

## Formal Specification of BSV's Elaboration and Dynamic Semantics

Rishiyur S. Nikhil

Bluespec, Inc., July 28, 2015

### Abstract

BSV is a High Level Hardware Design Language (HLHDL), intended for the full spectrum of digital hardware designs, from signal-processing accelerators to CPUs, SoCs and all kinds of Intellectual Property Blocks (IP Blocks). BSV has been in use commercially and in academia since the early 2000s.

Although BSV was inspired by formally-defined languages (specifically the Haskell functional programming language and Term Rewriting Systems), its formal elaboration and dynamic semantics have not been properly documented to date. So far, this has not been an issue, but there is now a growing interest in using BSV for formally verified hardware, because of its clean and high level semantics.

This document provides a formal specification of BSV elaboration and dynamic (execution) semantics. The specification is written as a Haskell program and is therefore executable. This document is accompanied by the full Haskell code as well as the (kernel) BSV source code for the fully-worked out examples which are described in this document.

## 1 Introduction and History

BSV [6] is a High Level Hardware Design Language (HLHDL), intended for the full spectrum of digital hardware designs, from signal-processing accelerators to CPUs, SoCs and all kinds of Intellectual Property Blocks (IP Blocks). BSV has been in use commercially and in academia since the early 2000s.

Acronyms

HDL Hardware Description Language

HLHDL High Level Hardware Description Language

HLS High Level Synthesis

HW Hardware

PL Programming Language SoC System on a Chip

SW Software

TRS Term Rewriting Systems

### 1.1 Historical context

From the beginning, BSV has been inspired by formally-defined languages, specifically Term Rewriting Systems [19, 30, 4] (TRSs) and the Haskell functional programming language [28]. It started

with research by Shen and Arvind at MIT to use Term Rewriting Systems to formally specify and verify complex concurrent hardware, such as deep processor pipelines and distributed coherent caches for multi-core systems [2, 29]. Similar approaches (using rewrite rules to specify complex concurrency) may also be found in TLA+ [20] and its use to verify cache coherence protocols [1, 18], in UNITY [7] and in Event-B [23].

From the specification research of Shen and Arvind emerged the idea of directly *synthesizing* hardware from TRS specifications. This idea was developed significantly by Hoe and Arvind [12, 13, 11] at MIT.

In 2000, activity on productizing these ideas moved from MIT into Sandburst Corporation. There, Lennart Augustsson et al. designed and implemented a new language called Bluespec [3]. Whereas the previous prototype from MIT had only rudimentary circuit-description facilities, focusing more on solving the problem of synthesis from TRSs, the new language borrowed heavily from Haskell to also provide a powerful circuit description language (also called "static elaboration" in HDL jargon). It had Haskell-like syntax, type definitions, polymorphism, type classes, higher-order functions, total absence of side effects, laziness, and a monadic structure for static elaboration. Although it was an industrial-strength language, it was only intended for internal use inside Sandburst. Nowadays we refer to this second-generation language as "Bluespec Classic".

In 2003, activity moved again, to Bluespec, Inc., with an aim to make it a publicly available Hardware Design Language (HDL). In this third generation, first released in 2004, its syntax was changed from Haskell's very spare syntax to something more similar to SystemVerilog [17] which is more familiar to hardware design engineers, the primary audience. The name of the language was changed to BSV (Bluespec SystemVerilog). Support for multiple clock domains was added, with statically typed clocks and resets and clock domain discipline. But apart from this syntax change and addition of clocks, the underlying language and its basic semantics has remained the roughly same (even today you can successfully compile and run BSV programs from 2004!).

## 1.2 Why no earlier formalization?

Like many industrial-strength languages, BSV is not a tiny language (although the language-definition part of the Reference Guide [6] is barely 150 pages, compared to more than 1300 pages for SystemVerilog [17]), and so defining formal semantics in gory detail is not trivial (and may not be readable by anyone!). BSV's semantics is described informally in the Reference Guide, in a number of books and papers (e.g., [24, 27, 25, 26]), and in Bluespec's BSV training materials. By and large this has been perfectly adequate for practical use.

There is now a strong emerging interest in *mechanized* formal verification of computing systems that include components implemented in hardware. The interest spans CPUs, memory systems and SoCs as well as hardware accelerators, hardware implementations of cryptography, and so on. There are many groups—commercial, non-profit research and academic—who are keen on using BSV for this purpose because it is both industrial-strength and has a very clean and high level semantics. This activity requires formal specification of BSV, hence this document.

### 1.3 Document overview

This document is accompanied separately by Haskell code that specifies the semantics. The Haskell code is executable, and comes with several example "kernel BSV" programs on which it can be run.

That Haskell code, and not this document, should be regarded as the official formal specification. This document should be regarded as a reading guide to that Haskell code. The Haskell code fragments shown here are from the accompanying Haskell code. In case they are out of sync, the accompanying Haskell code should be taken as definitive.

Note (July 27, 2015): This is the first public release of this document and the Haskell code. It is quite possible that both the semantics and this document need fixes and/or refinement. The author gladly welcomes comments and suggestion from readers.

A detailed table of contents follows next. Then, Sec. 3 provides some general commentary about our approach. Sec. 4 describes the concrete and abstract syntax of our kernel language. Sec. 5 talks briefly about parsing from concrete to abstract syntax. Sec. 6 describes static elaboration of the source program into a collection of module instances.

Sec. 7, "Dynamic Semantics", is the heart of this document. Readers who are experienced at reading Haskell and/or formal semantics of languages may wish to skip preceding sections and go there directly.

Sec. 8 presents a set of four example kernel BSV programs and their semantic interpretation (included with the accompanying Haskell code). The first two are explained in detail and the latter two are left as exercises for the reader (although we provide Haskell execution transcripts for all of them). Sec. 9 concludes with some comparisons to other hardware description languages and with a discussion of the relationship of this formal semantics to the full BSV language and its compilation with the *bsc* compiler.

