

# Calculus 6.5 Key Points

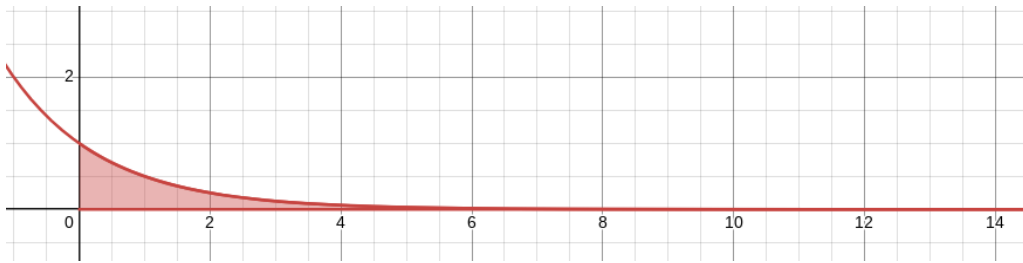
## Improper Integrals:

An improper integral is a definite integral with some form of unbounded area

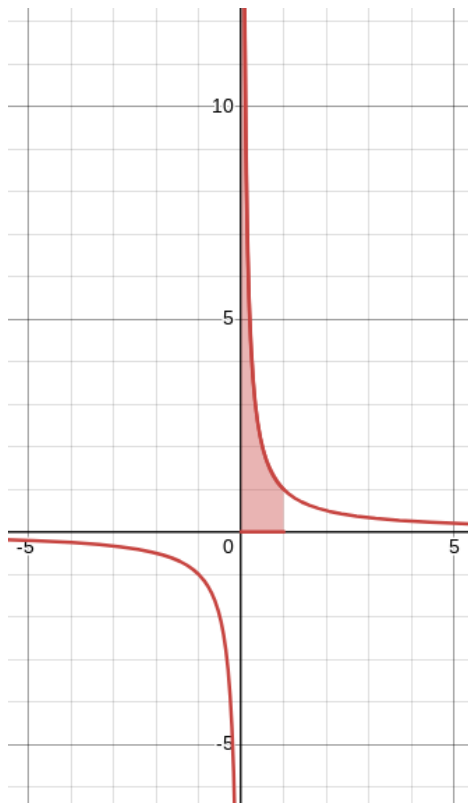
There are a couple of ways we can classify improper integrals:

- Whether the bounds or the integrand reaches infinity

- Infinite interval example:  $\int_0^{\infty} \left(\frac{1}{2}\right)^x dx$



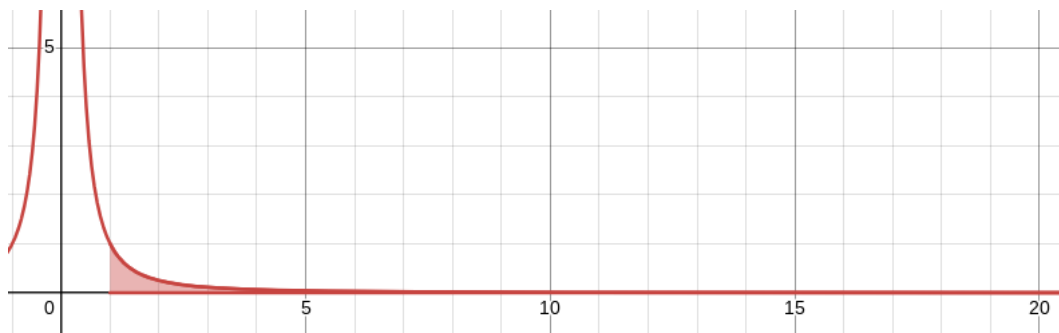
- Infinite integrand example:  $\int_0^1 \left(\frac{1}{x}\right) dx$



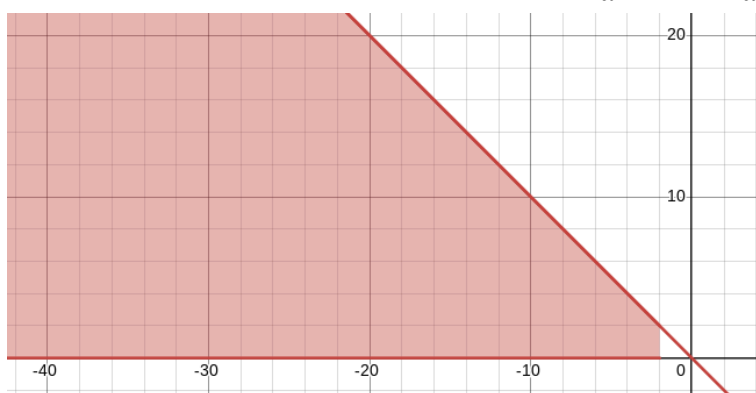
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- Whether the integral converges or diverges (when we take the limit of the improper integral to evaluate it, is the result equal to a finite value [the integral converges] or is the result infinity or DNE [the integral diverges]?)

○ Convergence example:  $\int_1^{\infty} \left(\frac{1}{x^2}\right) dx = \lim_{b \rightarrow \infty} \int_1^b \left(\frac{1}{x^2}\right) dx = 1$



○ Divergence example:  $\int_{-\infty}^{-2} -x dx = \lim_{a \rightarrow -\infty} \int_a^{-2} x dx = \infty$



Evaluating an improper integral example:

$$\int_1^{\infty} \left(\frac{1}{x^2}\right) dx = \lim_{b \rightarrow \infty} \int_1^b \left(\frac{1}{x^2}\right) dx = \lim_{b \rightarrow \infty} -\frac{1}{x} \Big|_1^b = \lim_{b \rightarrow \infty} \left(-\frac{1}{b}\right) - \left(-\frac{1}{1}\right) = (0) - (-1) = 1$$