Calculus 7.3 Key Points

Solving Differential Equations:

If we have a differential equation, such as $\frac{dy}{dx} = x$, here's how we can find our original equation (and solve for y):

- Separate the differentials

If we treat $\frac{dy}{dx}$ like a fraction, then we can multiply both sides by dx

$dx \bullet \frac{dy}{dx} = x \bullet dx$

$1\, dy = x\, dx$

- Integrate (and add $+\ C$!!!)

$\int 1\, dy = \int x\, dx$

$y + C_1 = \frac{1}{2}x^2 + C_2$

$y = \frac{1}{2}x^2 + C_2 - C_1$

$y = \frac{1}{2}x^2 + C$

Technically, when we integrate we are left with two different "$+\ C$"s, but if we subtract one constant from one side, we can combine the constants. By convention, we can add the constant to the right side of the equation when integrating differential equations.

- Solve for C based on initial condition (if given)

Solve for the value of C by plugging in x and y into our equation
In this case, we will say that $y(0) = 2$

$y = \frac{1}{2}x^2 + C$

$2 = \frac{1}{2}(0)^2 + C$

$2 = C \quad \Rightarrow \quad y = \frac{1}{2}x^2 + 2$

Note: For simplicity, we are treating $\frac{dy}{dx}$ as a fraction but it is NOT actually a fraction. We can only treat it as such while integrating because we can take advantage of a property that says $\int f(y)\frac{dy}{dx}\, dx = \int f(y)\, dy$

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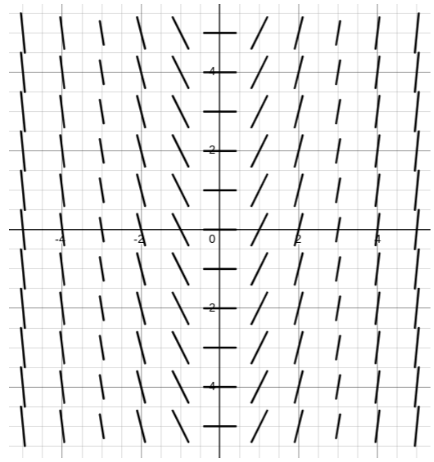
Slope Fields:

A Slope Field uses a function's derivative to represent its slopes

Here is the slope field using the equation $\frac{dy}{dx} = 2x$



If we take the antiderivative of this function, we get $y = x^2 + C$, whose slope gets increasingly steep as you get further from the vertex, as represented by the slope field.

To graph a slope field, plug the x- and y-coordinates at each coordinate point into your derivative equation.

Here are a couple more examples of slope fields:

$\frac{dy}{dx} = -\frac{x}{y}$	$\frac{dy}{dx} = 2x + y$