Calculus 7.2 Key Points

u-substitution:

$\int f(g(x)) \bullet g'(x)\, dx = \int f(u)du$, where $u = g(x)$ and $du = g'(x)\, dx$

Let's look at example of u-substitution, an integration technique to undo the chain rule and evaluate this integral:

$\int cos(x^2) \bullet 2x\, dx$

Let $u = x^2$
If we take the derivative of both sides of that equation, we get:
$du = 2x\, dx$

Now, let's substitute u and du into our integral problem

$\int cos(x^2) \bullet 2x\, dx = \int cos(u)\, du$

Now, we can easily integrate $cos(u)$, and substitute x back in after

$\int cos(u)\, du = sin(u) + c = sin(x^2) + c$

The same process applies to definite integrals, except you also have a choice to make regarding the bounds of integration

Example:

$\int_0^1 9x^2(6x^3 - 3)^4\, dx$

This time, we will need to divide both sides of the du equation by 2 because our integral has a 12 in the integrand while our du equation has a 24

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$u = 6x^3 - 3$

$du = 18x^2\, dx$

$\frac{du}{2} = 9x^3\, dx$

Now, we can substitute u and du into our integrand, but since we are now integrating with respect to du instead of dx, we have a choice: we can either change our bounds and continue using u for the rest of the problem OR we can list our bounds as general variables like a and b until we substitute x back in

Method 1:

Changing our bounds by plugging the old bounds into the u equation:

Upper Bound: $u = 6(1)^3 - 3 = 3$

Lower Bound: $u = 6(0)^3 - 3 =- 3$

$$\frac{1}{2}\int_{-3}^{3} u^4\, du = \frac{1}{2} \bullet \frac{u^5}{5} \Big|_{-3}^{3} = \frac{1}{2}\left(\frac{(3)^5}{5} - \frac{(-3)^5}{5}\right) = \frac{486}{10} = 48.6$$

Method 2:

Use a and b as general bounds until we substitute x back in

$$\frac{1}{2}\int_{a}^{b} u^4\, du = \frac{1}{2} \bullet \frac{u^5}{5} \Big|_{a}^{b} = \frac{1}{2} \bullet \frac{(6x^3-3)^5}{5} \Big|_{0}^{1} = \frac{1}{2}\left(\frac{(6(1)^3-3)^5}{5} - \frac{(6(0)^3-3)^5}{5}\right) = 48.6$$