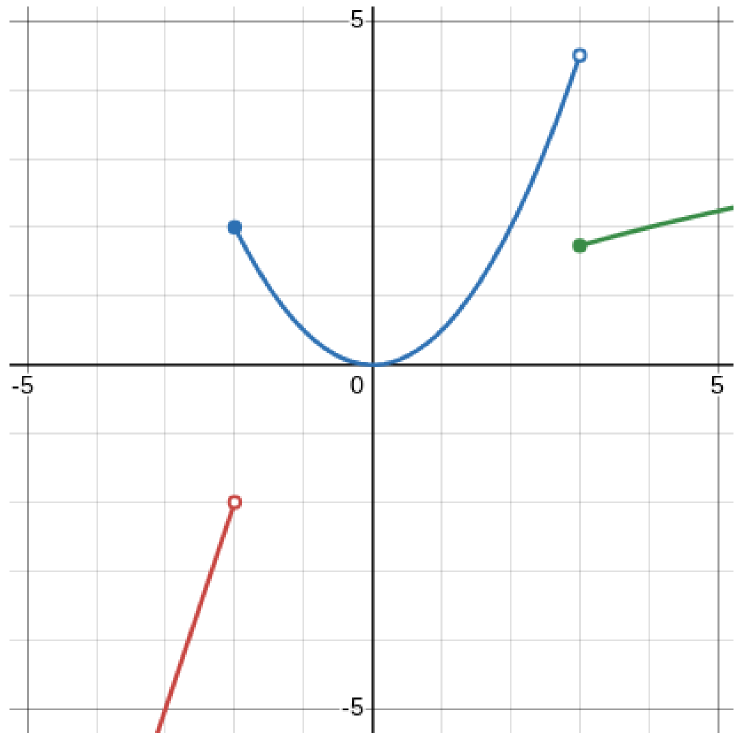
Calculus 1.2 Key Points

Piecewise Functions:

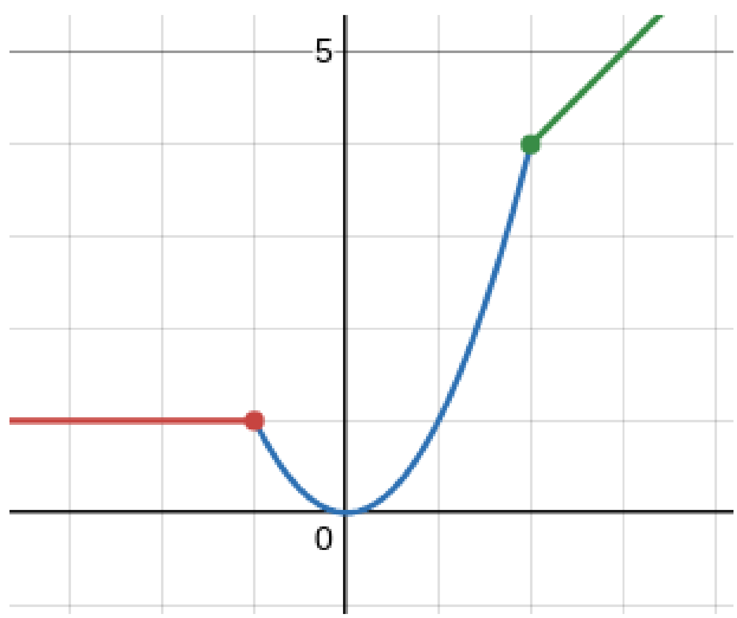
Piecewise functions are composed of multiple functions that typically have different, non-overlapping domains

$$f(x) = \begin{cases} 3x + 4, \ x < -2 \\ 0.5x^2, \ -2 \leq x < 3 \\ \sqrt{x}, \ x \geq 3 \end{cases}$$



Closed circles (•) indicate that the point is part of the function, while an open circle (◦) shows that the point is not part of the function

$$f(x) = \begin{cases} 1, \ x \leq -1 \\ x^2, \ -1 < x < 2 \\ x + 2, \ x \geq 2 \end{cases}$$



Because the segments of this piecewise function are connected without any gaps in between, this function is <u>continuous</u>. If you were to draw the graph of this function, you wouldn't have to lift your pencil while drawing it.

