Systems of Equations

What are system of equations?

Systems of equations are when you have two or more equations and you need to find a solution that satisfies both/all equations.

There are 3 main ways to solve systems of equations:

Graphing, Substitution, and Elimination

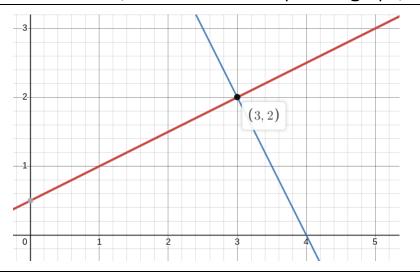
Graphing

To use graphing, graph both equations on a coordinate plane and look for the point where both lines intersect (Make sure to use a precise graph).

$$y = \frac{1}{2}x + \frac{1}{2}$$

$$y = -2x + 8$$

Solution: (3, 2)



Substitution

To use substitution, isolate a variable in one equation and then substitute the other side of the equation into the other equation to find one variable. Then, plug that number into one of the equations to find the other variable.

$$y = 3x - 1$$
$$2y + 14x = 18$$

$$y = 3x - 1$$

$$2y + 14x = 18$$

$$2(3x - 1) + 14x = 18$$

$$2(3x - 1) + 14x = 18$$

$$6x - 2 + 14x = 18$$

$$20x - 2 = 18$$

$$20x = 20$$

$$x = 1$$

$$y = 3x - 1$$

$$y = 3(1) - 1$$

$$y = 3 - 1$$

$$y = 2$$

Solution: (1, 2)

Systems of Equations

Elimination

To use elimination, set up the equations so that there are two of the same variables that have opposite coefficients. Then, cancel out those variables in one equation by adding the equations together and then solve for one variable. Then, plug that number into one of the equations to find the other variable.

$$4x + 7y = 12$$

$$8x + 6y = 16$$

$$-2 \cdot 4x + 7y = 12 \cdot -2$$

$$8x + 6y = 16$$

$$-8x - 14y = -24$$

$$8x + 6y = 16$$

$$-8x - 14y = -24$$

$$8x + 6y = 16$$

$$-8y = -8$$

$$y = 1$$

$$4x + 7y = 12$$

$$4x + 7(1) = 12$$

$$4x + 7 = 12$$

$$4x = 5$$

$$x = \frac{5}{4}$$

Solution: $(\frac{5}{4}, 1)$

Special Cases

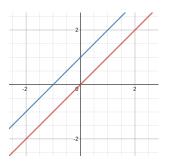
There are two special cases:

No Solution

Graphically, two parallel lines may never intersect

$$y = x$$

$$y = x + 1$$



Algebraically, there will be a false statement

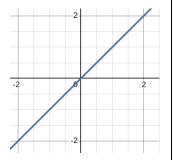
$$x = x + 1$$
$$0 = 1$$

Infinitely Many Solutions

Graphically, the lines will overlap

$$3y = 3x$$

$$y = x$$



Algebraically, there will be a statement that is always true

$$3(x) = 3x$$

$$3 = 3$$