## 2 Table Of Contents

1	Intr	roduction and History	1
	1.1	Historical context	1
	1.2	Why no earlier formalization?	2
	1.3	Document overview	2
2	Tab	le Of Contents	3
3	Pre	liminary comments	5
	3.1	Static elaboration and static vs. dynamic semantics	5
	3.2	TRSs, Linear Orders, and Clocks	5
	3.3	Haskell executability	6
	3.4	Kernel language only	6
	3.5	Rule firing constraints due to module sharing	6
	3.6	The semantics does not rely on any built-in primitives	7
	3.7	Schedule-construction is an orthogonal question and is not addressed here	7

4	Ker	rnel Language Concrete and Abstract Syntax	8	
	4.1	Programs	8	
	4.2	Bindings	9	
	4.3	Module Definitions	9	
	4.4	Rules	10	
	4.5	Method Definitions in Modules	11	
	4.6	Expressions	11	
5	Par	sing from concrete syntax to abstract syntax	13	
6	Static Elaboration			
	6.1	Module instances	14	
	6.2	The Static Elaboration function	15	
7	Dyr	namic Semantics	16	
	7.1	The central intuitions behind clocks and schedules	16	
	7.2	Formal description	17	
		7.2.1 Evaluating a Rule Expression	19	
		7.2.2 Checking whether the rule has intra-rule conflicts	22	
		7.2.3 Checking whether the rule has inter-rule conflicts	23	
		7.2.4 Checking whether the rule has hardware conflicts	23	
		7.2.5 Firing the rule (performing its actions)	24	
8	Examples			
	8.1	GCD	25	
		8.1.1 Executing the semantics	27	
	8.2	PipelineFIFO	31	
		8.2.1 Executing the semantics with schedule 1	33	
		8.2.2 Executing the semantics with schedule 2	34	
	8.3	BypassFIFO	36	
	8.4	PipeFwdBwd	36	
9	Conclusion and closing observations			
	9.1	Relationship to RTL and Chisel: Atomic transactions	37	
	9.2	Relationship to HLS (High Level Synthesis)	38	
	9.3	Relationship of this formal semantics to the full BSV language	38	
	9.4	Relationship of this formal semantics to the $bsc$ compiler	39	

## 3 Preliminary comments

Certain aspects of BSV's formal semantics may seem novel to those already familiar with semantics of programming languages or TRSs. We discuss them in the following sections.

## 3.1 Static elaboration and static vs. dynamic semantics

All SW programming languages have a dynamic control structure. Specifically, *frames* are pushed and popped on a stack as we dynamically call and return from functions, methods and exceptions.

In hardware, there is no such dynamic control structure. Hardware consists of physical electronics; they do not grow or shrink during execution!<sup>2</sup>

Nevertheless, you will find functions and procedures in all modern HDLs. In BSV you will also see extensive use of methods, loops, and recursion. These are merely linguistic devices for *circuit description*, i.e., they are unfolded, or inlined by the HDL compiler/interpreter to describe a circuit. It is almost like statically pre-building the possible tree of stack frames for a SW program just by looking at its text.

In SW programming languages, *static semantics* usually refers to syntax, scoping and other well-formedness checks, type-checking, and perhaps some simple syntax desugaring. The "output" of static semantics is still a program text that may be recognizably similar to the original program text. Everything after that is described by *dynamic semantics*, including dynamic control behavior like conditionals, loops, function calls and returns, recursion, exceptions, and so on.

In HDLs, static semantics covers the same topics as in SW PLs, but goes further to encompass static elaboration, i.e., the unfolding of linguistic constructs like function application, method calls, elaboration loops and recursion, and so on. The result of static elaboration is a hardware circuit. This hardware circuit may have a logical hierarchical structure, i.e., a top-level module containing instantiated sub-modules which, in turn, may contain next-level instantiated sub-modules, and so on. At the leaves of this hierarchy will be primitive modules such as registers, FIFOs, RAMs, and I/O devices. All the modules are connected by combinational circuits. In HDLs, dynamic semantics concerns the behavior of this statically elaborated circuit.

In this document, we will not discuss the part of BSV static semantics that coincides directly with Haskell (specifically types, polymorphism, type classes, type-checking and monadic elaboration). We will describe BSV's static elaboration, because modules, module instances, and module *sharing* are fundamental concepts.

## 3.2 TRSs, Linear Orders, and Clocks

Classical TRSs are non-deterministic, and have no notion of clocks. The basic dynamic semantics of TRSs is almost trivial:

• Given the current state (a "term"), find a part of the state (a subterm) that matches the left-hand side of one of the rewrite rules. (There may be many such matches; choose one non-deterministically.)

<sup>&</sup>lt;sup>1</sup>The only exceptions are ancient languages like early FORTRAN and perhaps a few languages for embedded programs.

<sup>&</sup>lt;sup>2</sup>Of course it is always possible to *represent* a dynamic stack in memory in a hardware system, but that is a level of abstraction above our current level of discourse, namely physical hardware.

- Replace that subterm by the right-hand side of the rule (with suitable variable substitutions), producing the next state. We call this a *rule firing*.
- Repeat and rinse.

The description above is sequential (one rule at a time) and may seem at odds with our earlier claim that this is a favorite way amongst researchers to model complex concurrency. The apparent paradox is resolved by realizing (a) that it is non-sequential in that there is no prescribed "control flow" or "program counter" to sequence rules; it is purely reactive—which rule can fire depends only on the instantaneous state, and (b) any actual concurrent execution that preserves the logical sequentiality of the semantics is acceptable (linearizability). Another way of saying this is that each rewrite is an atomic transaction on the state [21].

BSV goes further and refines TRS semantics to include *clocks*, which are fundamental to most digital HW<sup>3</sup>:

- Every BSV rule fires "within" a clock.<sup>4</sup>
- Many BSV rules may fire within a clock, but there will always be a *logical linear order* that explains their behavior.
- There will be practical constraints (described later) that preclude certain rules from firing within the same clock in a certain order.

## 3.3 Haskell executability

Our specification is given as a Haskell program, and is executable. It can be characterized as a high-level implementation-independent operational semantics.

## 3.4 Kernel language only

As is common in the literature, we will describe BSV's semantics for a *kernel* or subset of the language. Extending it to the full language would be routine but tedious to describe and to understand, without providing any useful additional illumination. Nevertheless, our kernel language is more comprehensive than previous toy descriptions of BSV semantics, and captures the essence of the actual BSV code behavior (specifically: module instances and module sharing). Further, it is *the* mental model that one uses in practice when developing and debugging BSV code.

## 3.5 Rule firing constraints due to module sharing

As mentioned earlier, we will be introducing constraints that prevent certain rules from firing within a clock in a certain order. Many of these constraints have functional motivations. For example, whether a FIFO logically enqueues before it dequeues, or vice versa, can affect its functionality in certain corner cases (when it is full or empty).

