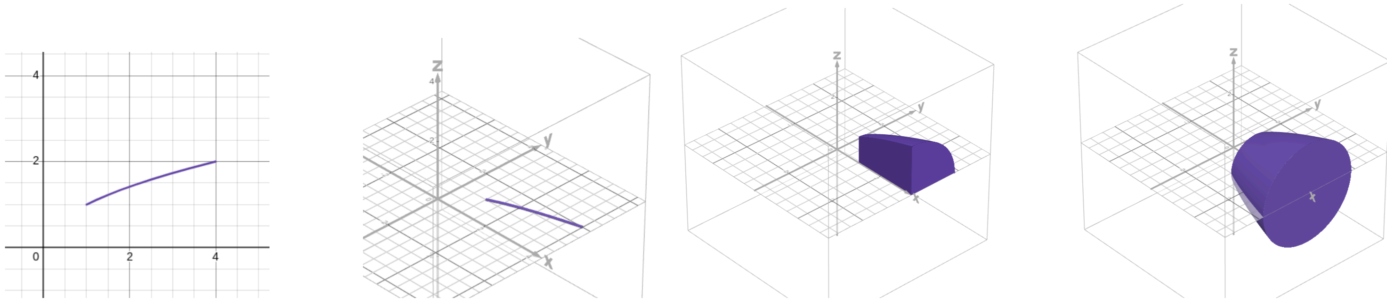
Calculus 8.1 Key Points

Disk Method:

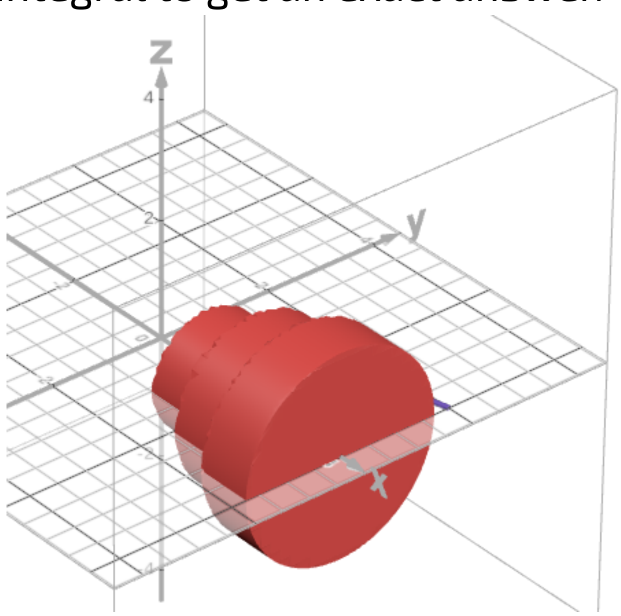
If we want to find the volume of a solid when a function is rotated about an axis, we can use the disk method



The general equation to find the volume of a solid using the disk method is

$$V = \int_{a}^{b} \pi(f(x))^2 \, dx$$

This equation is based off of the idea of summing up the volume of cylinders to approximate the volume, where each cylinder has a volume of $\pi r^2 h$. We can use an integral to get an exact answer.



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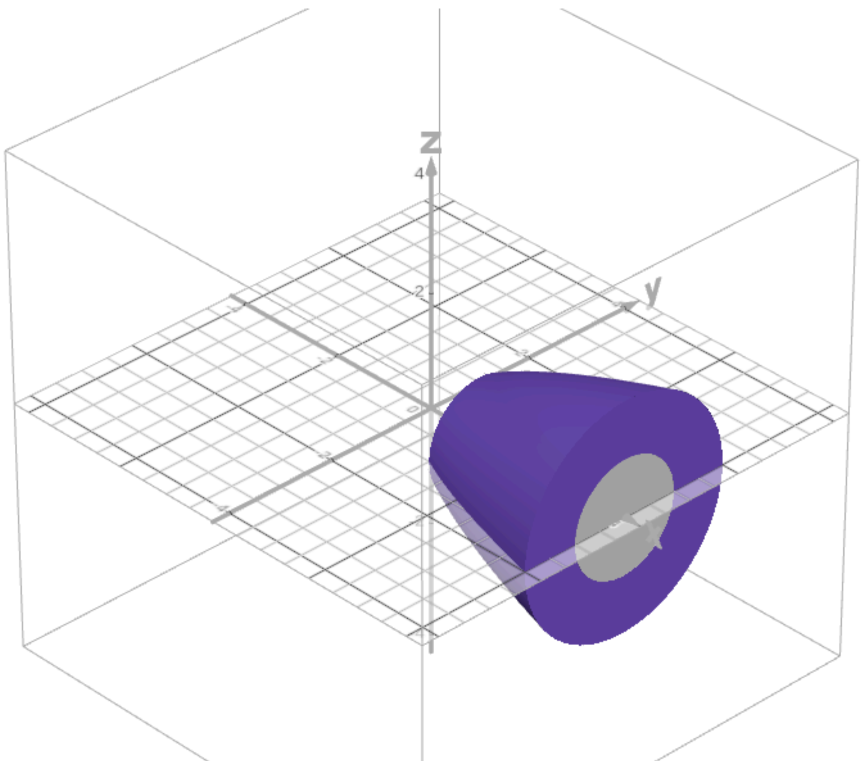
Washer Method:

To solve for the volume of a solid with a hole in the center, use the washer method
Take the volume of the outer solid and subtracts the volume of the missing space in the middle

$$V = \int_a^b \pi(f(x))^2\, dx - \int_a^b \pi(g(x))^2\, dx = \pi\int_a^b (f(x))^2 - (g(x))^2\, dx$$



Disk and Washer Methods Horizontally and Vertically:

Just like integrating horizontally, we can use the disk and washer methods to find the volume of solid by rotating a function about the y-axis. Just make sure that your integrand is in terms of y and you integrate with respect to y, or swap the variables to use x

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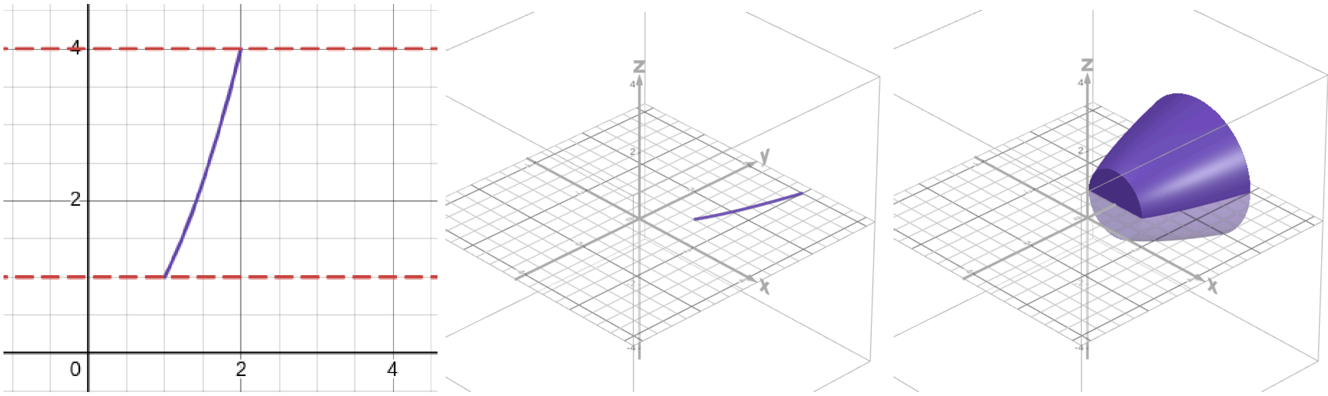

$$V = \int_1^4 \pi(\sqrt{y})^2\, dy$$

Disk and Washer Methods Axis of Rotation:

If we change the axis of rotation to not just be the x-axis or y-axis but instead a line like $y = -5$ or $x = 1$, then adjust your equations using a shift and then use the disk or washer method

For example, rotating the function $f(x) = \sqrt{x}$ around the line $y = -2$ from $x = 1$ to $x = 4$, that is the same as rotating $\sqrt{x} + 2$ around the line $y = 0$

Written with an integral:

$\int_1^4 \pi(\sqrt{x})^2\, dx$ with the axis of rotation as $y = -2$ is the same as

