Matching and Basis Rotation for the Intrinsic Unified Evolution Basis

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In this document we will explain how the matching and the basis rotation are performed in the Intrinsic Unified Evolution Basis.

1 Matching

Fist of all, we have to perform the matching between the basis vectors in the two different flavor schemes, i.e. the n_f and the n_f+1 flavor schemes. The matching of the gluon, light quark and heavy quark PDFs are the followings:

$$
\begin{aligned}
g^{(n_f+1)} &= A^S_{gg} g^{(n_f)} + A^S_{gq}\Sigma^{(n_f)}_{(n_f)} + A^S_{gH} h^{(n_f)} \\
l^{(n_f+1)} &= A^{ns}_{qq} l^{(n_f)} + \frac{1}{2n_f} A^S_{qg} g^{(n_f)} \quad \text{and the same for } \bar{l} \\
h^{(n_f+1)} &= \frac{1}{2} A^S_{Hg} g^{(n_f)}_{(n_f)} + \frac{1}{2} A^{ps}_{Hq}\Sigma^{(n_f)}_{(n_f)} + A_{HH} h^{(n_f)} \quad \text{and the same for } \bar{h}
\end{aligned}
$$

From the second relation we get that

$$
\begin{aligned}
l^{(n_f+1)}_+ &= A^{ns}_{qq} l^{(n_f)}_+ + \frac{1}{n_f} A^S_{qg} g^{(n_f)} \\
l^{(n_f+1)}_- &= A^{ns}_{qq} l^{(n_f)}_- \\
\Sigma^{(n_f+1)}_{(n_f)} &= A^S_{qg} g^{(n_f)} + A^{ns}_{qq}\Sigma^{(n_f)}_{(n_f)} \\
V^{(n_f+1)}_{(n_f)} &= A^{ns}_{qq} V^{(n_f)}_{(n_f)}
\end{aligned}
$$

while from the third relation we get that

$$
\begin{aligned}
h^{(n_f+1)}_+ &= A^S_{Hg} g^{(n_f)}_{(n_f)} + A^{ps}_{Hq}\Sigma^{(n_f)}_{(n_f)} + A_{HH} h^{(n_f)}_+ \\
h^{(n_f+1)}_- &= A_{HH} h^{(n_f)}_-
\end{aligned}
$$

The matching of the components $\Sigma_{\Delta(n_f)}$, $V_{\Delta(n_f)}$, T_i, V_i are diagonal. For the Vs this is trivial since they are composed by l_-. For $\Sigma_{\Delta(n_f)}$ instead: being it defined as

$$
\Sigma^{(n_f+1)}_{\Delta(n_f)} = \frac{n_d(n_f)}{n_u(n_f)} \sum_{i=1}^{n_u} u^{(n_f+1)}_{+\,i} - \sum_{i=1}^{n_d} d^{(n_f+1)}_{+\,i}
$$

the gluon contribution cancels, giving the relation

$$
\Sigma^{(n_f+1)}_{\Delta(n_f)} = A^{ns}_{qq}\Sigma^{(n_f+1)}_{\Delta(n_f)}
$$

The same holds for the T_i components.

Observe that this holds up to NNLO, since that at N^{3}LO we have to consider also the pure singlet components of the light quark matching (I think that we have to add $\frac{1}{2n_f} A^{ps}_{qq}\Sigma^{nf}_{(nf)}$ to the matching of $l^{(n_f+1)}$, but I'm not 100% sure. In this way the matching of l_- remains diagonal, as it should be, and the same hold for $\Sigma_{\Delta(n_f)}$ and T_i).

Up to second order the perturbative expansion of the matching terms is given by

$$
\begin{aligned}
A^S_{gg} &= 1 + a_s A^{S(1)}_{gg} + a_s^2 A^{S(2)}_{gg} \\
A^S_{gq} &= 1 + a_s^2 A^{S(2)}_{gq} \\
A^S_{gH} &= 1 + a_s A^{S(1)}_{gH} \\
A^S_{qg} &= 1 \\
A^{ns}_{qq} &= 1 + a_s^2 A^{ns(1)}_{qq} \\
A^S_{Hg} &= 1 + a_s A^{S(1)}_{Hg} + a_s^2 A^{S(2)}_{Hg} \\
A^{ps}_{Hq} &= 1 + a_s^2 A^{ps(2)}_{Hq} \\
A_{HH} &= 1 + a_s A^{(2)}_{HH}
\end{aligned}
$$

