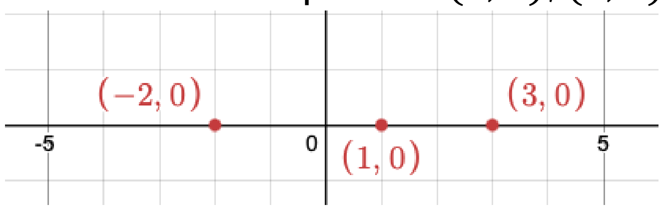
Precalculus 4.1 Key Points

Polynomial Functions in Factored Form:

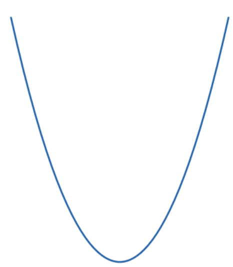
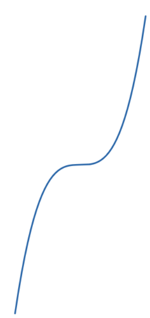
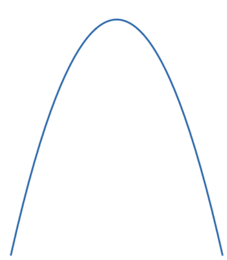
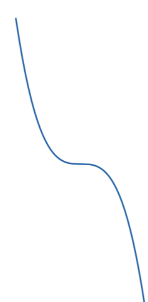
If you are given the factored form of a polynomial function, you can graph a rough sketch of the function using the polynomial's zeroes

For example, take $y = (x - 3)(x - 1)(x + 2)$

Notice that if you plug in 3, 1, or -2 in for x, then the expression will multiply to 0, so the three x-intercepts are $(3, 0)$, $(1, 0)$, and $(-2, 0)$



Now, we can use the <u>End Behavior</u> to sketch the rest of the graph. Here's a chart that you can use to figure out what the end behavior of a polynomial function is:

	Even Degree (x^2, x^4)	Odd Degree (x^3, x^5)
Positive Leading Coefficient $(3x^n, 8x^n)$		
Negative Leading Coefficient $(-2x^n, -7x^n)$		

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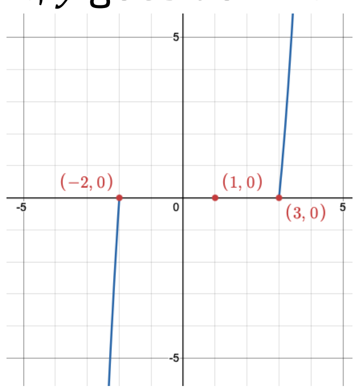
For our function, $y = (x - 3)(x - 1)(x + 2)$, we need to identify the degree and leading coefficient to determine the end behavior. One option is to multiply out each factor, which gives us $x^3 - 2x^2 - 5x + 6$
A faster way is to only multiply the x terms together, since the term with the largest exponent will be the one with the most x-terms being multiplied together: $x \bullet x \bullet x = x^3$, which is the same result

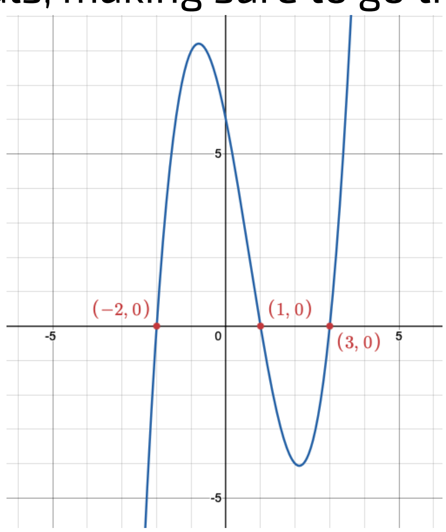
Since we have an odd degree with a positive leading coefficient, the end behavior for the polynomial function will be as follows:

- As x approaches negative infinity, y approaches negative infinity
- As x approaches positive infinity, y approaches positive infinity

In simple terms, as x goes left, y goes down and as x goes right, y goes up



Now, we can fill in the remainder of the graph with curves, based on the rough shape of polynomials, making sure to go through all of the zeroes



Your graph may not be exact if you are sketching by hand, but most of the time, a reasonable approximation should work

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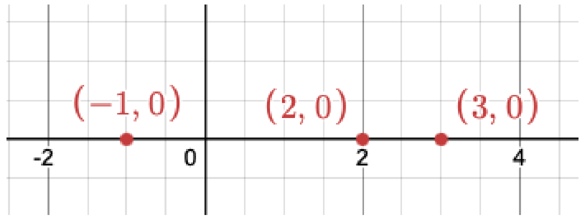
Let's do a slightly harder example: $y = -(x + 1)^2(x - 2)(x - 3)$

