Calculus 3.1 Key Points

Power Rule:

If a function $f(x) = x^n$, then $f'(x) = nx^{n-1}$

Examples:

$f(x) = x^3 \Rightarrow f'(x) = 3x^2$ $\quad$ $f(x) = 2x^6 \Rightarrow f'(x) = 12x^5$

$f(x) = x^1 \Rightarrow f'(x) = x^0 = 1$ $\quad$ $f(x) = x^{-3} \Rightarrow f'(x) = -3x^{-4}$

$f(x) = x^{0.5} \Rightarrow f'(x) = 0.5x^{-0.5}$ $\quad$ $f(x) = 3x^5 \Rightarrow f'(x) = 15x^4$

Warnings:

- The power rule only applies when a variable is being raised to a power (Ex: If $f(x) = \pi^3$, then $f'(x) \neq 3\pi^2$ but instead $f'(x) = 0$; Ex: If $f(x) = 2^x$, then $f'(x) \neq x \cdot 2^{x-1}$ but instead $f'(x) = 2^x \cdot ln(2)$
    - You don't need to know how to take the derivative of exponential functions like 2^x until chapter 6

Average Rate of Change:

Over the interval from $x = a$ to $x = a + h$, the average rate of change can be expressed as:

$$\frac{f(a+h)-f(a)}{(a+h)-a} = \frac{f(a+h)-f(a)}{h}$$

Instantaneous Rate of Change:

To turn average rate of change into instantaneous rate of change or instantaneous slope, take the limit as h approaches 0:

$$\lim_{h \to 0} \frac{f(a+h)-f(a)}{(a+h)-a} = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$$