Functional Secret Sharing, FSS

Part 1 Definition

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Secret sharing is a method of splitting a value into multiple shares, which can take
many forms. Here we focus on additive secret sharing so that the shares can be
recombined to restore the secret value. Any strict subset of shares cannot reveal
any information about the secret value.

We use$\ v[i]$ to indicate the ith share of v, use + as reconstruction: $v[1]+v[2] =
v$.

Function secret sharing has a additional requirement that the share of the
function$\ f_i$ can be calculate on input $\ x$ to get the result's shares $\
f(x)_i$, where $f(x) = \sum_i f(x)_i$.

For function as$\ f:\{0,1\}^n\rightarrow\{0,1\}^*$ and p parties, we have:
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- $Gen(1^\lambda, f) \rightarrow (f_1, ..., f_p)$: partition the function into a compact set of secret shares;
- $Eval([f_i], x) \rightarrow [f(x)]_i$: use secret shares and the secret share of input x and output $[f(x)]_i$;
- $Recover([f(x)]_i) \rightarrow f(x)$: recover f(x) from p secret shares.

Part 2 Motivation

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Suppose a client wants to run a function on data stored in the cloud, but does not
want to expose the function to the cloud server?
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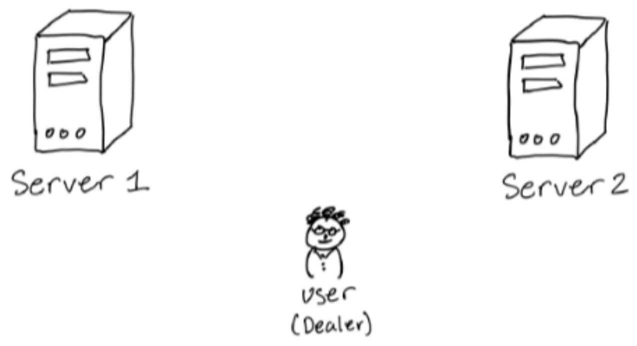
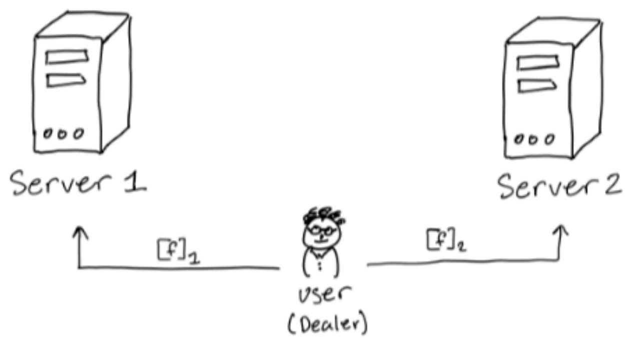
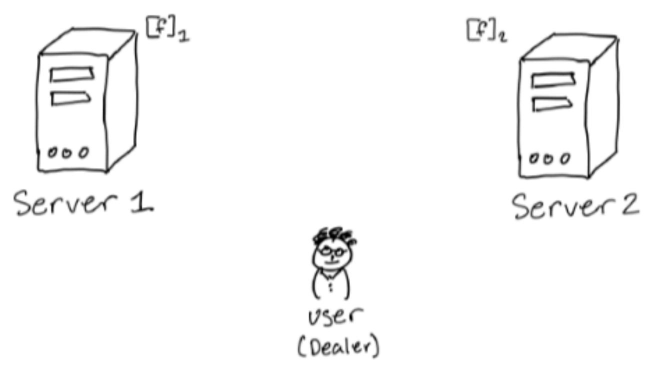
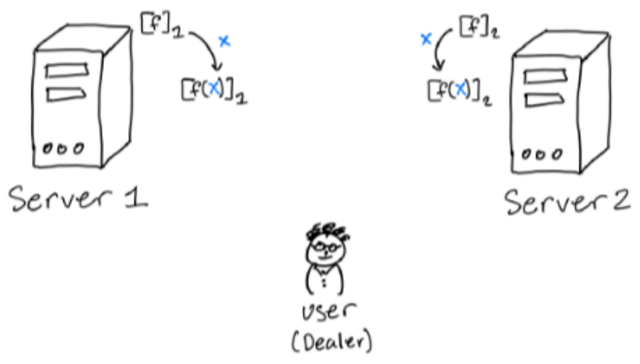
Solution: Assume no collusion between servers and use FSS to hide functions.