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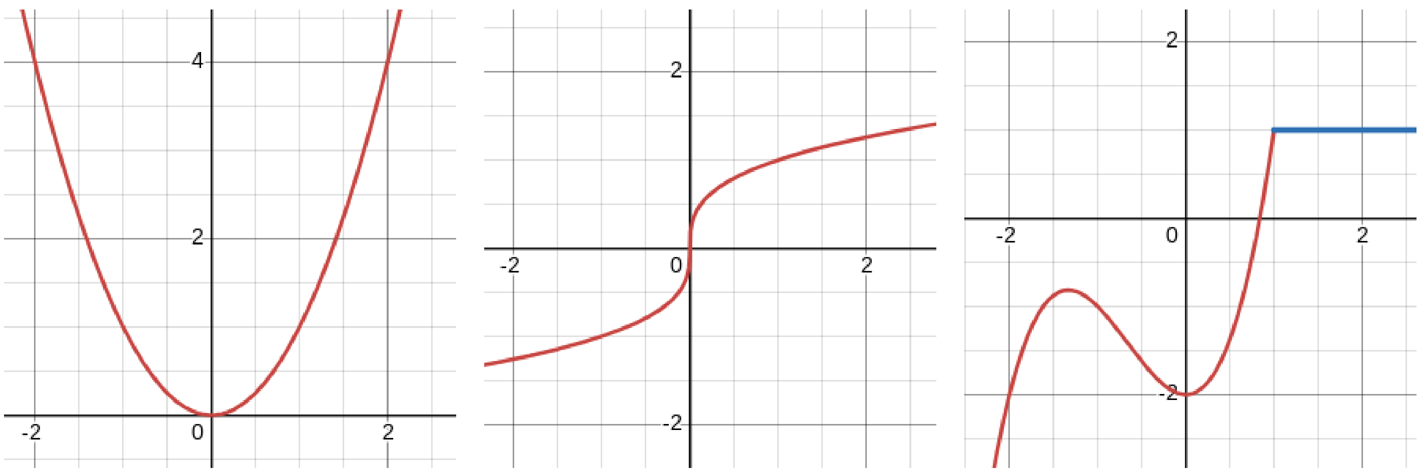
Continuity Intuition:

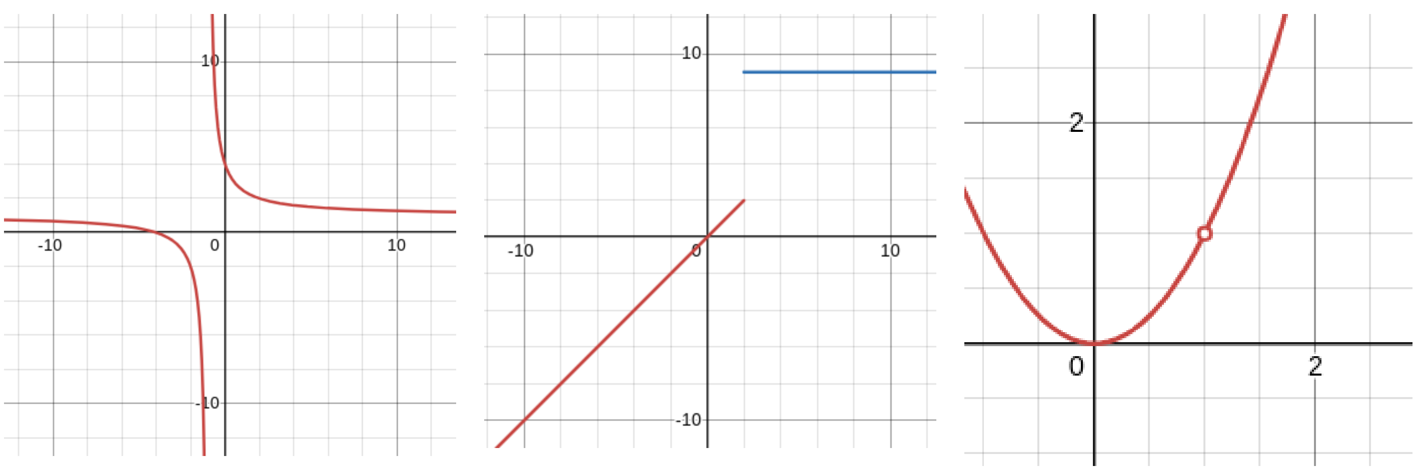
Continuity is the idea that all inputs over an interval have a defined output, without any holes, jumps, gaps, asymptotes, or other discontinuities

