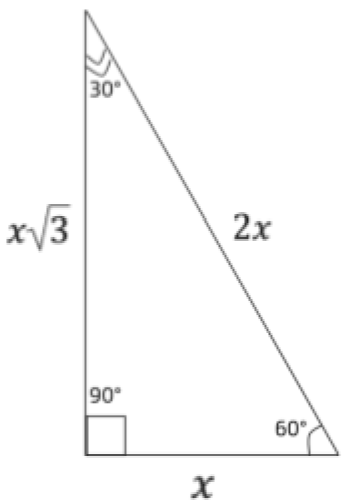
Precalculus 2.2 Key Points

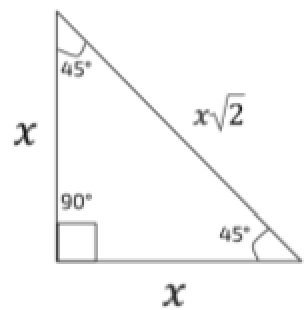
Special Right Triangles:

Special right triangles have a right angle (90 degrees) and special ratios between their sides. These ratios stay the same for any triangle of that type. There are two main types of special right triangles: the 30-60-90 triangle and the 45-45-90 triangle.

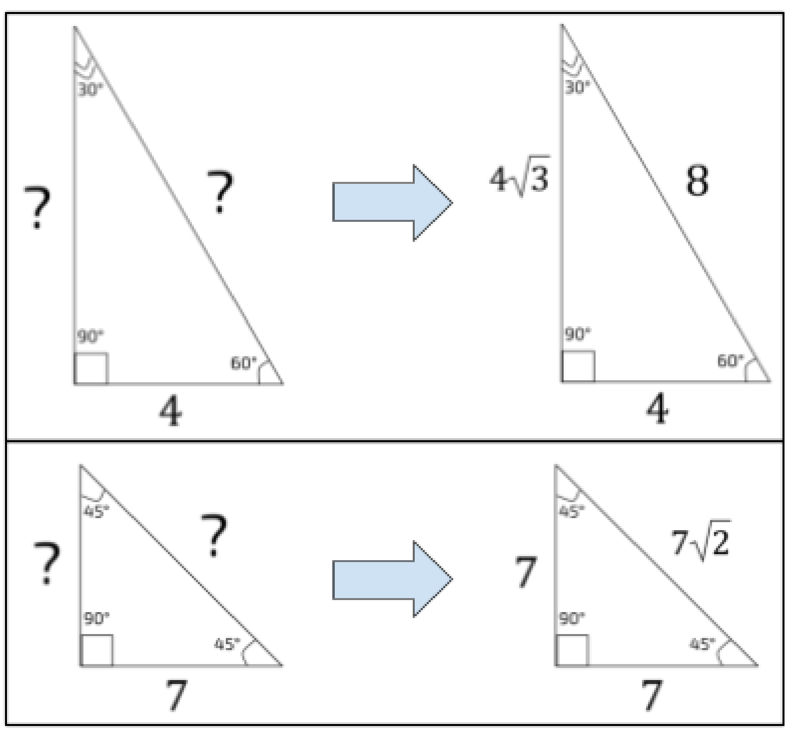
30-60-90



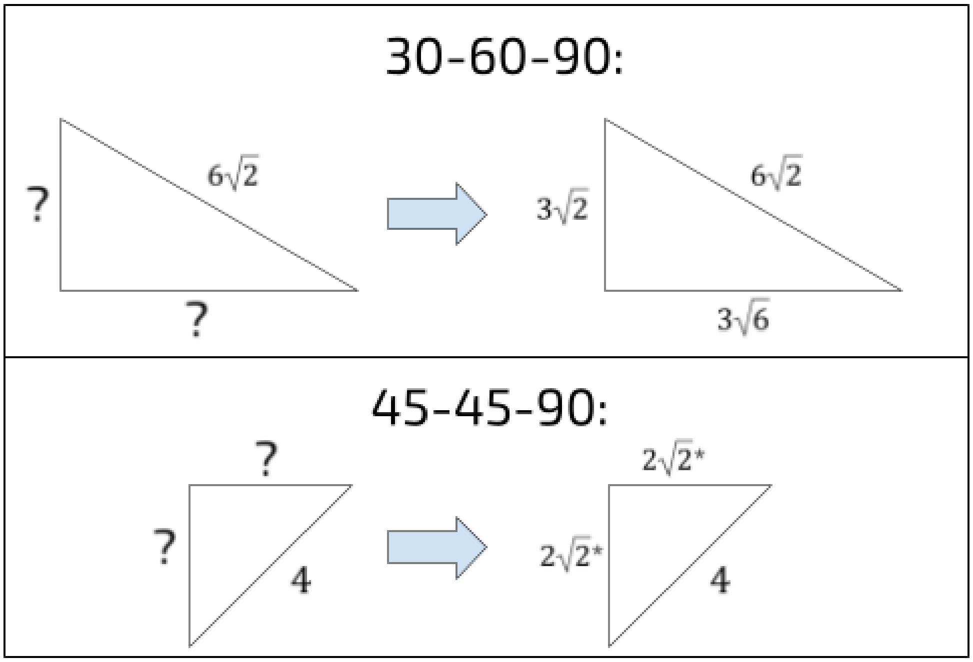
45-45-90



Examples



Precalculus 2.2 Key Points



*Rationalizing the denominator

<u>What is it?</u>

Rationalizing the denominator is the process of rewriting a fraction to remove the radical (square root) from the denominator

$\frac{4}{\sqrt{2}} \Rightarrow 2\sqrt{2}$

<u>How is it done?</u>

Multiplying the fraction by the radical over itself. This cancels out the square root and will leave an integer in the denominator.

$\frac{4}{\sqrt{2}} \bullet \frac{\sqrt{2}}{\sqrt{2}} \rightarrow \frac{4\sqrt{2}}{\sqrt{2}\bullet\sqrt{2}} \rightarrow \frac{4\sqrt{2}}{2} \rightarrow 2\sqrt{2}$

Since $\frac{\sqrt{2}}{\sqrt{2}} = 1$, this does not change the value of the original expression.