2 Basis Rotation

After the matching we have to perform the basis rotation from the basis with $\Sigma^{(n_f+1)}_{(n_f)}$, $\Sigma^{(n_f+1)}_{\Delta(n_f)}$, $h^{(n_f+1)}_+$ to the basis with $\Sigma^{(n_f+1)}_{(n_f+1)}$, $\Sigma^{(n_f+1)}_{\Delta(n_f+1)}$, $T_i^{(n_f+1)}$ (all the considerations that we will do for this basis rotation apply identically to the components V, V_Δ, h_-, V_i). Being all the PDFs in the n_f+1 flavor scheme, from now on we will drop the superscript (n_f+1).

2.1 Σ

For the Σ component the basis rotation is very simple, being

$$\Sigma_{(n_f+1)} = \Sigma_{(n_f)} + h_+$$

2.2 Σ_Δ

This component requires a bit more work: starting from

$$\begin{cases} \Sigma &= \Sigma_u + \Sigma_d \\ \Sigma_\Delta &= \frac{n_d}{n_u}\Sigma_u - \Sigma_d \end{cases}$$

we obtain that

$$\begin{cases} \Sigma_u &= \frac{n_u}{n_f}\Big(\Sigma + \Sigma_\Delta\Big) \\ \Sigma_d &= \frac{n_d}{n_f}\Sigma - \frac{n_u}{n_f}\Sigma_\Delta \end{cases}$$

Therefore, we find

$$\Sigma_{\Delta(n_f+1)} = \frac{n_u(n_f+1)}{n_d(n_f+1)}\Sigma_{u(n_f)} - \Sigma_{d(n_f)} + k(n_f)h_+$$

where

$$k(n_f) = \begin{cases} \frac{n_u(n_f+1)}{n_d(n_f+1)} & \text{if h=up-like} \\ -1 & \text{if h=down-like} \end{cases}$$

In the end we find that

$$\Sigma_{\Delta(n_f+1)} = \Big(\frac{n_d(n_f+1)}{n_u(n_f+1)}n_u(n_f) - n_d(n_f)\Big)\Sigma_{(n_f)} + \frac{n_f+1}{n_u(n_f+1)}\frac{n_u(n_f)}{n_f}\Sigma_{\Delta(n_f)} + k(n_f)h_+$$

2.3 T_i

In the end, we have to find the rotation for the T_i component: being

$$\begin{aligned} T_3^d &= d^+ - s^+ = \Sigma_{d(n_f)} - h_+ \quad \text{for } n_f = 3 \\ T_3^u &= u^+ - c^+ = \Sigma_{u(n_f)} - h_+ \quad \text{for } n_f = 4 \\ T_8^d &= d^+ + s^+ - 2b^+ = \Sigma_{d(n_f)} - 2h_+ \quad \text{for } n_f = 5 \\ T_8^u &= u^+ + c^+ - 2t^+ = \Sigma_{u(n_f)} - 2h_+ \quad \text{for } n_f = 6 \end{aligned}$$

Using the expressions of Σ_u and Σ_d as a function of Σ and Σ_Δ, we can write that

$$T_i = f_1(n_f)\Sigma_{(n_f)} + f_2(n_f)\Sigma_{\Delta(n_f)} + f_3(n_f)h_+$$

with

$$f_1(n_f) = \begin{cases} \frac{n_u(n_f)}{n_f} & \text{if } h \text{ is up-like } (n_f\text{=3,5}) \\ \frac{n_d(n_f)}{n_f} & \text{if } h \text{ is down-like } (n_f\text{=2,4}) \end{cases}$$

$$f_2(n_f) = \begin{cases} \frac{n_u(n_f)}{n_f} & \text{if } h \text{ is up-like} \\ -\frac{n_u(n_f)}{n_f} & \text{if } h \text{ is down-like} \end{cases}$$

$$f_3(n_f) = \begin{cases} -1 & \text{if } h \text{ is } s,\, c\ (n_f\text{=2,3}) \\ -2 & \text{if } h \text{ is } b,\, t\ (n_f\text{=4,5}) \end{cases}$$