Examples:

Continuous Functions



Discontinuous Functions



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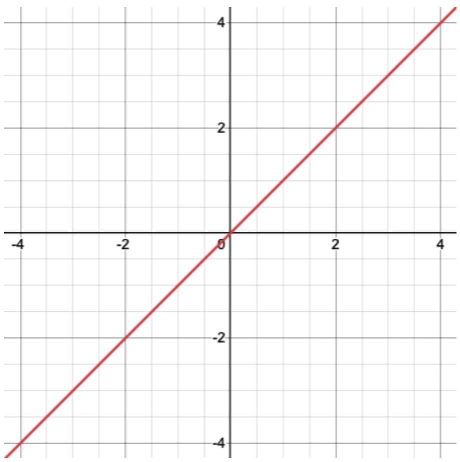
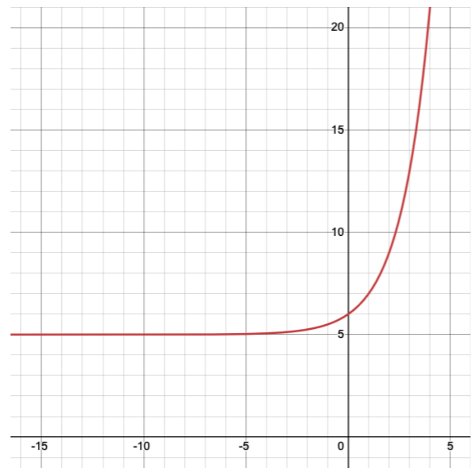
End Behavior:

End behavior describes how a function behaves as it approaches negative and positive infinity

$f(x) = x$	$f(x) = 2^x + 5$
	
As x approaches negative infinity, $f(x)$ approaches negative infinity. As x approaches (positive) infinity, $f(x)$ approaches (positive) infinity.	As x approaches negative infinity, $f(x)$ approaches 5. As x approaches (positive) infinity, $f(x)$ approaches (positive) infinity.
As $x \to -\infty$, $f(x) \to -\infty$ As $x \to \infty$, $f(x) \to \infty$	As $x \to -\infty$, $f(x) \to 5$ As $x \to \infty$, $f(x) \to \infty$

Asymptotes:

Asymptotes describe a line which a curve approaches but never reaches.