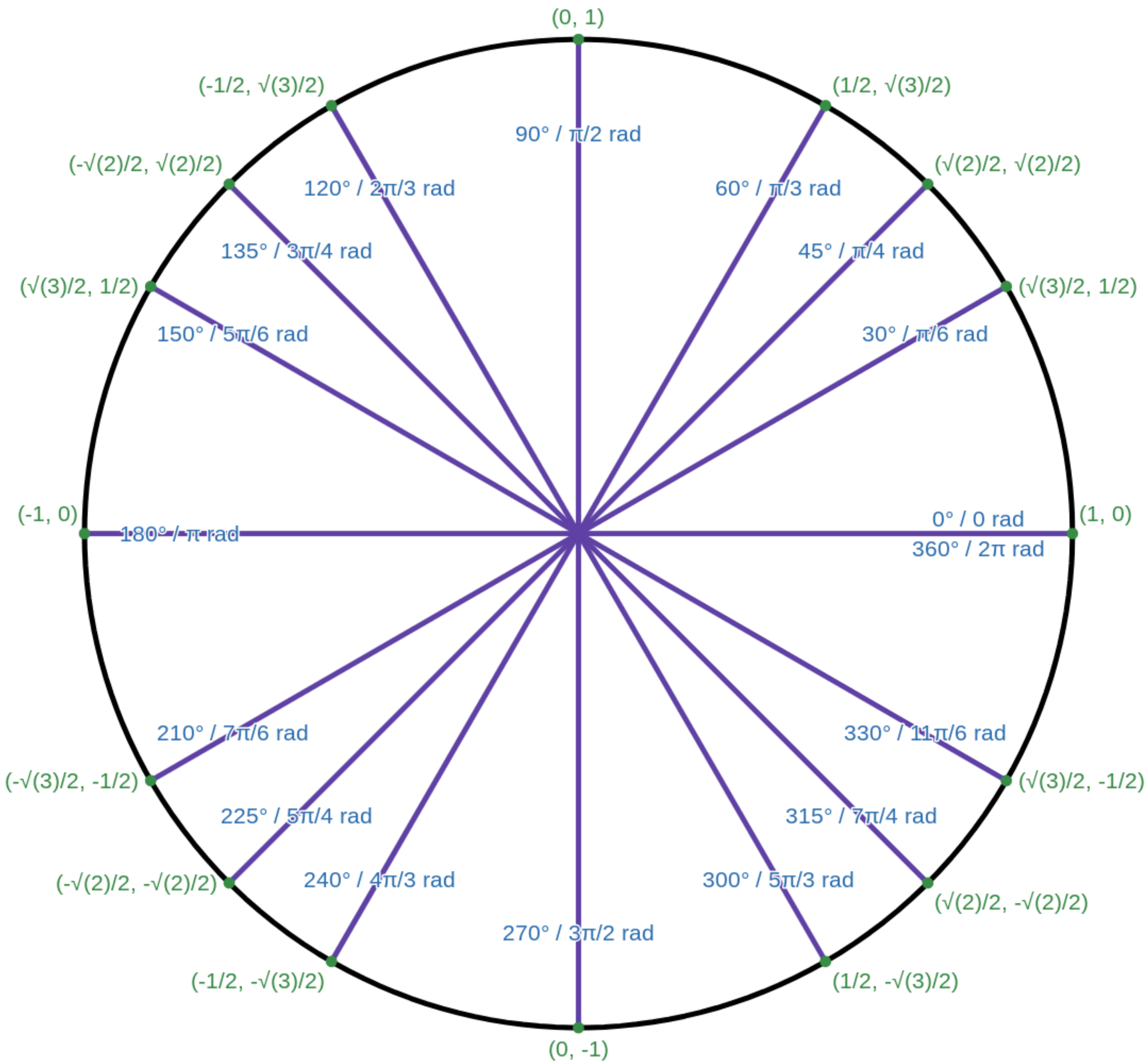
<u>Why is it used?</u>

Rationalizing the denominator makes the expression easier to work with in many cases, such as when finding a common denominator or simplifying expressions.

Precalculus 2.2 Key Points

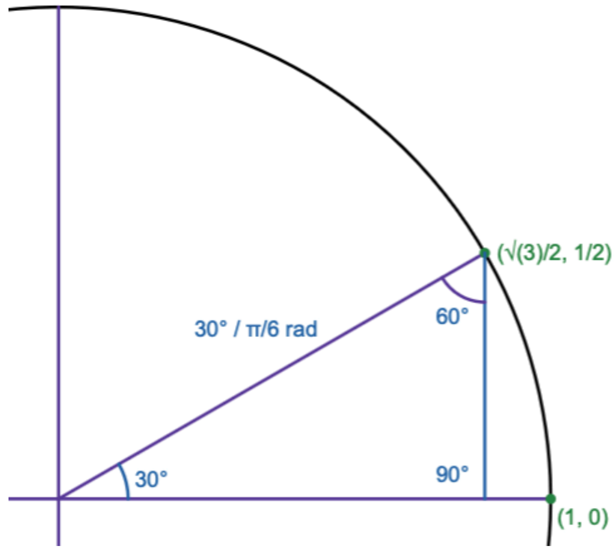
Unit Circle (Special Right Triangles, Sine, Cosine, Pythagorean Identity, Tangent, Reference Angle, Coterminal Angles):

A unit circle is a circle with a radius of 1. Below is a filled out unit circle, typically labeled with a few key angles(in degrees/radians) and points:

