Calculus 6.2 Key Points

Implicit Differentiation:

Let's find $\frac{dy}{dx}$ given the equation $2y = 4x^2 + 2x + 5$

One approach is to divide both sides by 2 and then take the derivative, like so:

$y = 2x^2 + x + \frac{5}{2}$

$\frac{dy}{dx} = 4x + 1$

Let's take an alternate approach to finding $\frac{dy}{dx}$

Rather than isolating y to begin, let's start by taking the derivative of both sides with respect to dx

$\frac{d}{dx}(2y) = \frac{d}{dx}(4x^2 + 2x + 5)$

Let's take a moment to review the chain rule, which says that:

If $h(x) = f(g(x))$, then $h'(x) = f'(g(x)) \bullet g'(x)$, or equivalently that $\frac{df}{dx} = \frac{df}{dg} \bullet \frac{dg}{dx}$

The derivative of an x-term like $\frac{d}{dx}(5x)$ is $5\frac{dx}{dx}$

Since $\frac{dx}{dx} = 1$, $\frac{d}{dx}(5x) = 5 \bullet 1 = 5$

But when we apply the chain rule with a y-term like $\frac{d}{dx}(5y)$, we now get $5\frac{dy}{dx}$

If we think of y as a composite function dependant on x, then we can say $\frac{d}{dx}(5y) = 5 \bullet \frac{d}{dx}(y(x)) = 5 \bullet \frac{dy}{dx} \bullet \frac{dx}{dx} = 5\frac{dy}{dx}$

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Returning to our example, here's how we continue the problem:

$\frac{d}{dx}(2y) = \frac{d}{dx}(4x^2 + 2x + 5)$

$2\frac{dy}{dx} = \frac{d}{dx}(4x^2 + 2x + 5)$

$2\frac{dy}{dx} = 8x\frac{dx}{dx} + 2\frac{dx}{dx}$

$2\frac{dy}{dx} = 8x + 2$

$\frac{dy}{dx} = 4x + 1$

With some problems, it is faster to use implicit differentiation than to isolate y and then take the derivative. Implicit differentiation is also helpful for taking the derivative of inverse functions.