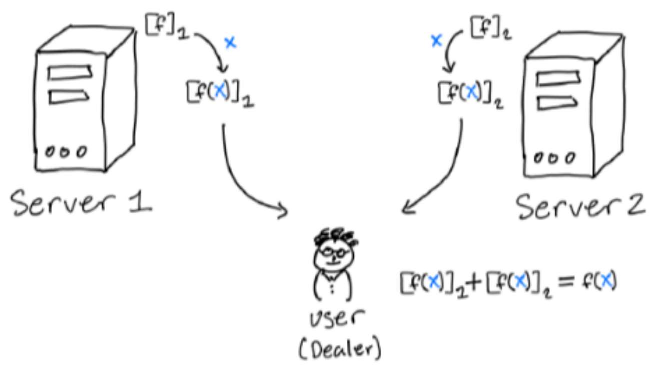
1. The client uses FSS to secretly share function f and distribute it to the cloud server.
2. the cloud server performs the calculation on the function f of the secret share and returns the result f(x) to the client.
3. the client recombines these shares locally to obtain f(x).











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In addition, FSS is very useful in building privacy-protecting a system:
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- Private reading distributed databases (private information retrieval): for example: private keyword searches on remote databases.
- Private write to distributed database (private information write): for example: anonymous communication.
- MPC: For example, generate preprocessing for multiparty computation (Silent OT extension).

Part 3 FSS and DPF

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The above introduces FSS, which is a general technique for splitting arbitrary
functions into secret shares so that these shares can be computed in a distributed
environment without revealing the function itself.Each participant holds a share of
the function, and through local computation and communication, can calculate the
results of the function on some input value, and finally combine these results to
restore the output of the function.

DPF (Distributed Point Function) is a specific form of FSS, specifically for sharing
point functions.

