Homework 01

CUNY MSDS DATA 609

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Problems

The below problems are taken from the text book:

A First Course in Mathematical Modeling, 5th Edition. Frank R. Giordano, William P. Fox, Steven B. Horton. ISBN-13: 9781285050904.

Exercise #10 Page Page 8.

Your grandparents have an annuity. The value of the annuity increases each month by an automatic deposit of 1% interest on the previous month's balance. Your grandparents withdraw $1000 at the beginning of each month for living expenses. Currently, they have $50,000 in the annuity. Model the annuity with a dynamical system. Will the annuity run out of money? When? Hint: What value will a_n have when the annuity is depleted?

Solution

Basically, the change in the amount increases by the amount paid in the form of interest and decreases by the amount withdraw each month.

Change $= \Delta a_n = a_{n+1} - a_n = 0.01 \cdot a_n - 1000$

The dynamical system will be:

$a_0 = 50000$

$a_{n+1} = a_n + 0.01 \cdot a_n - 1000$

The below table represent the values for the upcoming months.

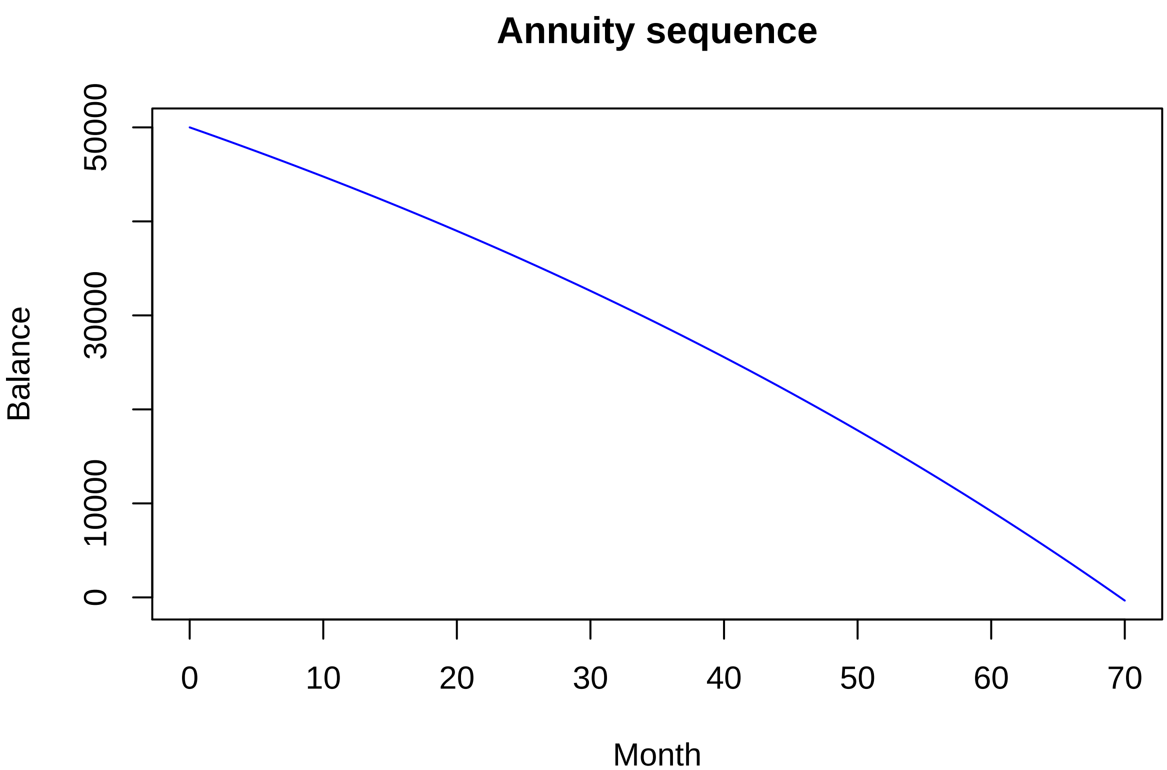
Let's see a few records and the graphical representation.

Month	Balance
0	50000
1	49500
2	48995
3	48485
4	47970
5	47450

Table 1: First records in the succession with no decimals

Month	Balance
65	4532
66	3577
67	2613
68	1639
69	655
70	-338

Table 2: Last records in the succession with no decimals



Answers

Will the annuity run out of money?

