Homework 2.1

CUNY MSDS DATA 609

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Problems

The below problems are taken from the text book:

A First Course in Mathematical Modeling, 5th Edition. Frank R. Giordano, William P. Fox, Steven B. Horton. ISBN-13: 9781285050904.

Exercise #2 Page 113.

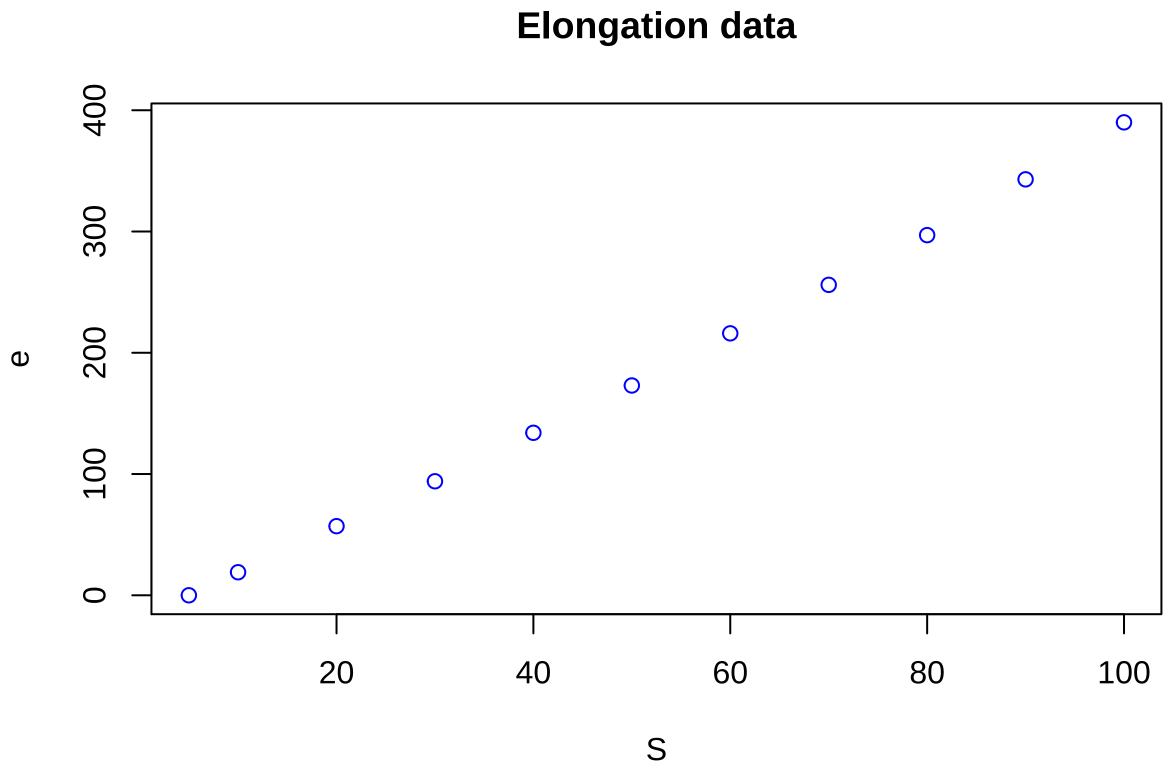
The following table gives the elongation e in inches per inc(in./in.) for a given stress S on a steel wire measured in pounds per square in (lb/in.^{2}). Test the model $e = c_1 S$ by plotting the data. Estimate $c1$ graphically.

S	e
5	0
10	19
20	57
30	94
40	134
50	173
60	216
70	256
80	297
90	343
100	390

Table 1: Elongation e with $S(x10^{-3})$ and $e(x10^5)$.

