters are (N, X^*, s, e, g, f_A, f_M) = (200, 1.0, 0.1, 0.5, 0.5, 1.0, 1.0). Random model with type I functional response was used.

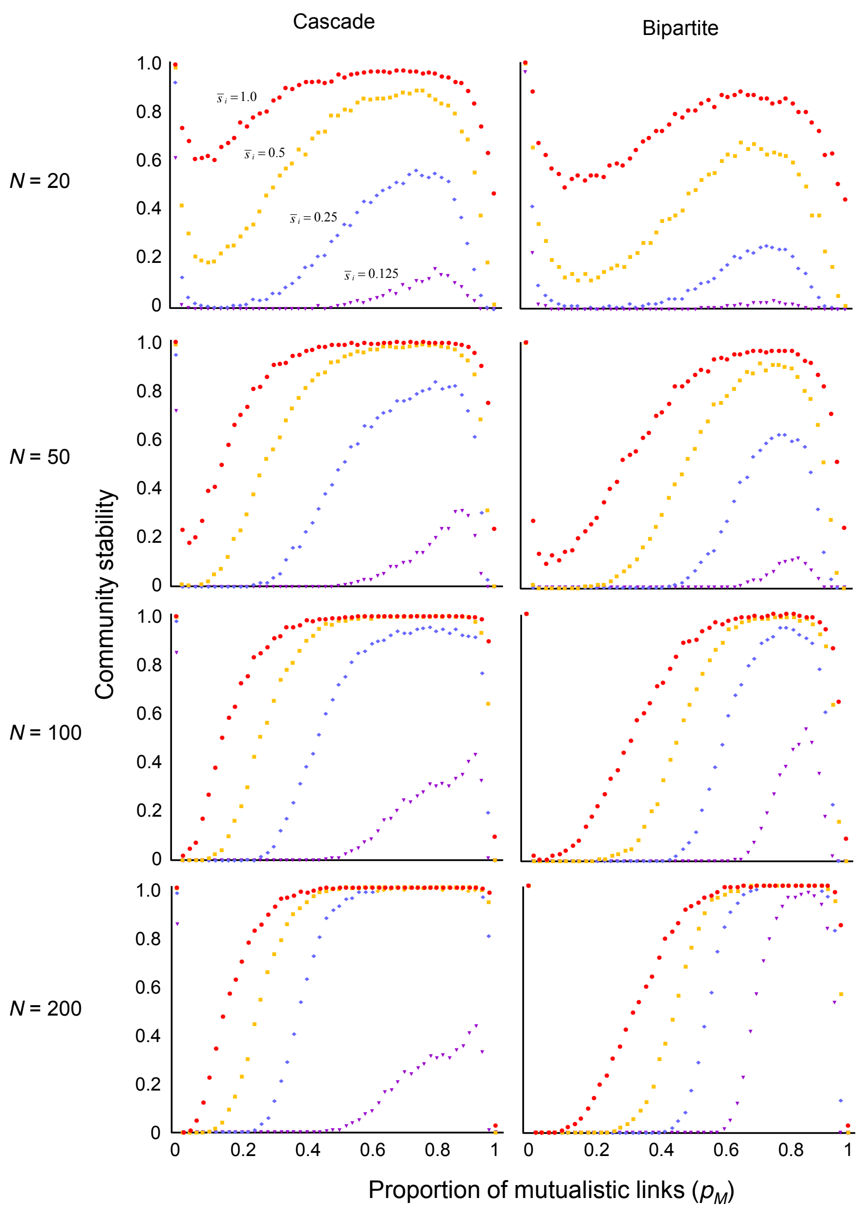

Fig. S2. Relationships between the proportion of mutualistic links (p_M), and stability with varying species richness (N) in models with cascade and bipartite networks. Type I functional response was used. To evaluate the effect of self-regulation intensity, s_i is set to a random value from $[0, 2\bar{s}_i]$ to have mean, $\bar{s}_i$. Different colours indicate different $\bar{s}_i$. P is set to 0.7.

