

- **CrypTFlow2: Practical 2-Party Secure Inference**
 - Part 0 Background and Motivation
 - Part 1 Preliminaries
 - 1.1 Oblivious Transfer
 - 1.2 OT-based MUX and B2A
 - :
 - :
 - Part 3 Millionaires' and DReLU Protocols (non-linear)
 - Millionaires' protocols
 - General idea
 - Protocol
 - Security Proof
 - Generalization
 - Optimization
 - Communication
 - Protocol for DReLU
 - General idea
 - Protocol
 - Optimization
 - ReLU
 - Maxpool layer
 - Part 4 Division and Truncation (linear)
 - 4.1 Expressing general division and truncation using arithmetic over secret shares
 - Theorem 4.1.1
 - 4.2 Truncation of Integer
 - 4.3 Division
 - 4.4 Truncation Optimization
 - Part 5 Secure Inference
 - Part 6 Evaluation
 - Part 7 Conclusion
 - Appendix I: Code-reading
 - `millionaire_with_equality.h`:

CrypTFlow2: Practical 2-Party Secure Inference

Part 0 Background and Motivation

The prediction of neural network inference under 2PC is always a difficult research problem. The first difficulty is the huge cost of nonlinear computation, such as comparison and division. The other is the loss of accuracy.

CrypTFlow2 proposes a new protocol for security comparison based on Oblivious Transfer (OT) and deeply optimizes the protocol. Furthermore, several operator protocols for neural networks are designed based on the comparison protocol, such as ReLU, Truncation, faithful Division (divisor is public), Avgpool, and Maxpool.

In addition, two versions of the ring \mathbb{Z}_L ($L = 2^l$) and \mathbb{Z}_n (n is any large integer) are designed in this paper, so as to adapt the linear layer computation for OT and Homomorphic Encryption (HE).

Finally, the code for this article has been open sourced, linked to:
<https://github.com/mpc-msri/EzPC/tree/master/SCI>

Part 1 Preliminaries

This paper is aimed at the semi-honest adversaries in two-party computation, mainly using the additive secret sharing under two parties, OT, OT constructed AND triples, Multiplexer, and B2A transformation, and HE, where secret sharing is already familiar, and HE is mainly used in the linear layer of neural networks. The core construction of this article is based on OT, and we mainly review the knowledge of OT here.

Notation :

- $w \leftarrow^S W$

For a set W , w is an element randomly selected from W .

- $[l]$


denotes the set of integers $0, \dots, l-1$

- $1\{b\}$

denote the indicator function that is 1 when $?$ is true and 0 when $?$ is false.

1.1 Oblivious Transfer

OT is very clear in terms of its function. The standard definition of 1-out-of-2 OT involves two parties: the sender S holds two bits of information, and the receiver R holds a selection bit. After the OT protocol is executed, R can get the information of the index corresponding to the selection bit, but cannot get other information, and S does not know which information R selects. It can be generalized to 1-out-of- n OT, that is, the receiver needs to secretly obtain a certain information from the n messages of the sender. Similarly, it is further extended to k -out-of- n OT, that is, out of n messages from the sender, the receiver secretly obtains k messages.

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In the OT protocol, the sender has all the data rights, the receiver has the option of a single data, the data exchange is completed inadvertently, while ensuring the privacy of the private data of both sides.

1.2 OT-based MUX and B2A

F_{MUX} :

- input: $\langle a \rangle^n$ and $\langle c \rangle^B$
- output: if $c = 1$, output $\langle a \rangle^n$; otherwise, output shares over 0

Based on $\binom{2}{1} - OT_\eta$, $x_0 = -r_0 + c \cdot \langle a \rangle_0^n$, $x_1 = -r_1 + c \cdot \langle a \rangle_1^n$. Therefore, $z = z_0 + z_1 = c \cdot a$. Since OT is used twice, the communication expense is $2(\lambda + 2\eta)$.

Algorithm 6 Multiplexer, Π_{MUX}^n :

Input: For $b \in \{0, 1\}$, P_b holds $\langle a \rangle_b^n$ and $\langle c \rangle_b^B$.

Output: For $b \in \{0, 1\}$, P_b learns $\langle z \rangle_b^n$ s.t. $z = a$ if $c = 1$, else $z = 0$.

- 1: For $b \in \{0, 1\}$, P_b picks $r_b \xleftarrow{\$} \mathbb{Z}_n$.
- 2: P_0 sets s_0, s_1 as follows: If $\langle c \rangle_0^B = 0$, $(s_0, s_1) = (-r_0, -r_0 + \langle a \rangle_0^n)$. Else, $(s_0, s_1) = (-r_0 + \langle a \rangle_0^n, -r_0)$.
- 3: P_0 & P_1 invoke an instance of $\binom{2}{1}$ -OT $_\eta$ where P_0 is the sender with inputs (s_0, s_1) and P_1 is the receiver with input $\langle c \rangle_1^B$. Let P_1 's output be x_1 .
- 4: P_1 sets t_0, t_1 as follows: If $\langle c \rangle_1^B = 0$, $(t_0, t_1) = (-r_1, -r_1 + \langle a \rangle_1^n)$. Else, $(t_0, t_1) = (-r_1 + \langle a \rangle_1^n, -r_1)$.
- 5: P_0 & P_1 invoke an instance of $\binom{2}{1}$ -OT $_\eta$ where P_1 is the sender with inputs (t_0, t_1) and P_0 is the receiver with input $\langle c \rangle_0^B$. Let P_0 's output be x_0 .
- 6: For $b \in \{0, 1\}$, P_b outputs $\langle z \rangle_b^n = r_b + x_b$.

F_{B2A} :

- Input: $\langle c \rangle^B$;
- output: $\langle d \rangle^n$, which satisfies $d = c$.

Since $d = \langle c \rangle_0^B + \langle c \rangle_1^B - 2\langle c \rangle_0^B \cdot \langle c \rangle_1^B$, the key is on the product term. Based on $\binom{2}{1}$ -OT $_\eta$, $y_1 = x + \langle c \rangle_0^B \cdot \langle c \rangle_1^B$. Therefore, $\langle d \rangle_0^n = \langle c \rangle_0^B + 2x$. Since OT is used twice, the communication expense is $\lambda + \eta$.

Algorithm 7 Boolean to Arithmetic, Π_{B2A}^n :

Input: P_0, P_1 hold $\langle c \rangle_0^B$ and $\langle c \rangle_1^B$, respectively, where $c \in \{0, 1\}$.

Output: P_0, P_1 learn $\langle d \rangle_0^n$ and $\langle d \rangle_1^n$, respectively, s.t. $d = c$.

- 1: P_0 & P_1 invoke an instance of $\binom{2}{1}$ -COT $_\eta$ where P_0 is the sender with correlation function $f(x) = x + \langle c \rangle_0^B$ and P_1 is the receiver with input $\langle c \rangle_1^B$. Party P_0 learns x and sets $y_0 = n - x$ and P_1 learns y_1 .
- 2: For $b \in \{0, 1\}$, P_b computes $\langle d \rangle_b^n = \langle c \rangle_b^B - 2 \cdot y_b$.

