Precalculus 1.2 Key Points

Composite Functions:

Composite functions are functions that are created by inputting one function into another function

For example, suppose $f(x) = 4x^2 + 1$ and $g(x) = x - 3$

A composite function can be created by inserting one of the functions into the other by replacing x with that function

$$f(x) = 4x^2 + 1 \qquad\qquad g(x) = x - 3$$

$$f(g(x)) = 4(x - 3)^2 + 1 \qquad\qquad g(f(x)) = (4x^2 + 1) - 3$$

To evaluate $f(g(1))$, simply plug 1 into $g(x)$ and plug the result into $f(x)$
Alternatively, plug 1 into the composite function $f(g(x))$

$g(1) = (1) - 3 = -2$ $f(g(1)) = f(-2) = 4(-2)^2 + 1 = 16 + 1 = 17$	$f(g(x)) = 4(x - 3)^2 + 1$ $f(g(1)) = 4((1) - 3)^2 + 1 = 17$

Note: An alternate notation for $f(g(x))$ is $(f \circ g)(x)$

Inverse Functions:

An inverse function, denoted by f^{-1}, is where the inputs and outputs are swapped

To find the inverse of a function, swap x and y and solve for y

$f(x) = 3x - 4 \quad \Rightarrow \quad y = 3x - 4 \quad \Rightarrow \quad x = 3y - 4$

$x + 4 = 3y$

$\frac{x+4}{3} = y \quad \Rightarrow \quad f^{-1}(x) = \frac{x+4}{3}$

Precalculus 1.2 Key Points

To verify that two functions are inverses of each other, use function composition and plug each function into the other. If both functions simplify to x, they are inverses

Example of inverses:

$$f(x) = 3x - 4$$

$$g(x) = \frac{x+4}{3}$$

$$f(g(x)) = 3(\frac{x+4}{3}) - 4 =$$
$$x + 4 - 4 =$$
$$x$$

$$g(f(x)) = \frac{(3x-4)+4}{3} =$$
$$\frac{3x}{3} =$$
$$x$$

Example of not inverses:

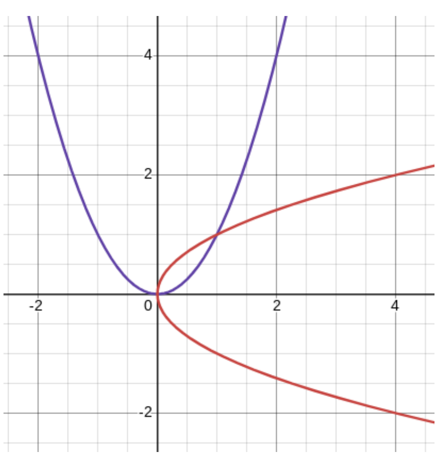
$$f(x) = 4x^3 - 2$$

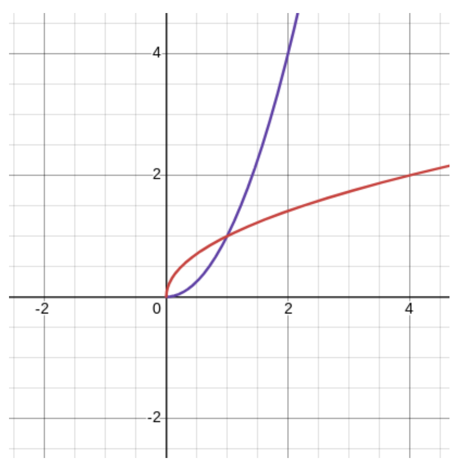
$$g(x) = \sqrt[3]{\frac{x+2}{2}}$$

$$f(g(x)) = 4(\sqrt[3]{\frac{x+2}{2}})^3 - 2 =$$
$$4(\frac{x+2}{2}) - 2 = 2x + 4 - 2 =$$
$$2x + 2$$

$$g(f(x)) = \sqrt[3]{\frac{(4x^3-2)+2}{2}} =$$
$$\sqrt[3]{\frac{4x^3}{2}} = \sqrt[3]{2x^3} =$$
$$x\sqrt[3]{2}$$

Not all inverses are functions, however. Take $f(x) = x^2$
The inverse equation is $y = \pm\sqrt{x}$, which is not a function since there are multiple outputs for inputs greater than 0. But if we restrict $f(x) = x^2$ to the domain $x \geq 0$, then our inverse equation becomes a function



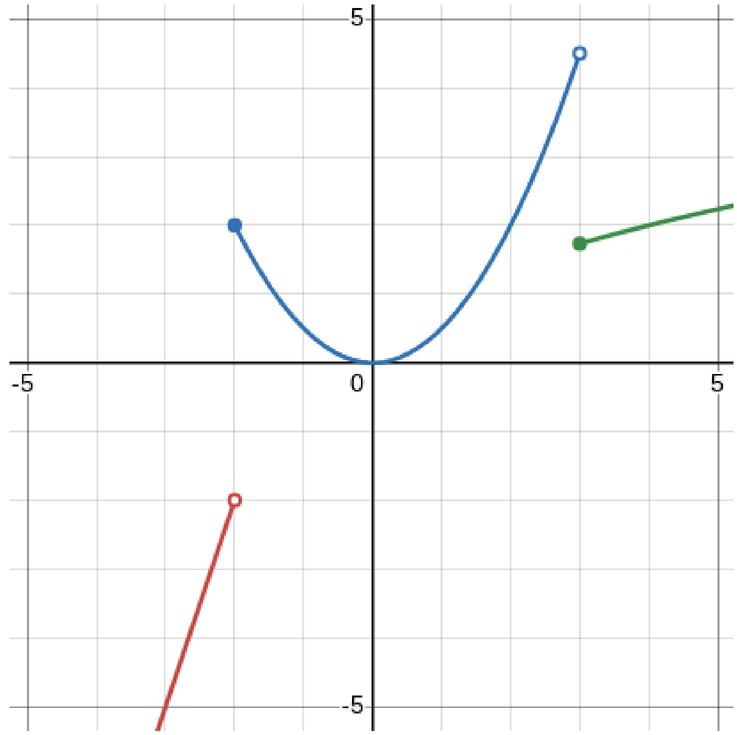


Precalculus 1.2 Key Points

Piecewise Functions:

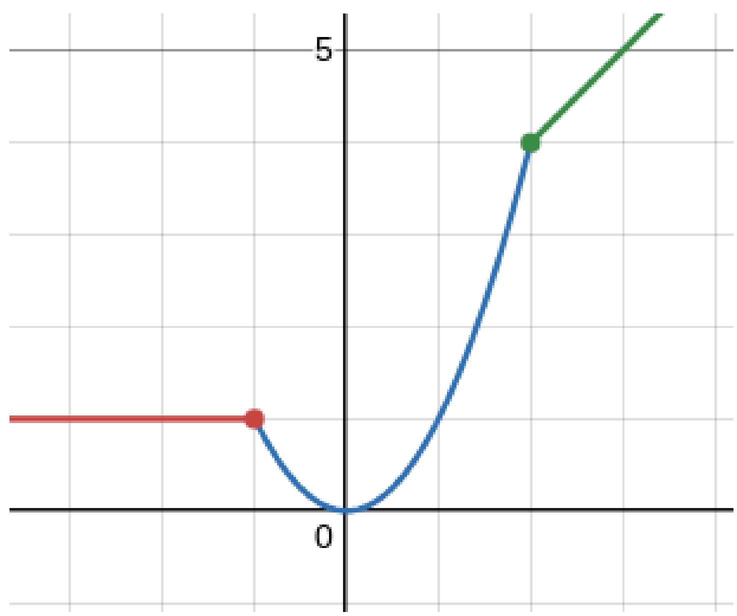
Piecewise functions are composed of multiple functions that typically have different, non-overlapping domains

$$f(x) = \begin{cases} 3x + 4, & x < -2 \\ 0.5x^2, & -2 \le x < 3 \\ \sqrt{x}, & x \ge 3 \end{cases}$$



Closed circles (•) indicate that the point is part of the function, while an open circle (◦) shows that the point is not part of the function

$$f(x) = \begin{cases} 1, & x \le -1 \\ x^2, & -1 < x < 2 \\ x + 2, & x \ge 2 \end{cases}$$



Because the segments of this piecewise function are connected without any gaps in between, this function is continuous. If you were to draw the graph of this function, you wouldn't have to lift your pencil while drawing it.

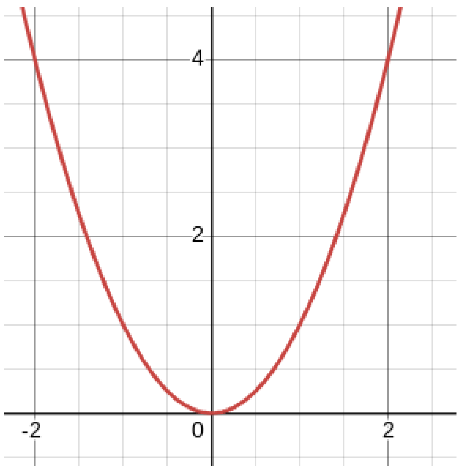
Precalculus 1.2 Key Points

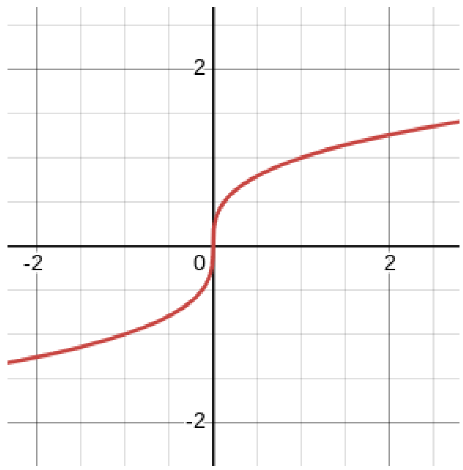
Continuity Intuition:

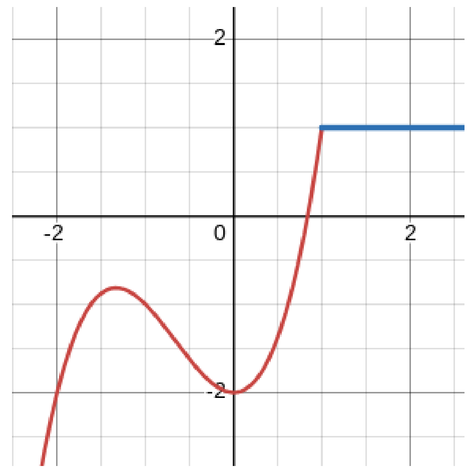
Continuity is the idea that all inputs over an interval have a defined output, without any holes, jumps, gaps, asymptotes, or other discontinuities

Examples:

Continuous Functions







Discontinuous Functions