

# Combining Rational Expressions

## What are Rational Expressions?

Rational expressions are fractions that contain polynomial expressions.

Some examples of rational expressions are:  $\frac{3x+5}{2x-4}$  and  $\frac{6x^2-4x+2}{7x-11}$

There are generally 2 types of problems for combining rational expressions: addition/subtraction problems and multiplication/division problems.

## Addition

Example 1:  $\frac{6x+5}{2x} + \frac{3x-2}{2x}$

In this problem, notice that both expressions have  $2x$  as a denominator (they have a common denominator). So, we can add them just like how we would add

$$\frac{2}{5} + \frac{4}{5} = \frac{2+4}{5} = \frac{6}{5}$$

$$\frac{6x+5}{2x} + \frac{3x-2}{2x} = \frac{(6x+5)+(3x-2)}{2x} = \frac{6x+3x+5-2}{2x} = \frac{9x+3}{2}$$

Example 2:  $\frac{x^2+x-6}{(x+4)(x-2)} + \frac{4x-11}{x+4}$

If the expressions do not have a common denominator, see if you can factor and cancel out some terms to get a common denominator

$$\frac{x^2+x-6}{(x+4)(x-2)} + \frac{4x-11}{x+4} = \frac{(x+3)(x-2)}{(x+4)(x-2)} + \frac{4x-11}{x+4} = \frac{x+3}{x+4} + \frac{4x-11}{x+4} = \frac{5x-8}{x+4}$$

Example 3:  $\frac{2x^2-13x+6}{x^2-5x-6} + \frac{9x^2-15x}{3x^2+3x}$

Sometimes, this requires you to cancel out factors in both expressions

$$\frac{2x^2-13x+6}{x^2-5x-6} + \frac{9x^2-15x}{3x^2+3x} = \frac{(2x-1)(x-6)}{(x+1)(x-6)} + \frac{3x(3x-5)}{3x(x+1)} = \frac{2x-1}{x+1} + \frac{3x-5}{x+1} = \frac{5x-6}{x+1}$$

# Combining Rational Expressions

Example 4:  $\frac{9x+1}{2x} + \frac{x-10}{4x}$

You might find that you cannot cancel out any factors but still do not have a common denominator. In this case, multiply the top and bottom of an expression to create a common denominator. For example, for fractions with integers, we could add them like  $\frac{5}{4} + \frac{7}{2} = \frac{5}{4} + (\frac{7}{2} \cdot \frac{2}{2}) = \frac{5}{4} + \frac{14}{4} = \frac{19}{4}$  Note that the total value of the expression has not changed because we multiplied by  $\frac{2}{2}$ , which is the same thing as multiplying by 1

$$\frac{9x+1}{2x} + \frac{x-10}{4x} = \frac{2(9x+1)}{4x} + \frac{x-10}{4x} = \frac{18x+2+x-10}{4x} = \frac{19x-8}{4x}$$

Example 5:  $\frac{7x+3}{3x-2} + \frac{9x-1}{2x+1}$

Sometimes, this requires you to multiply to change the denominators of both expressions. Hint: One method that will always create a common denominator is to multiply each expression by the denominator of the other expression.

$$\begin{aligned} \frac{7x+3}{3x-2} + \frac{9x-1}{2x+1} &= \left(\frac{2x+1}{2x+1} \cdot \frac{7x+3}{3x-2}\right) + \left(\frac{9x-1}{2x+1} \cdot \frac{3x-2}{3x-2}\right) = \frac{(7x+3)(2x+1)}{(2x+1)(3x-2)} + \frac{(9x-1)(3x-2)}{(2x+1)(3x-2)} = \\ &= \frac{(7x+3)(2x+1)+(9x-1)(3x-2)}{(2x+1)(3x-2)} = \frac{(14x^2+13x+3)+(27x^2-21x+2)}{(2x+1)(3x-2)} = \frac{41x^2-8x+5}{(2x+1)(3x-2)} \end{aligned}$$

Example 6:  $\frac{3+x}{5x} - \frac{7}{5x}$

For subtraction problems, you can combine expressions with common denominators in the same way as you do with addition problems (and use the same techniques to get the expressions to have a common denominator if they do not already), just subtract the numerator instead of adding.

$$\frac{3+x}{5x} - \frac{7}{5x} = \frac{3+x-7}{5x} = \frac{x-4}{5x}$$

Example 7:  $\frac{2x+5}{3x} - \frac{4x-6}{3x}$

Make sure to distribute the negative if there are multiple terms in numerator

$$\frac{2x+5}{3x} - \frac{4x-6}{3x} = \frac{(2x+5)-(4x-6)}{3x} = \frac{2x+5-4x+6}{3x} = \frac{-2x+11}{3x}$$

# Combining Rational Expressions

## Multiplication

Example 1:  $\frac{x+5}{2x} \cdot \frac{3x-1}{4}$

For multiplication problems, multiply the numerators together, and then multiply the denominators together

$$\frac{x+5}{2x} \cdot \frac{3x-1}{4} = \frac{(x+5)(3x-1)}{(2x)(4)} = \frac{3x^2+14x-5}{8x}$$

Note: You may be able to leave your answer in a form such as  $\frac{(x+5)(3x-1)}{8x}$  depending on the problem given

Example 2:  $\frac{4x+1}{6x} \cdot \frac{x^2+6x+8}{x+4}$

You may need to factor and cancel out some terms before (or after) multiplying

$$\frac{4x+1}{6x} \cdot \frac{x^2+6x+8}{x+4} = \frac{4x+1}{6x} \cdot \frac{(x+4)(x+2)}{x+4} = \frac{4x+1}{6x} \cdot \frac{x+2}{1} = \frac{(4x+1)(x+2)}{6x}$$

Example 3:  $\frac{5x+1}{9x} \div \frac{x-4}{7x}$

For division problems, combine the two expressions by multiplying by the reciprocal of the second expression

$$\frac{5x+1}{9x} \div \frac{x-4}{7x} = \frac{5x+1}{9x} \cdot \frac{7x}{x-4} = \frac{(5x+1)(7x)}{(9x)(x-4)} = \frac{x(5x+1)(7)}{x(9)(x-4)} = \frac{(5x+1)(7)}{(9)(x-4)} = \frac{35x+7}{9x-36}$$

Note: You may also see division problems written like  $\frac{\frac{5x+1}{9x}}{\frac{x-4}{7x}}$ , which is the same

thing as  $\frac{5x+1}{9x} \div \frac{x-4}{7x}$