<sup>&</sup>lt;sup>3</sup>Digital circuits can also be *asynchronous* or clock-free. It is a fascinating question whether a BSV-like language can be compiled to asynchronous hardware, but we do not explore that further here.

<sup>&</sup>lt;sup>4</sup>This is a design choice based on current tractability; a more general future approach may allow rules to use multiple clocks.

Other constraints arise due to *module sharing*. In digital hardware, a wire can be driven with only one, stable value ("0" or "1") during each clock. Consider two independent rules that each occasionally drive a particular input wire of a shared module. Those rules therefore cannot fire in the same clock. Our semantics will capture these constraints that arise due to module sharing.

## 3.6 The semantics does not rely on any built-in primitives

This semantics does not prescribe any particular predefined set of primitives (such as registers). For example, it is perfectly feasible with this semantics to define a version of BSV based entirely on FIFOs or RAMs, or even registers with different semantics. This semantics is, in effect, parameterized by a set of primitives. All it assumes is that each primitive is supplied with the primitive semantics of the methods when invoked (within an Action) and the parallel (intra-rule) and concurrent (inter-rule) ordering constraints on its methods.

Of course in practice our primitives include registers, Concurrent Registers, FIFOs, BRAMs and more. The accompanying Haskell code includes registers and Concurrent Registers (CRegs).

## 3.7 Schedule-construction is an orthogonal question and is not addressed here

We use the term *schedule* to mean a linear sequence of rule instances. In each clock, we will execute rule instances according to such a schedule (execution of a rule instance may be a no-op, because of conflicts or because the rule is not enabled).

It is important to distinguish two quite separate questions:

- How should one to construct a schedule from the rules in a BSV program? Typically, this
  is the job of a compiler which, may, for example, try to construct a schedule that maximizes
  concurrency.
- Given a schedule, what is a *correct execution* of the schedule?

In this document, we only address the latter question. A schedule is provided as an argument to the main semantic function, and may as well be provided by an oracle that selects the next rule instance to be considered at the very last moment (when we are ready to execute the next rule instance in the current clock). Thus, the semantics defined here can be seen as *universally quantified over all possible schedules* i.e., describing correct execution for any possible schedule.

In this semantics there is no such thing as a "wrong" schedule. Of course, from a practical point of view, some schedules are likely to be better than others because, for example, they admit more concurrency (more rules can fire in a clock). This semantics takes no position on how to construct such a schedule.

Our description here is therefore more general than the current bsc compiler for BSV, which:

- Only considers each rule once during each clock.
- Statically fixes a priority (linearization) of the rules.

In contrast, the semantic description here admits schedules that may vary from clock to clock, and may include duplicate rule execution within a clock.

## 4 Kernel Language Concrete and Abstract Syntax

The kernel language syntax is described below both as abstract syntax (a Haskell data type) and as concrete syntax. The parser reads concrete syntax from a text file and produces a Haskell data structure corresponding to the abstract syntax.

In the accompanying Haskell code, the file AST.hs defines the abstract syntax data types. The file Parser.hs implements the parser using Haskell's Parsec parser combinators.

Metasyntax: we use conventional BNF below for concrete syntax, with the following conventional extensions:

```
\{x\} means zero or more repetitions of x

x \dots x means zero or more repetitions of x

[x] means zero or one repetitions of x (i.e., x is optional)
```

Typewriter font is used for terminals, and *italics* for non-terminals.

## 4.1 Programs

A kernel BSV program is a list of module definitions followed by a schedule.

## Concrete syntax:

```
Program ::= Module Definition ... Module Definition Schedule
```

## Abstract syntax:

```
type Program = ([Binding], Schedule)
```

In the abstract syntax, each module definition is just a binding of the module name to a lambdaexpression whose arguments are the module parameters and whose body is a module expression.

There must be exactly one module definition called main, and it must have zero parameters. Static elaboration is performed by applying this niladic lambda expression (to an empty argument list).

Static elaboration will recursively instantiate modules; this is sometimes called the module hierarchy. Thus, each module instance has a unique hierarchical name, which is a sequence of identifiers that identify a path down this recursion, starting with main.

The hierarchical name of any module instance's component (such as a rule or method instance) is simply the hierarchical name of its enclosing module instance appended with component's name.

```
type Ide = String -- Identifiers
type RuleName = String -- Rule names
type MethName = String -- Method names
type HierName = [Ide] -- Hierarchical name
```

The schedule is a list of hierarchical names, each referring to a unique post-elaboration instance of a rule.

```
type Schedule = [HierName] -- list of rule instances
```

Having a schedule at the end of the source file is just a convenience that allows us easily to exercise the semantics. As described in Sec. 3.7, this semantics takes no position on schedule construction, just on correct execution of any given schedule. One could write variants of this Haskell code where the schedule is constructed from analysis of the module definitions, or uses an run time oracle, etc. Some of our examples contain multiple schedules at the end of the source file, and we comment-out all but one of them for each execution.

## 4.2 Bindings

A binding associates an identifier with an expression/value.

## Concrete syntax:

```
Binding ::= let Ide = Expr
```

**Abstract syntax:** 

```
type Binding = (Expr, Expr)
```

The lhs uses Expr instead of Ide to allow for possible future extension where it could be an array selection, struct selection or pattern.

## 4.3 Module Definitions

The word "module" is used in quite different ways in different languages and often concerns mechanisms for separate compilation and namespace management, without any serious role in dynamic semantics. In BSV, as in many HDLs, a module is a fundamental dynamic semantic entity, representing a hardware unit with an interface. Source programs contain module expressions which evaluate to modules. Elaborated programs contain module instances. A single module expression may result in multiple module instances. A module instance may be *shared*, which can affect dynamic semantics.

A module expression is an expression which contains a sequence of bindings, rules, and interface method definitions.

## Concrete syntax:

The optional "# ( *Ide* , ... , *Ide* )" on the first line represents parameters to the module. If missing parameter, it is treated as #(), the empty argument list.

In the abstract syntax, such a module definition binding is converted into a Binding that binds the *Ide* (name of module definition) to a lambda-expression whose arguments are the parameters and body is a module-expression:

```
Binding: (Ide, (Lambda [Ide ... Ide] module-expression))
```

**Abstract syntax** for module-expressions:

```
data Expr = ... | ModuleExpr [Binding] [Rule] [MethDef] -- module-expression
```

(Expr is defined in more detail later.)

