Calculus 5.3 Key Points

Optimization:

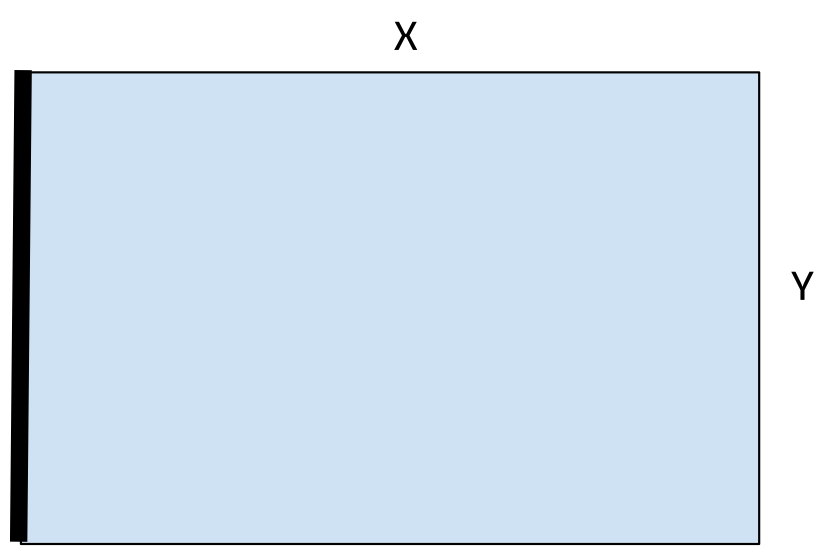
There are many, many different types of optimization problems, which you will often see in real-world situations.
As a general rule:

1. Identify that function for the quantity you want to maximize/minimize (This could be area, volume, cost, etc.)
2. Solve for where the derivative of that equation equals zero to find where that function has minimums/maximums
3. ANSWER THE QUESTION. Make sure you go back and find the answer to the original question and see if your solution makes sense based on the context of the problem.

Optimization Example:

1. You want to build an enclosed pasture. One side of the pasture already has a wall, so you will only need to fill in the other three sides. You have 60 meters of fencing available. Find the dimensions of the pasture that will give you the maximum possible area for the enclosure.

Solution: Let's start by making a diagram



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If we set one side of the enclosure to x and the other side to y, then the area of our enclosure (which we want to optimize) can be represented by the equation $A(x, y) = xy$

We want to optimize the area for this equation, but we have two variables in it, and we want to take the derivative with only one variable in our equation. We can rewrite one of our variables in terms of the other, using the fact that we have 60 meters of fencing.

Note: I will rewrite y in terms of x, but rewriting x in terms of y will also yield the same answer.

Let's solve for y from our perimeter function:

$2x + y = 60$

$y = 60 - 2x$

Now, we can substitute this back into our area equation:

$A(x, y) = xy \Rightarrow A(x) = x \bullet (60 - 2x) = 60x - 2x^2$

Then, we can take the derivative of the area function, which gives us

$A'(x) = 60 - 4x$

We can set this equal to zero to solve for our maximum:

$60 - 4x = 0$

$-4x = -60$

$x = 15$

Now that we have solved for one of the sides, let's return to our perimeter equation to solve for y:

$2(15) + y = 60$

$y = 30$

Answer: The enclosure's dimensions are 15 meters by 30 meters.