Calculus 7.4 Key Points

Euler's Method:

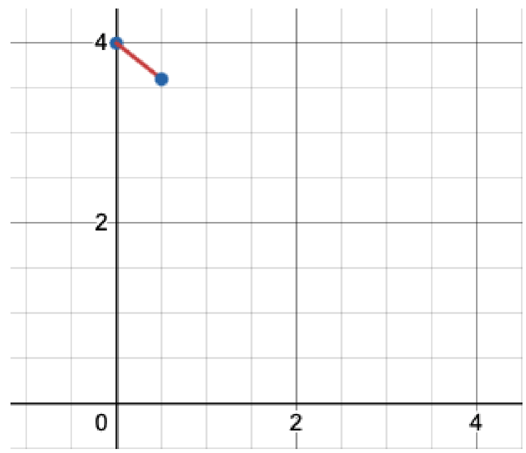
If you are given a differential equation and an initial condition, you may be able to solve it by integrating. However, many differential equations are inseparable, such as $\frac{dy}{dx} = 0.1x - 0.2y$. One process for approximating solutions is Euler's Method using a chain of tangent lines.

Suppose we have the differential equation $\frac{dy}{dx} = 0.1x - 0.2y$ and the initial condition $y(0) = 4$. We want to find the approximate of the function at $x = 2$ using a step size of 0.5.

At $(0, 4)$, the slope is $0.1(0) - 0.2(4) = -0.8$
We can approximate the value of the function at $x = 0.5$ using a tangent line with a slope of -0.8 and the initial point of $(0, 4)$.

$x_0 = 0,\ y_0 = 4$

$x_1 = 0.5,\ y_1 = 4 + 0.5(-0.8) = 3.6$

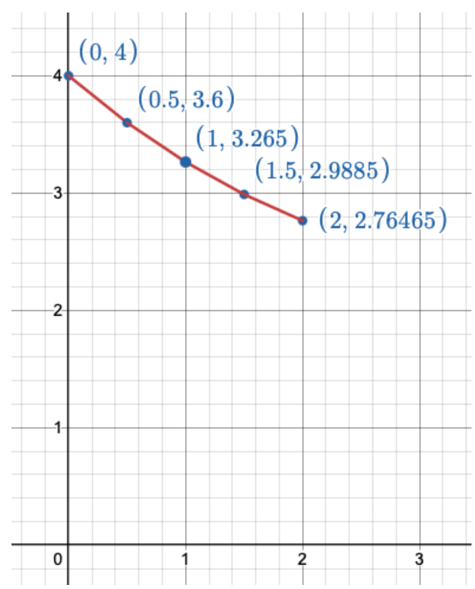


Take the new point, $(0.5, 3.6)$, and continue using tangent line approximations until you reach the desired point:

$x_2 = 1,\ y_2 =$
$3.6 + 0.5(0.1(0.5) - 0.2(3.6)) = 3.265$

$x_3 = 1.5,\ y_3 =$
$3.265 + 0.5(0.1(1) - 0.2(3.265)) = 2.9885$

$x_4 = 2,\ y_4 =$
$2.9885 + 0.5(0.1(1.5) - 0.2(2.9885)) = 2.76465$



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Integration by Parts:
When the integrand is a product, you can use integration by parts.

The formula for integration by parts is:

$\int f'(x)g(x)dx = f(x)g(x) - \int f(x)g'(x)\, dx$

It can also be written as $\int udv = uv - \int vdu$ or $\int u\, v'dx = uv - \int v\, u'\, dx$

To integrate $\int x\, ln(x)\, dx$, let $f'(x) = x$ and $g(x) = ln(x)$. Then, use the formula to solve:

$\int x\, ln(x)\, dx = \frac{x^2}{2} \bullet ln(x) - \int \frac{x^2}{2} \bullet \frac{1}{x}\, dx = \frac{x^2 \bullet ln(x)}{2} - \int \frac{x}{2}\, dx =$
$\frac{x^2 \bullet ln(x)}{2} - \frac{x^2}{4} + C$

You may need to repeat the process multiple times to solve

Integration by Partial Fractions:
When the integrand is a fraction with a factorable denominator, you can use integration by partial fractions.

If you have the integrand $\frac{px+q}{(x-a)(x-b)}$, it can be rewritten as $\frac{A}{x-a} + \frac{B}{x-b}$

To integrate $\int \frac{x-4}{x^2+2x-15}\, dx$, begin by factoring the bottom:

$\int \frac{x-4}{x^2+2x-15}\, dx = \int \frac{x-4}{(x+5)(x-3)}\, dx = \int \frac{A}{(x+5)} + \frac{B}{(x-3)}\, dx$

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Let's focus on the fractions. If we multiply both sides of the following equation by the original denominator, we can set up a systems of equations to solve for A and B

$(x + 5)(x - 3)[\frac{x-4}{(x+5)(x-3)}] = [\frac{A}{(x+5)} + \frac{B}{(x-3)}](x + 5)(x - 3)$

$x - 4 = A(x - 3) + B(x + 5)$

Now, let $x = 3$. This will eliminate the A variable, allowing us to solve for the value of B. Afterwards, we can repeat the process to solve for A.

$3 - 4 = A(3 - 3) + B(3 + 5)$

$- 1 = 8B$

$B =- \frac{1}{8}$

$- 5 - 4 = A(- 5 - 3) + B(- 5 + 5)$

$- 9 =- 8A$

$A = \frac{9}{8}$

Alternatively, you can set up a systems of equations to solve for A and B:

$A + B = 1$

$- 3A + 5B =- 4$

$A = \frac{9}{8}, B =- \frac{1}{8}$

Now, integrate each part of the integral separately:

$\int \frac{\frac{9}{8}}{(x+5)} + \frac{-\frac{1}{8}}{(x-3)}\, dx = \int \frac{\frac{9}{8}}{(x+5)} dx + \int \frac{-\frac{1}{8}}{(x-3)}\, dx = \frac{9}{8}\int \frac{1}{(x+5)} dx - \frac{1}{8}\int \frac{1}{(x-3)}\, dx =$

$\frac{9}{8} ln|x + 5| - \frac{1}{8} ln|x - 3| + C$

Depending on how the denominator can be factored, you may be able to rewrite the integral in a different form such as $\frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$ or $\frac{A}{x-a} + \frac{B}{(x-a)^2}$