There are 3 main types of asymptotes: Horizontal, Vertical, Slant/Oblique

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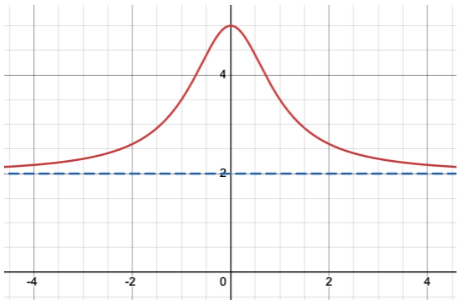
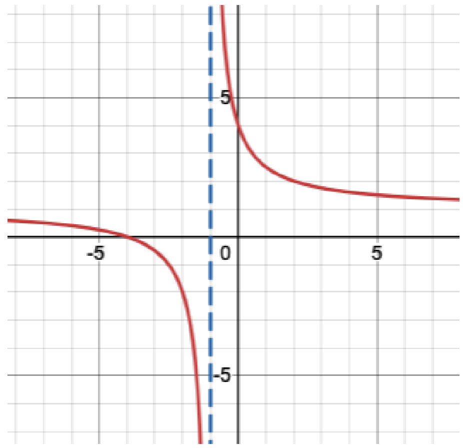
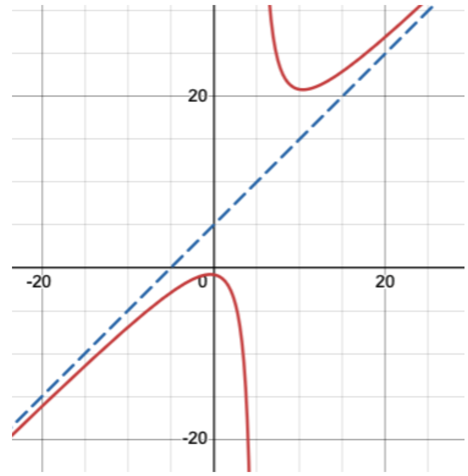
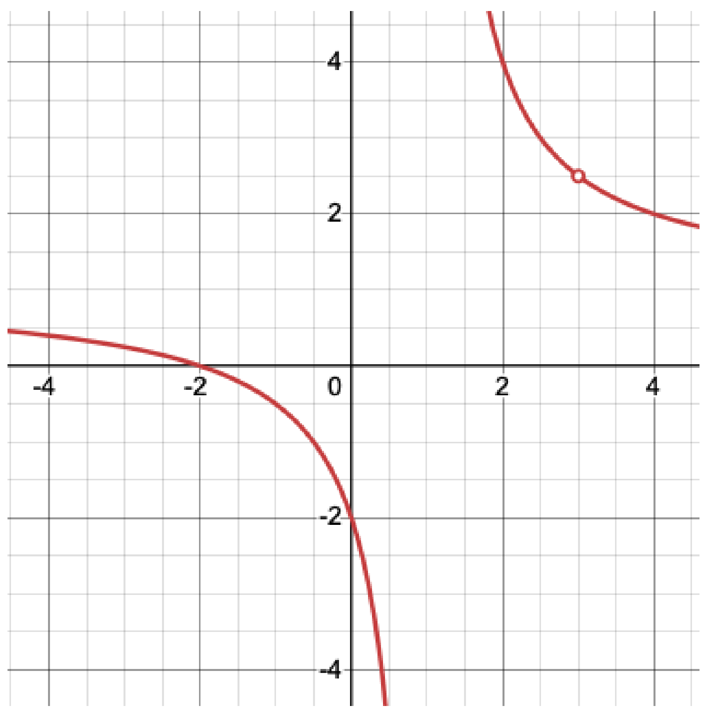
Horizontal	Vertical	Slant
		

Holes:

Holes are single points where a function is undefined, represented by an open circle on a graph.



$f(x) = \frac{x^2 - x - 6}{x^2 - 4x + 3}$

To find the hole, factor the function.

$f(x) = \frac{(x-3)(x+2)}{(x-3)(x-1)} = \frac{x+2}{x-1}$

Any factors that cancel out are holes, so there will be a hole when $x = 3$. To find the y value of the point, plug 3 into the function after the factors are canceled out.