## 4.4 Rules

Rules are defined inside modules.

## Concrete syntax:

```
Rule ::= rule \ Ide \ ( [ \ Expr \ ] \ ) \ ; Stmt ... Stmt endrule
```

The Expr is the rule condition (a boolean expression). "Stmt ... Stmt" is a list of zero or more statements constituting the rule body. In the abstract syntax, the rule condition and body are combined into a "When" expression (described in more detail later in Sec. 4.6), and the statements are combined into a Block.

```
Rule Ide (When Expr (Block [Stmt \dots Stmt]))
```

## **Abstract syntax:**

## 4.5 Method Definitions in Modules

Methods are defined inside modules.

## Concrete syntax:

The *MethodKind* specifies whether this is a value method (V), Action method (A) or ActionValue method (AV). Value methods are pure (no side-effects). Action methods just have a side effect, and do not return any value. ActionValue methods have a side effect and return a value.

This is followed by an *Ide*, the name of the method, and the arguments of the methods in parentheses. This is optionally followed by the keyword **if** and the method condition, a boolean *Expr*. A missing method condition is treated as **if** (True). Then we have the method body, which is a sequence of statements.

In the abstract syntax, this is converted into a MethDef that contains the method kind, the method name, and a When-expression. whose condition is the method condition and whose body is a lambda-expression. The lambda-expression's arguments are the method arguments, and the body is a Block containing the method's statements:

```
MethDef {methKind = "A/V/AV", methName = Ide, methBody = When Expr
Lambda [Ide \ldots Ide]
Block [Stmt \ldots Stmt]
```

### **Abstract syntax:**

## 4.6 Expressions

Finally, we can describe all the different kinds of expressions.

#### Concrete syntax:

```
Expr ::= ( Expr ) Parenthesized expression Ide Identifier ( ) Void literal IntegerLiteral integer literal
```

StringLiteralboolean literal True | False if ( Expr ) Expr else Exprwhile ( Expr ) ExprExpr . Ide Method value begin StmtStmt end

string literal

conditional

loop

block

## Abstract syntax:

```
data Expr = EIde String
                                    -- Identifier
          | Void
                                    -- Constant void
          | ConstI Int
                                    -- Constant integer
                                    -- Constant string
          | ConstS String
          | PrimFn Opcode
                                    -- Constant primitive function value
          | Lambda [Expr] Expr
                                    -- args have form 'Elde x'
          | App [Expr]
                                    -- apply e0 to e1, e2, ...
          | If Expr Expr Expr
                                    -- Conditional
          | While Expr Expr
                                    -- Loop
          | MethodVal Expr MethName -- Method value: Expr is a module instance
          | Block [Stmt]
                                    -- Block
          | When Expr Expr
                                    -- When guard-cond value
          | ModuleExpr [Binding] [Rule] [MethDef]
          -- The following only appears during static elaboration
          | ModuleInstRef Int
                                -- index into global IntMap of module instances
          -- The following only appear during dynamic execution
          | Unavail
                                -- due to a method condition being False
          | Closure Env Expr
                                -- value of a lambda
data Stmt = StmtExpr Expr
          | StmtBinding Binding
```

The first several alternatives are quite conventional (identifiers, constants, lambda-expressions, applications, conditionals, loops). As usual, lambda-expressions will evaluate to closures at run time. A Block-expression contains a list (sequence) of statements, each of which can be either a let-binding or an expression (typically a method-call or function-call). The final value of the Blockexpression is the value of the last item: Void if it is a binding, and the expression value if it is an expression.

A When-expression has a guard (a boolean expression) and a body (an expression). At run time, a When-expression evaluates to the special value Unavail if the guard expression evaluates to False or is itself Unavail. The body is only executed if the guard evaluates to True.

During static elaboration, a ModuleExpr creates a new module instance which has a unique integer identifier (more about this later), and is replaced by a ModuleInstRef j value that refers to it.

A method value contains an expression and a method name. At run time, the expression evaluates to a particular module instance reference, and the method name selects a method in that module instance. In an application, the function value may be a primitive function value, a standard lambda-closure, or a method value.

## 5 Parsing from concrete syntax to abstract syntax

The Haskell file Parser.hs defines a parser from concrete syntax into abstract syntax, using the standard Haskell Parsec library. The main function is:

```
type Program = ([Binding], Schedule)
parseProgramFromString :: String -> Program
```

Its input is a String representing a complete program (typically taken from a source file), and its output is a pair: the list of top-level bindings (module definition bindings), and the schedule (list of hierarchical names identifying rule instances).

This is a vanilla use of Haskell's Parsec library, and is not particularly interesting for a discussion about semantics of the abstract syntax, so we do not discuss it further here.

## 6 Static Elaboration

The Haskell code for static elaboration is in the file StaticElab.hs.

The source program is parsed into a list of bindings and a schedule. Each binding associates an identifier to a lambda-expression whose body is a module-expression, as described in Sec. 4.3. The schedule is not relevant for static elaboration; it will only play a role in dynamic semantics, discussed in Sec. 7.

One of the bindings in a source program must bind the distinguished name main to module with no parameters (more specifically, to a niladic lambda-expression whose body is a module-expression). The *static elaboration* step applies this lambda expression (to zero arguments) to produce a statically elaborated program, which is (abstractly) a description of a concrete hardware module hierarchy with behavior expressed as rules.

The process of static elaboration is recursive: applying main will result in a module instance; the bindings in this module, when evaluated, will create other module instances. The bindings inside those module instances may, in turn, bind further module instances, and so on, recursively. A particular source-program module definition may be instantiated multiple times. Ultimately we have a hierarchy (i.e., a tree) of module instances, with an instance of main at the root.

#### 6.1 Module instances

A module instance is either an instance of a primitive module (built-in) or is an instance of a user-defined module in the source program (the value of a module-expression):

(Mnemonics: PMI = Primitive Module Instance, UMI = User Module Instance).

Here we show "Registers" and "Concurrent Registers" as primitives, but these can be replaced or extended with an arbitrary set of primitives such as FIFOs, RAMs, and so on. Each primitive has its own representation of internal state. For example, the Int value for Registers represents the current register data contents. For Concurrent Registers, the first Int represents the number of its concurrent ports, and the second Int represents its current data contents.

