

Derivative Tools

Limit Definition of Derivative

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h)-f(x)}{h} \text{ or } \lim_{x \rightarrow a} \frac{f(x)-f(a)}{x-a}$$

Power Rule

$$f(x) = a \cdot x^n$$

$$f'(x) = a \cdot n \cdot x^{n-1}$$

Sum Rule

$$h(x) = f(x) + g(x)$$

$$h'(x) = f'(x) + g'(x)$$

Product Rule

$$h(x) = f(x) \cdot g(x)$$

$$h'(x) = (f'(x) \cdot g(x)) + (f(x) \cdot g'(x))$$

Quotient Rule

$$h(x) = \frac{f(x)}{g(x)}$$

$$h'(x) = \frac{(f'(x) \cdot g(x)) - (f(x) \cdot g'(x))}{(g(x))^2}$$

Chain Rule

$$h(x) = f(g(x))$$

$$h'(x) = f'(g(x)) \cdot g'(x)$$

Exponential

$$f(x) = a^{g(x)}$$

$$f'(x) = a^{g(x)} \cdot \ln(a) \cdot g'(x)$$

Logarithm

$$f(x) = \log_a(x)$$

$$f'(x) = \frac{1}{x \cdot \ln(a)} \text{ or } \frac{\log_a(e)}{x}$$

Derivative Tools

Trigonometric

$f(x) = \sin(x), f'(x) = \cos(x)$	$f(x) = \cos(x), f'(x) = -\sin(x)$
$f(x) = \tan(x), f'(x) = \sec^2(x)$	$f(x) = \cot(x), f'(x) = -\cot^2(x)$
$f(x) = \sec(x),$ $f'(x) = \sec(x) \cdot \tan(x)$	$f(x) = \csc(x),$ $f'(x) = -\csc(x) \cdot \cot(x)$

Inverse Trigonometric

$f(x) = \sin^{-1}(g(x)),$ $f'(x) = \frac{g'(x)}{\sqrt{1-g^2(x)}}$	$f(x) = \cos^{-1}(g(x)),$ $f'(x) = -\frac{g'(x)}{\sqrt{1-g^2(x)}}$
$f(x) = \tan^{-1}(g(x)),$ $f'(x) = \frac{g'(x)}{1+g^2(x)}$	$f(x) = \cot^{-1}(x), f'(x) = -\frac{g'(x)}{1+g^2(x)}$
$f(x) = \sec^{-1}(g(x)),$ $f'(x) = \frac{g'(x)}{ g(x) \cdot \sqrt{g^2(x)-1}}$	$f(x) = \csc^{-1}(g(x)),$ $f'(x) = -\frac{g'(x)}{ g(x) \cdot \sqrt{g^2(x)-1}}$

Implicit Differentiation (Chain Rule)
 Example: If $x + f(x) = 10$, then $1 + f'(x) = 0$