Defining

$$\Sigma_u = \sum_{k=1}^{n_u} u_k^+, \quad \Sigma_d = \sum_{k=1}^{n_d} d_k^+$$
$$V_u = \sum_{k=1}^{n_u} u_k^-, \quad V_d = \sum_{k=1}^{n_d} d_k^-$$

our basis is

$$\begin{aligned}
&g\\
&\gamma\\
\Sigma &= \Sigma_u + \Sigma_d\\
\Delta_\Sigma &= \frac{n_d}{n_u}\Sigma_u - \Sigma_d\\
V &= V_u + V_d\\
\Delta_V &= \frac{n_d}{n_u}V_u - V_d\\
T_3^d &= d^+ - s^+\\
V_3^d &= d^- - s^-\\
T_3^u &= u^+ - c^+\\
V_3^u &= u^- - c^-\\
T_8^d &= d^+ + s^+ - 2b^+\\
V_8^d &= d^- + s^- - 2b^-\\
T_8^u &= u^+ + c^+ - 2t^+\\
V_8^u &= u^- + c^- - 2t^-
\end{aligned}$$

- Singlet sector:

$$\mu^2 \frac{d}{d\mu^2}\begin{pmatrix} g\\ \gamma\\ \Sigma\\ \Delta_\Sigma\end{pmatrix} = \begin{pmatrix} P_{gg} + n_f\langle e^2\rangle\tilde{P}_{gg} & n_f\langle e^2\rangle\tilde{P}_{g\gamma} & P_{gq} + \langle e^2\rangle\tilde{P}_{gq} & \nu_u e_-^2\tilde{P}_{gq}\\ n_f\langle e^2\rangle\tilde{P}_{\gamma g} & n_f\langle e^2\rangle\tilde{P}_{\gamma\gamma} & \langle e^2\rangle\tilde{P}_{\gamma q} & \nu_u e_-^2\tilde{P}_{\gamma q}\\ 2n_f(P_{qg} + \langle e^2\rangle\tilde{P}_{qg}) & 2n_f\langle e^2\rangle\tilde{P}_{q\gamma} & P_{qq} + \langle e^2\rangle\Big(\tilde{P}_+ + \langle e^2\rangle(\tilde{P}_{qq} - \tilde{P}_+)\Big) & \nu_u e_-^2\Big(\tilde{P}_+ + \langle e^2\rangle(\tilde{P}_{qq} - \tilde{P}_+)\Big)\\ 2n_f\nu_d e_-^2\tilde{P}_{qg} & 2n_f\nu_d e_-^2\tilde{P}_{q\gamma} & \nu_d e_-^2\Big(\tilde{P}_+ + \langle e^2\rangle(\tilde{P}_{qq} - \tilde{P}_+)\Big) & P_+ + e_\Delta^2\tilde{P}_+ + \nu_u\nu_d(e_-^2)^2(\tilde{P}_{qq} - \tilde{P}_+)\end{pmatrix}\begin{pmatrix} g\\ \gamma\\ \Sigma\\ \Delta_\Sigma\end{pmatrix}$$

with

$$\begin{aligned}
\langle e^2\rangle &= \frac{n_u e_u^2 + n_d e_d^2}{n_f}\\
e_\Delta^2 &= \frac{n_u e_d^2 + n_d e_u^2}{n_f}\\
e_-^2 &= e_u^2 - e_d^2\\
\nu_u &= \frac{n_u}{n_f}\\
\nu_d &= \frac{n_d}{n_f}
\end{aligned}$$

- Valence sector:

$$\mu^2\frac{d}{d\mu^2}\begin{pmatrix} V\\ \Delta_V\end{pmatrix} = \begin{pmatrix} P_V + \langle e^2\rangle\tilde{P}_- & \nu_u e_-^2\tilde{P}_-\\ \nu_d e_-^2\tilde{P}_- & P_- + e_\Delta^2\tilde{P}_-\end{pmatrix}\begin{pmatrix} V\\ \Delta_V\end{pmatrix}$$

- Decoupled sector:

$$\mu^2\frac{d}{d\mu^2}T_{3/8}^{u/d} = (P_+ + e_i^2\tilde{P}_+)T_{3/8}^{u/d}$$
$$\mu^2\frac{d}{d\mu^2}V_{3/8}^{u/d} = (P_- + e_i^2\tilde{P}_-)V_{3/8}^{u/d}$$