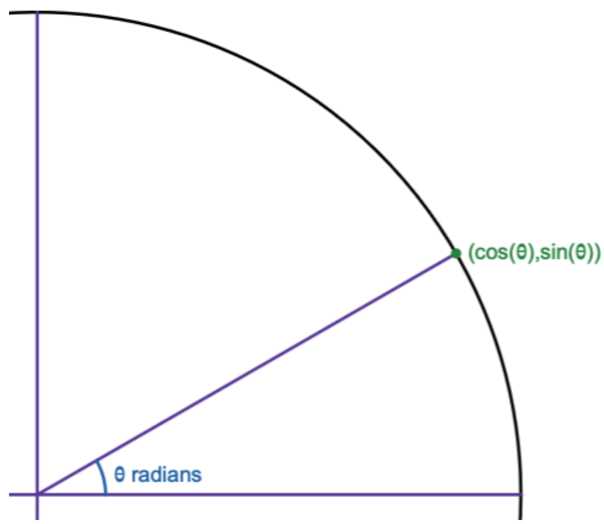


Precalculus 2.2 Key Points

Notice that values the unit circle can be determined used the special right triangles for a circle with a radius of 1



Also, using special right triangles allows us to express each point on the unit circle as $(cos(\theta),\ sin(\theta))$, where θ is the angle in radians and $cosine$ and $sine$ replace x and y, respectively

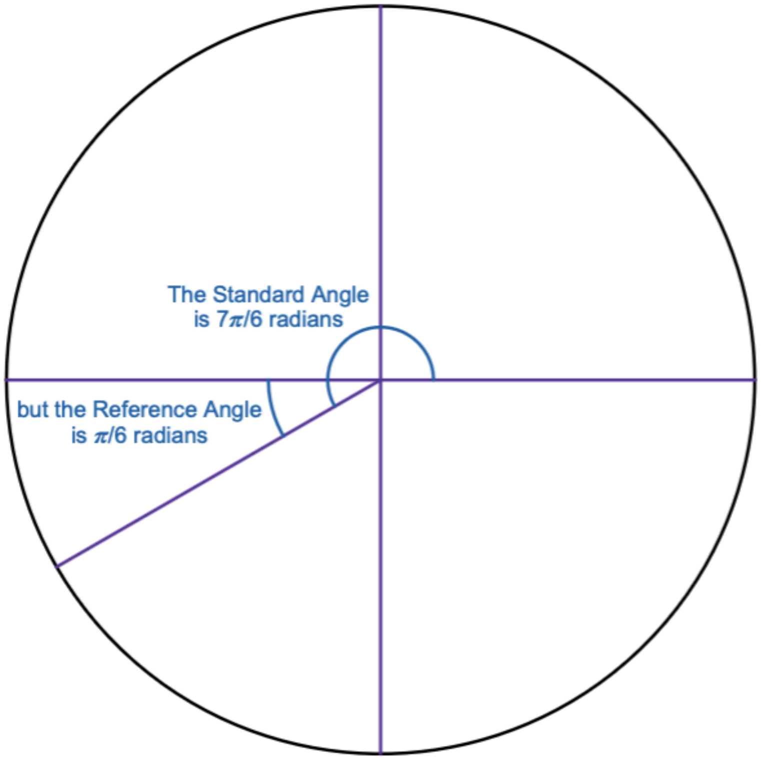


If we substitute $sine$ and $cosine$ into the formula for a unit circle, $x^2 + y^2 = 1$, we get the Pythagoraen Identity: $(cos(\theta))^2 + (sin(\theta))^2 = 1$, also written as $sin^2(\theta) + cos^2(\theta) = 1$

The tangent of theta, $tan(\theta)$, which is equal to $\frac{sin(\theta)}{cos(\theta)}$ or $\frac{y}{x}$ in this case. $tan(\theta)$ is also equal to the slope of the terminal ray of the angle

Precalculus 2.2 Key Points

While the Standard Angle is the angle between the x-axis in the positive direction and the terminal side (the line that you are measuring the angle of), the Reference Angle is the smallest angle between the x-axis (the positive or negative side) and the terminal side.



