机器学习导论作业 1

2020 年 10 月 3 日

1

(1)

$$F_X(x) = \begin{cases} 0, & x \le 0 \\ \frac{1}{2}x, & 0 < x \le 1 \\ \frac{1}{2}, & 1 < x \le 2 \\ \frac{1}{6}(x+1), & 2 < x \le 5 \\ 1, & x > 5 \end{cases} \tag{1}$$

(2)

$F_Y(y) = P(\frac{1}{X^2} \le y) = P(X > \frac{1}{\sqrt{y}}) + P(X < -\frac{1}{\sqrt{y}})(y > 0)$
$= F_X(-\frac{1}{\sqrt{y}}) + 1 - F_X(\frac{1}{\sqrt{y}}) = 1 - F_X(\frac{1}{\sqrt{y}})$

$$F_Y(y) = \begin{cases} 1 - \frac{1}{6}(\frac{1}{\sqrt{y}} + 1), & \frac{1}{25} \le y < \frac{1}{4} \\ \frac{1}{2}, & \frac{1}{4} \le y < 1 \\ 1 - \frac{1}{2\sqrt{y}}, & y \ge 1 \\ 0, & else \end{cases} \tag{2}$$

$f_Y(y) = F_Y(y)' = f_X(\frac{1}{\sqrt{y}}) * (\frac{1}{2})y^{-\frac{3}{2}}$

$$f_Y(y) = \begin{cases} \frac{1}{4}y^{-\frac{3}{2}}, & y > 1 \\ \frac{1}{12}y^{-\frac{3}{2}}, & \frac{1}{25} \le y \le \frac{1}{4} \\ 0 & else \end{cases} \tag{3}$$

(3)

$E(Z) = \int_{z=0}^{\infty} Pr\{Z \ge z\}dz = \int_{z=0}^{\infty}(1 - F_Z(z))dz$
$E(Z) = \int_{z=0}^{\infty} zf(z)dz = \int_{z=0}^{\infty} zdF_Z(z)$
$= zF_Z(z)|_{z=\infty} - zF_Z(z)|_{z=0} - \int_{z=0}^{\infty} F_Z(z)dz$
$= z|_{z=\infty} - \int_{z=0}^{\infty} F_Z(z)dz = \int_{z=0}^{\infty} dz - \int_{z=0}^{\infty} F_Z(z)dz$
$= \int_{z=0}^{\infty}(1 - F_Z(z))dz$
得证
验证: $E(X) = \int_0^{\infty}(1 - F_X(x))dx = \int_0^1(1 - \frac{1}{2}x)dx + \int_1^2 \frac{1}{2}dx + \int_2^5 \frac{1}{6}(5 - x)dx$
$= \frac{3}{4} + \frac{1}{2} + \frac{5}{2} - \frac{7}{4} = 2$
$E(X) = \int_0^{\infty} xf(x)dx = \int_0^1 \frac{1}{2}xdx + \int_2^5 \frac{1}{6}xdx = \frac{1}{4} + \frac{7}{4} = 2$

$E(Y) = \int_0^{\infty}(1 - F_Y(y))dy = \int_{\frac{1}{25}}^{\frac{1}{4}} \frac{1}{6}(\frac{1}{\sqrt{y}} + 1)dy + \int_{\frac{1}{4}}^{1} \frac{1}{2}dy + \int_1^{\infty} \frac{1}{2}y^{-\frac{1}{2}}dy + \frac{1}{25}$
$= \frac{27}{200} + \frac{3}{8} + \int_1^{\infty} \frac{1}{2}y^{-\frac{1}{2}}dy + \frac{1}{25} = \frac{11}{20} + \int_1^{\infty} \frac{1}{2}y^{-\frac{1}{2}}dy$
$= y^{\frac{1}{2}}|_{y=\infty} - \frac{9}{20}$
$E(Y) = \int_0^{\infty} yf_Y(y)dy = \int_1^{\infty} \frac{1}{4}y^{-\frac{1}{2}}dy + \int_{\frac{1}{25}}^{\frac{1}{4}} \frac{1}{12}y^{-\frac{1}{2}}dy$
$= \frac{1}{2}y^{\frac{1}{2}}|_{y=\infty} - \frac{9}{20}$
E(Y) 不存在, 无法验证

2

(1)

