

Equation of a Line

Overview

This worksheet will help you find the equation of a line given an input and output table. Which turns a chart full of "x" and "y" values into linear equation form written as $y=mx+b$. This method is almost identical to finding the equation of a line from a graph.

x	y
0	2
1	4
2	6
3	8
4	10

Input & Output Chart Patterns

For this lesson we will be using the table to the left as an example. Before we begin calculating the difference between points, one specific input and output pair should stand out (0,2). This point is our y-intercept (where the line crosses the y-axis). That means the y-value of our coordinate will be the "b" in $y=mx+b$. Don't worry if your chart does not list the y-intercept, you will still be able to find the entire location of the line.

Finding the Equation of a Line

- Find the rate of change between two points, remember to use the equation $\frac{y_2 - y_1}{x_2 - x_1}$. For example, the points (1,4) and (2,6), remember its best to subtract the lesser value from the greater value. $\frac{6-4}{2-1} = \frac{2}{1}x$. Now we have our slope, $\frac{2}{1}x$ or $2x$. That means for every +2 units on the y-axis, we have +1 unit(s) on the x-axis.
- Next we have to find our y-intercept. Remember if you see any point with a zero in the x-value [in this case (0,2)], the y-value will be our "b" in the equation.
- Combine the slope and our y-intercept into the form of $y = mx + b$
- Now that we have our slope, $y = 2x + 2$, it is important to double check to make sure that your slope is correct. To check your slope just plug in any pair of points from the table. After you have substituted one of your input points (x) into the equation it will equal the output point (y), your output and the y-coordinate of the selected point pair will match.

Slope:
Change in "y"
over
Change in "x"

	x	y	
	0	2	
+1 ↓	1	4	↓ +2
+1 ↓	2	6	↓ +2
+1 ↓	3	8	↓ +2
+1 ↓	4	10	↓ +2

Practice Problems

1. Find the equation of the line, given the following table:

x	-2	-1	0	1	2
y	9	6	3	0	-3

Solution:

Identify 2 points that can be used to solve for the slope, such as (1,0) and (2, -3)

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 0}{2 - 1} = \frac{-3}{1}x$$

$$y = -3x + b$$

Now, we need to solve for b and there are a couple different ways we can approach this

- This can be found by plugging in any point on the line, such as (1,0)
 - $0 = -3(1) + b$
 - $0 = -3 + b$
 - $3 = b$
- Alternatively, you could observe the point in the table that has an x-value of zero, which means that it lies on the y-axis, and the y-value of the point is 3

Either way, the final solution is $y = -3x + 3$

2. Find the equation of the line, given the following table:

x	5	8	11	14	17
y	-6	-2	2	6	10

Solution:

Note: Reference the above problem for more explanation on the steps to solve

Two points for slope: (11,2) and (14,6)

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 2}{14 - 11} = \frac{4}{3}x$$

$$y = \frac{4}{3}x + b$$

Plugging in a point to solve for b: (11,2)

$$2 = \frac{4}{3}(11) + b$$

$$2 = -\frac{44}{3} + b$$

$$\frac{6}{3} + \frac{44}{3} = b$$

$$\frac{50}{3} = b$$

$$\text{Final solution: } y = \frac{4}{3}x + \frac{50}{3}$$