Solution

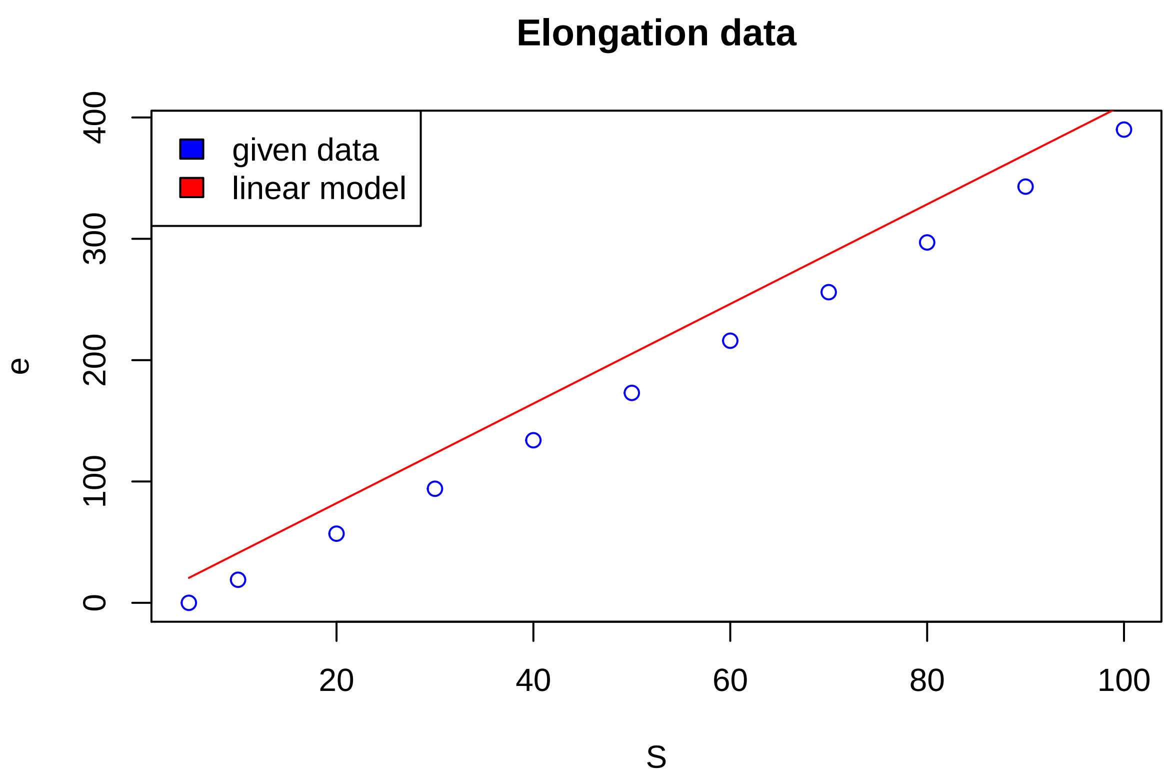
Let's plot our original data and see how it looks.



From the above, we can appreciate that the data is fairly linear. Hence, we could express it in the form $e = c_1 S$ as provided above.

Where $c_1 = \frac{390-0}{100-5} = 4.105263$

thus, it returns the following model $e = 4.105263S$



From what we can see, our linear model is not very accurate but it is beyond our current problem.

Exercise #2.a Page 121.

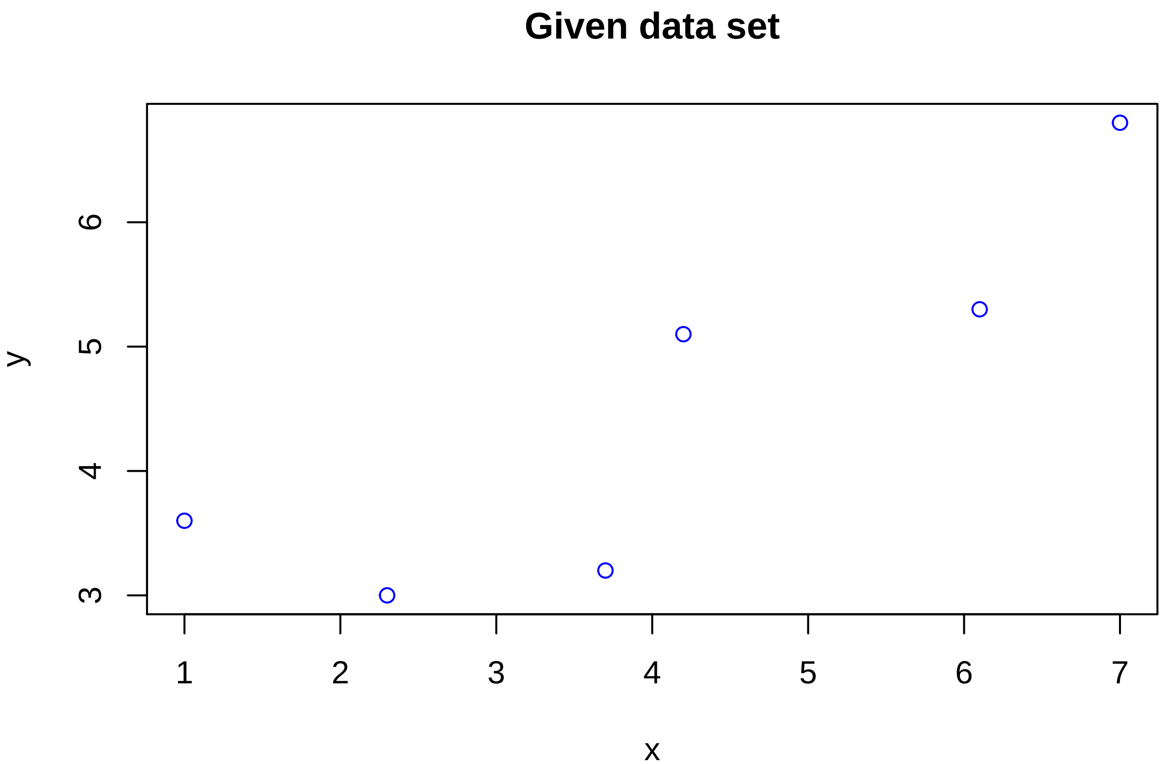
For each of the following data sets, formulate the mathematical model that minimizes the largest deviation between the data and the line $y = ax + b$. If a computer is available, solve for the estimates of a and b.

y	x
3.6	1.0
3.0	2.3
3.2	3.7
5.1	4.2
5.3	6.1
6.8	7.0

Table 2: Problem 2.a data set.

