Homework 3.2

CUNY MSDS DATA 609

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Problems

The below problems are taken from the text book:

A First Course in Mathematical Modeling, 5th Edition. Frank R. Giordano, William P. Fox, Steven B. Horton. ISBN-13: 9781285050904.

Exercise #6 Page 264

Solve using graphical analysis.

Maximize $10x + 35y$

subject to

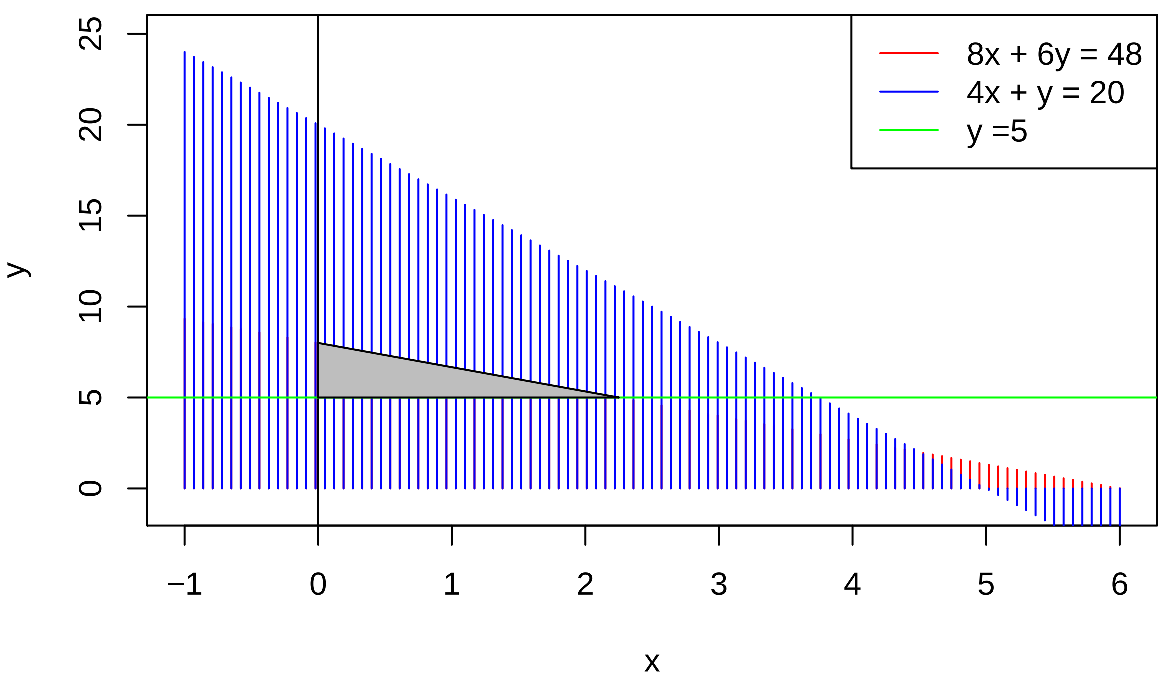
$8x + 6y \leq 48$ (board-feet of lumber)

$4x + y \leq 20$ (hour of carpentry)

$y \geq 5$ (demand)

$x, y \geq 0$ (non-negativity)

Solution



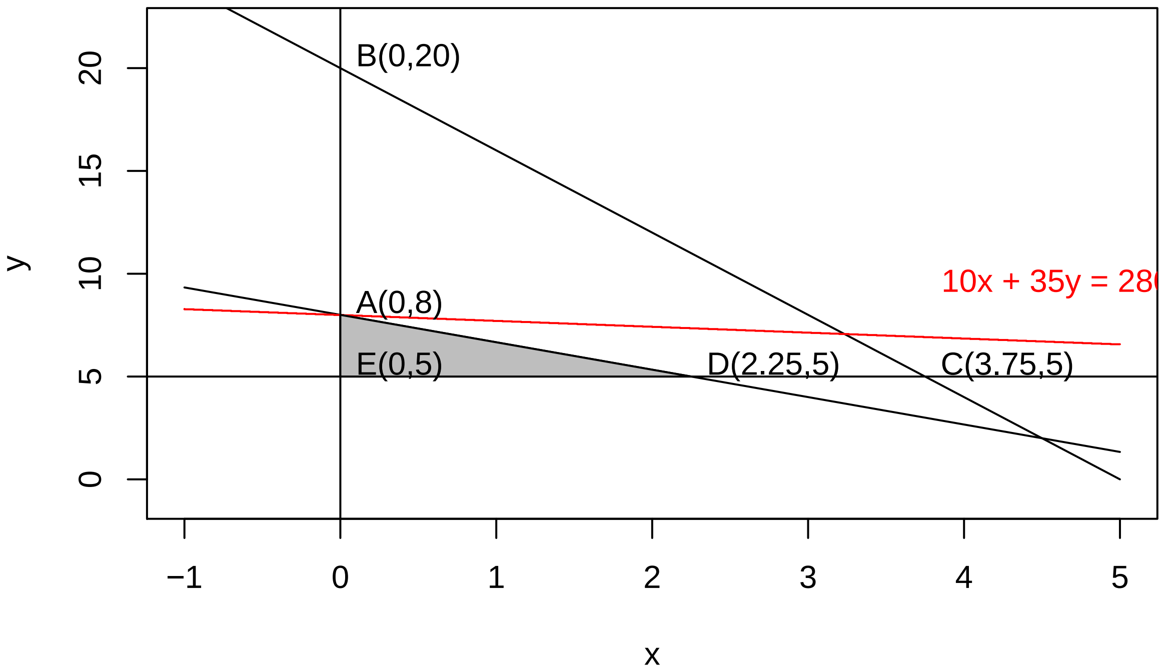
From the above graph, we can see that we have 3 main points in the y axis as follows $A(0,8)$, $B(0,20)$, $E(0,5)$ and then two more intersections with $y = 5$ as follows $C(\frac{15}{4}, 5)$ and $D(\frac{18}{8}, 5)$ and by evaluating such points on the objective function, we obtain the following results:

```
x <- c(0,0,18/8)
y <- c(48/6,5,5)
f.df <- data.frame(x,y)
f.df$`10x + 35y` <- 10*f.df$x + 35*f.df$y

f.df
```

```
##      x y 10x + 35y
## 1 0.00 8     280.0
## 2 0.00 5     175.0
## 3 2.25 5     197.5
```

