Calculus 4.1 Key Points

Integral as the Limit of a Riemann sum:

$$\lim_{n \to \infty} \sum_{i=an}^{bn} \frac{1}{n} f\left(\frac{i}{n}\right) = \int_a^b f(x)dx$$

The basic idea is that the area under the curve of a function can be found by summing close to an infinite number of rectangles,

Think about the left side of this formula as trying to find the area under the curve from a to b, where the width of each rectangle is infinitely small, $\frac{1}{n}$.

The right side of this formula depicts this summation using an integral.

Definite Integral Properties:

$$\int_a^b f(x)dx = -\int_b^a f(x)dx$$

Reverse Interval: If an integral has a lower bound of "a" and an upper bound of "b" then flipping the bounds so that the lower bound is "b" and the upper bound is "a" will be the negative of the original integral

$$\int_a^a f(x)dx = 0$$

Zero: An integral with the same lower and upper bounds will be equal to zero

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$$\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$$

Additive Interval: Adding two integrals together where the lower bound of one integral is the upper bound of another can be combined into one integral to calculate the area under the curve of the combined areas of the smaller integrals

$$\int_a^b [f(x) \pm g(x)]dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$$

Sum/Difference: If an integral can be written as the sum or difference of multiple parts, then the integral can be split up by integrating each part separately and then adding them together

$$\int_a^b k \bullet f(x)dx = k \bullet \int_a^b f(x)dx$$

Constant Multiple: If an integral can be written as the product of a constant and a function, then the constant can be moved outside of the integral

$$\int_a^b f(x)dx = \int_{a+c}^{b+c} f(x-c)dx$$

Shift: If a function is shifted horizontally by a constant, the bounds of the integral must also change for the original integral to remain equal to the new integral