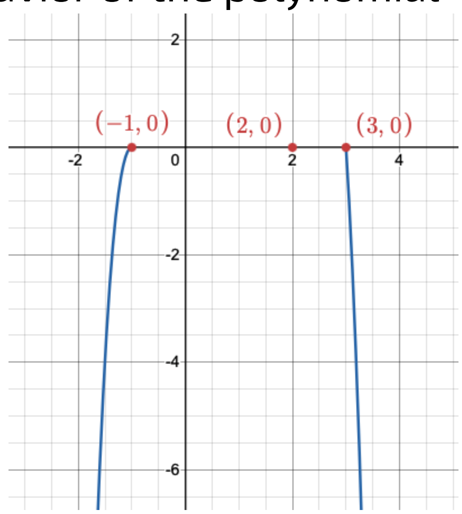
Again, we can start by determining our zeroes by seeing what numbers make each factor equal 0, which are -1, 2, and 3



Notice a couple of features about this equation:

- There is a negative out front
- We have $(x + 1)^2 = (x + 1)(x + 1)$

Thus, the term with the degree is: $-x \bullet x \bullet x \bullet x = -x^4$, which we can use to graph the end behavior of the polynomial



Lastly, to fill in the remainder of the graph, use the following rules:

- If the factor has an odd power, it will go through the x-intercept
- If the factor has an even power, it will "bounce" on the x-intercept

We can write the equation as $y = -(x + 1)^2(x - 2)^1(x - 3)^1$

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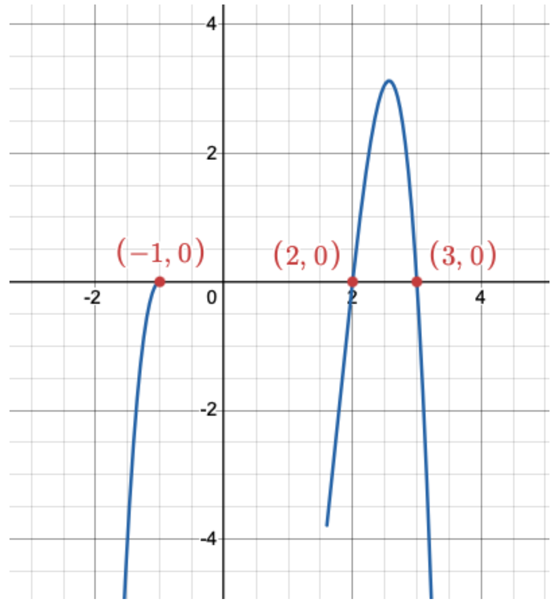
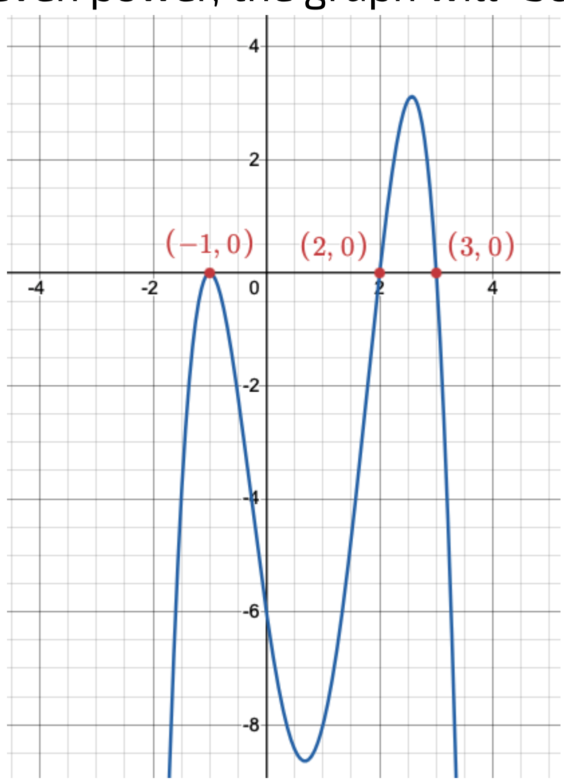
$(x - 2)$ and $(x - 3)$ have odd powers, so our graph will go through those points:



Since $(x + 1)^2$ has an even power, the graph will "bounce" at that point:



We can say that $(x + 1)^2$ has a <u>multiplicity</u> of 2, while $(x - 2)$ and $(x - 3)$ both have a multiplicity of 1