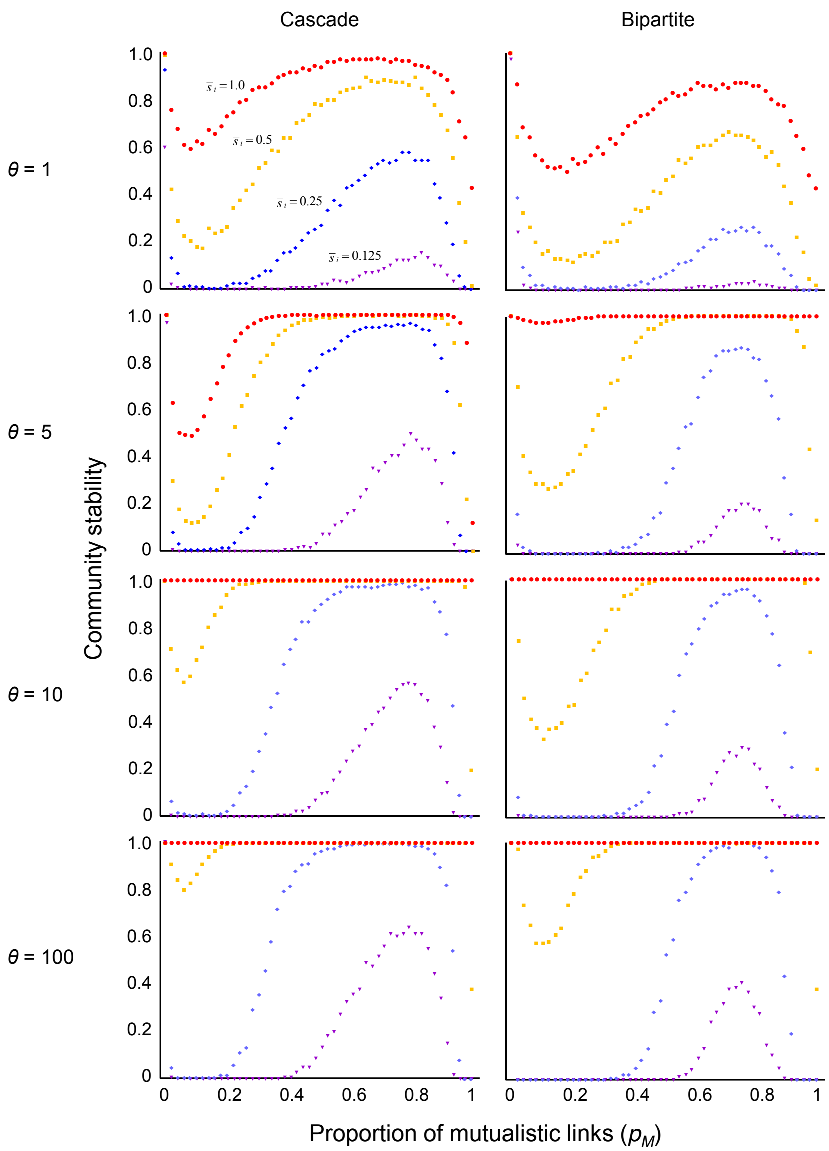

Fig. S3. Relationships between the proportion of mutualistic links (p_M) and stability with varying parameter variability in cascade and bipartite models with type I functional response. In these simulations, parameters (e_{ij}, g_{ij}) and equilibrium densities (X_i*) are randomly chosen from Beta distribution, $\beta(\theta,\theta)$, where θ indicates the variability in the distribution. The distribution is uniform for θ = 1; and the distribution becomes to have stronger central tendency as θ increases. s_i is set to a random value from Beta distribution, $2\overline{s_i}\ \beta(\theta,\theta)$ to have mean, $\overline{s_i}$. Different colours indicate different $\overline{s_i}$. P and N are set to 0.7 and 20, respectively.