Part 3 Millionaires' and DReLU Protocols (non-linear)

Millionaires' protocols

General idea

Input : P_0 has $x \in \{0, 1\}^1$ and P_1 has $y \in \{0, 1\}^1$

output : P_0, P_1 learn shares of $\{x > y\}$

for $x = x_1 \parallel x_0$, $y = y_1 \parallel y_0$:

$$\{x > y\} = \{x_1 < y_1\} \oplus (\{x_1 = y_1\} \wedge \{x_0 < y_0\})$$

$$\{x = y\} = (\{x_1 = y_1\} \wedge \{x_0 = y_0\})$$

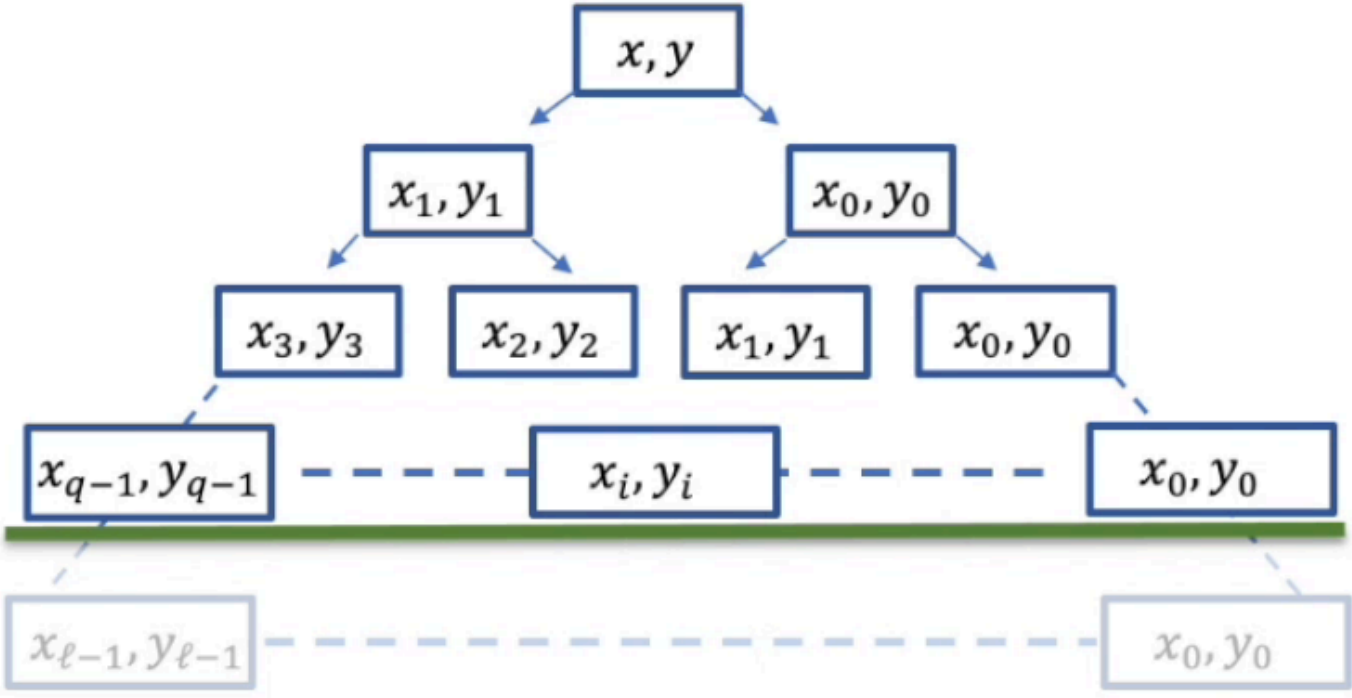
In GSV07, we conduct this comparison until the single bit-leaves

In CRTF2, however, we stop at $q = \lceil \log m \rceil$ leaves; $x_i, y_i \in \{0, 1\}^m$ and conduct $2 \times \binom{M}{1} - \text{OT}_1$ on each leaf; $M = 2^m$

This change is efficient only for small bitlengths, such as $m=4$

Communications for $m = 4$: $6\lambda \rightarrow 2\lambda$

After more optimizations: $2\lambda/ \rightarrow \lambda/$



Protocol

As shown in the picture below. Let $M = 2^m$ and consider a simple case $q = 1/m$, where q is to the power of 2. By recursing $\log_2 q$ times, we obtain a tree with q leaves. Each leaf has m bits, i.e. $x = x_{q-1} || \dots || x_0$ and $y = y_{q-1} || \dots || y_0$, where $x_i, y_i \in \{0, 1\}^m$. By doing so, both parties can take advantage of Matrix-Mult and use its comparison function and equivalence function. After calculating those leaves, we can calculate AND and XOR recursively to the root node in order to get the final output.

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$$P_0, P_1, \dots, \tilde{o}, \sqcup, \dots, h, \dots, \lambda, \dots$$
$$\begin{aligned} & \mathbb{E}[\mathbf{h}(\mathbf{P}_0) \mathbf{x}(\mathbf{q}) \ddot{\mathbf{q}}(\mathbf{P}_1) \\ & \mathbf{y}(\mathbf{q}) \ddot{\mathbf{q}}(\mathbf{y}) \ddot{\mathbf{q}}(\mathbf{y}) \hat{\mathbf{J}}(\mathbf{C}) \mathbf{m}(\mathbf{y}) \end{aligned}$$
$$j \in \{0, 1, \dots, m-1\} \quad M = 2^m \quad j = 0, 1, \dots, q-1 \quad \mathbb{K} \quad P_0$$

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1. In steps 9 and 10, OT is used to combine two 1-bit $\binom{M}{1} - OT$ into a 2-bit $\binom{M}{1} - OT$. Because P_1 's input is still y_j
2. In steps 14 AND 16, we use AND. Use related AND, that is, the AND protocol with the same input, to save expenses.
3. For the entire binary tree, the least significant fork of the equation is never used, so it can be omitted.