A=rain today,B=rain tomorrow
$P(B|A) = \frac{P(AB)}{P(A)} = \frac{0.25}{0.3} = \frac{5}{6}$

(2)

$P(G|\neg H) = \frac{P(G\neg H)}{P(\neg H)} = \frac{P(\neg H|G)P(G)}{1-P(H)} = \frac{(1-P(H|G))P(G)}{1-P(H)}$
$= \frac{P(G)-P(HG)}{1-P(H)}$

(3)

令 A 代表第一个球是白色,B 代表第一个球是黑色,C 代表第二个球是白色
$P(C) = P(C|A)P(A) + P(C|B)P(B) = \frac{w}{w+b} * \frac{w+d-1}{w+b+d-1} + \frac{b}{w+b} * \frac{w}{w+b+d-1} = \frac{w}{w+b}$
得证

3

(1)

由于 $x_\perp$ 是在 y 上的投影, 可以令 $x_\perp = (a, \sqrt{3}a)^T$
由 $xy = x_\perp y$
$2\sqrt{3} = a + 3a \rightarrow a = \frac{\sqrt{3}}{2}$
$x_\perp = (\frac{\sqrt{3}}{2}, \frac{3}{2})^T$

(2)

$x - x_\perp = (\frac{\sqrt{3}}{2}, -\frac{1}{2})^T$
$y(x - x_\perp) = 0$
因此 $y \perp (x - x_\perp)$

(3)

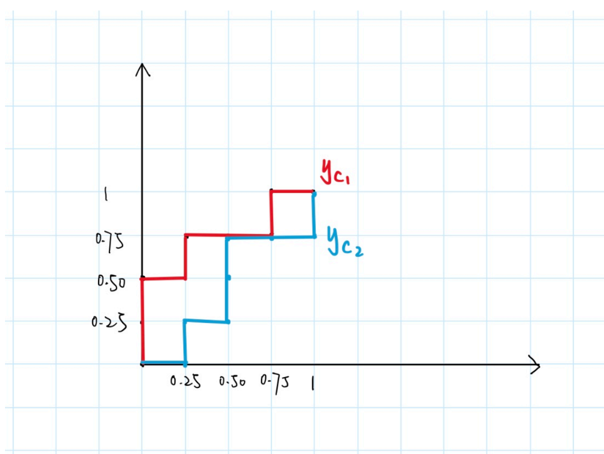
$||x - x_\perp|| = 1$
$||x - \lambda y|| = |(\sqrt{3} - \lambda, 1 - \sqrt{3}\lambda)^T| = \sqrt{3 - 2\sqrt{3}\lambda + \lambda^2 + 1 - 2\sqrt{3}\lambda + 3\lambda^2}$
$= 2\sqrt{\lambda^2 - \sqrt{3}\lambda + 1} = 2\sqrt{(\lambda - \frac{\sqrt{3}}{2})^2 + \frac{1}{4}} \geq 1$
得证

4

原假设 $H_0 : \mu_0 \leq 0.5$
令 $T = \frac{\overline{X}-\mu_0}{S/\sqrt{n}}\ t(n-1)$
拒绝域为 $T = \frac{\overline{X}-\mu_0}{S/\sqrt{n}} > t_\alpha(n-1)$
$S = \sqrt{\frac{1}{49}(0.7^2 * 15 + 0.3^2 * 35)} = 0.463$
而 T=$\frac{0.7-0.5}{0.463/7} = 3.024, t_0.05(49) = 1.676$
T 落在拒绝域内, 原假设被拒绝
因此可以认为 $\mu > 0.5$, 也即硬币不均匀偏向人头的一边

5

(1)



$AUC_{C_1} = 0.75, AUC_{C_2} = 0.4375$

(2)

C_1 :

真实类别/预测类别	1	0
1	3	1
0	1	3

$P = \frac{TP}{TP+FP} = \frac{3}{4}, R = \frac{TP}{TP+FN} = \frac{3}{4}$
$F_1 = \frac{2PR}{P+R} = \frac{3}{4}$

$C_2:$

真实类别/预测类别	1	0
1	3	1
0	3	1

$P=\frac{TP}{TP+FP}=\frac{1}{2}, R=\frac{TP}{TP+FN}=\frac{3}{4}$

$F_1=\frac{2PR}{P+R}=\frac{3}{5}$