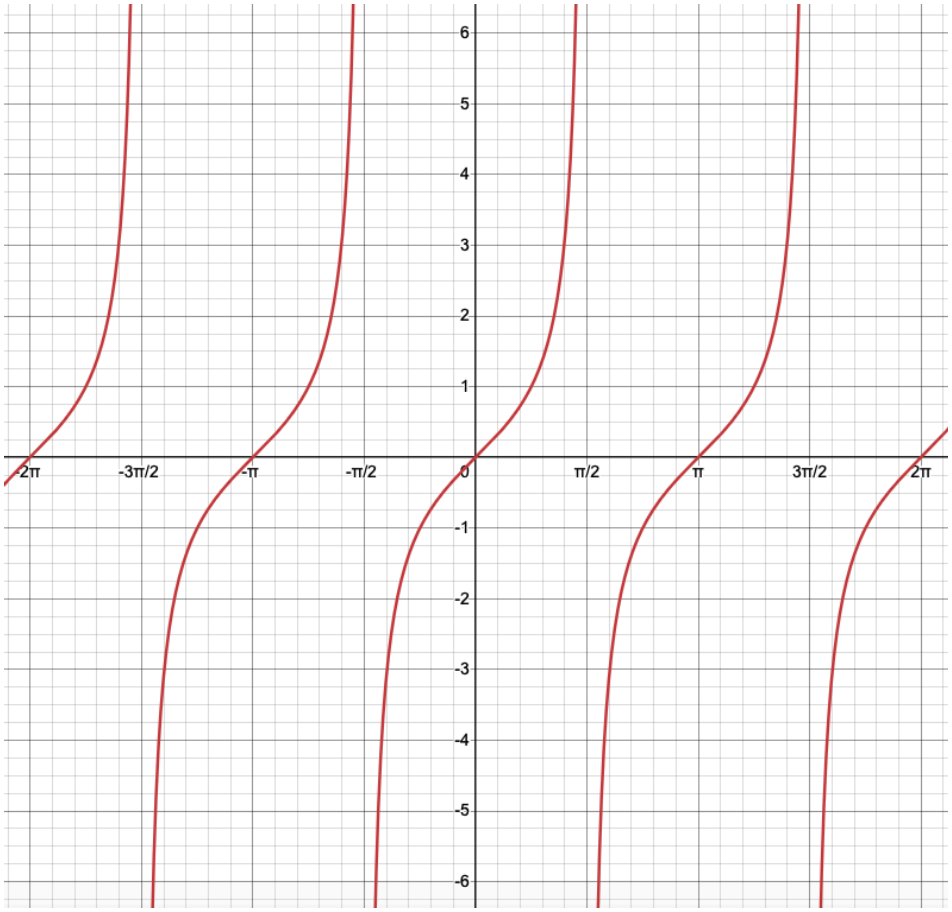
Precalculus 2.3 Key Points

Graphing Tangent:

Like $sin(x)$ and $cos(x)$, the graph of $tan(x)$ can be found created from the values of the unit circle. Notice that $tan(x)$ has vertical asymptotes



We will still graph $tangent$ using 5 key points, like with $sine$ and $cosine$, but there will be a few differences

$y = 2\, tan(\frac{1}{3}(x - \pi)) + 3$

Let's identify our a, b, c, and d

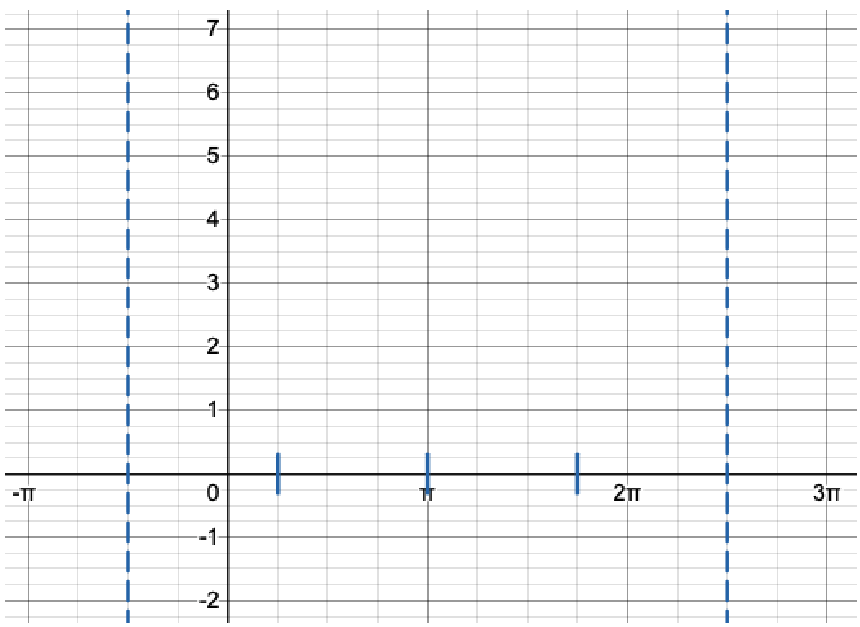
$a = 2, b = \frac{1}{3}, c = \pi, d = 3$