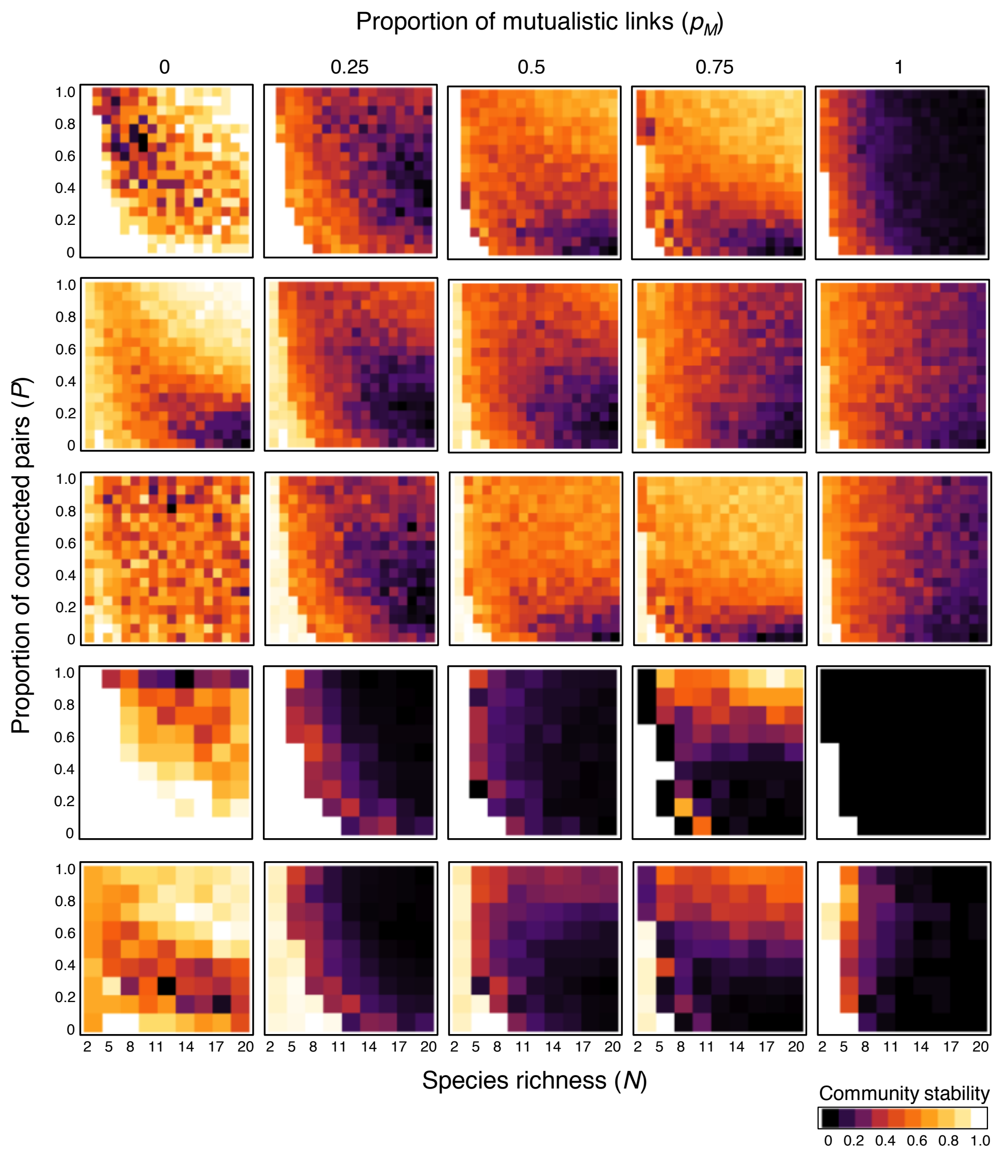

Fig. S4. Complexity-stability relationships with varying proportions of mutualistic links (p_M) in five models, random with type I functional response, random with type II functional response, cascade with type II functional response, bipartite with type I, and bipartite with type II.

