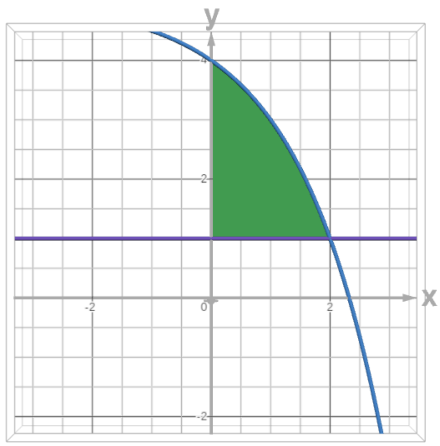
Calculus 8.3 Key Points

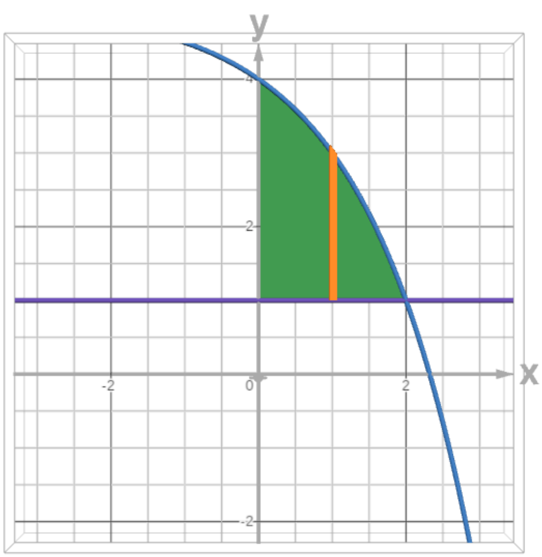
Cross Sections:
Cross sections help us find the volume of a three dimensional figure that is formed when we take some area between curves and use it as a “base” for creating a 3 dimensional figure.

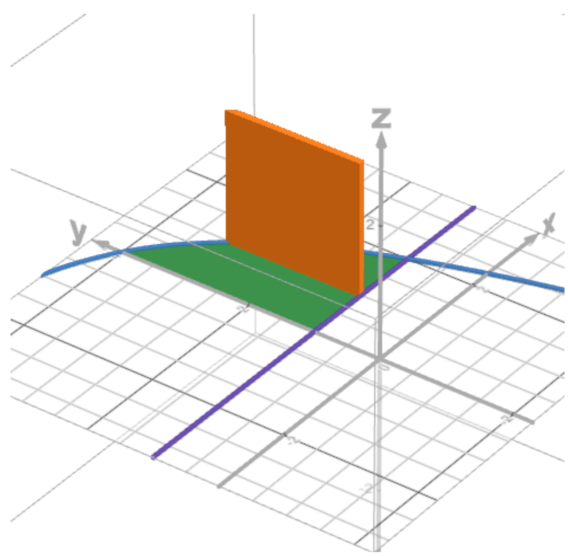
Take the following area bounded by $y = 1$, $y = -2^x + 5$, and the y-axis:



Suppose we want to find the volume of the figure using the bounded region with square cross-sections perpendicular to the x-axis

We’ve identified our region, and now we need to create <u>square cross-sections perpendicular to the x-axis</u>. One side of the square will be the distance between the top and bottom functions (and at a $90°$ angle to the x-axis), while the other will be the same length, extended “up” in 3d





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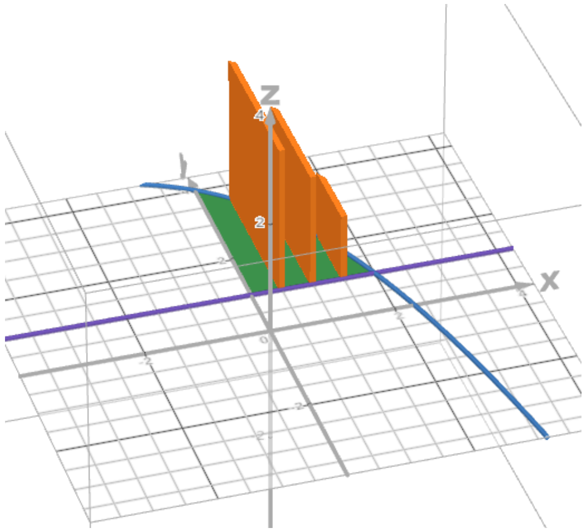
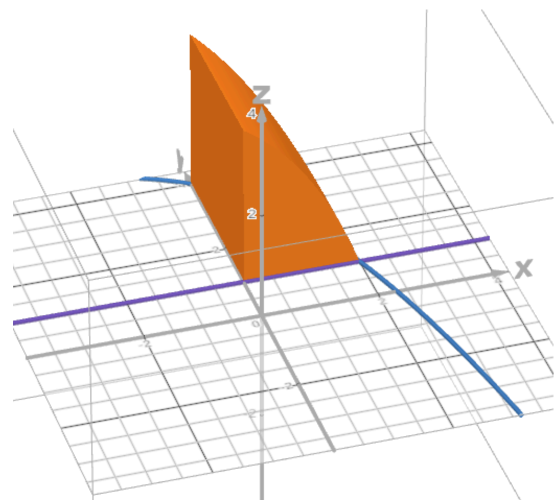
We want to fill the entire bounded area with these cross sections, so we can sum up more and more squares until the entire bounded area is covered.





Let's write an integral to find the volume of the entire solid.

Our bounds will be 0 to 2 because that is where our functions intersect

The area of each square cross section is s^2, where $s = (-2^x + 5) - (1) = -2^x + 4$

If we use each square as a rectangular prism with a depth of dx, we can find the volume of the solid by summing up all of these prisms

$$V = \int_0^2 ((-2^x + 4)^2)\, dx \approx 8.120\, un^3$$

Cross-sections can also be shapes other than squares, such as rectangles, triangles, and semicircles. They volume of solids using these shapes can be found with a similar process, just by modifying the integrand to reflect the area of the shape being used for the cross section

If using cross sections perpendicular to the y-axis, make sure the functions used in your integrand are in terms of y