When two angles have the same initial and terminal sides but different angle measurements, they are Coterminal Angles. Their angles will differ by some factor of 2π radians or $360°$

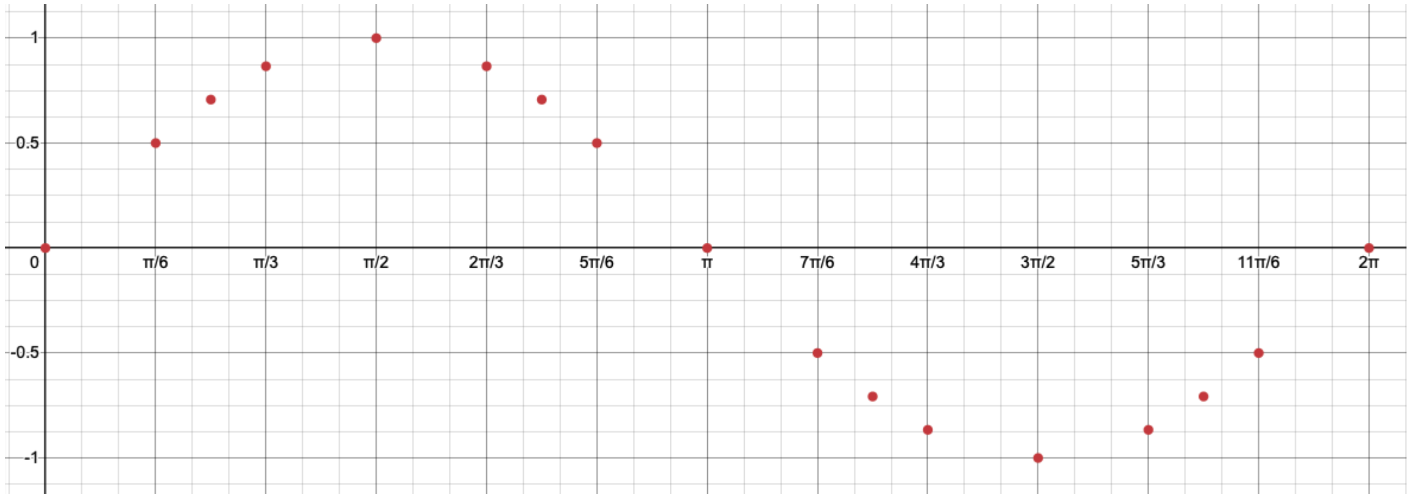
For example, $-\frac{3}{2}\pi,\ \frac{1}{2}\pi,\ and \frac{5}{2}\pi$ are coterminal angles

Graphing Sine and Cosine:

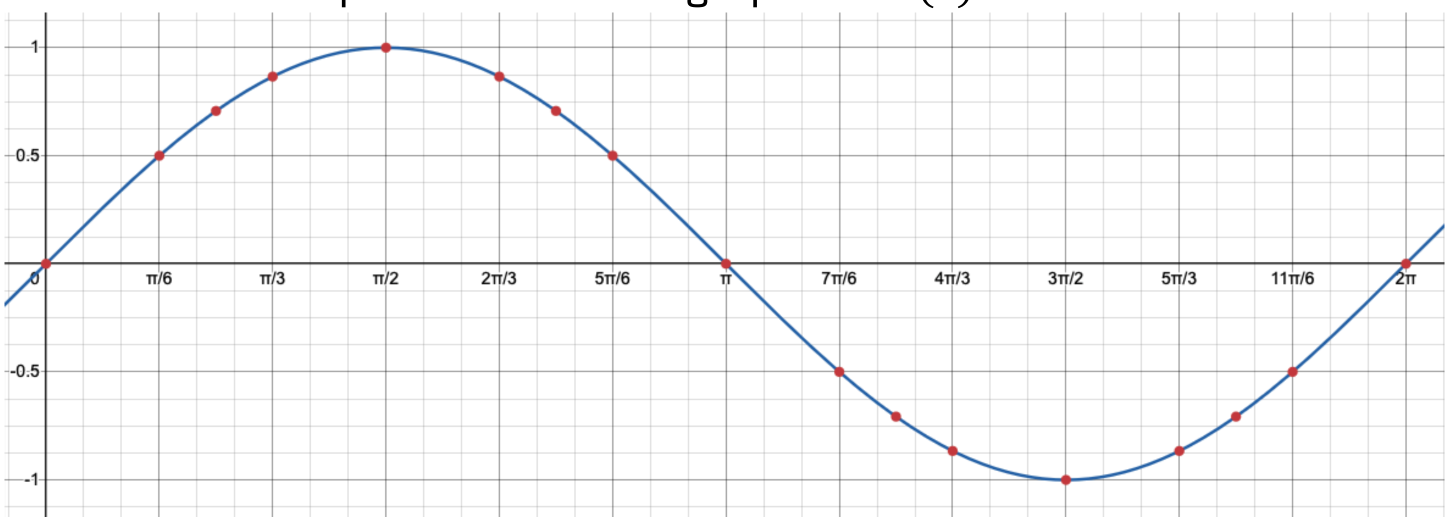
We can create a graphs of $sin(x)$ and $cos(x)$ using the values on the unit circle. For example, the angle $\pi/6$ radians results in an x-value of $\frac{\sqrt{3}}{2}$ and a y-value of $\frac{1}{2}$, so $cos(\frac{\pi}{6}) = \frac{\sqrt{3}}{2}$ and $sin(\frac{\pi}{6}) = \frac{1}{2}$