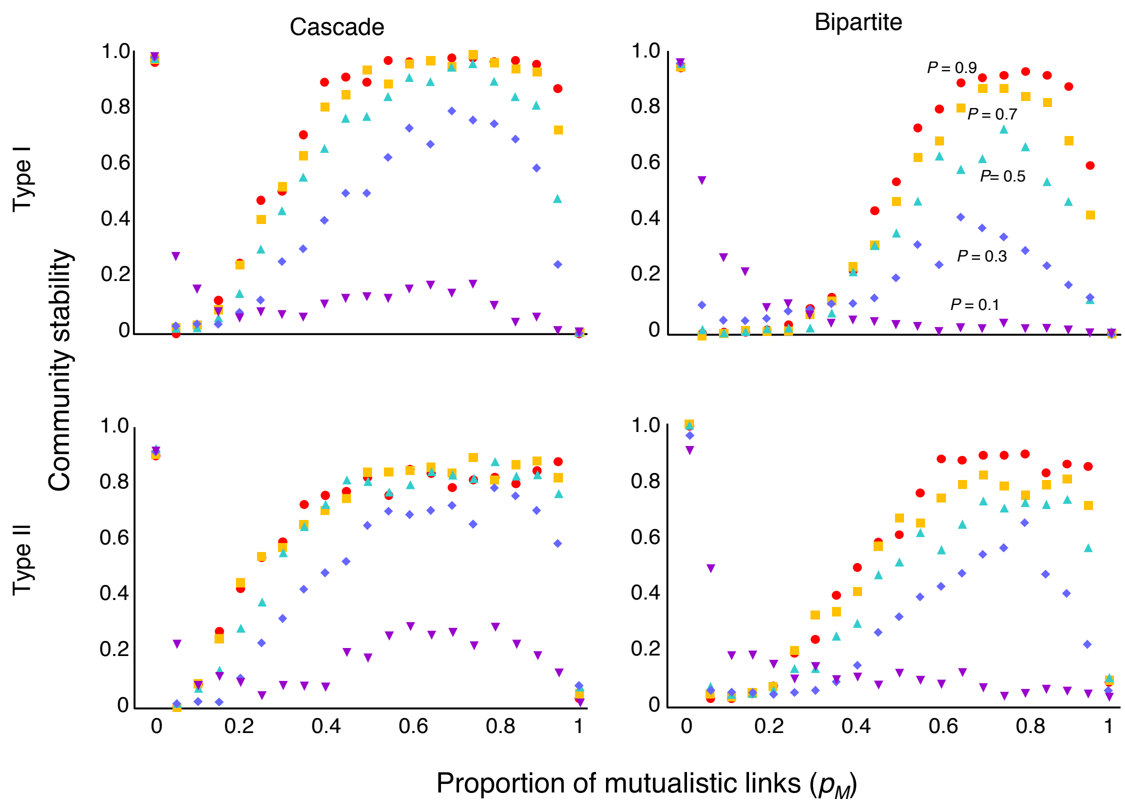

Fig. S5. Relationships between the proportion of mutualistic links (p_M) and stability with varying proportion of connected pairs (P) in the presence of interspecific competition among basal species. The four models are combinations of different network structures (cascade or bipartite), and functional responses (type I or II). The competition occurs between all basal species pairs. The competition coefficients a_{ij} and a_{ji} are randomly chosen from uniform distribution $(0, -1)$. Colours indicate different values of P. N is set to 50.