$\frac{3+2}{3-1} = \frac{5}{2} = 2.5$

The hole is at $(3, 2.5)$

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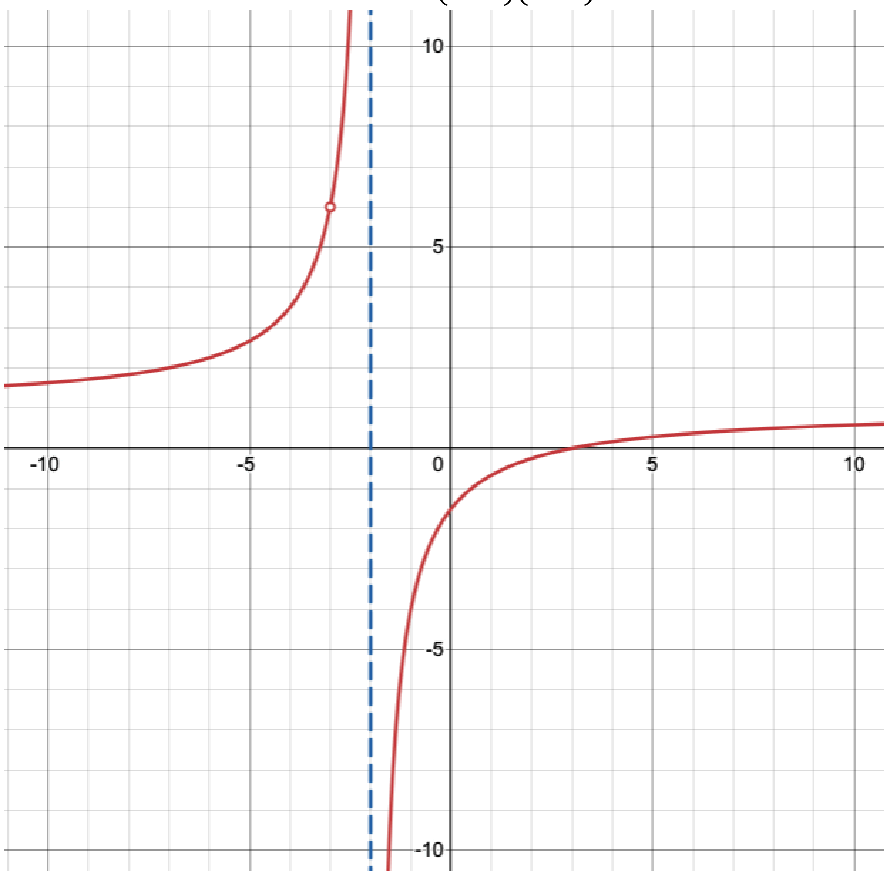
Approach Statements:

Approach statements describe how a function behaves around various locations, typically including the ends of the function's domain and any discontinuities.

$$f(x) = \frac{x^2-9}{(x+3)(x+2)}$$



For the approach statement for this function, we need to describe how the function behaves at negative and positive infinity, as well as its hole at $(-3, 6)$ and vertical asymptote at $x = -2$

As $x \to -\infty$, $f(x) \to 1$
As $x \to -3$, $f(x) \to 6$
As $x \to -2^-$, $f(x) \to \infty$
As $x \to -2^+$, $f(x) \to -\infty$
As $x \to \infty$, $f(x) \to 1$

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Composite Functions:

Composite functions are functions that are created by inputting one function into another function

For example, suppose $f(x) = 4x^2 + 1$ and $g(x) = x - 3$

A composite function can be created by inserting one of the functions into the other by replacing x with that function

$f(x) = 4x^2 + 1$ $\qquad$ $g(x) = x - 3$

$f(g(x)) = 4(x - 3)^2 + 1$ $\qquad$ $g(f(x)) = (4x^2 + 1) - 3$

To evaluate $f(g(1))$, simply plug 1 into $g(x)$ and plug the result into $f(x)$
Alternatively, plug 1 into the composite function $f(g(x))$

$g(1) = (1) - 3 =- 2$ $f(g(1)) = f(- 2) = 4(- 2)^2 + 1 =$ $16 + 1 = 17$	$f(g(x)) = 4(x - 3)^2 + 1$ $f(g(1)) = 4((1) - 3)^2 + 1 = 17$

Note: An alternate notation for $f(g(x))$ is $(f \circ g)(x)$

Inverse Functions:

An inverse function, denoted by f^{-1}, is where the inputs and outputs are swapped

To find the inverse of a function, swap x and y and solve for y

$f(x) = 3x - 4 \quad \Rightarrow \quad y = 3x - 4 \quad \Rightarrow \quad x = 3y - 4$

$x + 4 = 3y$

$\frac{x+4}{3} = y \quad \Rightarrow \quad f^{-1}(x) = \frac{x+4}{3}$

Calculus 1.2 Key Points

To verify that two functions are inverses of each other, use function composition and plug each function into the other. If both functions simplify to x, they are inverses