Communication

Layer	Protocol	Comm. (bits)	Rounds
Millionaires' on $\{0, 1\}^\ell$	GC [62, 63]	$4\lambda\ell$	2
	GMW ⁴ /GSV [29, 32]	$\approx 6\lambda\ell$	$\log \ell + 3$
	SC3 ⁵ [21]	$> 3\lambda\ell$	$\approx 4 \log^* \lambda$
	This work ($m = 4$)	$< \lambda\ell + 14\ell$	$\log \ell$
Millionaires' example $\ell = 32$	GC [62, 63]	16384	2
	GMW/GSV [29, 32]	23140	8
	SC3 [21]	13016	15
	This work ($m = 7$)	2930	5
	This work ($m = 4$)	3844	5

Protocol for DReLU

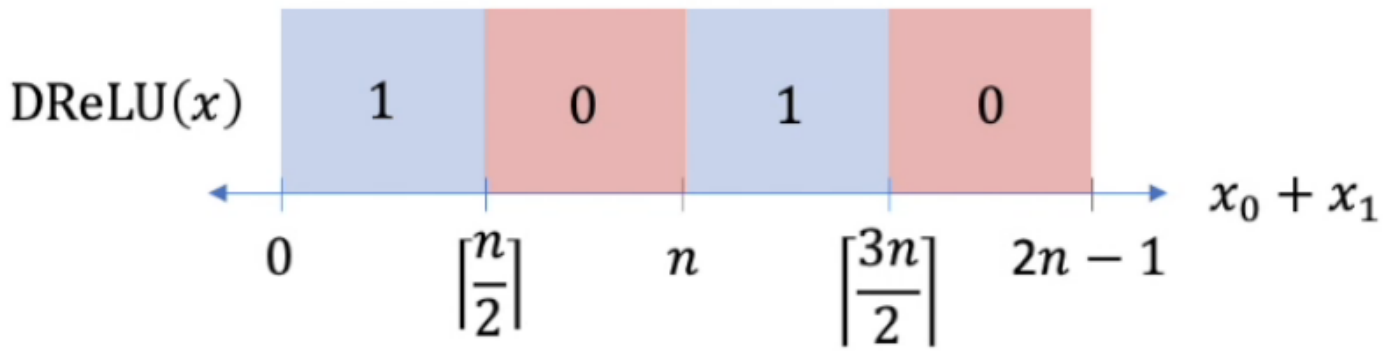
General idea

Over \mathbb{Z} , $\text{DReLU}(x) = \begin{cases} 0, & x < 0 \\ 1, & x \geq 0 \end{cases}$

Over \mathbb{Z}_n , $\text{DReLU}(x) = \begin{cases} 0, & x \in [\lceil \frac{n}{2} \rceil, n) \\ 1, & x \in [0, \lceil \frac{n}{2} \rceil) \end{cases}$

Input: P_0 has $x_0 \in [0, n)$ and P_1 has $x_1 \in [0, n)$; s.t. $x_0 + x_1 = x \bmod n$

Output: P_0, P_1 learn shares of $\text{DReLU}(x)$



Computing DReLU(x):

- Simple: 3 calls to Millionaires'
- Optimized: 2 comparisons
- In particular: for $n = 2^l$, only needs 1 comparison

$$lt: x_0 + x_1 \geq \lceil \frac{n}{2} \rceil \Leftrightarrow x_0 + \lfloor \frac{n}{2} \rfloor > n - 1 - x_1$$

$$wrap: x_0 + x_1 \geq n \Leftrightarrow x_0 > n - 1 - x_1$$

$$rt: x_0 + x_1 \geq \lceil \frac{3n}{2} \rceil \Leftrightarrow x_0 + \lfloor \frac{n}{2} \rfloor > 2n - 1 - x_1$$

Protocol

Over Z , $x_0, x_1 \in \{2^l\}$

We can use 2's complement to encode a_0, a_1 , where $a = a_0 \oplus a_1$

Algorithm 2 ℓ -bit integer DReLU, $\Pi_{\text{DReLU}}^{\text{int}, \ell}$:

Input: P_0, P_1 hold $\langle a \rangle_0^L$ and $\langle a \rangle_1^L$, respectively.

Output: P_0, P_1 get $\langle \text{DReLU}(a) \rangle_0^B$ and $\langle \text{DReLU}(a) \rangle_1^B$.

- 1: P_0 parses its input as $\langle a \rangle_0^L = \text{msb}_0 || x_0$ and P_1 parses its input as $\langle a \rangle_1^L = \text{msb}_1 || x_1$, s.t. $b \in \{0, 1\}$, $\text{msb}_b \in \{0, 1\}$, $x_b \in \{0, 1\}^{\ell-1}$.
 - 2: P_0 & P_1 invoke an instance of $\mathcal{F}_{\text{MILL}}^{\ell-1}$, where P_0 's input is $2^{\ell-1} - 1 - x_0$ and P_1 's input is x_1 . For $b \in \{0, 1\}$, P_b learns $\langle \text{carry} \rangle_b^B$.
 - 3: For $b \in \{0, 1\}$, P_b sets $\langle \text{DReLU} \rangle_b^B = \text{msb}_b \oplus \langle \text{carry} \rangle_b^B \oplus b$.
-

It is obvious that the result of DReLU can be deduced by the MSB of a . (the result is the inversion of the MSB for a)

Step1: we can divide a_0 and a_1 into 2 parts

Step2: online, we only compare $2^{l-1} - 1 - x_0$ and x_1 by Millionaires'

Step3: offline, we generate $[DReLU]_b = msb_b \oplus [carry]_b \oplus b$

The last b is used as an inversion of the MSB for a .

Over \mathbb{Z}_n ,

Algorithm 3 Simple Integer ring DReLU, $\Pi_{DReLU}^{ring,n}_{simple}$:

Input: P_0, P_1 hold $\langle a \rangle_0^n$ and $\langle a \rangle_1^n$, respectively, where $a \in \mathbb{Z}_n$.

Output: P_0, P_1 get $\langle DReLU(a) \rangle_0^B$ and $\langle DReLU(a) \rangle_1^B$.

- 1: P_0 & P_1 invoke an instance of \mathcal{F}_{MILL}^η with $\eta = \lceil \log n \rceil$, where P_0 's input is $(n - 1 - \langle a \rangle_0^n)$ and P_1 's input is $\langle a \rangle_1^n$. For $b \in \{0, 1\}$, P_b learns $\langle wrap \rangle_b^B$ as output.
 - 2: P_0 & P_1 invoke an instance of $\mathcal{F}_{MILL}^{\eta+1}$, where P_0 's input is $(n - 1 - \langle a \rangle_0^n)$ and P_1 's input is $((n - 1)/2 + \langle a \rangle_1^n)$. For $b \in \{0, 1\}$, P_b learns $\langle lt \rangle_b^B$ as output.
 - 3: P_0 & P_1 invoke an instance of $\mathcal{F}_{MILL}^{\eta+1}$, where P_0 's input is $(n + (n - 1)/2 - \langle a \rangle_0^n)$ and P_1 's input is $\langle a \rangle_1^n$. For $b \in \{0, 1\}$, P_b learns $\langle rt \rangle_b^B$ as output.
 - 4: For $b \in \{0, 1\}$, P_b invokes \mathcal{F}_{MUX}^2 with input $(\langle lt \rangle_b^B \oplus \langle rt \rangle_b^B)$ and choice $\langle wrap \rangle_b^B$ to learn $\langle z \rangle_b^B$.
 - 5: For $b \in \{0, 1\}$, P_b outputs $\langle z \rangle_b^B \oplus \langle lt \rangle_b^B \oplus b$.
-

In the first 3 steps, we obtain the share of lt , $wrap$, and rt

Step 4: if $wrap = 1$, $z = lt \text{ XOR } rt$; else, $z = 0$, we can interpret as, $[0,0,1,0]$

Step 5: $P_b = z \text{ XOR } lt \text{ XOR } b$, we can interpret as, $[1,0,1,0]$

Optimization

1. First let P_1 adjust the input of step 1, 2, 3 to be consistent, so that the leaf nodes in F_{Mill} can be calculated together (step 9 & 10 algorithm 1). The specific method is to add $\frac{n-1}{2}$, to P_1 's input of step 1&3 in algorithm 3;
2. Further, the P_0 reduces the execution of step 2 or step 3 according to its input. That is, if $\langle a \rangle_0^n > \frac{n-1}{2}$, then $it = 1$ in step 2; Otherwise, $rt = 0$ in step 3.