Now, we will find the x-values of our 5 key "points". The third x-value will be our c-value, which is π. The period of a $tangent$ function is $\frac{\pi}{b}$, which will be $\frac{\pi}{\frac{1}{3}} = 3\pi$

Since we are starting from the third point, the first and fifth points will each be the $\frac{period}{2}$ away from the third, which in our case is $\frac{3\pi}{2}$ away. So, the x-value of the first point is $\pi - \frac{3\pi}{2} =- \frac{\pi}{2}$ and the x-value of the fifth point is $\pi + \frac{3\pi}{2} = \frac{5\pi}{2}$

The second and fourth points will be in the middle of the first and third, and the third and fifth, respectively, or $\frac{period}{4}$ away from the third. So, the second point's x-value is $\pi - \frac{3\pi}{4} = \frac{\pi}{4}$ and the fourth point's x-value is $\pi + \frac{3\pi}{4} = \frac{7\pi}{4}$

Look back at the graph of the *tangent* function above. Instead of being points on the graph, the x-values of our first and fifth key points are our asymptotes.



The y-value for our third point is the d-value in our equation, which is 3. The y-value for our second and fourth points will be a units away from the y-value of our third point, so our second point will have a y-value of $3 - 2 = 1$ and our third point $3 + 2 = 5$

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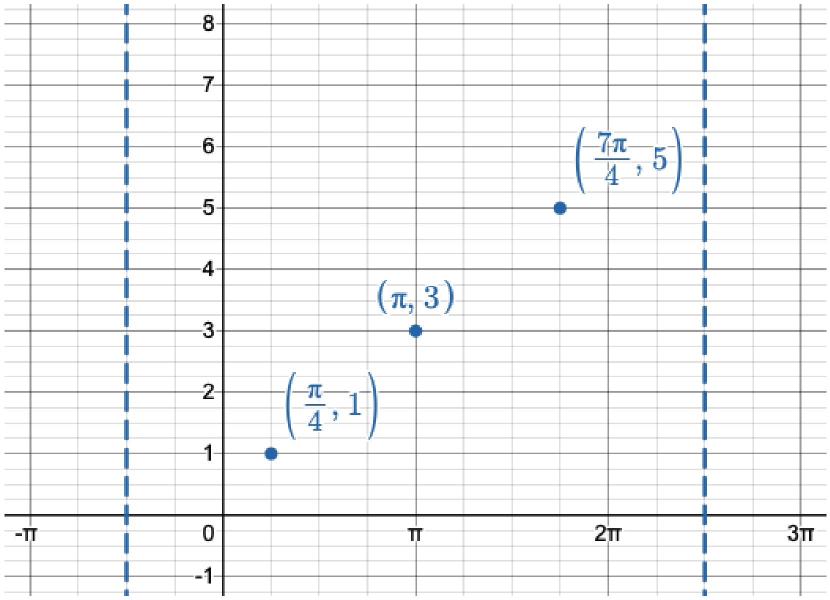
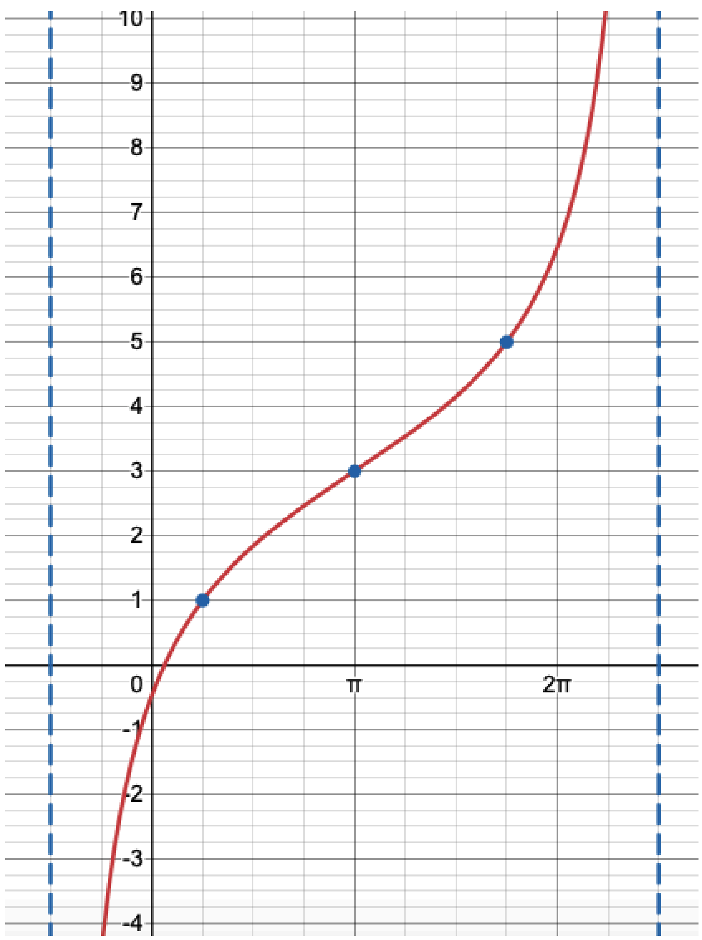


