AP Calculus Limits

To evaluate $\lim\limits_{x \to a} f(x)$, use the following steps:

First, directly plug a into $f(x)$

- If $f(a) = b$ is a real number, then you have probably found the limit. Make sure the function is continuous around $x = a$
- If $f(a) = \frac{b}{0}$, where b is a non-zero number, then the limit is probably an asymptote. Consider what happens on the left and right sides of the asymptote
- If $f(a) = \frac{0}{0}$, then the limit is in indeterminate form

If the limit is in **indeterminate form**, there are several methods you can use to evaluate the limit, depending on the specific function:

- Factoring (e.g. $\lim\limits_{x \to 3} \frac{x^2-9}{x-3} = \lim\limits_{x \to 3} \frac{(x+3)(x-3)}{x-3} = \lim\limits_{x \to 3} x + 3 = 6$)
- Conjugates (e.g. $\lim\limits_{x \to 4} \frac{\sqrt{x}-2}{x-4} = \lim\limits_{x \to 4} \frac{\sqrt{x}-2}{x-4} \bullet \frac{\sqrt{x}+2}{\sqrt{x}+2} =$ $\lim\limits_{x \to 4} \frac{x-4}{(x-4)(\sqrt{x}+2)} = \lim\limits_{x \to 4} \frac{1}{(\sqrt{x}+2)} = \frac{1}{4}$)
- Trig Identities (e.g. $\lim\limits_{x \to \frac{\pi}{4}} \frac{cos(2x)}{cos(x)-sin(x)} = \lim\limits_{x \to \frac{\pi}{4}} \frac{cos^2(x)-sin^2(x)}{cos(x)-sin(x)} =$ $\lim\limits_{x \to \frac{\pi}{4}} \frac{(cos(x)+sin(x))(cos(x)-sin(x))}{cos(x)-sin(x)} = \lim\limits_{x \to \frac{\pi}{4}} cos(x) + sin(x) = \sqrt{2}$)
- L'Hôpital's Rule— $\lim\limits_{x \to a} \frac{f(x)}{g(x)} = \lim\limits_{x \to a} \frac{f'(x)}{g'(x)}$
- Squeeze Theorem—If $h(x) \leq f(x) \leq g(x)$ and $\lim\limits_{x \to a} h(x) = \lim\limits_{x \to a} g(x) = L$, then $\lim\limits_{x \to a} f(x) = L$
- Approximations—If no other method works, you can use a table or graph to determine the function's behavior around $x = a$.

After rewriting the limit, go back and reevaluate the limit at $x = a$. You may need to repeat this process multiple times. It is possible that the limit does not exist.