Example of inverses:

$$f(x) = 3x - 4$$

$$g(x) = \frac{x+4}{3}$$

$$f(g(x)) = 3(\frac{x+4}{3}) - 4 =$$
$$x + 4 - 4 =$$
$$x$$

$$g(f(x)) = \frac{(3x-4)+4}{3} =$$
$$\frac{3x}{3} =$$
$$x$$

Example of not inverses:

$$f(x) = 4x^3 - 2$$

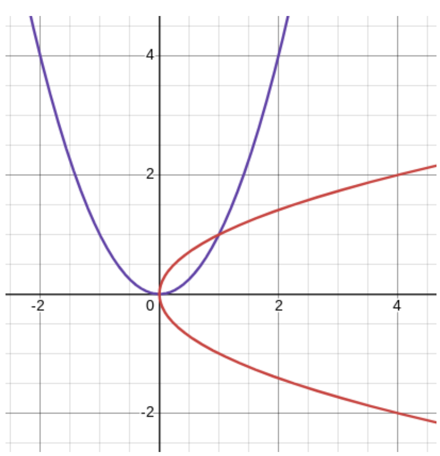
$$g(x) = \sqrt[3]{\frac{x+2}{2}}$$

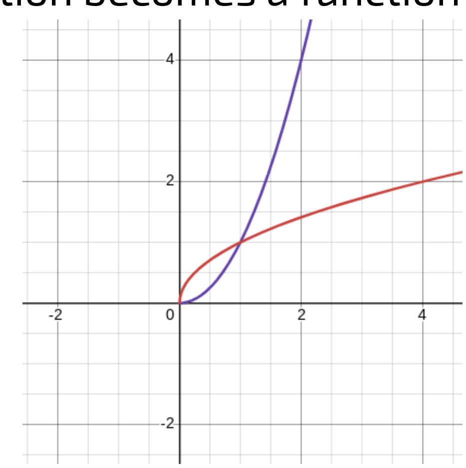
$$f(g(x)) = 4(\sqrt[3]{\frac{x+2}{2}})^3 - 2 =$$
$$4(\frac{x+2}{2}) - 2 = 2x + 4 - 2 =$$
$$2x + 2$$

$$g(f(x)) = \sqrt[3]{\frac{(4x^3-2)+2}{2}} =$$
$$\sqrt[3]{\frac{4x^3}{2}} = \sqrt[3]{2x^3} =$$
$$x\sqrt[3]{2}$$

Not all inverses are functions, however. Take $f(x) = x^2$

The inverse equation is $y = \pm\sqrt{x}$, which is not a function since there are multiple outputs for inputs greater than 0. But if we restrict $f(x) = x^2$ to the domain $x \geq 0$, then our inverse equation becomes a function



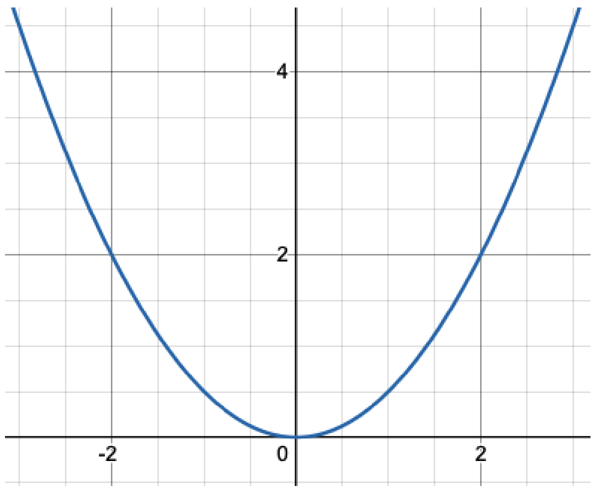


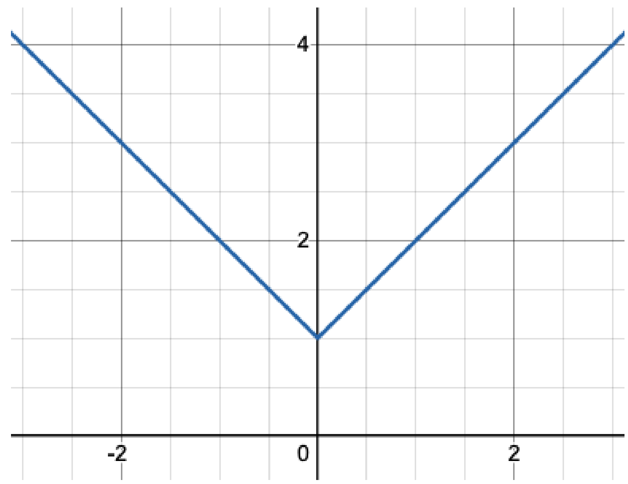
Researched & Developed by: Pi-Rates Add to your knowledge