Precalculus 2.2 Key Points

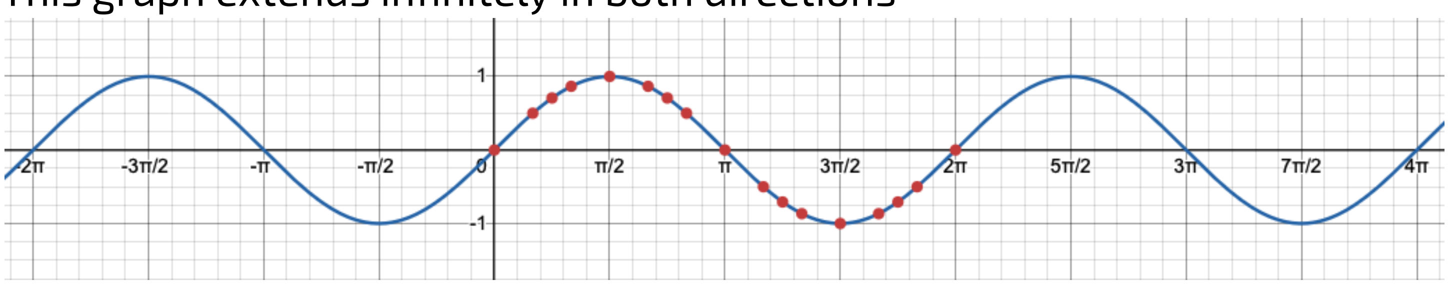
Let's begin by graphing these key points on a coordinate plane for $sin(x)$, where x is the angle and y is the value of $sine$ at that angle. Note that the scale for x-axis uses π.