Yes, the annuity will run out of money.

When?

The annuity will run out of money in about 70 months from now on.

What value will a_n have when the annuity is depleted?

a_n will have a value of - \$338.09. That means that on their final withdrawal they will take home \$1000 - \$338.09 = \$ 661.91 since they gained some interest in the last month.

Exercise #9 Page Page 17.

The data in the accompanying table show the speed n (in increments of 5 mph) of an automobile and the associated distance a_n in feet required to stop it once the brakes are applied. For instance, $n = 6$ (representing $6 \times 5 = 30$ mph) requires a stopping distance of $a_6 = 47$ ft.

n	Distance
1	3
2	6
3	11
4	21
5	32
6	47
7	65
8	87
9	112
10	140
11	171
12	204
13	241
14	282
15	325
16	376

Table 3: n vs stopping distance

a.

Calculate and plot the change Δa_n versus n. Does the graph reasonably approximate a linear relationship?

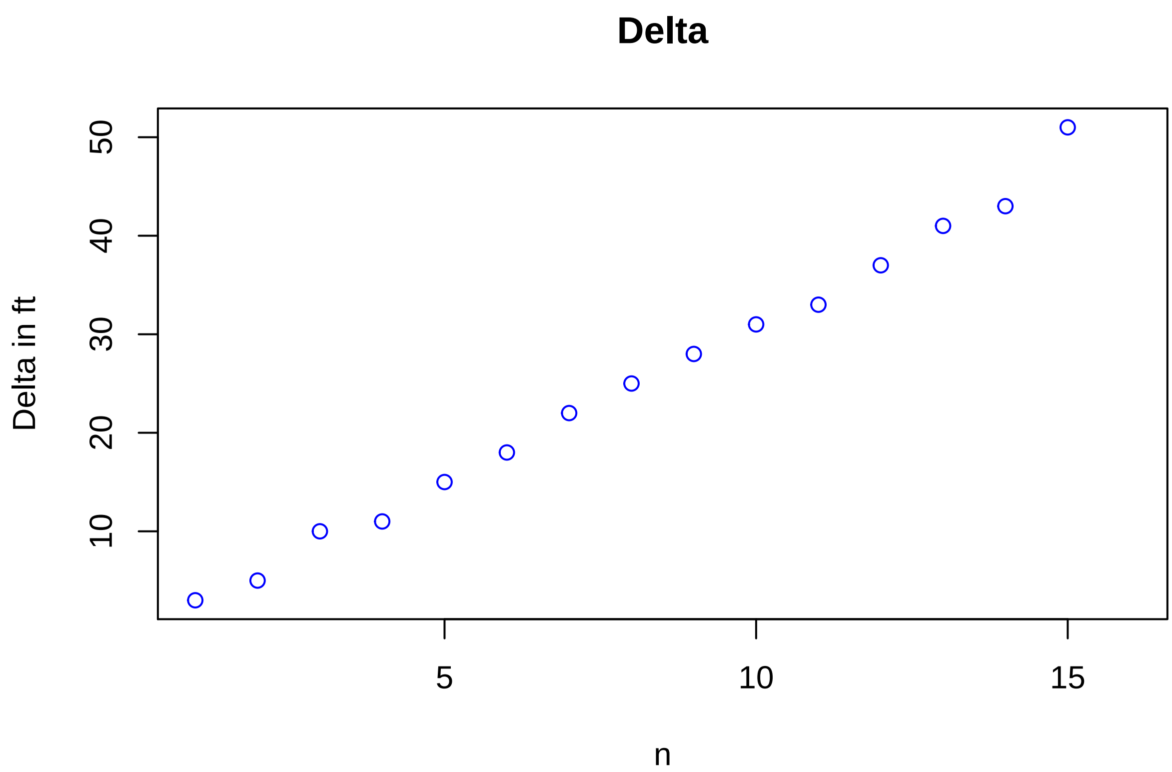
Solution

Now, the next table shows $\Delta a_n = a_{n+1} - a_n$.

n	Distance	Delta
1	3	3
2	6	5
3	11	10
4	21	11
5	32	15
6	47	18
7	65	22
8	87	25
9	112	28
10	140	31
11	171	33
12	204	37
13	241	41
14	282	43
15	325	51
16	376	

Table 4: Speed, stopping distance and delta

Let's plot our Delta vs our n values.



