**Properties of Alternative Dirichlet-Multinomial Forms**

**For Fitting Composition Data**

**Goal**

* Evaluate the relative performance of three alternative composition likelihood models:
  + Multinomial – the large sample limit of the Saturated Dirichlet-multinomial
  + Linear Dirichlet-multinomial
  + Saturated Dirichlet-multinomial
* Support better selection of composition likelihoods in integrated fishery stock assessment models.
* Use function-level diagnostics to assess estimator quality over the probability simplex for observed composition data from a simulated fishery system.

**Problem**

* Traditional modeling of age or size composition data often fails to account for:
  + **Overdispersion** beyond the multinomial assumption.
  + **Residual correlation** between composition bins.
  + **Intracluster correlation** among composition data samples.
* This can lead to biased estimates, underestimated uncertainty, and poor model fit in stock assessments that use the multinomial distribution to model composition data.

**Solution**

* Implement a **functional analysis framework in C++ and R** to:
  + Sample observed proportions from a simulated fishery system over appropriate subsets of the **probability simplex**.
  + Evaluate each distribution using:
    - Log-likelihood values
    - Stochasticity and variance metrics
    - Sensitivity diagnostics from **gradients and Hessians**
    - Residual correlation structure
    - Effective sample size estimation
    - Bias in recovered proportions
    - Convergence and speed metrics
    - Accuracy and precision of predictions.
* Provide **practical guidance** on the selection of composition model likelihoods based on robustness, informativeness, and computational stability.