Jason Lamphere

Assignment 4

**Part 1 Integer Properties:**

(5 points) Euclids in ZIP

(1 point) Use modular arithmetic to show that n(n+1)(2n+1) is divisible by 6 for any integer n.

let n = 1, 1(1+1)(2(1)+1) = 6%6 = 0

let n = 2, 2(2+1)(2(2)+1)= 30%6 = 0

let n = 3, 3(3+1)(2(3)+1)= 84%6 = 0

**(1 point) Determine 8^-1 (mod 45)**

Can’t mod a decimal .125

**(1 point) Use Caesar Cipher with a shift of +3 to encrypt the message BYE BYE, BIRDIE**

EBH EBH, ELUGLH

**(1 point) Use Caesar Cipher with a shift of -12 to decrypt EAXAZSNMNK.**

SOLONGBABY

**(1 point) Translate the following message (NOW IS THE TIME) into a list of numerals, one character at a time, in the usual way and apply the encryption algorithm 5P^2 + P mod 26.**

0,6,24,4,16,0,4,16,18,22,4, 16, 4,6

**(2 point) Convert the following number to binary, octal, and hexadecimal.**

a) 42 , 101010, 52, 2A

b) 123, 1111011, 173, 7B

**Part 2 Counting:**

**(5 points) This is the method traditionally used in some Russian villages to see which of the girls in the village are to be married next year! You take three blades of grass, folded in two and hold them in your hand so that that six ends are hanging down. A young girl ties the ends together in pairs. If, on release, a large loop is formed, the girl will be married next year.**

**1. What are the possible outcomes for this experiment?**

3\*3 = 9

**2. Find the probability of getting the large loop.**

1/3

**3. If a Russian village has 30 young girls and they all go through this ritual, how many do you estimate will be predicted to marry next year?**

10

**4. What happens if either 4 or 5 blades of grass are used? What is the probability of now obtaining one large loop?**

4=1/4 5 = 1/5

**(2 points) How many eight-character passwords can be formed with the 26 letters in the English alphabet, each of which can be uppercase or lowercase, and the 10 digits? How many of them do not have repeated character?**

P(36, 8), (36,8) = 1 (28,8) = 2, (20, 8) = 3, (12, 8) = 4, distrubute reamaing 4\*4 ways = 16 ways

**(1 point) A binary string is a sequence of digits chosen from 0 to 1. How many binary strings of length 16 contain exactly seven 1s?**

16choose7

**(3 points) Let S = {1,2,3,4,5,6}**

a) How many subsets are there total? 2^6 = 64

b) How many subsets are there of cardinality 4? 48

c) How many subsets of cardinality 4 contain {2, 3, 5} as a subset? 3

**Part 3 Advanced Counting:**

**(1 point) Compute (12 choose 3 ) by hand.**

12!/3!(9!)= (12\*11\*10)/6 = 220

**(1 point) How many subsets of {1, 2, …, 23} have five elements?** 23Choose5

**(1 point) Use the binomial theorem to expand (3x-5y)^4 .**

(3x)^4-(5y)^0+ (3x)^3-(5y)^1+(3x)^2-(5y)^2+(3x)^1-(5y)^3+(3x)^0-(5y^4)

**(1 point) What is the coefficient of x^9 in the expansion of (x+1)^14 + x^3 (x+2)^15**

(x+1)^5+x^9(x+2)^6

(1 point) Come up with a problem that would meet the Pidgeonhole principle. Make it complicated. The best on gets extra credit.

There are infinity holes, with infinity +1 balls distrubuted in said holes evenly such that no hole has more than 1 ball or a fraction of a ball.

ceiling(n+1/n)

**Part 4 Probability:**

(1 point) Rolling three dice. Your score is obtained by adding together the three face values. What is your probability of getting a 10?

Each of the following has 6 permutations.

* 1, 3, 6
* 1, 4, 5
* 2, 3, 5

Each of the following has 3 permutations.

* 2, 2, 6
* 2, 4, 4
* 3, 3, 4

e 6⋅3+3⋅3=276⋅3+3⋅3=27 27/216

**1/8**

**(1 point) A family has two children. Given that one of the children is a boy, and that he was born on a Tuesday, what is the probability that both children are a boys?**

½ the first boy is decided. The probabily that 2 are boys is equal to the probablity the next child is a boy which is 1/2