A point function is a very simple function of the form:
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$$DPF(x) = \begin{cases} v, & \textit{if } x = x_0 \\ 0, & \textit{otherwise} \end{cases}$$

```
DPF allows multiple participants to calculate the function without leaking 2o and v,
and recombine to get the correct point function output when needed. Due to the
simplicity of the point function, DPF is generally more efficient and has less
computational overhead, and is especially suitable for applications where a specific
location or value needs to be queried, such as private information retrieval (PIR).
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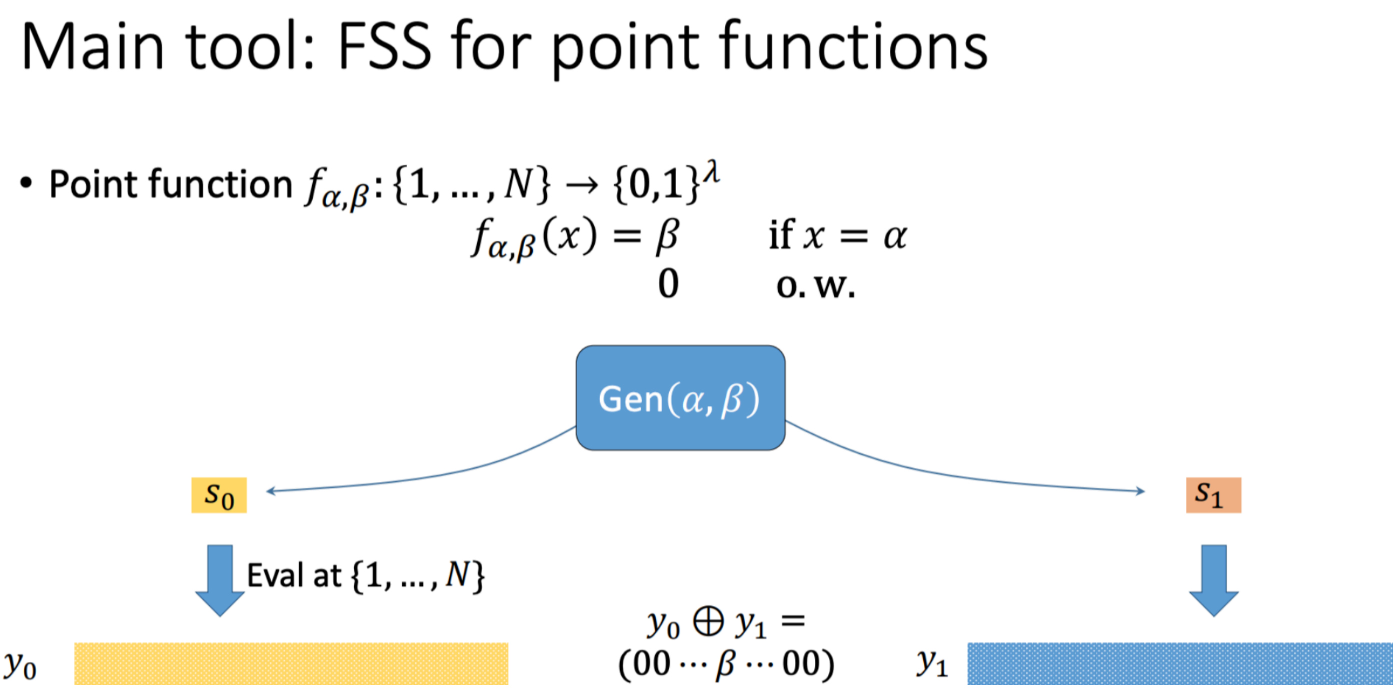