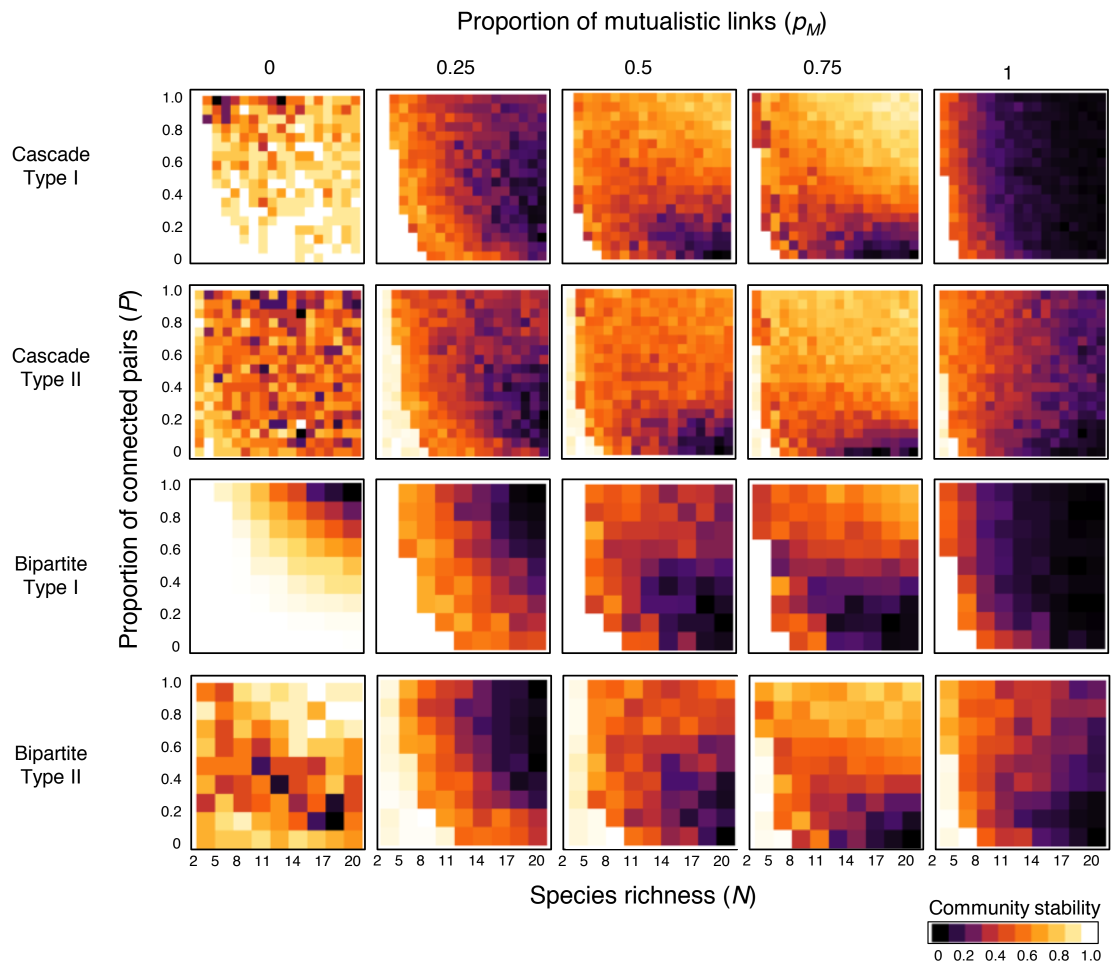

Fig. S6. Complexity-stability relationships with varying proportions of mutualistic links (p_M) in the presence of interspecific competition among basal species. Cascade and bipartite networks, and type I and II functional responses were used.

