Homework 5.2

CUNY MSDS DATA 609

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Contents

1 Problems

The below problems are taken from the text book:

A First Course in Mathematical Modeling, 5th Edition. Frank R. Giordano, William P. Fox, Steven B. Horton. ISBN-13: 9781285050904.

1.1 Exercise #3 Page 469

The following data were obtained for the growth of a sheep population introduced into a new environment on the island of Tasmania (adapted from J. Davidson, "On the Growth of the Sheep Population in Tasmania," *Trans. R. Soc. S. Asutralia* 62(1938): 342-346).

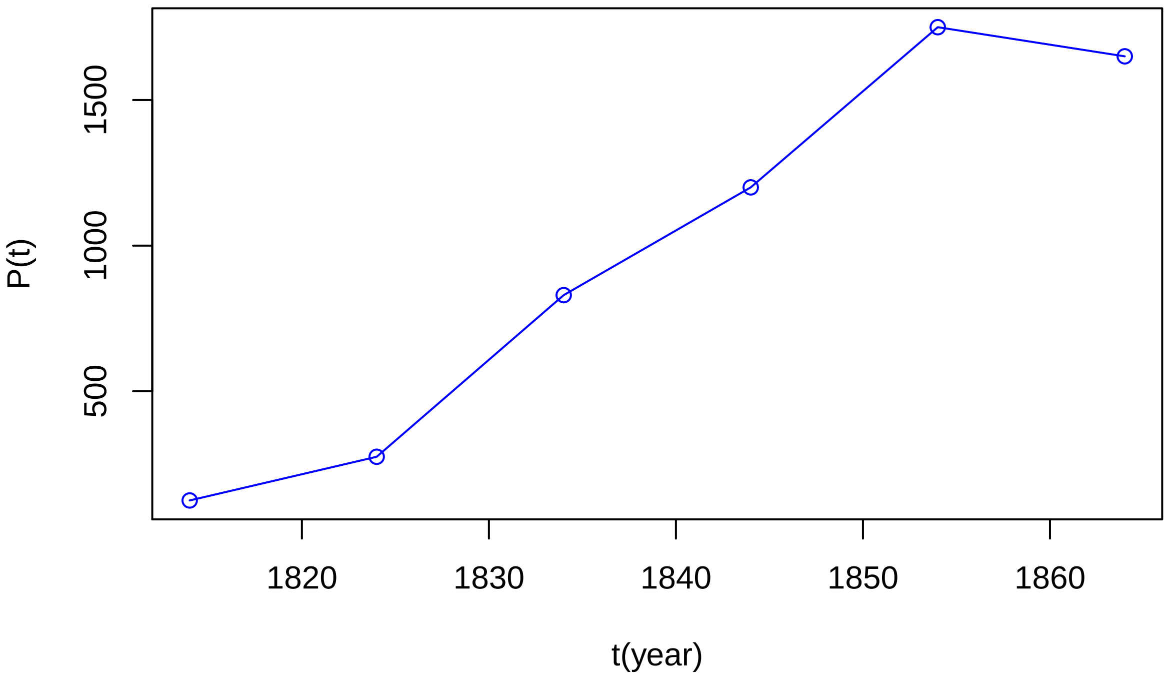
t(year)	1814	1824	1834	1844	1854	1864
P(t)	125	275	830	1200	1750	1650

1.1.1 Estimates

Make an estimate of M by graphing $P(t)$.

1.1.1.1 Solution

First, lets plot our data.



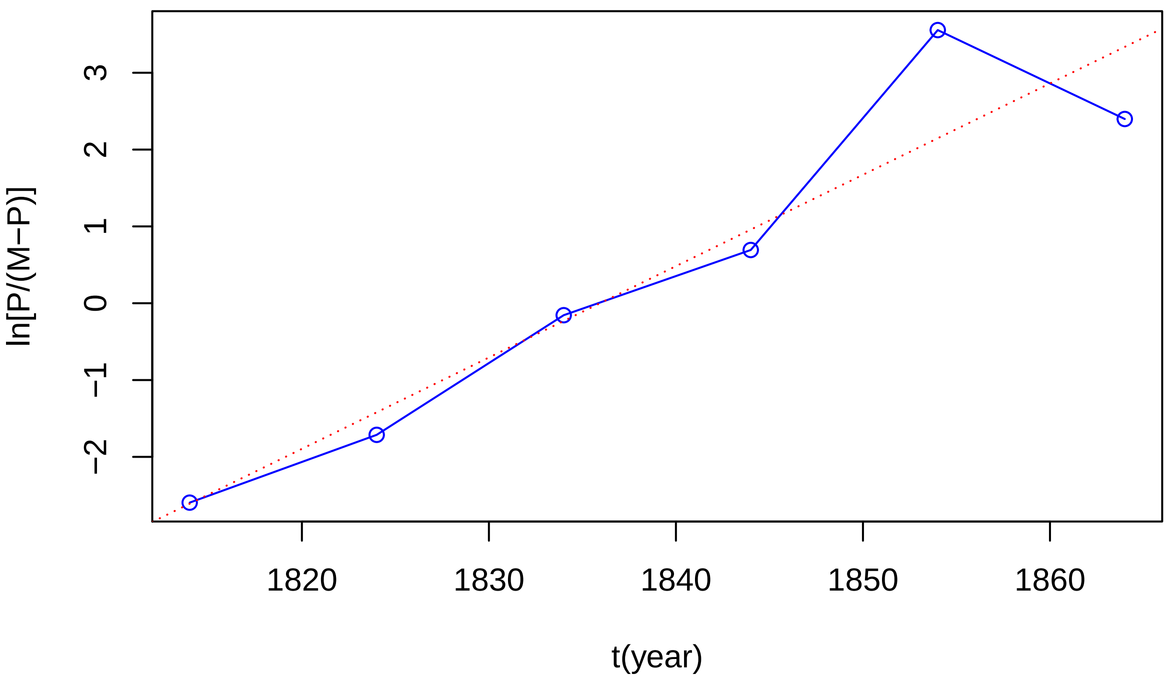
From the graph, we can observe how there seems to be a trend and the maximum value M seems no to exceed $M = 1800$.