Static elaboration is a classical lambda-calculus "Eval" of the source program. This is supplied with an initial environment that binds the names of constructors for primitive modules to primitive functions:

When applied to arguments (during static elaboration), these primitive functions create (instantiate) PMI module instances.

When module-expressions are evaluated, they create UMI module instances. User-defined module instances are closures—the Env component represents the bindings in the module, which may be used by in its rules and method definitions.

For the dynamic semantics the module hierarchy is not important: the only important thing is that a rule in one module instance may invoke a method in another module instance, and thus must have a way to refer to another module instance. Thus, during static elaboration, we give each module instance a unique integer name, and we collect all module instances in a mapping (a Haskell IntMap object) that maps each unique integer name to a pair—the module instance's hierarchical name, and its instance value. We call this mapping a System:

```
type System = Intmap (HierName, ModuleInst)
```

Each time we evaluate a "ModuleExpr [Binding] [Rule] [MethDef]" expression, we create a new ModuleInst in the global system, and replace the expression with a ModuleInstRef Int value, which is a reference to the module instance.

```
data Expr = ... | ModuleInstRef Int -- index into global IntMap of module instances
```

### 6.2 The Static Elaboration function

Static elaboration is carried out by this function:

```
staticElab :: [Binding] -> (System, Expr)
staticElab bindings = eval [] ["main"] DI.empty (App [EIde "main"])
   where
    ...
```

The argument is the list of bindings from the source file produced by the parser. It merely calls the eval function, to be described next, giving it:

- an initially empty environment: [],
- an initial hierarchical name: ["main"]
- an initially empty map of module instances: DI.empty,
- and an initial application: [main] (main applied to no arguments).

Static elaboration is performed by a classical mutually recursive Eval-Apply pair performing evaluation and application (locally inside the staticElab function):

```
eval :: Env -> HierName -> System -> Expr -> (System, Expr)
app :: HierName -> System -> Expr -> [Expr] -> (System, Expr)
```

In both eval and app:

- The HierName argument is the current position in the module instance hierarchy, i.e, the list of module instance names above this recursive point.
- The System argument is the collection of module instances accumulated so far.

and the result is a pair: the updated System, and an Expr representing an evaluated value. In eval:

- The Env argument is the current environment binding identifiers to values.
- The Expr argument is the expression being evaluated.

Most of the clauses in eval are classical (identifiers, constants, primitive functions, lambdas, applications). The only unusual clause is for module definitions:

A module-expression contains bindings bs of identifiers to expressions (typically module instantiations), rules rs and method definitions rs. Here, we leave rs and rs untouched. We convert rs into an environment rs by evaluating each right-hand side expression to a value. We do this sequentially, using rs that values from earlier bindings are in scope for later bindings. We create a new user-defined module instance (rs instance). Finally, rs extendSystem adds this module instance to the system, allocating a new unique integer identifier rs, returning the new extended system and rs moduleInstRef rs as the value of the expression.

## In app:

- The Expr argument is the value of a function to be applied (either a primitive, or the closure of a lambda expression if user-defined).
- The [Expr] argument is a list of argument values for the function.

Most of the clauses in app are classical (primitive functions and lambda-closures). The only unusual clauses are for primitive module constructors. In each case, we simply extend the incoming system with new primitive module instances:

## 7 Dynamic Semantics

Having parsed and elaborated a program, we are ready to turn to the central focus of this document—the dynamic semantics of rules.

## 7.1 The central intuitions behind clocks and schedules

Hardware execution is a sequence of clocks. In each clock we execute a sequence of rules<sup>5</sup>. We call a sequence of rules a *schedule*.

 $<sup>^5</sup>$ We remind that the sequence of rules is only a logical concept to explain semantics and understand behavior. It may not be discernible at all in the clocked digital hardware that is the output of the bsc compiler.

As explained in Sec. 3.7, the semantics takes a schedule as a parameter, and takes no position on how that schedule is constructed, it just explains correct execution of the given schedule.

Thus, the central question answered by the semantics is:

In the current clock, given that we have already performed rule executions  $r_0, r_1, ..., r_{j-1}$  so far in this clock, and the schedule has determined that we must next attempt rule  $r_j$ ,

What is a correct execution of  $r_i$ ?

This step is repeated (within this clock) until the schedule offers no more rules for this clock.

And all this is repeated forever for multiple clocks.

Correct execution of a rule is straightforward, and is summarized informally in Fig. 1. Internalizing this will make it easy to understand the formal description that follows. This is also (roughly) the mental model used every day by the practicing BSV programmer to understand BSV program execution and the schedules produced by the *bsc* compiler.

## 7.2 Formal description

The code for the dynamic semantics is in the file Semantics.hs. We skip over the first few definitions, exec\_n\_clocks, findRule and initEnv because they are about specifying schedules (here, we arbitrarily read a schedule at the end of each source file) and repeated execution of a schedule across clocks, etc., which, as explained earlier, are not the central focus of this document.

Instead, we pick it up at the following function:

The system argument is the IntMap from integers to module instances.

The ruleSeq argument is a list of rule instances representing the schedule. Each is a triple of a hierarchical name that uniquely identifies the rule, an environment representing the bindings in the module instance where this rule is found, and a Rule-expression.<sup>6</sup>.

<sup>&</sup>lt;sup>6</sup> There is no assumption that the schedule is constant across clocks. In fact, an alternative formulation may replace the ruleSeq argument by an oracle function on the current system state and list of rules fired so far in a clock, i.e., it can take a last-minute instantaneous decision on which rule should be considered next.