Does the graph reasonably approximate a linear relationship?

Yes, the graph reasonably approximate a linear relationship.

b.

Based on your conclusions in part (a), find a difference equation model for the stopping distance data. Test your model by plotting the errors in the predicted values against n. Discuss the appropriateness of the model.

Solution

In order to solve this, we can basically calculate our stopping distance as follows:

$Distance_{n+1} = Distance_n + Delta_n$

Since we already know the values for the $Distance_n$; we could find a formula for Delta in terms of n since we concluded that is somehow linear.

That is:

$Delta = k \cdot n + b$

We could calculate k by finding the slope that passes in between the points $(n = 5, Delta = 15)$ and $(n = 6, Delta = 18)$.

$k \approx \frac{18-15}{6-5} \approx 3$

Now, if we want to substitute to find b, we could use as follows:

let's use the point $(n = 1, Delta = 3)$ and by replacing in the following formula:

$Delta = k \cdot n + b$

$3 = 3 \cdot 1 + b$

$0 = b$

Hence our final model formula for the Delta will be as follows:

$Delta_n = 3n$

Now that we have our $Delta_n$ in terms of n; we can proceed as follows:

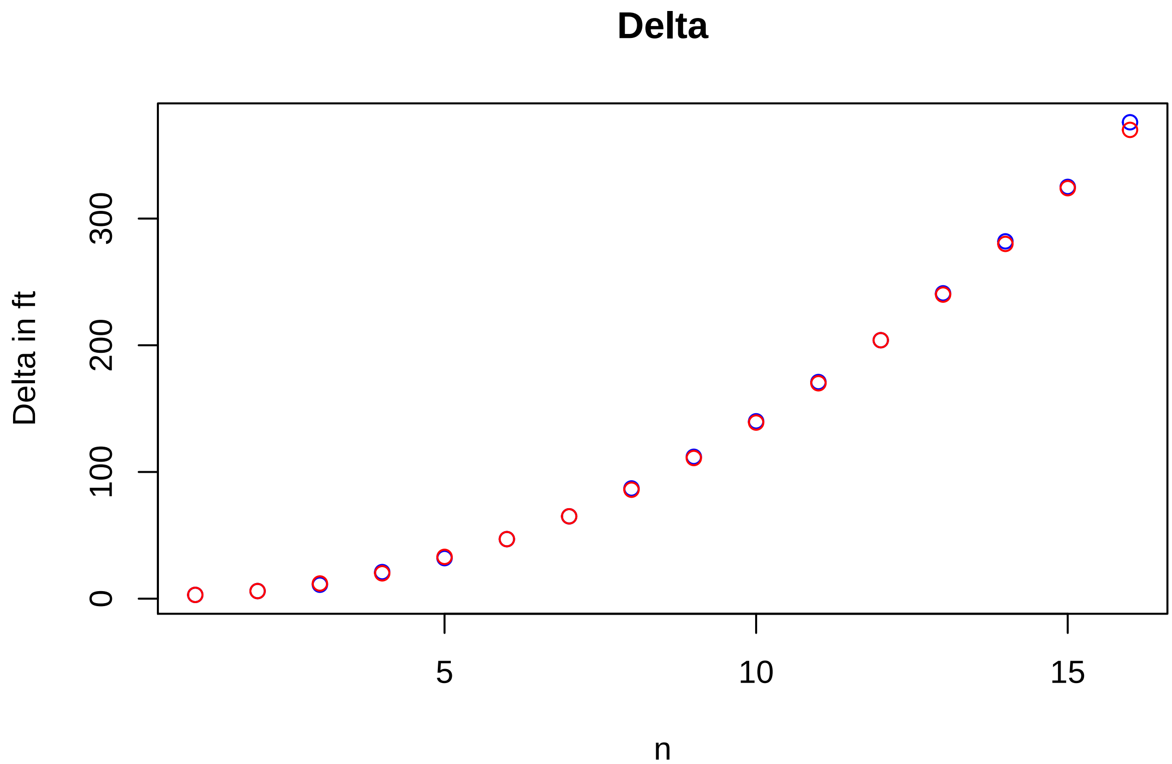
$Predicted\,Distance_{n+1} = Distance_n + Delta_n$