Solution

Let's have a visual representation of the data set.



The idea here is to minimize our largest deviation in our data set and predicted values; hence, we could perform as follows:

Using Chebyshev criterion

That is:

$r - (3.6 - (1.0a + b)) \geq 0$, $r + (3.6 - (1.0a + b)) \geq 0$

$r - (3.0 - (2.3a + b)) \geq 0$, $r + (3.0 - (2.3a + b)) \geq 0$

$r - (3.2 - (3.7a + b)) \geq 0,\ r + (3.2 - (3.7a + b)) \geq 0$

$r - (5.1 - (4.2a + b)) \geq 0,\ r + (5.1 - (4.2a + b)) \geq 0$

$r - (5.3 - (6.1a + b)) \geq 0,\ r + (5.3 - (6.1a + b)) \geq 0$

$r - (6.8 - (7.0a + b)) \geq 0,\ r + (6.8 - (7.0a + b)) \geq 0$

The above, is the formulation of the mathematical model that minimizes the largest deviation between the data and the line $y = ax + b$.

Finding the estimates of a and b.

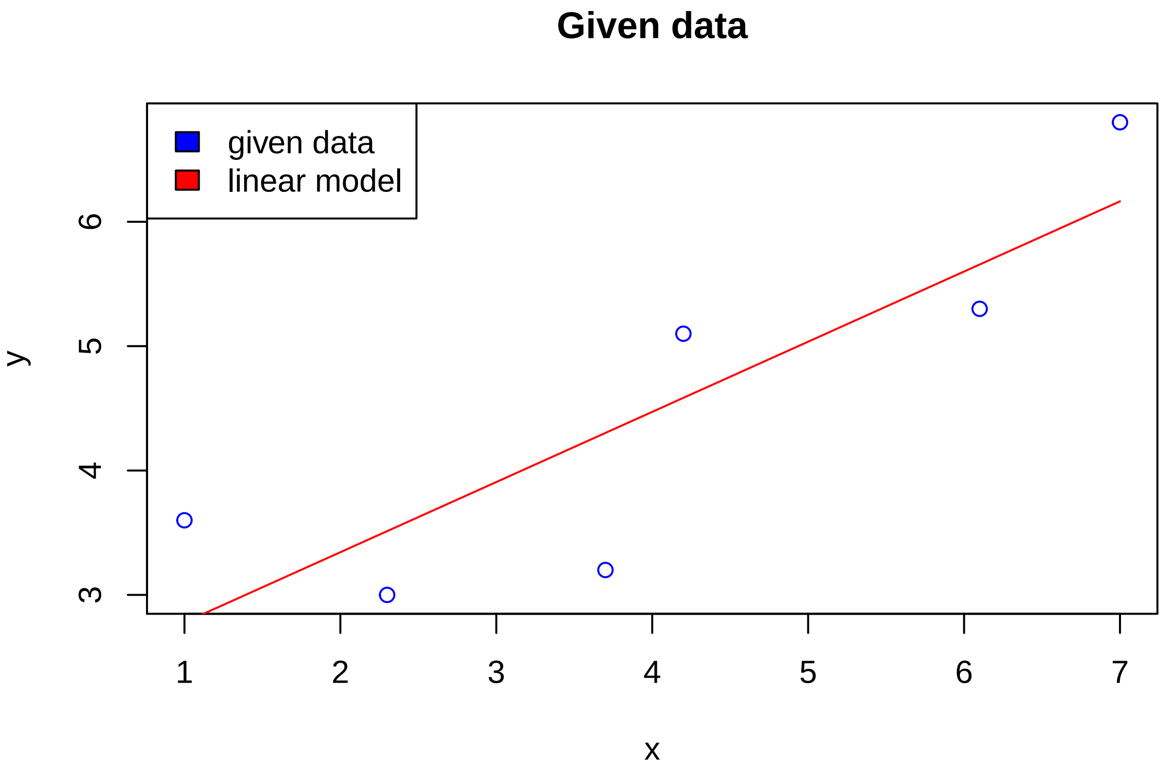
Using least squares criterion.

```
lm.y <- lm(t.df$y~t.df$x)
summary(lm.y)
```

```
##
## Call:
## lm(formula = t.df$y ~ t.df$x)
##
## Residuals:
##       1       2       3       4       5       6
##  0.8209 -0.5126 -1.1025  0.5154 -0.3567  0.6355
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   2.2149     0.7737   2.863   0.0458 *
## t.df$x        0.5642     0.1703   3.313   0.0296 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.8586 on 4 degrees of freedom
## Multiple R-squared:  0.7329, Adjusted R-squared:  0.6661
## F-statistic: 10.98 on 1 and 4 DF,  p-value: 0.02957
```

From th above, we can conclude that our function employing the R^2 method is:

$y = 2.2149 + 0.5642x$



END.