Markers law

NNrico

September 2024

1 Introduction

2 Proof

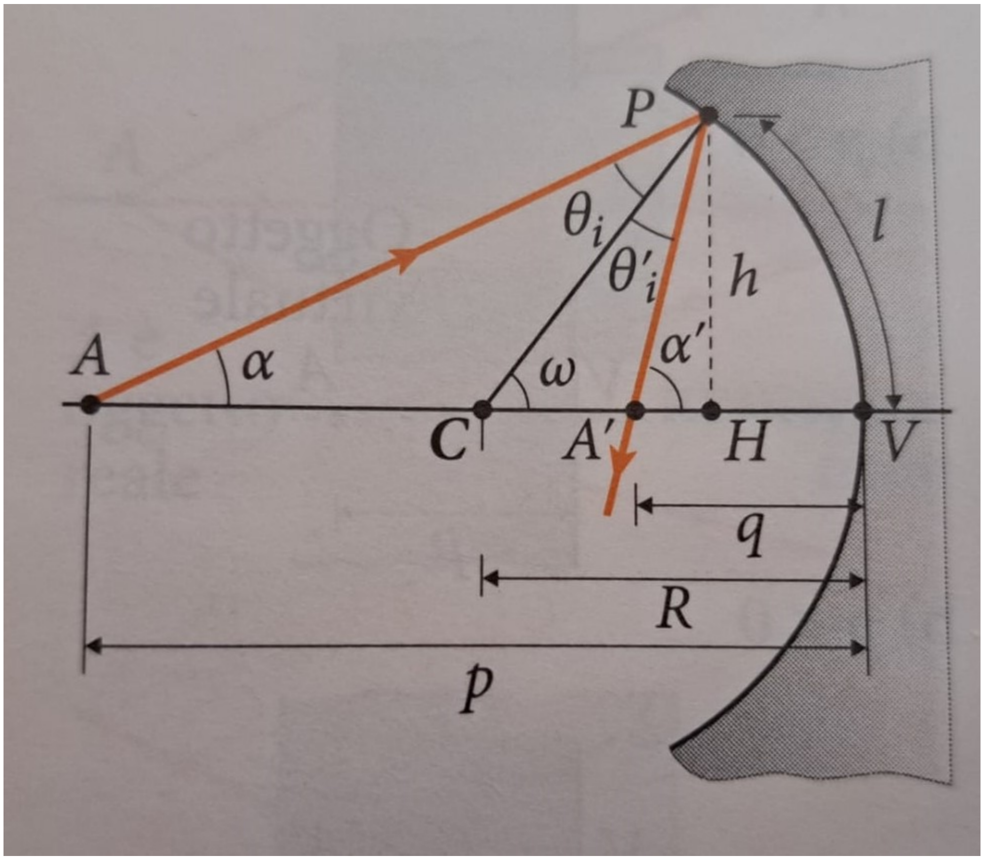

Photo taken from C. Mencuccini V. Silvestrini, Fisica elettromagnetismo ed ottica, edizione 1 p. 560

From reflection we have

$$\theta_i = \theta_i' \tag{1}$$

From triangles angle summations we obtain

$$\alpha + \alpha' = 2\omega \tag{2}$$

Taking the tan of LHS and RHS

$$tan(\alpha + \alpha') = tan(2\omega) \tag{3}$$

We use this trigonometric relation

$$tan(\alpha + \beta) = \frac{tan(\alpha) + tan(\beta)}{1 - tan(\alpha)tan(\beta)} \tag{4}$$

To obtain

$$\frac{tan(\alpha) + tan(\alpha')}{1 - tan(\alpha)tan(\alpha')} = tan(\alpha + \alpha') = tan(2\omega) = \frac{2tan(\omega)}{1 - tan(\omega)^2} \tag{5}$$

We also have this relations

$$tan(\alpha) = \frac{h}{AH} \quad tan(\alpha') = \frac{h}{A'H} \quad tan(\omega) = \frac{h}{CH} \tag{6}$$

We use (6) in (5)

$$\frac{\frac{h}{AH} + \frac{h}{A'H}}{1 - \frac{h}{AH}\frac{h}{A'H}} = \frac{2\frac{h}{CH}}{1 - \frac{h}{CH}\frac{h}{CH}} \tag{7}$$

We also have this relations

$$A'C = R - q = CH - A'H \quad CH^2 = R^2 - h^2 \quad AC = p - R = AH - CH \tag{8}$$

We use (8) in (7)

$$\frac{\frac{1}{p-R+\sqrt{R^2-h^2}} + \frac{1}{q-R+\sqrt{R^2-h^2}}}{1 - \frac{h^2}{(p-R+\sqrt{R^2-h^2})(q-R+\sqrt{R^2-h^2})}} = \frac{2\frac{1}{\sqrt{R^2-h^2}}}{1 - \frac{h^2}{R^2-h^2}} \tag{9}$$

Assuming h ¡¡ R we write the taylor series of h in 0 we have

$$\frac{2}{R} + h^2\frac{3}{R^3} + o(h^3) = \frac{1}{p} + \frac{1}{q} + h^2(\frac{p^2 + 2pR + q^2 + 2qR}{2Rp^2q^2}) + o(h^3) \tag{10}$$