# Correct execution of a rule in a clock, given that we have previously executed certain rules in this clock. (Informal Description)

- We first "evaluate" the rule (which is a When-expression). This is mostly a conventional lambda-calculus Eval-Apply pair, with a few variations:
  - In any When-expression (typically rule and method bodies), if the guard condition evaluates to False, the When-expression evaluates to the special value Unavail. In fact, the guard condition may itself evaluate to Unavail, in which case also the When-expression's value is Unavail.
  - We collect a record of every method called, whether on a primitive module instance or on a user-defined module instance.
  - We collect but do not perform applications of primitive Action method values to their argument values. Note, therefore, that, so far, rule evaluation has no side effects on the system state.
- After rule evaluation, we decide whether the rule will fire or not. Any of the following conditions will stop a rule from firing.
  - There is at least one *intra-rule conflict*, i.e., the rule invokes two methods  $m_1$  and  $m_2$  that are not allowed to be invoked within the same Action (simultaneously/instantaneously/parallel). These prohibitions are axiomatic properties of methods on primitive modules, and are given in a fixed intra-rule conflict table.
  - There is at least one *inter-rule conflict*, i.e., a rule executed earlier in this clock invokes a method  $m_1$  and this rule invokes a method  $m_2$ , and they are not allowed to be executed in that order. These prohibitions are axiomatic properties of methods on primitive modules, and are given in a fixed inter-rule conflict table.
  - The rule evaluates to Unavail. Either the rule condition expression, or the condition of some method (either in the rule condition expression or its body), evaluates to False.
- Finally, if we decide that the rule will fire, we execute the primitive Actions that we collected during rule evaluation. This transforms the system state. Conceptually, all these actions are performed simultaneously and instantaneously, i.e., in parallel.

Figure 1: The central idea in rule semantics: correct rule execution

The result is a pair. The Bool component indicates whether any rule fired in this clock. The System component is the updated system state (which will be the same as the system argument if the rule did not fire).

The exec\_1\_clock function is straightforward, just using foldM to apply the execRule function over the rules in the schedule, carrying along three values: a Bool indicating whether any rule fired, the current system state, and mcsPrev, the collection of methods called in rules that have fired so far in this clock.

A "method-called" is simply a method value, which is an expression:

```
type MC = Expr -- MethodVal (ModuleInstRef j) MethName
```

## 7.2.1 Evaluating a Rule Expression

The central function of this semantics is this one, which is a direct implementation of the informal explanation of Fig. 1.

```
execRule :: System -> [MC] -> (HierName, Env, Rule) -> IO (Bool, System, [MC])
execRule system mcsPrev (hname, env, Rule name e) =
 do -- Eval this rule
     (v,actions,mcsThis) <- evalExpr system env e</pre>
    let intra_cfs = intraRuleConflicts system mcsThis
    let inter_cfs = interRuleConflicts system mcsPrev mcsThis
     let hw_cfs
                = hwConflicts
                                         system mcsPrev mcsThis
     -- has intra-rule method conflicts
     if not (intra_cfs == []) then return (False, system, mcsPrev)
     -- has inter-rule method conflicts
     else if not (inter_cfs == []) then return (False, system, mcsPrev)
     -- has hardware method conflicts
     else if not (hw_cfs == []) then return (False, system, mcsPrev)
     -- Rule is not enabled (rule cond, or some method cond, is false)
    else if (v == Unavail) then return (False, system, mcsPrev ++ mcsThis)
     -- Rule is enabled (rule-cond is True and all method-conds are True)
     -- Perform the rule's actions, updating the system state
     else do system' <- doActions system actions
            return (True, system', mcsPrev ++ mcsThis)
```

The mcsPrev argument is a collection of methods called in rules that have previously fired in this clock. The third argument is a rule instance (its hierarchical name, an environment representing

the bindings in its surrounding module, and the Rule-expression itself). The output is a boolean indicating whether the rule fired, the updated system state (same as input system state if the rule did not fire), and an updated collection of methods called (mcsPrev augmented by methods called in this rule, if any).

The evalExpr function evaluates an expression to a value, which is just an expression restricted to certain forms:

The evalExpr function is a classical lambda-calculus Eval-Apply pair, with a few variations.

```
evalExpr :: System -> Env -> Expr -> IO (Val, [Action], [MC])
evalExpr system env e = eval e
    where
    eval :: Expr -> IO (Val, [Action], [MC])

... clauses for classical eval of identifiers, contants, ...
... primitive functions, lambdas ...
... conditionals, loops, blocks ...
...
```

We omit here the classical clauses (identifiers, constants, conditionals, lambdas, ...). If the expression is (MethodVal e methName), then e evaluates to a module-instance reference of the form ModuleInstRef j where j is an index into the system IntMap and selects a module instance.

```
eval (MethodVal e methName) =
   do (miref, [], mcs) <- evalExpr system env e
   return (MethodVal miref methName, [], mcs)</pre>
```

Since this evaluation cannot have side-effects, the pattern-match asserts [] for the actions collected. Evaluating a When-expression can return True, False or Unavail. The boolean values are encoded as ConstI 1 and ConstI 0.

```
eval (When econd ebody) =
   do (vcond, [], mcs) <- evalExpr system env econd
   if ((vcond == Unavail) || (vcond == (ConstI 0)))
   then return (Unavail, [], mcs)
   else do (vbody, as', mcs') <- evalExpr system env ebody
        if (vbody == Unavail) then return (Unavail, [], mcs)
        else return (vbody, as', mcs++mcs')</pre>
```

Evaluating an application is classical: evaluate the list of expressions representing the function and the arguments, and then apply the function value to the argument values except, if any of those values are Unavail, just return Unavail.

```
eval (App es) =
    do (vs, as, mcs) <- evalList es
    if any (== Unavail) vs then
        return (Unavail, as, mcs)
        else do (v, as', mcs') <- apply system vs
            return (v, as++as', mcs++mcs')
...</pre>
```

The apply function is classical for primitive functions and closures. For applying method values, we use the applyMethod function.

```
apply :: System -> [Expr] -> IO (Val, [Action], [MC])
apply system (PrimFn opcode : vargs) =
    do (y, as) <- applyPrimFn (PrimFn opcode) vargs
    return (y, as, [])

apply system (Closure env (Lambda xs e) : vargs) =
    evalExpr system (env ++ zip xs vargs) e

apply system (mv @ (MethodVal miref methname): vargs) =
    applyMethod system mv vargs</pre>
```

The applyPrimFn function is just a direct implementation of each primitive function's semantics. We show the clause for Plus here:

```
applyPrimFn :: Val -> [Val] -> IO (Val, [Action])
...
applyPrimFn (PrimFn Plus) [ConstI x, ConstI y] = return (ConstI (x+y), [])
...
```

The PrimDisplay function is an Action, so we just collect it for later execution after deciding that the rule will fire:

```
applyPrimFn (PrimFn PrimDisplay) [ConstI x] =
   return (Void, [App [PrimFn PrimDisplay, ConstI x]])
applyPrimFn (PrimFn PrimDisplay) [ConstS s] =
   return (Void, [App [PrimFn PrimDisplay, ConstS s]])
...
```

The applyMethod function records the method called. Then, for methods of primitive modules (like registers), it just implements the primitive's semantics for value methods, or collects the application for Action/ActionValue methods.

