Derivative Tools

Limit Definition of Derivative

$$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h} \text{ or } \lim_{x \to a} \frac{f(x)-f(a)}{x-a}$$

Power Rule

$$f(x) = a \bullet x^n$$

$$f'(x) = a \bullet n \bullet x^{n-1}$$

Sum Rule

$$h(x) = f(x) + g(x)$$
$$h'(x) = f'(x) + g'(x)$$

Product Rule

$$h(x) = f(x) \bullet g(x)$$
$$h'(x) = (f'(x) \bullet g(x)) + (f(x) \bullet g'(x))$$

Quotient Rule

$$h(x) = \frac{f(x)}{g(x)}$$

$$h'(x) = \frac{(f'(x) \bullet g(x)) - (f(x) \bullet g'(x))}{(g(x))^2}$$

Chain Rule

$$h(x) = f(g(x))$$
$$h'(x) = f'(g(x)) \bullet g'(x)$$

Exponential

$$f(x) = a^{g(x)}$$

$$f'(x) = a^{g(x)} \bullet ln(a) \bullet g'(x)$$

Logarithm

$$f(x) = log_a(x)$$

$$f'(x) = \frac{1}{x \bullet ln(a)}$$

Derivative Tools

Trigonometric	
$f(x) = sin(x), f'(x) = cos(x)$	$f(x) = cos(x), f'(x) =- \ sin(x)$
$f(x) = tan(x), f'(x) = sec^2(x)$	$f(x) = cot(x), f'(x) =- \ cot^2(x)$
$f(x) = sec(x), f'(x) = sec(x) \bullet tan(x)$	$f(x) = csc(x), f'(x) =- \ csc(x) \bullet cot(x)$

Inverse Trigonometric	
$f(x) = sin^{-1}(g(x)), f'(x) = \dfrac{g'(x)}{\sqrt{1-g^2(x)}}$	$f(x) = cos^{-1}(g(x)), f'(x) =- \dfrac{g'(x)}{\sqrt{1-g^2(x)}}$
$f(x) = tan^{-1}(g(x)), f'(x) = \dfrac{g'(x)}{1+g^2(x)}$	$f(x) = cot^{-1}(x), f'(g(x)) =- \dfrac{g'(x)}{1+g^2(x)}$
$f(x) = sec^{-1}(g(x)), \dfrac{g'(x)}{|g(x)|\bullet\sqrt{g^2(x)-1}}$	$f(x) = csc^{-1}(g(x)), f'(x) =- \dfrac{g'(x)}{|g(x)|\bullet\sqrt{g^2(x)-1}}$

Implicit Differentiation (Chain Rule)

Example: If $x + f(x) = 10$, then $1 + f'(x) = 0$