From the above table, after evaluating our critic points in $10x + 35y$, we found out that the maximum value is 280.



Exercise #6 Page 264

Solve using the method of Algebraic Solutions.

Maximize $10x + 35y$

subject to

$8x + 6y \leq 48$ (board-feet of lumber)

$4x + y \leq 20$ (hour of carpentry)

$y \geq 5$ (demand)

$x, y \geq 0$ (non-negativity)

Solution

First, we convert the inequalities to equations by adding new necessary non negative “slack” variables z_1, z_2 and z_3. If either z_1, z_2 or z_3 is negative, then the constrain is not satisfied. Thus the problem becomes:

Maximize $10x_1 + 35y_1$

subject to

$8x_1 + 6y_1 + z_1 = 48$

$4x_1 + y_1 + z_2 = 20$

$y_1 + z_3 = 5$

$x_1, y_2, z_1, z_2, z_3 \geq 0$ (non-negativity)

We now consider the entire set of 5 variables x_1, y_1, z_1, z_2, z_3 to determine the possible intersection point in the x_1x_2 plane.

Let’s begin by assigning the variables x_1 and x_2 the value of zero; resulting in the following set of equations:

$z_1 = 48$

$z_2 = 20$

Which is could be a feasible intersection point $A(0, 0)$ because all four variables are non negative; but the second constrain is violated by 5 units, indicating that the intersection point $A(0, 0)$ is infeseable.

For the second intersection point we choose the variable x_1 and z_1 and set them to zero, yielding the system:

$6y_1 = 48$

$y_1 + z_2 = 20$

That has solution $y_1 = 8$ and $z_2 = 12$, which is feasible intersection point $B(0, 8)$.

For the third intersection point we choose x_1 and z_2 and set them to zero, yielding the system:

$6y_1 + z_1 = 48$

$y_1 = 20$

That has solution $y_1 = 20$ and $z_1 = -72$. Thus, the first constrain is violated by 72 units and the second constrain is violated by 77 units, indicating that the intersection point $C(0, 20)$ is infeseable.

In a similar fashion choosing z_1 and z_2 and setting to zero, yielding the system:

$8x_1 + 6y_1 = 48$

$4x_1 + y_1 = 20$

That has solution $x_1 = \frac{9}{2}$ and $y_1 = 2$ Thus, the second constrain is violated by 3 units, indicating that the intersection point $D(\frac{9}{2}, 2)$ is infeseable.

For the fourth intersection point we choose $y_1 = 5$ and $z_1 = 0$, yielding the system:

$8x_1 + 30 = 48$

$5 + z_3 = 5$

That has solution $x_1 = \frac{18}{8}$ and $z_3 = 0$ which is feasible intersection point $E(\frac{18}{8}, 5)$.

For the fifth intersection point we choose $y_1 = 5$ and $z_2 = 0$, yielding the system:

$4x_1 + 5 = 20$

$5 + z_3 = 5$

That has solution $x_1 = \frac{15}{4}$ and $z_3 = 0$ which is feasible intersection point $F(\frac{15}{4}, 5)$.

For the sixth intersection point we choose, $x_1 = 0$ and $y_1 = 5$, yielding the system:

$30 + z_1 = 48$

$5 + z_2 = 20$

$z_3 = 5$

That has solution $z_1 = 18$, $z_2 = 15$ and $z_3 = 5$ which is feasible intersection point $G(0, 5)$.

In summary, of the six possible intersection points in the x_1x_2 plane, the following were found to be feasible.

$B(0, 8)$

$E(\frac{18}{8}, 5)$

$F(\frac{15}{4}, 5)$

$G(0, 5)$

```
x <- c(0,18/8, 15/4, 0)
y <- c(8,5,5,5)
f.df <- data.frame(x,y)
f.df$`10x + 35y` <- 10*f.df$x + 35*f.df$y

f.df
```

```
##      x y 10x + 35y
## 1 0.00 8     280.0
## 2 2.25 5     197.5
## 3 3.75 5     212.5
## 4 0.00 5     175.0
```

Answer The maximum generated will be with $x = 0$ and $y = 8$.

End.