Calculus 1.2 Key Points

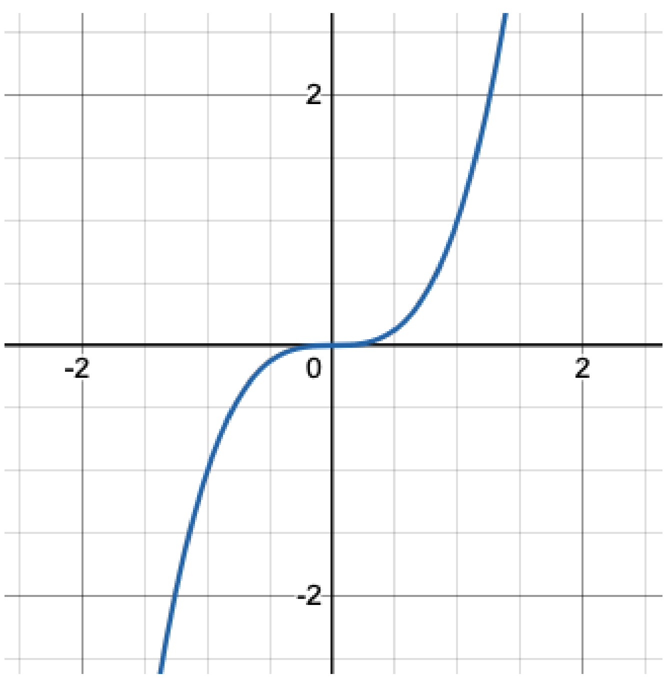
Even and Odd Functions:

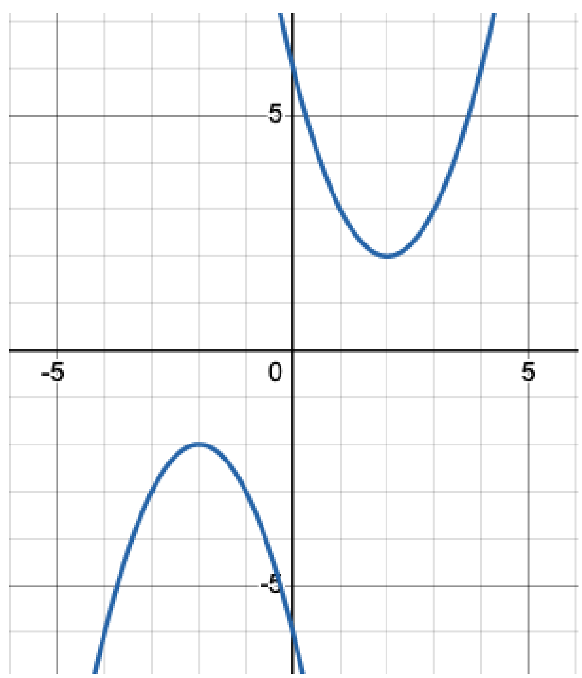
Even functions are symmetrical about the y-axis and flipping the graph across the y-axis will result in the same graph





Odd functions are symmetric to the origin and rotating the graph by $180°$ about the origin will result in the same graph





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To determine whether a function is even, odd, or neither algebraically, plug in $-x$ into the function $f(x)$. If the result is $f(x)$, then the function is even. If the result is $-f(x)$, then the function is odd. Otherwise, it is neither even nor odd.

Examples:

$f(x) = x^2$ $f(-x) = (-x)^2$ $f(-x) = (-x)(-x)$ $f(-x) = x^2$ $f(-x) = f(x)$ This function is even	$f(x) = x^3$ $f(-x) = (-x)^3$ $f(-x) = (-x)(-x)(-x)$ $f(-x) = -x^3$ $f(-x) = -f(x)$ This function is odd	$f(x) = 3x + 1$ $f(-x) = 3(-x) + 1$ $f(-x) = -3x + 1$ $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$ This function is neither even nor odd