Now, we can draw our final graph of $y = 2\,tan(\frac{1}{3}(x - \pi)) + 3$ using the general shape of $tangent$



Precalculus 2.3 Key Points

Graphing Inverse Trigonometric Equations:

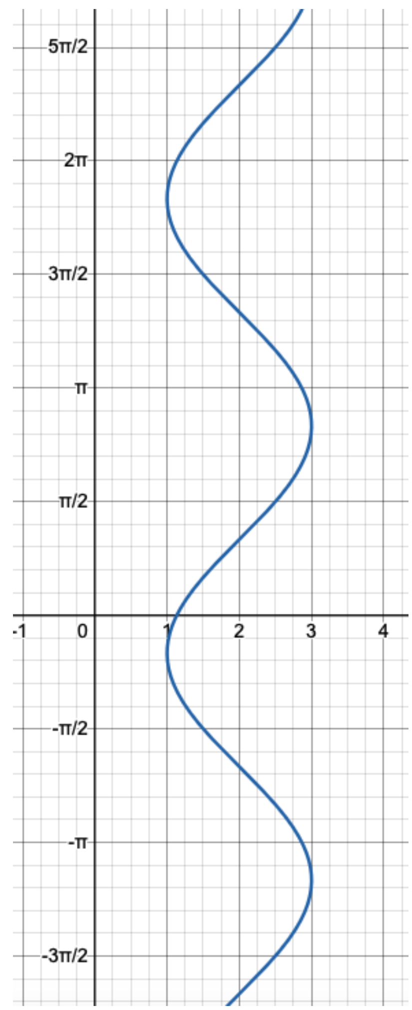
To graph inverse trigonometric functions, start by graphing the regular trigonometric function, then swap the inputs and outputs.

Take $y = sin(x - \frac{\pi}{3}) + 2$, which has the following graph:

For our inverse graph, we will swap the x and y values:

If you are using key point to graph, then swap the x and y values once you have found the key points

To make the inverse graph a function, you can restrict the domain or range of the equation



Precalculus 2.3 Key Points

Solving Trigonometric Equations:

To solve trigonometric equations, isolate the trig function and then use the unit circle or special right triangle values to solve

Let's do some examples:

1. $4sin(x) + 11 = 13$, where $\frac{\pi}{2} > x > 0$

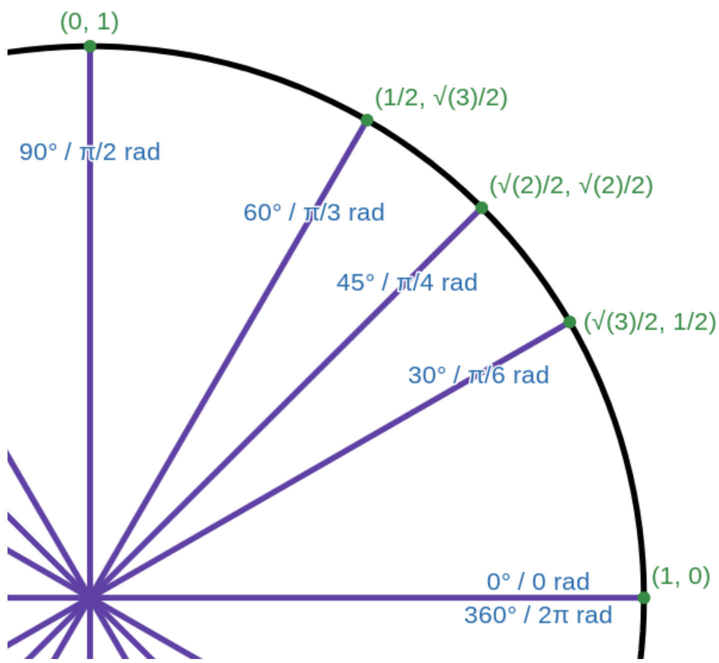
First, we will isolate $sin(x)$

$4sin(x) = 2$

$sin(x) = \frac{1}{2}$

Now, we want to look at the unit circle. Where does the *sine* of an angle equal $\frac{1}{2}$?

Hopefully you identified the angle as $\frac{\pi}{6}$ because the y-value of the angle $\frac{\pi}{6}$ is $\frac{1}{2}$



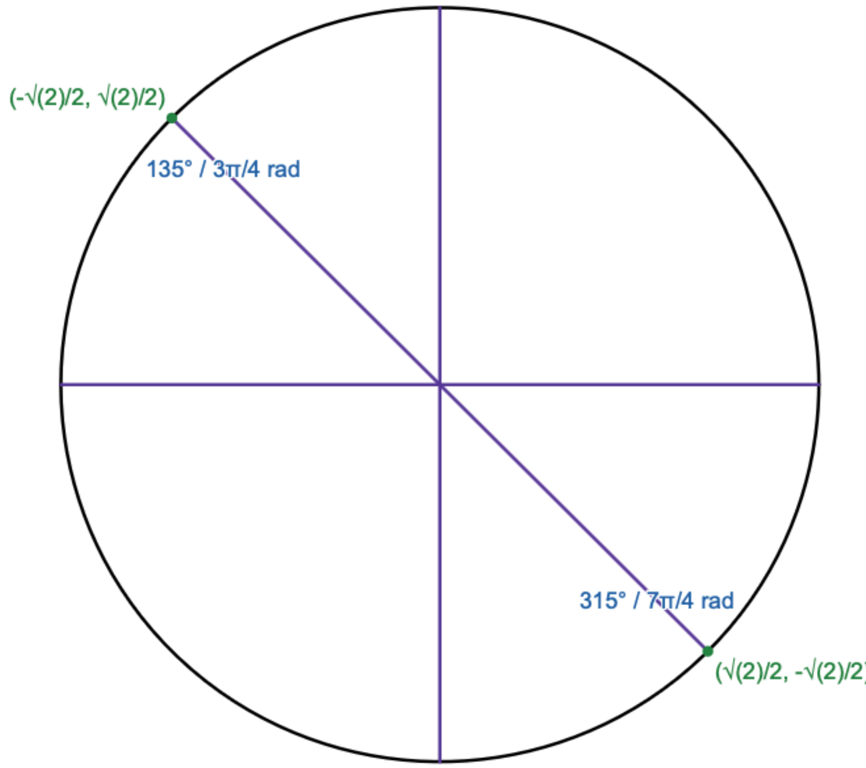
So, the solution to the equation is $x = \frac{\pi}{6}$

It's important to pay attention to the domain restriction because there would be infinite solutions without it, in which case you would have to write a general solution, such as $x = \frac{\pi}{6} + 2\pi n$, where n is any integer

Precalculus 2.3 Key Points

2. $tan(x) =- 1$, where $0 > x > 2\pi$

In this problem, we are looking for the angle where the slope of the terminal side is equal to -1



Within the angles of 0 and 2π radians, there are two angles that produce a terminal side with a slope of -1, at $\frac{3\pi}{4}$ and $\frac{7\pi}{4}$ radians.

Thus, this problem has two solutions: $x = \frac{3\pi}{4}$ or $x = \frac{7\pi}{4}$

3. $cos^{-1}(x) = \frac{\pi}{2}$

In this problem, we are looking for where the inverse *cosine* of x equals 0. In this case, rather than looking for an angle, we are looking for an x-value

Let's take the *cosine* of both sides. On the left hand side, the *cosine* and inverse *cosine* cancel and we are left with x.

$x = cos(\frac{\pi}{2})$

Using the unit circle or special right triangles, we can determine that $cos(\frac{\pi}{2}) = 0$, so the solution to our equation is $x = 0$