Let's connect the points to create a graph of $sin(x)$

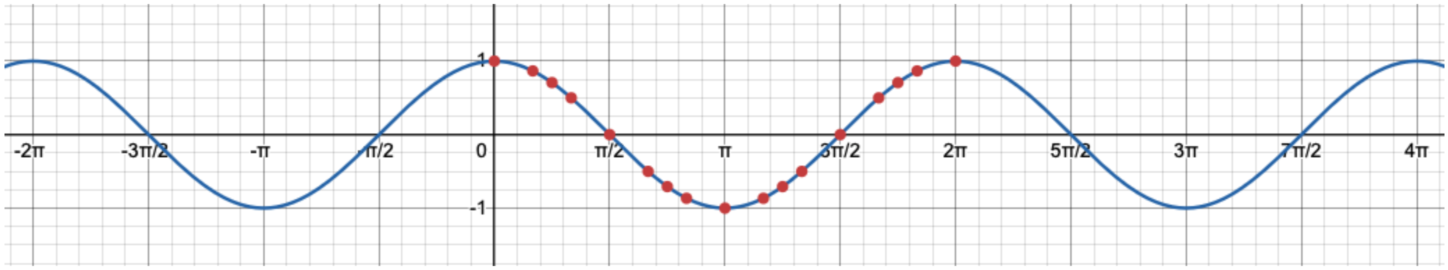


This graph extends infinitely in both directions



Precalculus 2.2 Key Points

We can also use a similar process to create a graph of $cos(x)$



Let's go over some vocabulary related to sinusoidal functions

Midline: The horizontal line between the function's maximum/minimum
Amplitude: The distance between the maximum/minimum and the midline
Period: The horizontal distance needed to complete one complete cycle

For a standard $sin(x)$ or $cos(x)$, the midline is $y = 0$, the amplitude is 1, and the period is 2π

Now that we understand the general graph of sinusoidal functions, let's find out how to graph transformed functions, using only five points:

The general equation is $y = a\ sin(b(x - c)) + d$, where
a: vertical stretch/compression
b: horizontal stretch/compression
c: horizontal shift
d: vertical shift

Let's graph $y = 3sin(2x - \pi) + 1$
First, we need to put it into the form above by factoring 2 from the inside of the $sine$ function

$y = 3sin(2x - \pi) + 1 = 3sin(2(x - \frac{\pi}{2})) + 1$

Now, let's identity each of the letters:

$a = 3, b = 2, c = \frac{\pi}{2}, d = 1$

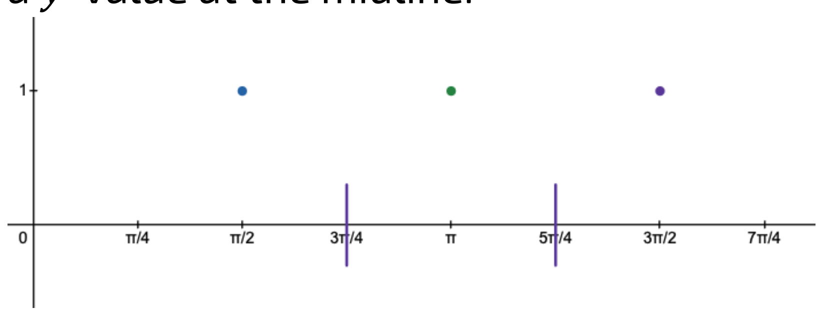
Precalculus 2.2 Key Points

Note: We will show one way to continue, but you do not have to graph using the exact same way that we do and you can do the steps in a slightly different order if you wish

Let's find the x-values of our 5 points:
The first x-value will be the value of c, because that is our horizontal shift
The fifth x-value will the the value of c plus the value of the period. The period is equal to $\frac{2\pi}{b}$
The second, third, and fourth points will be an equal x-distance between the first and fifth points