Application of user-defined methods is similar to application of closures. A user-defined method is in fact a closure—it is a lambda-expression whose environment is the bindings in its enclosing module instance. We evaluate the lambda-expression to a closure value, and apply it to the arguments.

```
-- User-defined modules

(UMI env rules methDefs, _, _) ->

do -- Select the method by name, getting its body and kind

let [(mbody, mkind)] =

= [(mbody', mkind')| md <- methDefs

, let MethDef mkind' methname' mbody' = md
, methname == methname']

-- mbody is a lambda expr, evaluating to closure v

(v, actions, mcs') <- evalExpr system env mbody

-- let actions' = thisActions ++ actions

let actions' = if (mkind == "V") then []

else (thisActions ++ actions)

if (v == Unavail) then return (Unavail, actions', mcs++mcs')
else do (v', actions'', mcs'') <- apply system (v:vargs)

return (v', actions'++actions'', mcs++mcs'++mcs'')
```

## 7.2.2 Checking whether the rule has intra-rule conflicts

After evaluating the rule (which has no side-effects, since we only collect its Actions, we don't perform them yet), we check for conflicts. Checking whether the collection of methods-called within a rule has any intra-rule conflicts is done with this function:

```
intraRuleConflicts :: System -> [MC] -> [(MC,MC)]
intraRuleConflicts system mcs = ...
```

It checks each method-called in mcs with every other method-called in mcs, and returns a list of all pairs that conflict. This check is just a test for presence in the fixed intraRuleConflictTable. An example entry in this table is shown below.

## 7.2.3 Checking whether the rule has inter-rule conflicts

Checking whether the collection of methods-called within a rule has any inter-rule conflicts with any method called in rules previously fired in this clock is done with this function:

```
interRuleConflicts :: System -> [MC] -> [MC] -> [(MC,MC)]
interRuleConflicts system mcsPrev mcsThis = ...
```

It checks each method-called in mcsPrev with each method-called in mcsThis, and returns a list of all pairs that conflict. This check is just a test for presence in the fixed interRuleConflictTable. An example entry in this table is shown below.

## 7.2.4 Checking whether the rule has hardware conflicts

Checking whether there are any hardware conflicts is done with this function:

```
hwConflicts :: System -> [MC] -> [MC] -> [(MC,MC)]
hwConflicts system mcsPrev mcsThis = ...
```

It checks each method-called in mcsThis with each other method-called in mcsThis, and it checks each method-called in mcsThis with each method-called in mcsPrev. Note that hardware-conflicts between methods-called in mcsPrev would have already been checked while executing earlier rules. It returns a list of all pairs that conflict.

A hardware conflict captures the fact that a hardware wire can only be driven by one value in each clock. Thus an Action or ActionValue method always has a hardware conflict with itself (the "enable" input wire can only be driven once). A Value method that has any arguments always has a hardware conflict with itself (the argument input wires can only be driven once). To put it another way, only a niladic Value method does not have a hardware conflict with itself. This is captured in the following function.

```
hwConflict :: System -> MC -> MC -> Bool
hwConflict system (MethodVal (ModuleInstRef j) methname)
                   (MethodVal (ModuleInstRef j2) methname2)
                        | j /= j2
                                                = False
                                                            -- different modules
                        | methname /= methname2 = False
                                                            -- different methods
                        | otherwise
                                                = b
  where
      (hname, mi) = (system IntMap.! j)
      b = case mi of
                             -> (methname == "_write") -- Action method
              (PMIReg _)
               ... other primitives ...
              (UMI _ _ _)
                                   -- Method on user-defined module
                  let
                                        = select_UMI_method methname
                      Lambda args body = methBody md
                      niladic_Value_method = (
                                                   (methKind md == "V")
                                               && (length args == 0))
                  in
                      not niladic_Value_method
```

### 7.2.5 Firing the rule (performing its actions)

Finally, after evaluating a rule, checking that it has no inter-rule, intra-rule or hardware conflicts, and checking that its rule and method conditions are True (not Unavail), we can fire the rule, i.e., perform its actions to update the system state.

The function doActions just uses foldM to iterate over all the actions, repeatedly updating the system state:

```
doActions :: System -> [Expr] -> IO System
doActions system actions = foldM f system actions
   where f ...
```

Each action is executed by doAction. Methods of primitive modules just implement the primitive's semantics, updating the module instance state to a new module instance state (like the register \_write shown below). Methods of user-defined modules are ignored here: they were originally collected for checking hardware conflicts, but they were also applied to their arguments to produce (recursively), any underlying primitive-module actions. Ultimately, only primitive-module actions affect system state.

And we're done with this rule (execRule). Function exec\_1\_clock will call execRule again for the next rule in the schedule, until there are no more rules in the schedule. Then, exec\_n\_clocks will call exec\_1\_clock again for the next clock, and so on.

## 8 Examples

In this section we describe some examples that are in the Progs/ directory. Each example is in a source file with extension .kbsv (for kernel BSV).

## 8.1 GCD

It seems obligatory with hardware-design languages to start with a GCD example (Greatest Common Divisor). The code is in the file GCD.kbsv. Here is the source code for the GCD module.

```
module mkGCD;
1
       let zero = 0;
2
       let x = mkReg (zero);
3
       let y = mkReg (zero);
4
       let busy = mkReg (zero);
5
6
     rules
7
       rule swap (
                      x._read()
                                     > y._read()
```

```
&& busy._read () == 1
9
                     && y._read ()
                                        ! = 0);
10
             x._write (y._read ());
11
             y._write (x._read ())
12
        endrule
13
14
        rule subtract (
                            x._read ()
                                            <= y._read ()
15
                         && busy._read () == 1
16
                         && y._read ()
                                            ! = 0);
17
             y._write (y._read () - x._read ())
18
        endrule
19
20
      methods
21
        method A start (num1, num2) if (busy._read () == 0);
22
             x._write (num1);
23
             y._write (num2);
24
             busy._write (1)
25
        endmethod
26
27
        method AV getResult () if ((busy._read () == 1) && (y._read () == 0));
28
             busy._write (0);
29
             x._read ()
30
        endmethod
31
    endmodule
32
```

L.2 (line 2) is just a trivial binding giving the name zero to the constant integer 0, just to demonstrate that you can have conventional let-bindings of names to expressions. The next three lines instantiate three registers with initial (reset) value 0, and bind x, y and busy to those instances.