Based on the above result; in order to obtain our final model formula, we could write as follows:

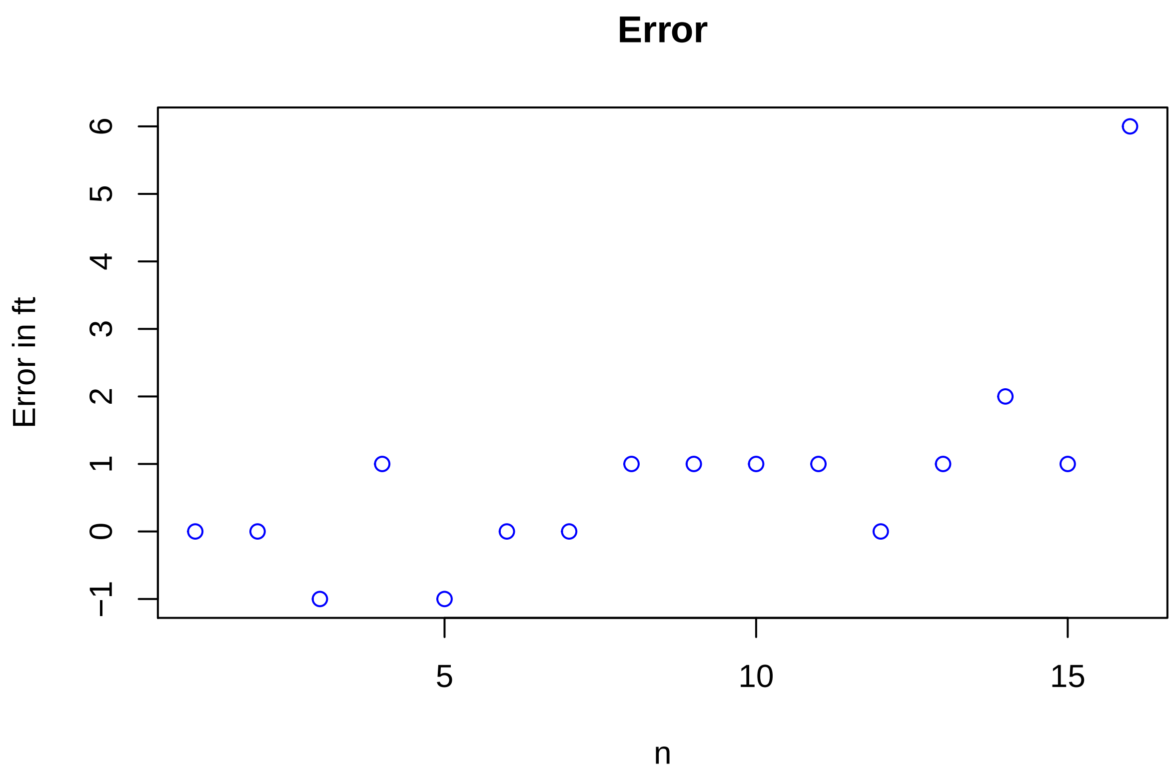
$Predicted\,Distance_{n+1} = Distance_n + 3n$

n	Distance	Delta	Predicted	Error
1	3	3	3	0
2	6	5	6	0
3	11	10	12	-1
4	21	11	20	1
5	32	15	33	-1
6	47	18	47	0
7	65	22	65	0
8	87	25	86	1
9	112	28	111	1
10	140	31	139	1
11	171	33	170	1
12	204	37	204	0
13	241	41	240	1
14	282	43	280	2
15	325	51	324	1
16	376		370	6

Table 5: Speed, stopping distance, delta and predicted values



The above plot, represents the real value in blue color and the predicted value in red.



The above table represent the difference in between the actual reading and the predicted value employing the difference equation model for the stopping distance data.

In regards of the appropriateness of the model; it seems to be very accurate with what seems to be an outlier when the speed is very high; this could be understandable and seems to predict the distance pretty good; perhaps could be improved using other methods.

END.