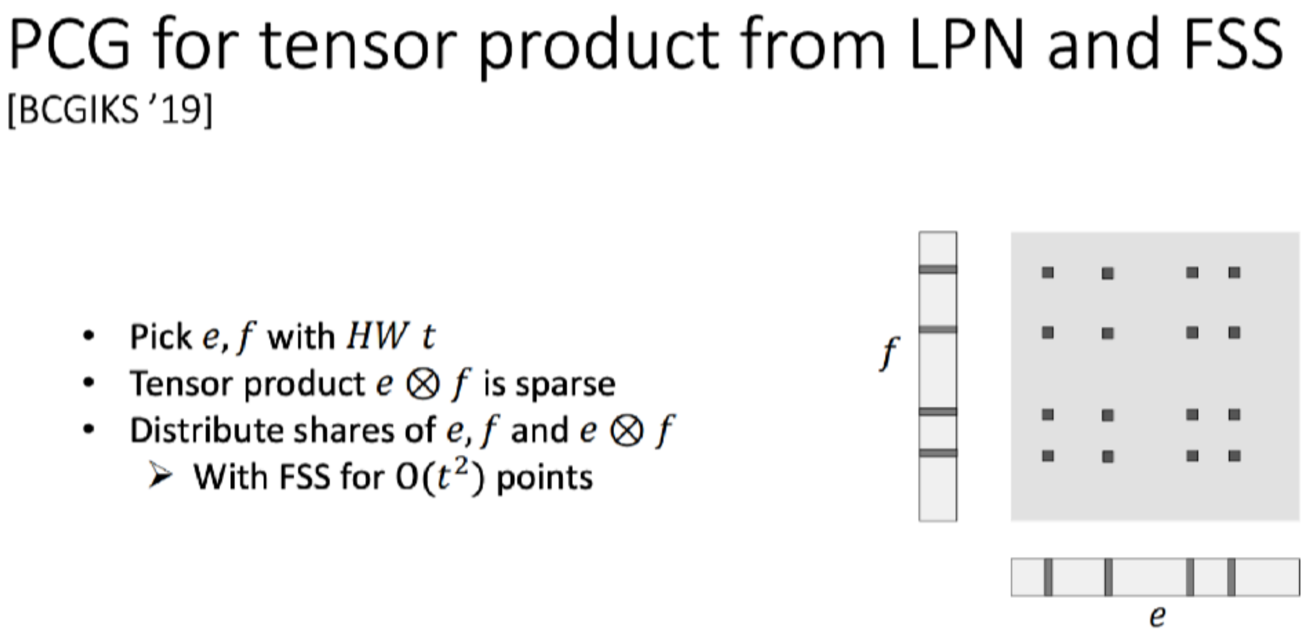


Main tool: FSS for point functions

- Point function $f_{\alpha,\beta}: \{1,\dots,N\} \to \{0,1\}^\lambda$

$$f_{\alpha,\beta}(x) = \begin{cases} \beta & \text{if } x = \alpha \\ 0 & \text{o.w.} \end{cases}$$

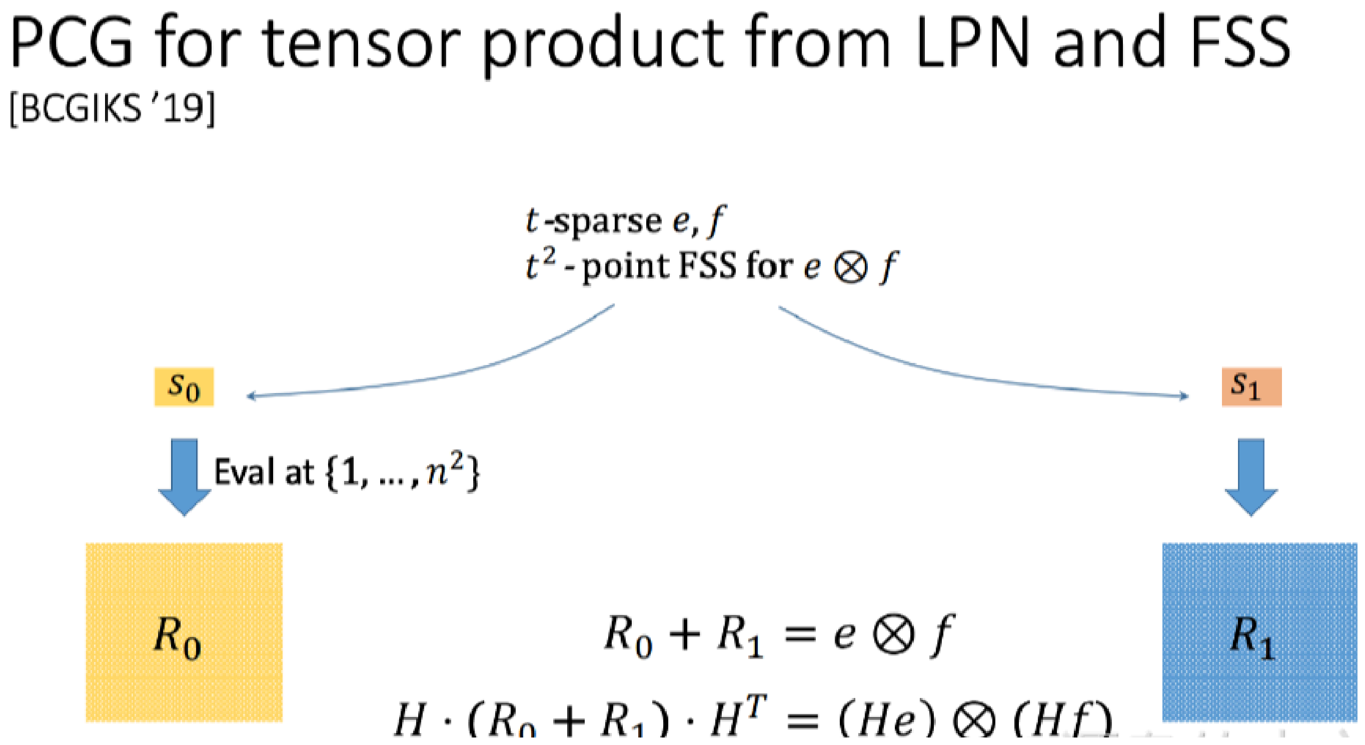
$\mathsf{Gen}(\alpha,\beta)$
s_0
s_1
Eval at $\{1,\dots,N\}$
y_0
$y_0 \oplus y_1 = (00\cdots\beta\cdots00)$
y_1



PCG for tensor product from LPN and FSS

[BCGIKS ’19]

- Pick e, f with $HW\ t$
- Tensor product $e \otimes f$ is sparse
- Distribute shares of e, f and $e \otimes f$
  - ➢ With FSS for $O(t^2)$ points



PCG for tensor product from LPN and FSS

