Calculus 4.5 Key Points

Newton's Method:

Newton's Method is a method for approximating the x-intercepts of a function

- First, guess a point that is near the root of the function
- Then, find the line tangent to the function at that point. Solve for where that tangent line intersects the x-axis. This is your new approximation for the x-intercept
- Continue repeating this process to until you get an accurate enough answer

To make these calculations easier, you can use the following formula to find the next approximation for a root:

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Example: $\frac{1}{4}x^3 - 2x - 1$

Let's guess $x = 3 \quad \Rightarrow \quad \frac{1}{4}(3)^3 - 3 - 1 = \frac{11}{4} = 2.75$

In this case, $x_1 = \frac{11}{4}, f(x_1) = \frac{11}{4}, f'(x_1) = \frac{23}{4}$

So, $x_2 = 3 - \frac{\frac{11}{4}}{\frac{23}{4}} = \frac{58}{23} \approx 2.522$

Let's see how close x_2 is to the root

$x = \frac{58}{23} \quad \Rightarrow \quad (\frac{58}{23})^3 - \frac{58}{23} - 1 = \frac{5929}{12167} \approx 0.487$

0.487 is much closer to 0 than 2.75, but we can do better.

If we repeat this process again, we get closer approximations:

$x_3 \approx 2.392$ and $f(x_3) \approx 0.0311, x_4 \approx 2.383$ and $f(x_4) \approx 0.000160$

Note: Newton's Method does not work well for all functions