1.1.2 Plots

Plot $ln[P/(M-P)]$ against t. If a logistic curve seems reasonable, estimate rM and t^*.

1.1.2.1 Solution

Let's see our plot:



From the above plot, there series of values, seems not to be approximated by a linear model. Hence, a logistic curve seems to be appropriate.

Now, since

$$ln\frac{P}{M-P} = rMt + C$$

We could estimate the slope rM as follows:

```
##
## Call:
## lm(formula = df$ln ~ df$t)
##
## Residuals:
##        1        2        3        4        5        6
##  0.01387 -0.29299  0.07498 -0.26514  1.40793 -0.93866
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -218.31833   38.23277   -5.71  0.00465 **
## df$t           0.11891    0.02079    5.72  0.00462 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.8697 on 4 degrees of freedom
## Multiple R-squared:  0.8911, Adjusted R-squared:  0.8638
## F-statistic: 32.72 on 1 and 4 DF,  p-value: 0.004623
```

From the above results, we have:

$\beta_0 = -218.31833$

$\beta_1 = 0.11891$

Resulting in:

$$ln\frac{P}{M-P} = \beta_0 + \beta_1 t$$

Which means that $\beta_0 = C$ and $\beta_1 = rM$

$\therefore C = -218.31833$

$\therefore rM = 0.11891$

Now, in order to find t^*, we perform as follows:

$$t^* = -\frac{C}{rM}$$

$$t^* = -\frac{-218.31833}{0.11891}$$

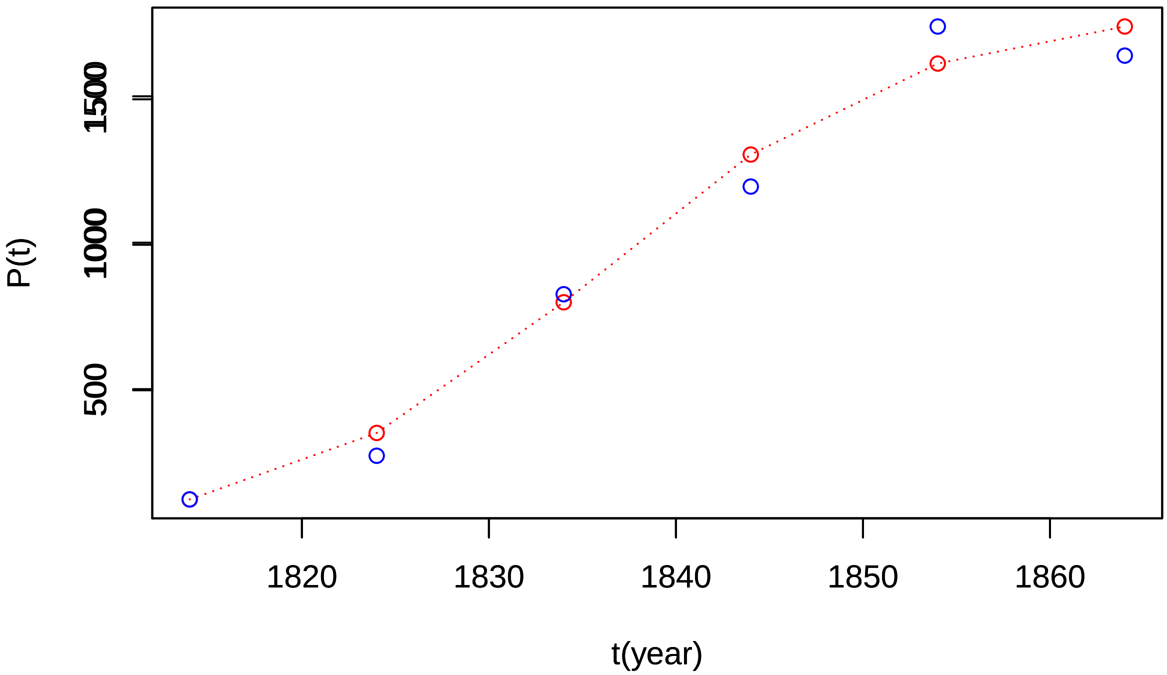
$t^* = 1835.9413542.$

An alternative, will be by employing the following formula:

$$t^* = t_0 - \frac{1}{rM} ln \frac{P_0}{M - P_0}$$

$$t^* = 1814 - \frac{1}{0.11891} ln \frac{125}{1800 - 125}$$

$t^* = 1835.8247155$.



1.2 Exercise #6 Page 478

Suggest other phenomena for which the model described in the text might be used.

1.2.1 Solution

Let's see what is our Problem identification:

How can the doses and the time between doses be adjusted to maintain a safe but effective concentration of the drug in the blood?

1.2.2 Suggested phenomena

In my case, I think that it could be used for colony of bacteria growing in an environment with limited resources; let's say with lack of food, or space constraints on the size of the colony. In this case, I believe that it is not reasonable to expect the colony to grow exponentially; making the colony unable to grow larger than some maximum population P.