$\int_1^4 \pi(\sqrt{x})^2 + 2\, dx$ with the axis of rotation as $y = 0$

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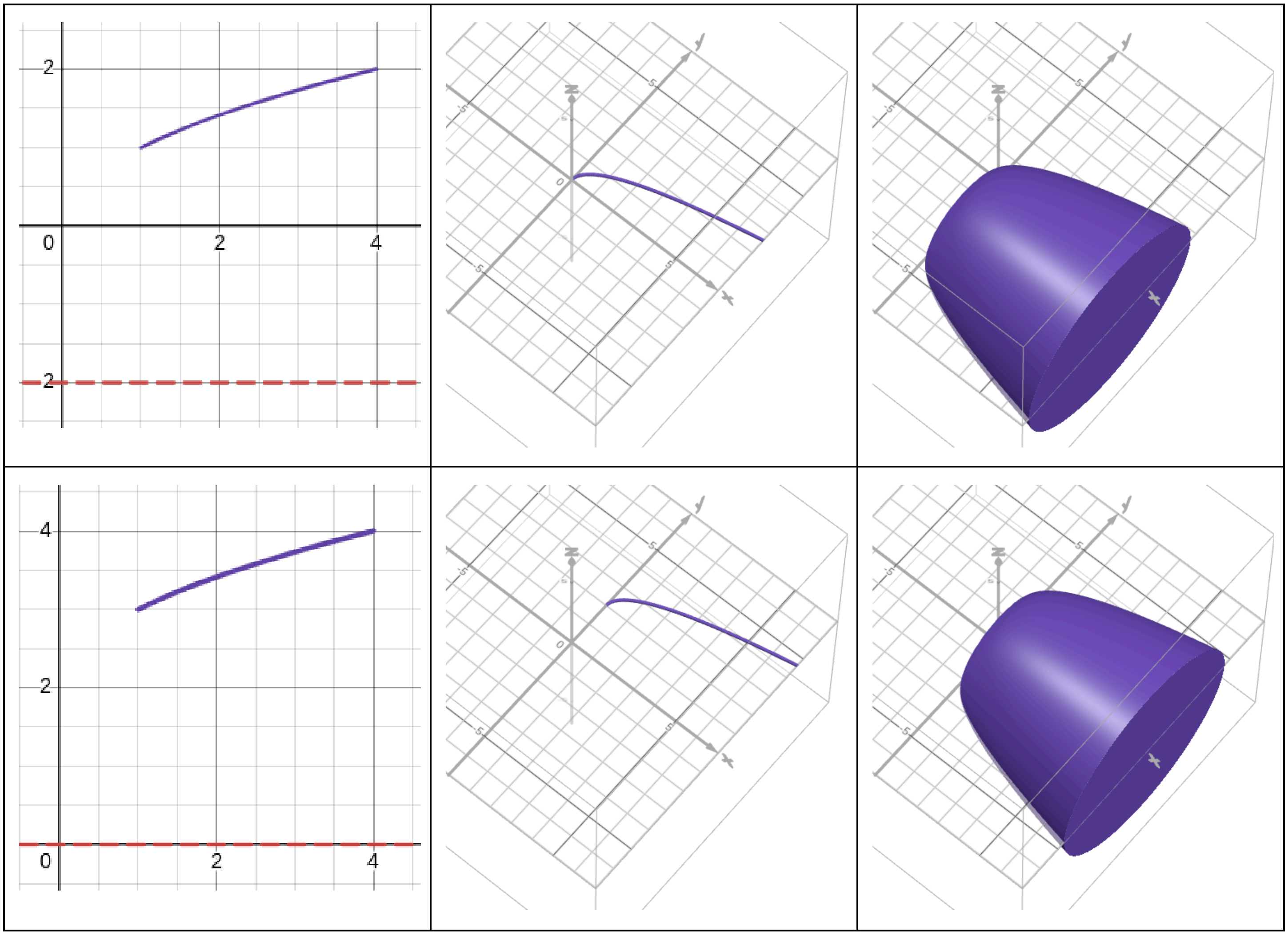

Then, use the disk or washer method to find the volume of the solid that is now being rotated about the x-axis or y-axis: $\int_{1}^{4} \pi(\sqrt{x}+\ 2)^{2}\,dx\ =\frac{229\pi}{6}$