Continuity

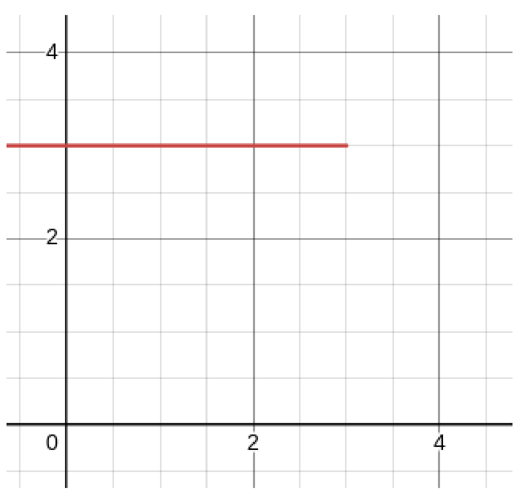
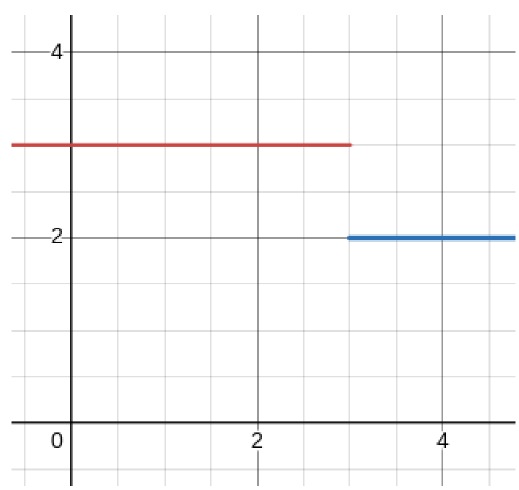
Here are the three conditions for continuity at a point

- $\lim_{x \to a} f(x)$ exists
- $f(a)$ exists
- $\lim_{x \to a} f(x) = f(a)$

Let's look deeper into each condition and some cases of discontinuous functions

★ $\lim_{x \to a} f(x)$ exists

For the limit of $f(x)$ as x approaches a to exist, the left and right side limits must also exist. They must also be equal to each other for the overall limit to exist.

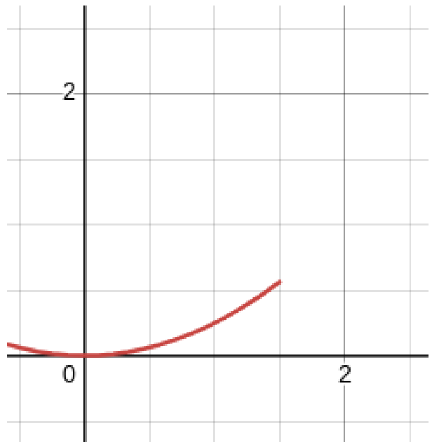
Example: If we wanted to observe limit as x approaches 3, we would see that the limit is undefined, as it has no right side limit	Example 2: For $\lim_{x \to 3} f(x)$ to exist, the left and right limits must be equal, which is not true
	

Continuity

★ $f(a)$ exists

The function must have a defined output at $x = a$

Example: This function is not defined at $x = 2$, so it is not continuous at $x = 2$



★ $\lim_{x \to a} f(x) = f(a)$

Assuming that the first two conditions have already been met, the third conditions says that the limit of $f(x)$ as x approaches a must be equal to the output of $f(x)$ at $x = a$

Example: At $x = 3$, This function passes the first two conditions ($\lim_{x \to 3} f(x) = 2.5$ and $f(3) = 4$), but because $\lim_{x \to a} f(x) \neq f(a)$, this function is not continuous