Correctness can be easily deduced if we take the input of P_1 in step 14 into the OT protocol from step 5 to step 12.

Since $\langle z \rangle_0^B$ is completely random, the safety can be proved by $(F_{Mill}^{\eta+1}, \binom{4}{1} - OT)$ -hybird model.

Algorithm 4 calls $F_{Mill}^{\eta+1}$ twice and $\binom{4}{1} - OT$ once. Total communication $< \frac{3}{2}\lambda(\eta + 1) + 28(\eta + 1) + 2\lambda + 4$, which is better than algorithm 3.

Algorithm 4 Optimized Integer ring DReLU, $\Pi_{\text{DReLU}}^{\text{ring},n}$:

Input: P_0, P_1 hold $\langle a \rangle_0^n$ and $\langle a \rangle_1^n$, respectively, where $a \in \mathbb{Z}_n$. Let $\eta = \lceil \log n \rceil$.

Output: P_0, P_1 get $\langle \text{DReLU}(a) \rangle_0^B$ and $\langle \text{DReLU}(a) \rangle_1^B$.

- 1: P_0 & P_1 invoke an instance of $\mathcal{F}_{\text{MILL}}^{\eta+1}$, where P_0 's input is $(3(n-1)/2 - \langle a \rangle_0^n)$ and P_1 's input is $(n-1)/2 + \langle a \rangle_1^n$. For $b \in \{0, 1\}$, P_b learns $\langle \text{wrap} \rangle_b^B$ as output.
- 2: P_0 sets $x = (2n-1 - \langle a \rangle_0^n)$ if $\langle a \rangle_0^n > (n-1)/2$, else $x = (n-1 - \langle a \rangle_0^n)$.
- 3: P_0 & P_1 invoke an instance of $\mathcal{F}_{\text{MILL}}^{\eta+1}$, where P_0 's input is x and P_1 's input is $((n-1)/2 + \langle a \rangle_1^n)$. For $b \in \{0, 1\}$, P_b learns $\langle \text{xt} \rangle_b^B$ as output.
- 4: P_0 samples $\langle z \rangle_0^B \xleftarrow{s} \{0, 1\}$.
- 5: **for** $j = \{00, 01, 10, 11\}$ **do**
- 6: P_0 parses j as $j_0 || j_1$ and sets $t_j = 1 \oplus \langle \text{xt} \rangle_0^B \oplus j_0$.
- 7: **if** $\langle a \rangle_0^n > (n-1)/2$ **then**
- 8: P_0 sets $s'_j = t_j \wedge (\langle \text{wrap} \rangle_0^B \oplus j_1)$.
- 9: **else**
- 10: P_0 sets $s'_j = t_j \oplus ((1 \oplus t_j) \wedge (\langle \text{wrap} \rangle_0^B \oplus j_1))$
- 11: **end if**
- 12: P_0 sets $s_j = s'_j \oplus \langle z \rangle_0^B$
- 13: **end for**
- 14: P_0 & P_1 invoke an instance of $\binom{4}{1}$ -OT₁ where P_0 is the sender with inputs $\{s_j\}_j$ and P_1 is the receiver with input $\langle \text{xt} \rangle_1^B || \langle \text{wrap} \rangle_1^B$. P_1 sets its output as $\langle z \rangle_1^B$.
- 15: For $b \in \{0, 1\}$, P_b outputs $\langle z \rangle_b^B$.

https://blog.csdn.net/m0_37908414

ReLU

Relu = DRelu * a. We can use DRelu's and Mux's protocol to caculate Relu

Algorithm 8 ℓ -bit integer ReLU, $\Pi_{\text{ReLU}}^{\text{int},\ell}$:

Input: P_0, P_1 hold $\langle a \rangle_0^L$ and $\langle a \rangle_1^L$, respectively.

Output: P_0, P_1 get $\langle \text{ReLU}(a) \rangle_0^L$ and $\langle \text{ReLU}(a) \rangle_1^L$.

- 1: For $b \in \{0, 1\}$, P_b invokes $\mathcal{F}_{\text{DReLU}}^{\text{int},\ell}$ with input $\langle a \rangle_b^L$ to learn output $\langle y \rangle_b^B$.
- 2: For $b \in \{0, 1\}$, P_b invokes $\mathcal{F}_{\text{MUX}}^L$ with inputs $\langle a \rangle_b^L$ and $\langle y \rangle_b^B$ to learn $\langle z \rangle_b^L$ and sets $\langle \text{ReLU}(a) \rangle_b^L = \langle z \rangle_b^L$.

https://blog.csdn.net/m0_57993411

Maxpool layer

A pairwise comparison is used to select the largest method. Choose the two largest numbers first, and then compare them in turn.

For x, y are both at P_0, P_1 as the number of secret shares, first locally calculate the respective secret share $x - y = w$. Input the two secret shared w into the DRelu protocol to see if w is greater than 0, and the result is set to v . Using a data selector, $t = wv$. Finally each output $z = y + t$.

Consider that when $v = 0$, $t = 0$, the output $z = y$.

Consider $v = 1$, $t = w = x - y$ The output $z = y + x - y = x$.

Part 4 Division and Truncation (linear)

4.1 Expressing general division and truncation using arithmetic over secret shares

Define $\text{idiv} : \mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Z}$ as signed integer division, the quotient is prone to $-\infty$, the remainder have the same sign as divisor. Furthermore, define:

$$\text{rdiv}(a, d) = \text{idiv}(a_u - 1 \{a_u \geq \lceil n/2 \rceil\} \cdot n, d) \bmod n,$$

Here $a_u \in \{0, \dots, n-1\}$ is the unsigned exhibition for $a \in \mathbb{Z}_n$, $0 < d < n$.

