

Calculus 5.1 Key Points

Relationship of Position-Velocity-Acceleration:

Position: The location of an object - Units include meters (m), feet (ft), miles (mi)

Velocity: The rate of change of position - Units include meters per second ($\frac{m}{s}$), feet per minute ($\frac{ft}{min}$), miles per hour ($\frac{mi}{h}$)

Acceleration: The rate of change of velocity - Units include meters per second per second ($\frac{\frac{m}{s}}{s}$ or $\frac{m}{s^2}$), feet per minute squared ($\frac{ft}{min^2}$), miles per hour squared ($\frac{mi}{h^2}$)

Derivative of
Position \Rightarrow Velocity \Rightarrow Acceleration

Integral of
Acceleration \Rightarrow Velocity \Rightarrow Position

Optimization:

There are many, many different types of optimization problems, which you will often see in real-world situations.

As a general rule:

1. Identify that function for the quantity you want to maximize/minimize (This could be area, volume, cost, etc.)
2. Solve for where the derivative of that equation equals zero to find where that function has minimums/maximums
3. ANSWER THE QUESTION. Make sure you go back and find the answer to the original question and see if your solution makes sense based on the context of the problem.

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First & Second Derivative Tests:

If the derivative of a function equals 0 at a point, here's how we can determine if it is a local minimum or maximum:

- First Derivative Test
 - First derivative left of point is negative and right of point is positive ➡ Local Minimum
 - First derivative left of point is positive and right of point is negative ➡ Local Maximum
- Second Derivative Test:
 - Second derivative is positive at point ➡ Local Minimum
 - Second derivative is negative at point ➡ Local Maximum

Extreme Value Theorem:

If a function $f(x)$ is continuous over a closed interval $[a, b]$, then $f(x)$ has a global minimum and maximum over that interval

For example, look at the following continuous function on the interval $[0, 4]$



Since this function is continuous on this closed interval, we know that the function must have a global minimum and maximum on this interval, which it does. The interval's global minimum is at $(4, -0.226)$ and global maximum is at $(0.648, 2.498)$.