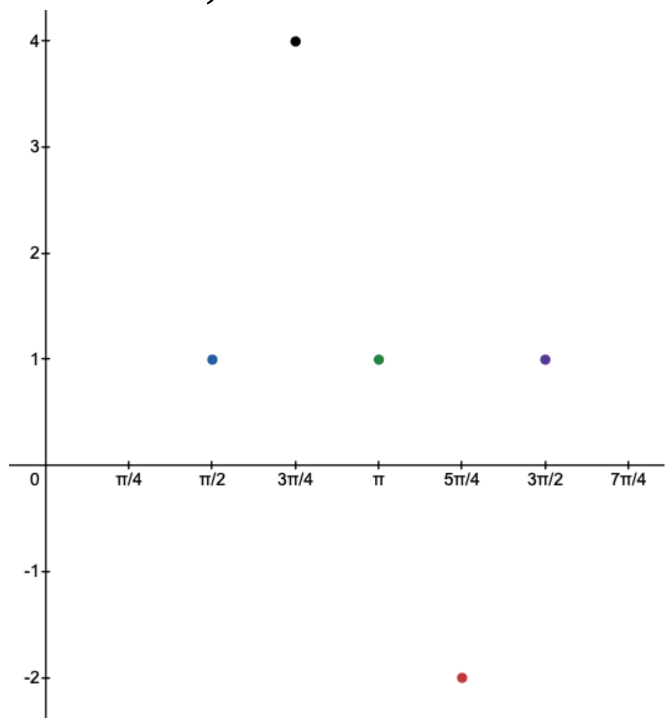
In our problem, $c = \frac{\pi}{2}$ and our period is $\frac{2\pi}{2} = \pi$, so the x-value of the first point is $\frac{\pi}{2}$ and the x-value of the fifth point is $\frac{\pi}{2} + \pi = \frac{3\pi}{2}$.
The distance between each x-value will be the period divided by 4, which is $\frac{\pi}{4}$ in our case. The second point's x-value will be c plus $\frac{\pi}{4}$, or $\frac{3\pi}{4}$. The third is $\frac{4\pi}{4}$ or π and the fourth is $\frac{5\pi}{4}$

Next, let's identify the midline, which is equal to the value of our vertical shift, d, which is 1. Since this is a *sine* function, the first, third, and fifth points will have a y-value at the midline.

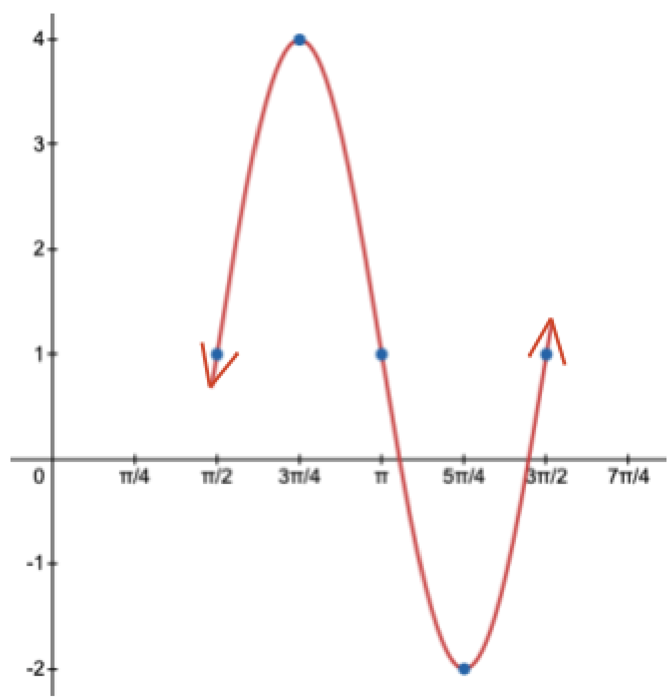


Precalculus 2.2 Key Points

Because our a-value is 3, our graph will be vertically stretched by a factor of three. The second point's y-value will be above the midline by a units, while the fourth point's y-value will be below the midline by a units (if a is negative, then the second point will be below the midline and the fourth point will be above the midline).



Now, we can connect these points to create our graph of $y = 3sin(2x - \pi) + 1$



Precalculus 2.2 Key Points

The process for graphing a *cosine* function is similar, but with a few differences. Instead of the first, third, and fifth points being at the midline, the second and fourth points will be at the midline. The first point and fifth points will be a units above the midline and the third point will be a units below the midline (unless a is negative, then the graph will be vertically flipped/reflected). The process for finding the horizontal and vertical shifts, and the period, will be the same as with the *sine* example we did.

For the equation, $y =- 2\,cos(\frac{1}{2}(x + \pi)) - 4$,

$a =- 2$, $b = \frac{1}{2}$, $c =- \pi$, $d =- 4$

The amplitude is 2 (amplitude is always positive), the period is $\frac{2\pi}{\frac{1}{2}} = 4\pi$, and the midline is $y =- 4$

Here is the graph with 5 key points: