Kruskal's Algorithm for MST

Part (a): Required Algorithms

To find the Minimum Spanning Tree (MST) using Kruskal's Algorithm, we need the following steps:

1. **Input Preparation**:
    - Collect all edges of the graph along with their weights. Each edge is represented as a tuple(u, v, weight), where u and v are the vertices and `weight` is the edge weight.
2. **Sort Edges by Weight**:
    - Sort all edges in non-decreasing order of their weights. This allows us to add edges with the smallest weights first.
3. **Union-Find Data Structure**:
    - **Find Operation**: Determines which set (or subset) a vertex belongs to. This helps to identify if adding an edge would form a cycle.
    - **Union Operation**: Merges two subsets into one. This is used to add an edge and merge two disjoint sets of vertices.
4. **Build the MST**:
    - Start with an empty MST.
    - Iterate over the sorted edges and add each edge to the MST if it does not form a cycle (checked using the Union-Find structure).
    - Stop when the MST contains V-1 edges, where V is the number of vertices, since an MST for V vertices always contains V-1 edges.

Part (b): Algorithm Analysis

Time Complexity

1. **Sorting Edges**:
    - Sorting the edges takes O(E log E), where E is the number of edges.
2. **Union-Find Operations**:
    - Each **find** and **union** operation takes nearly constant time due to path compression and union by rank, making it effectively O(logV), where V is the number of vertices.
    - Since we perform these operations for each edge, the total time complexity for Union-Find operations is O(ElogV).

Overall Time Complexity:

- Combining the time for sorting and the Union-Find operations, the overall time complexity is $O(E\log E+E\log V)$, which simplifies to $O(E\log E)$, as $E \le V^2$

Space Complexity

- **Graph Representation**:
    - Storing the graph requires $O(E + V)$ space to store edges and vertices.
- **Union-Find Data Structure**:
    - The Union-Find structure requires $O(V)$ space to store the parent and rank arrays for V vertices.

Overall Space Complexity:

- The total space complexity is $O(E+V)$.