[BCGIKS ’19]

t-sparse e, f
t^2-point FSS for $e \otimes f$

$$R_0 + R_1 = e \otimes f$$

$$H \cdot (R_0 + R_1) \cdot H^T = (He) \otimes (Hf)$$

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The distributed point function contains two algorithms (Gen and Eval):
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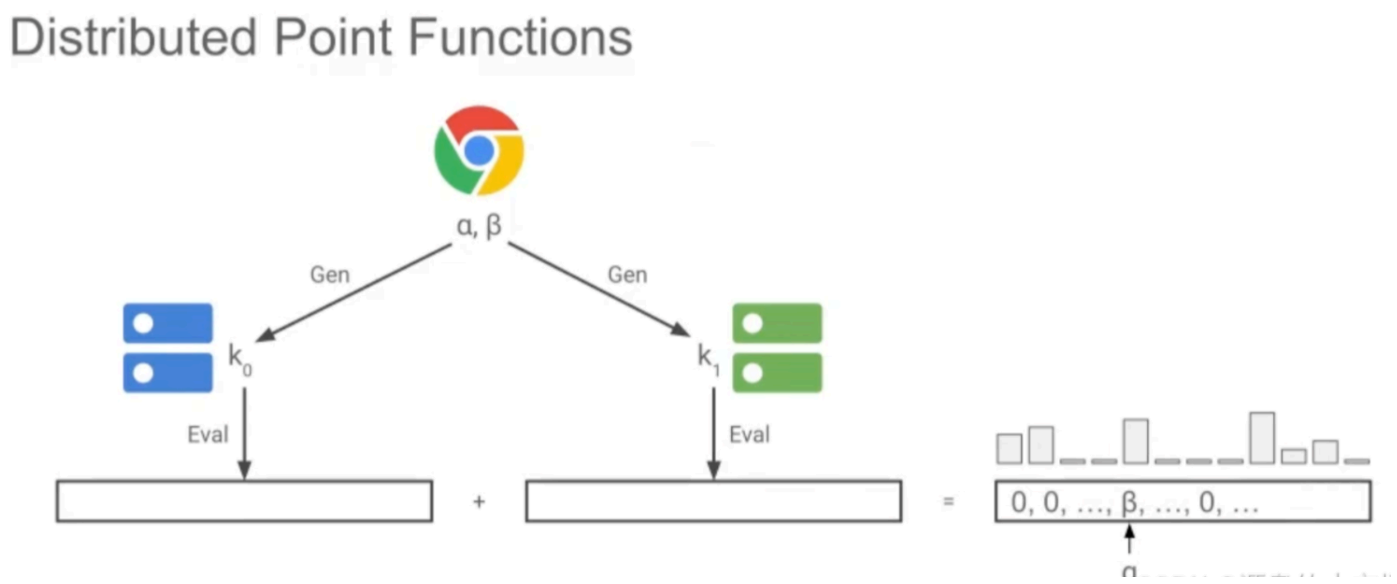
- $Gen(y) \rightarrow (k_a, k_i)$: generate secret keys for y;
- $Eval(k_a, x') \rightarrow y'$: assess DPF on input x.



Part 4 Reference

[1] https://sachaservanschreiber.com/slides/pacls.pdf

[2] https://cris.csa.iisc.ac.in/MPCWorkshop/ppts/Talk3_3.pdf

[3] https://slideplayer.com/slide/15682146/