Calculus 4.4 Key Points

Area Between Two Curves:

The area between two curves can be found by integrating the top function and subtracting the integral under the bottom function from that

$$\int_a^b f(x)dx - \int_a^b g(x)dx \quad \text{or} \quad \int_a^b [f(x) - g(x)]dx$$

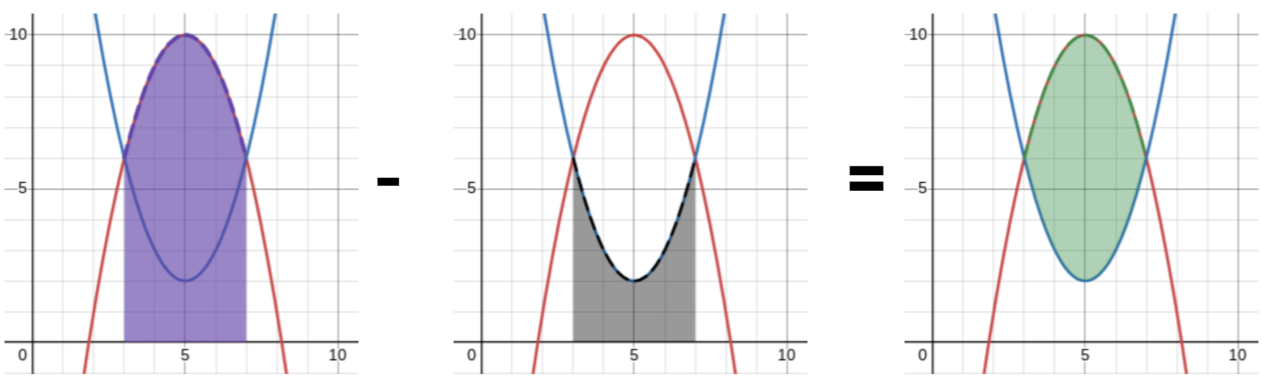
For example, let $f(x) = -(x-5)^2 + 10$ and $g(x) = (x-5)^2 + 2$
The area between these two curves can be expressed as:

$$\int_3^7 [-(x-5)^2 + 10]dx - \int_3^7 [(x-5)^2 + 2]dx \quad \text{or}$$

$$\int_3^7 [(-(x-5)^2 + 10) - ((x-5)^2 + 2)]dx$$

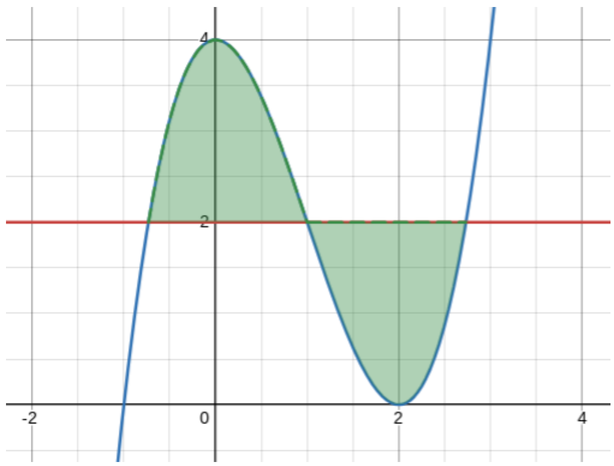
Visually:

Area under top curve - Area under bottom curve = Area between curves



Calculus 4.4 Key Points

Area Between Two Curves That Cross Over:
If two curves cross over, then you will need to use multiple integrals and switch which function is the top function.

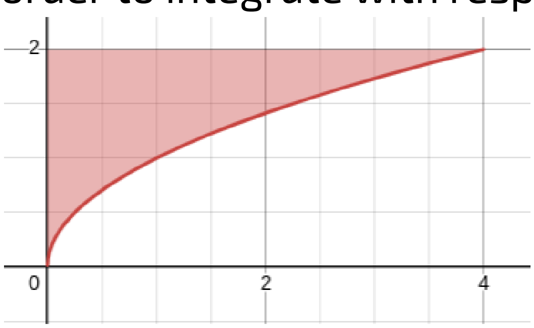


The area between the two curves represented above is approximately equal to:

$$\int_{-0.732}^{1} [(x^3 - 3x^2 + 4) - (2)]dx + \int_{1}^{2.732} [(2) - (x^3 - 3x^2 + 4)]dx$$