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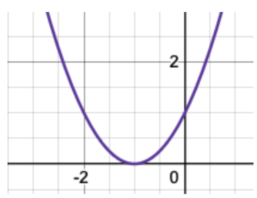
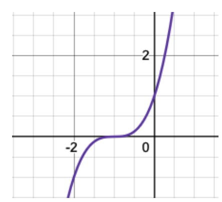
Polynomial Factor Theorem:

If a polynomial function has an x-intercept at $x = c$, then $x - c$ is a factor of that polynomial. For example, if you know that $(3, 0)$ is a point on the graph of a polynomial, then $(x - 3)$ is a factor of that polynomial

Writing Equations of Polynomial Functions:

If you are given a graph of a polynomial function and asked to find its equation, then try to work your way backwards. Here are some tips for some things that are helpful to consider:

- x-intercepts: Because of the polynomial factor theorem, you can use the x-intercepts of the graph to determine polynomial's factors
- Multiplicity: If a function bounces or passes through a point with a cubic-like pattern (shown below), then its factor may have an exponent greater than 1

 Bounces: $(x + 1)^2$	 Cubic-like pattern: $(x + 1)^3$

- End Behavior: Make sure that the end behavior of your equation lines up with the path of the graph
- Other points: If you are given or can identify other points on the graph, such as a y-intercept, you can scale your graph to include that point. For example, let's say you wanted your polynomial to pass through the point $(0, 6)$ and your current equation is
  $y = (x + 1)(x - 2)(x - 1)$
  If we plug in 0, we get $(1)(-2)(-1) = 2$
  We can multiply this number by 3 to get the desired number, 6
  So, if we multiply the original polynomial equation by 3, it will line up with the given y-intercept

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Imaginary Numbers:

i is defined as a number whose square equals -1, so $i = \sqrt{-1}$

Imaginary Numbers: Uses i or $\sqrt{-1}$ are called
Real Numbers: Non-imaginary numbers, such as $3, -\frac{4}{5}, \sqrt{\pi}$
Complex Numbers: Uses Real and/or Imaginary Numbers. For example, $3 + 4i$

- All real and imaginary numbers are also complex numbers

If a pair of complex numbers has imaginary numbers with opposite signs, they are referred to as <u>complex conjugates</u>. For example, $6 + 7i$ and $6 - 7i$ are conjugates

Here is what i raised to a few powers looks like:
$i^1 = i$
$i^2 = \sqrt{-1} \bullet \sqrt{-1} = -1$
$i^3 = i^2 \bullet i = -1 \bullet i = -i$
$i^4 = i^2 \bullet i^2 = -1 \bullet -1 = 1$
$i^5 = i^4 \bullet i = 1 \bullet i = i$
This cycle will continue, rotating between $i, -1, -i$ and 1

Fundamental Theorem of Algebra:

The Fundamental Theorem of Algebra says that a polynomial with a degree of n will have n complex roots. For example, $x^3 - 3x^2 + x - 2$ will have 3 complex roots.

Complex roots are made of real roots, which are the zeroes/x-intercepts on a graph, and imaginary roots.

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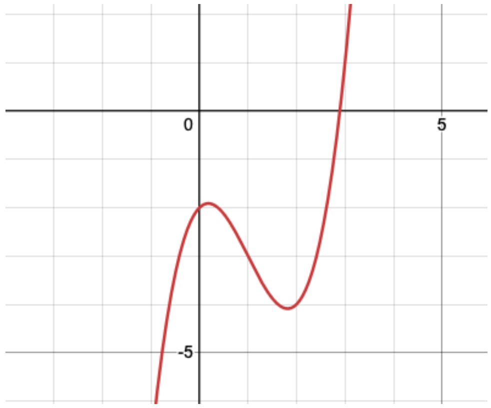
Since $x^3 - 3x^2 + x - 2$ has 1 real root, it will have 2 imaginary roots



Rational Roots Theorem:

Suppose we have the polynomial $4x^3 - 9x^2 - 3x + 8$
Let the constant $8 = q$ and the leading coefficient $4 = p$

The possible rational zeroes of the polynomial are $\pm \frac{factors\ of\ p}{factors\ of\ q}$

For this polynomial, the factors of $p = 4$ are 1, 4, and 2. The factors of $q = 8$ are 1, 8, 2, and 4

Thus, the possible rational zeroes of this polynomial are any positive or negative combination of the following $\pm \frac{1,4,2}{1,8,2,4}$, which are

$\pm\ 1, \pm \frac{1}{8}, \pm \frac{1}{2}, \pm \frac{1}{8}, \pm \frac{1}{4}, \pm\ 4, \pm\ 2$