Theorem 4.1.1

Let $a \in \mathbb{Z}_n$ has its share as $(a_0, a_1) \in \mathbb{Z}_n^2$, $n = n^1 \cdot d + n^0$. ($n^0, n^1, d \in \mathbb{Z}_n$ and $0 \leq a_0, a_1 < d$). Denote $n' = \lceil \frac{n}{2} \rceil \in \mathbb{Z}$. Define $corr, A, B, C$ as follows:

$$corr = \begin{cases} -1, & (a_u \geq n') \wedge (a_0 < n') \wedge (a_1 < n') \\ 1, & (a_u < n') \wedge (a_0 \geq n') \wedge (a_1 \geq n') \\ 0, & otherwise \end{cases}$$

$$A = a_0^0 + a_1^0 - (1\{a_0 \geq n'\} + 1\{a_1 \geq n'\}) \cdot n^0$$

$$B = idiv(a_0^0 - 1\{a_0 \geq n'\} \cdot n', d) + idiv(a_1^0 - 1\{a_1 \geq n'\} \cdot n', d)$$

$$C = 1\{A < d\} + 1\{A < 0\} + 1\{A < -d\}.$$

Therefore we have:

$$rdiv(\langle a \rangle_0^n, d) + rdiv(\langle a \rangle_1^n, d) + (corr \cdot n^1 + 1 - C - B) =_n rdiv(a, d)$$

Proof

- decompose $rdiv(\langle a \rangle_i^n, d)$;

$$\begin{aligned} rdiv(\langle a \rangle_i^n, d) &= _n idiv(a_i - 1\{a_i \geq n'\} \cdot n, d) \\ &= _n idiv(a_i^1 + a_i^0 - 1\{a_i \geq n'\} \cdot (n^1 \cdot d + n^0), d) \\ &= _n a_i^1 - 1\{a_i \geq n'\} \cdot n^1 + idiv(a_i^0 - 1\{a_i \geq n'\} \cdot n^0, d). \end{aligned}$$

- $a_u = a_0 + a_1 - w \cdot n$, where $w = 1\{a_0 + a_1 \geq n\}$;

$$\begin{aligned} a_u &= a_0 + a_1 - w \cdot n \\ &= (a_0^1 + a_1^1 - w \cdot n^1) \cdot d + (a_0^0 + a_1^0 - w \cdot n^0) \\ &= (a_0^1 + a_1^1 - w \cdot n^1 + k) \cdot d + (a_0^0 + a_1^0 - w \cdot n^0 - k \cdot d), \end{aligned}$$

- Since $\leq a_0^0 + a_1^0 - w \cdot n^0 - k \cdot d < d$

$$\begin{aligned} \text{rdiv}(a, d) &= {}_n a_0^1 + a_1^1 - w \cdot n^1 + k - 1\{a \geq n'\} \cdot n^1 \\ &\quad + \text{idiv}(a_0^0 + a_1^0 - w \cdot n^0 - k \cdot d - 1\{a \geq n'\} \cdot n^0, d) \\ &= {}_n a_0^1 + a_1^1 - w \cdot n^1 - 1\{a \geq n'\} \cdot n^1 \\ &\quad + \text{idiv}(a_0^0 + a_1^0 - w \cdot n^0 - 1\{a \geq n'\} \cdot n^0, d). \end{aligned}$$

- Then we have:

$$\begin{aligned} c &= {}_n \text{rdiv}(a, d) - \text{rdiv}(\langle a \rangle_0^n, d) - \text{rdiv}(\langle a \rangle_1^n, d) \\ &= (1\{a_0 \geq n'\} + 1\{a_1 \geq n'\} - w - 1\{a \geq n'\}) \cdot n^1 \\ &\quad + \text{idiv}(a_0^0 + a_1^0 - w \cdot n^0 - 1\{a \geq n'\} \cdot n^0, d) \\ &\quad - (\text{idiv}(a_0^0 - 1\{a_0 \geq n'\} \cdot n^0, d) + \text{idiv}(a_1^0 - 1\{a_1 \geq n'\} \cdot n^0, d)) \\ &= {}_n c^1 \cdot n^1 + c^0 - B. \end{aligned}$$

CSDN @

- Let $A'_i = \text{idiv}(a_0^0, a_1^0 - i \cdot n^0, d)$;

#	$1\{a_0 \geq n'\}$	$1\{a_1 \geq n'\}$	$1\{a_u \geq n'\}$	w	c^1	c^0
1	0	0	0	0	0	A'_0
2	0	0	1	0	-1	A'_1
3	0	1	0	1	0	A'_1
4	0	1	1	0	0	A'_1
5	1	0	0	1	0	A'_1
6	1	0	1	0	0	A'_1
7	1	1	0	1	1	A'_1
8	1	1	1	1	0	A'_2

Table 8: Truth table for the correction terms c^0 and c^1 in the proof of division theorem (Appendix C).

From this table we know that $c^1 = \text{corr}$. Therefore $c = {}_n \text{corr} \cdot n^1 + c^0 - B$.

Let $C_0 = 1\{A < d\}$, $C_1 = 1\{A < 0\}$, $C_2 = 1\{A < -d\}$, then $C = C_0 + C_1 + C_2$

Row 1 corresponds to $A = a_0^0 + a_1^0$;

Row 8 corresponds to $A = a_0^0 + a_1^0 - 2 \cdot n^0$;

Other rows correspond to $c^0 = \text{idiv}(A, d)$;

It is obvious that $-2 \cdot d + 2 \leq A \leq 2 \cdot d - 2$, therefore $c^0 \in \{-2, -1, 0, 1\}$

Therefore, $C = n \cdot corr \cdot n^1 + (1 - C) - B$

Corollary 4.2: truncation for l -bit integers can be simplified to:

$$(a_0 \gg s) + (a_1 \gg s) + corr \cdot 2^{l-s} + 1 \{a_0^0 + a_1^0 \geq 2^s\} =_L (a \gg s)$$

4.2 Truncation of l -bit Integer

$F_{\text{Trunc}}^{\text{int}, l, s}$ is a function that performs on l -bit integers and truncate its lower s bits. The result is exactly the same as cleartext.

In the algorithm, step 1-15 calculate $corr$. Step 16 calculate $1 \{a_0^0 + a_1^0 \geq 2^s\}$, and conduct F_{B2A} at step 17.

The complicated part is the verification of $corr$, especially the construction of OT in step 15. By taking P_1 's input $\text{rang}_1^m | x_1$ into step 15, we can verify its correctness. Since the calculation of $corr$ is completely random, the safety can be proved by $(F_{\text{DReLU}}^{\text{int}, l}, \begin{pmatrix} 4 \\ 1 \end{pmatrix} - OT, F_{\text{Mill}}^s, F_{\text{B2A}}^L)$ -hybird model.

Since $(F_{\text{DReLU}}^{\text{int}, l}, \begin{pmatrix} 4 \\ 1 \end{pmatrix} - OT, F_{\text{Mill}}^s, F_{\text{B2A}}^L)$ are each conducted once, the communication is less than $\lambda l + 2\lambda + 191 + \text{communication for } F_{\text{Mill}}^s$, which is mainly based on s .

Algorithm 5 Truncation, $\Pi_{\text{Trunc}}^{\text{int}, \ell, s}$:

Input: For $b \in \{0, 1\}$, P_b holds $\langle a \rangle_b^L$, where $a \in \mathbb{Z}_L$.

Output: For $b \in \{0, 1\}$, P_b learns $\langle z \rangle_b^L$ s.t. $z = a \gg s$.

- 1: For $b \in \{0, 1\}$, let $a_b, a_b^0, a_b^1 \in \mathbb{Z}$ be as defined in Corollary 4.2.
 - 2: For $b \in \{0, 1\}$, P_b invokes $\mathcal{F}_{\text{DReLU}}^{\text{int}, \ell}$ with input $\langle a \rangle_b^L$ to learn output $\langle \alpha \rangle_b^B$. Party P_b sets $\langle m \rangle_b^B = \langle \alpha \rangle_b^B \oplus b$.
 - 3: For $b \in \{0, 1\}$, P_b sets $x_b = \text{MSB}(\langle a \rangle_b^L)$.
 - 4: P_0 samples $\langle \text{corr} \rangle_0^L \xleftarrow{\$} \mathbb{Z}_{2^\ell}$.
 - 5: **for** $j = \{00, 01, 10, 11\}$ **do**
 - 6: P_0 computes $t_j = (\langle m \rangle_0^B \oplus j_0 \oplus x_0) \wedge (\langle m \rangle_0^B \oplus j_0 \oplus j_1)$ s.t. $j = (j_0 || j_1)$.
 - 7: **if** $t_j \wedge 1\{x_0 = 0\}$ **then**
 - 8: P_0 sets $s_j =_L -\langle \text{corr} \rangle_0^L - 1$.
 - 9: **else if** $t_j \wedge 1\{x_0 = 1\}$ **then**
 - 10: P_0 sets $s_j =_L -\langle \text{corr} \rangle_0^L + 1$.
 - 11: **else**
 - 12: P_0 sets $s_j =_L -\langle \text{corr} \rangle_0^L$.
 - 13: **end if**
 - 14: **end for**
 - 15: P_0 & P_1 invoke an instance of $\binom{4}{1}\text{-OT}_\ell$, where P_0 is the sender with inputs $\{s_j\}_j$ and P_1 is the receiver with input $\langle m \rangle_1^B || x_1$ and learns $\langle \text{corr} \rangle_1^L$.
 - 16: P_0 & P_1 invoke an instance of $\mathcal{F}_{\text{MILL}}^s$ with P_0 's input as $2^s - 1 - a_0^0$ and P_1 's input as a_1^0 . For $b \in \{0, 1\}$, P_b learns $\langle c \rangle_b^B$.
 - 17: For $b \in \{0, 1\}$, P_b invokes an instance of $\mathcal{F}_{\text{B2A}}^L$ ($L = 2^\ell$) with input $\langle c \rangle_b^B$ and learns $\langle d \rangle_b^L$.
 - 18: P_b outputs $\langle z \rangle_b^L = (\langle a \rangle_b^L \gg s) + \langle \text{corr} \rangle_b^L \cdot 2^{\ell-s} + \langle d \rangle_b^L, b \in \{0, 1\}$.
-

$F_{\text{Div}}^{\text{ring},n,d}$ stands for division on general ring. This protocol is similar to truncation protocol. Since $-3d+2 \leq A-d, A, A+d \leq 3d-2$, $C = (\text{DReLU}(A-d) \oplus 1) + (\text{DReLU}(A+d) \oplus 1)$ can be calculated in terms of $\delta = \lceil \log_2^{6d} \rceil$ -bit. Before calculate C, A needs to be calculated first. Therefore, calculate A on \mathbb{Z}_n and $\mathbb{Z}_{\Delta} (\Delta = 2^\delta)$ simultaneously.

Algorithm 9 Integer ring division, $\Pi_{\text{DIV}}^{\text{ring}, n, d}$:

Input: For $b \in \{0, 1\}$, P_b holds $\langle a \rangle_b^n$, where $a \in \mathbb{Z}_n$.

Output: For $b \in \{0, 1\}$, P_b learns $\langle z \rangle_b^n$ s.t. $z = \text{rdiv}(a, d)$.

- 1: For $b \in \{0, 1\}$, let $a_b, a_b^0, a_b^1 \in \mathbb{Z}$ and $n^0, n^1, n' \in \mathbb{Z}$ be as defined in Theorem 4.1. Let $\eta = \lceil \log(n) \rceil$, $\delta = \lceil \log 6d \rceil$, and $\Delta = 2^\delta$.
- 2: For $b \in \{0, 1\}$, P_b invokes $\mathcal{F}_{\text{DReLU}}^{\text{ring}, n}$ with input $\langle a \rangle_b^n$ to learn output $\langle \alpha \rangle_b^B$. Party P_b sets $\langle m \rangle_b^B = \langle \alpha \rangle_b^B \oplus b$.
- 3: For $b \in \{0, 1\}$, P_b sets $x_b = 1\{\langle a \rangle_b^n \geq n'\}$.
- 4: P_0 samples $\langle \text{corr} \rangle_0^n \xleftarrow{\$} \mathbb{Z}_n$ and $\langle \text{corr} \rangle_0^\Delta \xleftarrow{\$} \mathbb{Z}_\Delta$.
- 5: **for** $j = \{00, 01, 10, 11\}$ **do**
- 6: P_0 computes $t_j = (\langle m \rangle_0^B \oplus j_0 \oplus x_0) \wedge (\langle m \rangle_0^B \oplus j_0 \oplus j_1)$ s.t. $j = (j_0 || j_1)$.
- 7: **if** $t_j \wedge 1\{x_0 = 0\}$ **then**
- 8: P_0 sets $s_j =_n -\langle \text{corr} \rangle_0^n - 1$ and $r_j =_\Delta -\langle \text{corr} \rangle_0^\Delta - 1$.
- 9: **else if** $t_j \wedge 1\{x_0 = 1\}$ **then**
- 10: P_0 sets $s_j =_n -\langle \text{corr} \rangle_0^n + 1$ and $r_j =_\Delta -\langle \text{corr} \rangle_0^\Delta + 1$.
- 11: **else**
- 12: P_0 sets $s_j =_n -\langle \text{corr} \rangle_0^n$ and $r_j =_\Delta -\langle \text{corr} \rangle_0^\Delta$.
- 13: **end if**
- 14: **end for**
- 15: P_0 & P_1 invoke an instance of $\binom{4}{1}$ -OT $_{\eta+\delta}$ where P_0 is the sender with inputs $\{s_j || r_j\}_j$ and P_1 is the receiver with input $\langle m \rangle_1^B || x_1$. P_1 sets its output as $\langle \text{corr} \rangle_1^n || \langle \text{corr} \rangle_1^\Delta$.
- 16: For $b \in \{0, 1\}$, P_b sets $\langle A \rangle_b^\Delta =_\Delta a_b^0 - (x_b - \langle \text{corr} \rangle_b^\Delta) \cdot n^0$.
- 17: For $b \in \{0, 1\}$, P_b sets $\langle A_0 \rangle_b^\Delta =_\Delta \langle A \rangle_b^\Delta - b \cdot d$, $\langle A_1 \rangle_b^\Delta = \langle A \rangle_b^\Delta$, and $\langle A_2 \rangle_b^\Delta =_\Delta \langle A \rangle_b^\Delta + b \cdot d$.
- 18: **for** $j = \{0, 1, 2\}$ **do**
- 19: For $b \in \{0, 1\}$, P_b invokes $\mathcal{F}_{\text{DReLU}}^{\text{int}, \delta}$ with input $\langle A_j \rangle_b^\Delta$ to learn output $\langle \gamma_j \rangle_b^B$. Party P_b sets $\langle C'_j \rangle_b^B = \langle \gamma_j \rangle_b^B \oplus b$.
- 20: For $b \in \{0, 1\}$, P_b invokes an instance of $\mathcal{F}_{\text{B2A}}^n$ with input $\langle C'_j \rangle_b^B$ and learns $\langle C_j \rangle_b^n$.
- 21: **end for**

- 22: For $b \in \{0, 1\}$, P_b sets $\langle C \rangle_b^n = \langle C_0 \rangle_b^n + \langle C_1 \rangle_b^n + \langle C_2 \rangle_b^n$.
- 23: For $b \in \{0, 1\}$, P_b sets $B_b = \text{idiv}(a_b^0 - x_b \cdot n^0, d)$.
- 24: P_b sets $\langle z \rangle_b^n =_n \text{rdiv}(\langle a \rangle_b^n, d) + \langle \text{corr} \rangle_b^n \cdot n^1 + b - \langle C \rangle_b^n - B_b$, for $b \in \{0, 1\}$.

Correctness verification is the same to truncation's. Safety check is based on $(\begin{pmatrix} 4 \\ 1 \end{pmatrix} - \text{OT}_{\{\eta + \delta\}}, F_{\{\text{DReLU}\}^{\{\text{ring}, n\}}}, F_{\{\text{B2A}\}^n})$ -hybird model.

The protocol calls $(\begin{pmatrix} 4 \\ 1 \end{pmatrix} - \text{OT}_{\{\eta + \delta\}})$ and $F_{\{\text{DReLU}\}^{\{\text{ring}, n\}}}$ once, $F_{\{\text{DReLU}\}^{\{\delta\}}}$ and $F_{\{\text{B2A}\}^n}$ 3 times. Total communication is less than $(\frac{3}{4}\lambda + 34) \cdot (\eta + 2\delta)$ bit.

Improvement on Avgpool is listed below:

Layer	Protocol	Comm. (bits)	Rounds
Avgpool_d \mathbb{Z}_{2^ℓ}	GC [62, 63]	$2\lambda(\ell^2 + 5\ell - 3)$	2
	This work	$< (\lambda + 21) \cdot (\ell + 3\delta)$	$\log(\ell\delta) + 4$
Avgpool_d \mathbb{Z}_n	GC [62, 63]	$2\lambda(\eta^2 + 9\eta - 3)$	2
	This work	$< (\frac{3}{2}\lambda + 34) \cdot (\eta + 2\delta)$	$\log(\eta\delta) + 6$
Avgpool_{49} $\mathbb{Z}_{2^\ell}, \ell = 32$	GC [62, 63]	302336	2
	This work	5570	10
Avgpool_{49} $\mathbb{Z}_n, \eta = 32$	GC [62, 63]	335104	2
	This work	7796	14

Table 3: Comparison of communication with garbled circuits for Avgpool_d . We define $\eta = \lceil \log n \rceil$ and $\delta = \lceil \log(6 \cdot d) \rceil$. For concrete bits of communication we use $\lambda = 128$. Choice of $d = 49$ corresponds to average pool filter of size 7×7 .

4.4 Truncation Optimization

For scenarios where $2 \cdot n^0 \leq d = 2^s$ is satisfied, $A \geq -d$ always stands. Therefore $A \geq -d$ in the calculation of C can be omitted. Further decrease by $\delta = \lceil \log_2(4d) \rceil$.

Part 5 Secure Inference

The inference process of neural network model is carried out as follows

1. Linear layer: Call multiplication based on OT or multiplication based on HE, adjust according to the scene;
2. ReLU: invokes the ReLU protocol
3. Avgpool: Call division protocol
4. Truncation: Since the output of ReLU is non-negative, we can reduce calculation expense
5. Maxpool and Argmax: Millionaire's and F_{MUX} in order

Part 6 Evaluation

In this paper, they implement OT based on EMP and chooses efficient AES. The HE is SEAL. They integrate those into CryptFlow system. Firstly, the ReLU is calculated by comparing with the GC-based method, raising it by 2-25%.

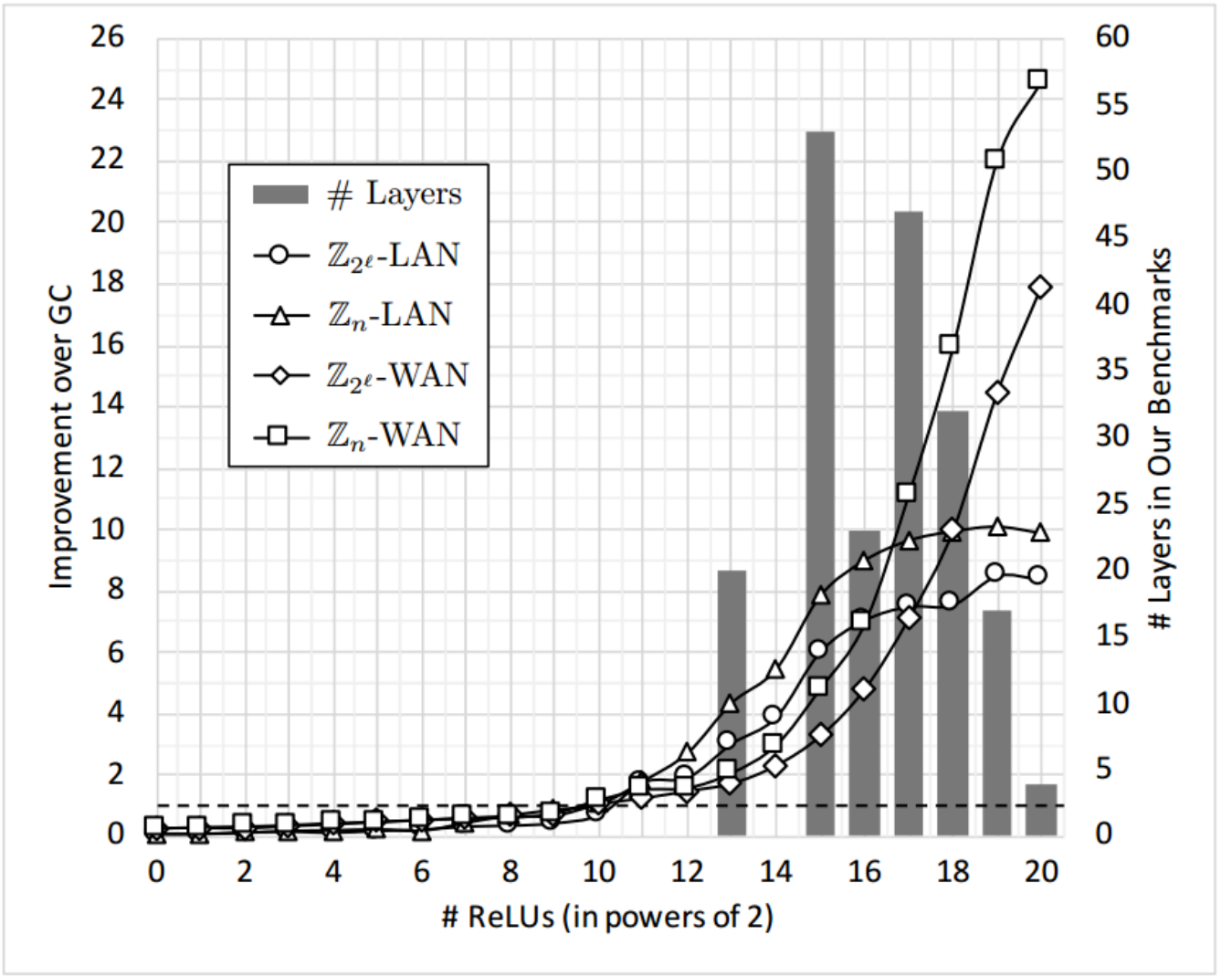


Figure 1: The left y-axis shows $(\frac{GC \text{ Time}}{Our \text{ Time}})$. The right y-axis shows the total number of ReLU layers corresponding to each layer size in our benchmark set. The legend entries denote the input domain and the network setting.

Next, the ReLU in the real network is promoted as follows. In the local area network, improve the calculation 8-9 \times , in the wide area network to improve the time 18-21 \times , improve the communication 7-9 \times .

Benchmark	Garbled Circuits			Our Protocols		
	LAN	WAN	Comm	LAN	WAN	Comm
SqueezeNet	26.4	265.6	7.63	3.5	33.3	1.15
ResNet50	136.5	1285.2	39.19	16.4	69.4	5.23
DenseNet121	199.6	1849.3	56.57	24.8	118.7	8.21

(a) over \mathbb{Z}_{2^ℓ}

Benchmark	Garbled Circuits			Our Protocols		
	LAN	WAN	Comm	LAN	WAN	Comm
SqueezeNet	51.7	525.8	16.06	5.6	50.4	1.77
ResNet50	267.5	2589.7	84.02	28.0	124.0	8.55
DenseNet121	383.5	3686.2	118.98	41.9	256.0	12.64

(b) over \mathbb{Z}_n

Table 4: Performance comparison with Garbled Circuits for ReLU layers. Runtimes are in seconds and comm. in GiB.

For Avgpool, time is improved 51 \times .

Benchmark	Garbled Circuits			Our Protocol		
	LAN	WAN	Comm	LAN	WAN	Comm
SqueezeNet	0.2	2.0	36.02	0.1	0.8	1.84
ResNet50	0.4	3.9	96.97	0.1	0.8	2.35
DenseNet121	17.2	179.4	6017.94	0.5	3.5	158.83

(a) over \mathbb{Z}_{2^ℓ}

Benchmark	Garbled Circuits			Our Protocol		
	LAN	WAN	Comm	LAN	WAN	Comm
SqueezeNet	0.2	2.2	39.93	0.1	0.9	1.92
ResNet50	0.4	4.2	106.22	0.1	1.0	3.82
DenseNet121	19.2	198.2	6707.94	0.6	4.4	214.94

(b) over \mathbb{Z}_n

Table 9: Performance comparison of Garbled Circuits with our protocols for computing Avgpool layers. Runtimes are in seconds and communication numbers are in MiB.

Compared with Delphi, it is still greatly improved in both nonlinear layer and online calculation.

Benchmark	Metric	Linear	Non-linear		
			Delphi	Ours	Improvement
MiniONN	Time	10.7	30.2	1.0	30.2×
	Comm.	0.02	3.15	0.28	12.3×
ResNet32	Time	15.9	52.9	2.4	22.0×
	Comm.	0.07	5.51	0.59	9.3×

Table 5: Performance comparison with Delphi [49] for non-linear layers. Runtimes are in seconds and comm. in GiB.

Benchmark	Linear	Non-linear		
		Delphi	Ours	Improvement
MiniONN	< 0.1	3.97	0.32	12.40×
ResNet32	< 0.1	6.99	0.63	11.09×

Table 6: Performance comparison with Delphi [49] for on-line runtime in seconds.

Finally, for large networks, predictions can be made in 10min (LAN) and 20min (WAN).

Benchmark	Protocol	LAN	WAN	Comm
SqueezeNet	SCI _{OT}	44.3	293.6	26.07
	SCI _{HE}	59.2	156.6	5.27
ResNet50	SCI _{OT}	619.4	3611.6	370.84
	SCI _{HE}	545.8	936.0	32.43
DenseNet121	SCI _{OT}	371.4	2257.7	217.19
	SCI _{HE}	463.2	1124.7	35.56

Table 7: Performance of CRYPTFLOW2 on ImageNet-scale benchmarks. Runtimes are in seconds and comm. in GiB.

Part 7 Conclusion

This paper is one of the best two-party computational comparison protocols and is the basis of SIRNN. In this paper, an efficient calculation method of accurate

truncation and division is presented. This ideas are very enlightening for later work.

Appendix I: Code-reading

millionaire_with_equality.h:

MillionaireWithEquality

IO
L
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- IO *io: h IO
- sci::OTPack<IO> *otpack: h sci::OTPack<IO> Oblivious Transfer OT
- TripleGenerator<IO> *triple_gen: h TripleGenerator<IO> 養
- MillionaireProtocol<IO> *mill: h MillionaireProtocol<IO> 願
- bool del_mill: h mill
- int party: party
- int l, r, log_alpha, beta, beta_pow: 1, r, log_alpha, beta, beta_pow beta beta
- int num_digits, num_triples, log_num_digits: num_digits num_triples
- uint8_t mask_beta, mask_r: mask_beta mask_r

MillionaireWithEquality
L
願

MillionaireWithEquality

- party: party
- io: h IO

- `otpack: h sci::OTPack<IO>` Oblivious Transfer OT
- `mill_in: h w MillionaireProtocol<IO>` `nullptr`
- `bitlength: h` 8 32
- `radix_base: h` MILL_PARAM

`mill_in nullptr` `h MillionaireProtocol<IO>` `del_mill` `mill_in` `mill`

`mill` `triple_gen` `triple_gen` `configure` `bitlength` `radix_base`

`MillionaireWithEquality` `MillionaireProtocol<IO>`

`bitlen_lt_beta` `Transfer` OT

- `res_cmp: h uint8_t`
- `res_eq: h uint8_t`
- `data: h uint64_t`
- `num_cmps: h`
- `bitlength: h`
- `greater_than: h`
- `radix_base: h`

`N` `mask` `N` 2 `bitlength` `mask` `N` 1

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Multi-Party Computation (MPC) vs. Oblivious

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