Integrating Horizontally:

To integrate horizontally, just make sure that you use the correct equation and integrate with respect to dy or swap the variables in order to integrate with respect to dx



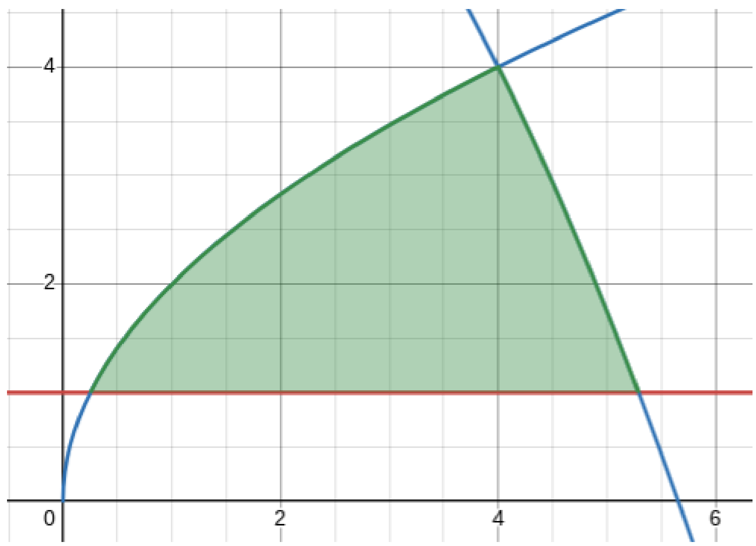
The area under the curve $x = y^2$ from $y = 0$ to $y = 4$ can be represented as $\int_0^4 y^2 dy$ or $\int_0^4 x^2 dx$

Calculus 4.4 Key Points

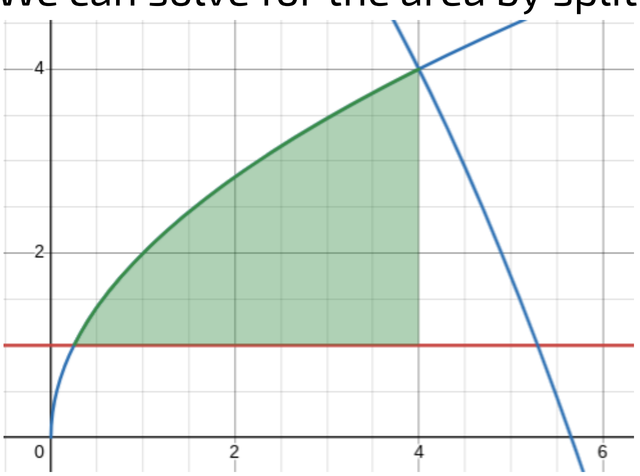
Area Between Three Curves:

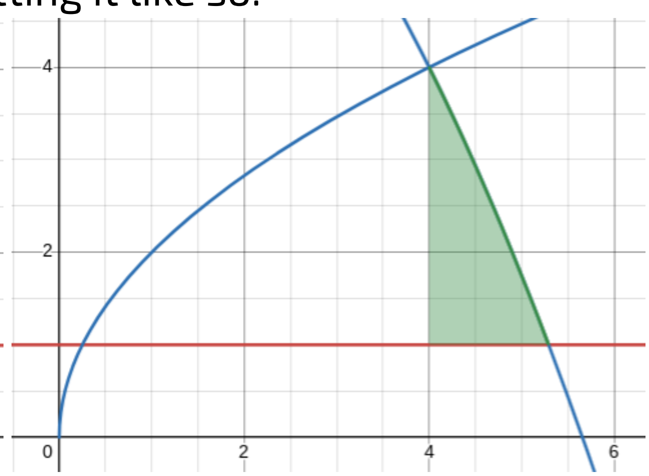
To find the area between three curves, use two integrals after splitting up the area

Let's try to find the area between the curves $y = 1$, $y = 2\sqrt{x}$, and $y = -0.25x^2 + 8$



We can solve for the area by splitting it like so:





$$\int_{0.25}^{4} [(2\sqrt{x}) - (1)]dx + \int_{4}^{5.2915} [(-0.25x^2 + 8) - (1)]dx$$