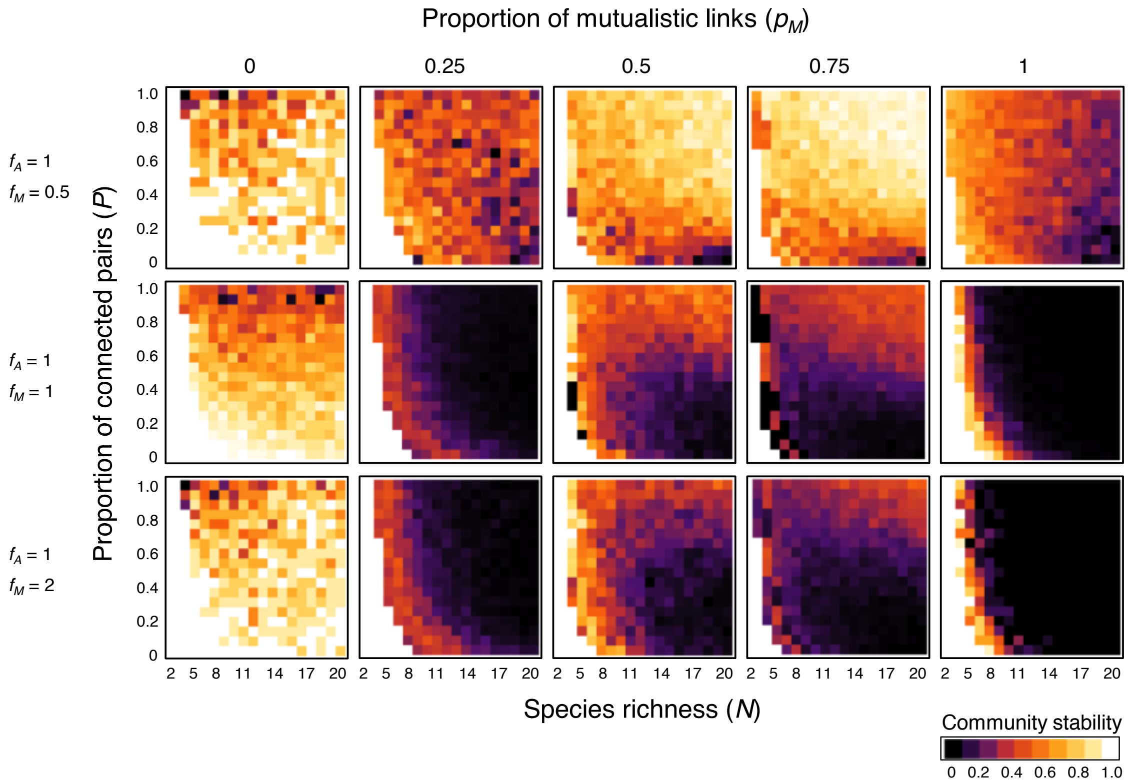

Fig. S7. Complexity-stability relationships with varying proportions of mutualistic links (p_M) and its response to varying the relative strength of mutualistic interactions (f_M). Cascade model with type I functional response was used.

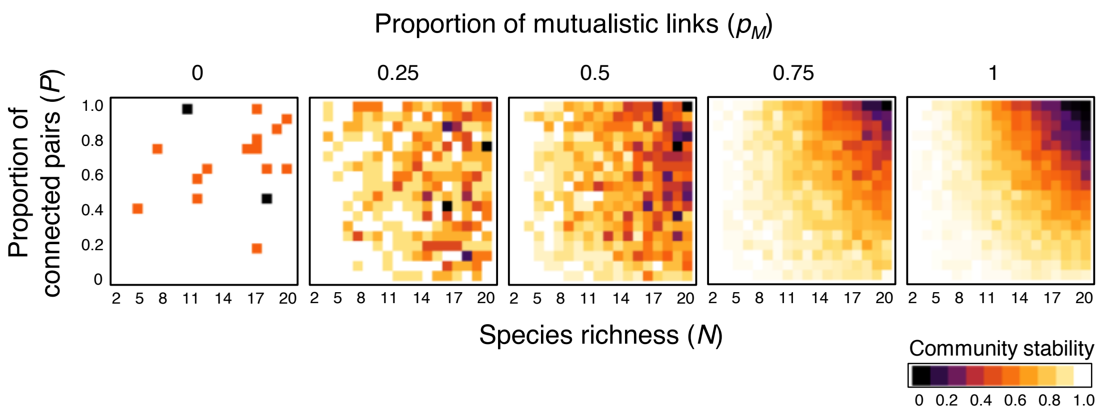

Fig. S8. Complexity-stability relationships with varying proportions of mutualistic links (p_M) in the absence of the negative relationship between the number of interaction and the interaction strength. Cascade model with type I functional response was used. The interaction strengths are randomly determined from uniform distribution $(0, 0.1)$.