L.8-L.13 define the swap rule. The rule condition tests if x > y, if we are busy and if  $y \ne 0$ . Note, we're using the  $\_read()$  method on a register to retrieve its value (in BSV these method calls can be omitted and bsc automatically fills them in). The body of the rule, L.11-L.12 effectively swap the x and y values. Here we use the  $\_write()$  method to store a value into a register (in BSV we typically use the more convenient infix  $\le syntax$ ).

L.15-L.19 define the subtract rule. Here the rule condition tests if  $x \leq y$  (and busy and  $y \neq 0$ ). The rule body assigns y - x to y.

L.22-L26 define the start method. The method condition ensures it can only be invoked when we are not busy. The method body stores the arguments into x and y, and store True (coded as integer 1) into busy.

L.28-L.32 define the getResult method. The method condition ensures it can only be invoked when we are busy and y = 0. It returns the value of x and writes False (0) into busy, so that the start method can now be invoked again.

Here is a "main" module to test our GCD module.

```
module main;
let state = mkReg (0);
```

```
let gcd = mkGCD ();
3
4
      rules
5
        rule init (state._read () == 0);
6
             gcd.start (24, 16);
             state._write (1)
8
        endrule
9
10
        rule finish (state._read () == 1);
11
             let z = gcd.getResult ();
12
             $display ("The GCD is ");
13
             $display (z);
14
             state._write (2)
15
        endrule
16
      methods
17
    endmodule
18
```

It instantiates a state register initialized to 0, and it instantiates the GCD module. Rule init can fire when state is 0; it invokes gcd.start(24,16) and writes 1 to state. Rule finish can fire when state is 1; it retrieves a result from gcd, displays it, and writes 2 to state. When state is 2 no rules will enabled, and this will terminate the simulation.

The final piece of source text is a schedule:

```
schedule
[main, init]
[main, finish]
[main, gcd, swap]
[main, gcd, subtract]
```

Each line is a hierarchical name. For example, the last line represents the subtract rule inside the gcd instance inside the main instance. In this example, the choice of schedule is not important, since the rule conditions ensure that only one of them can fire in a clock anyway.

### 8.1.1 Executing the semantics

We execute the semantics as follows:

```
$ runghc Main 100 Progs/GCD.kbsv
```

(This command is also available in the Makefile.) "runghc" is the standard Glasgow Haskell Compiler command to execute a Haskell program. "Main" refers to the file Main.hs which is the top-level of the Haskell code provided, which invokes all the parsing, static elaboration and semantic functions discussed in this document. "100" limits the execution to a maximum of 100 (simulated) clock cycles. Finally, we provide the source file name.

The variable debugLevel in the file Utils.hs controls the verbosity of the output. When set to 0, it should only produce the output of \$display functions. We set it to 1 and captured the output in the file log\_GCD. It consists of three sections: a rendering of the abstract syntax tree (AST) produced by parsing, a rendering of the elaborated program produced by static elaboration and, finally, a clock-by-clock trace of the execution. We skip the AST here, since it is not very interesting.

The first part of the elaborated program looks like this:

```
---- Elaborated program
      0 ["main","state"]
                              PMIReg 0
2
      1 ["main", "gcd", "x"]
                                PMIReg 0
3
      2 ["main", "gcd", "y"]
                                PMIReg 0
4
      3 ["main", "gcd", "busy"]
                                    PMIReg 0
5
      4 ["main", "gcd"]
6
        UMI
             zero = 0
8
             x = ModuleInstRef 1
9
             y = ModuleInstRef 2
10
             busy = ModuleInstRef 3
11
          RULES
12
             rule swap when (And(Gt(x._read(),y._read()),Eq(busy._read(),1)),
13
                              Neq(y._read(),0));
14
               begin
15
                 x._write(y._read());
16
                 y._write(x._read());
17
               end
18
             rule subtract when (And(Leq(x._read(),y._read()),Eq(busy._read(),1)),
19
                                       Neq(y._read(),0));
20
               begin
21
                 y._write(Minus(y._read(),x._read()));
22
               end
23
          METHODS
24
             method A start when (Eq(busy._read(),0));
25
               (lambda (num1, num2)
26
                 begin
27
                   x._write(num1);
28
                   y._write(num2);
29
                   busy._write(1);
30
                 end)
31
             endmethod
32
             method AV getResult when (And(Eq(busy._read(),1),Eq(y._read(),0)));
33
               (lambda ()
34
                 begin
35
                   busy._write(0);
36
                   x._read();
37
                 end)
38
             endmethod
39
```

EUMI

This is the Haskell IntMap that maps integers (unique names) to module instances. For example, "3" refers to the register primitive module instance (PMI) with hierarchical name main.gcd.busy, i.e., the busy register inside the gcd module instance which is inside the main module instance. The initial (reset-value) data in that register is 0.

"4" refers to the GCD user-defined module instance (UMI). In the module, you can see that busy is bound to the value ModuleInstRef 3, i.e., the register instance discussed in the previous para. If we had two instances of the GCD module, each would have its own entry in the IntMap. Inside, the value bound to the name busy would different in the two GCD instances, each referring to its own register PMI, its own copy of busy. The rules and method definitions in the UMI are the same as in the original AST (this is true in all UMIs—only the bindings are evaluated, the rules and method definitions are left untouched).

The rest of the elaborated program is the UMI for main, and the finally we have the value of the expression "main()" which is the root of static elaboration.

```
5 ["main"]
        UMI
2
             state = ModuleInstRef 0
3
             gcd = ModuleInstRef 4
          RULES
5
             rule init when (Eq(state._read(),0));
6
               begin
                 gcd.start(24,16);
                 state._write(1);
9
               end
10
             rule finish when (Eq(state._read(),1));
               begin
12
                 let z = gcd.getResult()
13
                 $display("The GCD is ");
14
                 $display(z);
15
                 state._write(2);
16
               end
17
          METHODS
18
        EUMI
    value of main(): ModuleInstRef 5
20
```

After static elaboration, we see a clock-by-clock trace of execution. In each clock, it says what happened with each rule in the schedule. For example, clock 0:

```
1 .... Clock 0
2 Rule ["main","init"] fired
3 Rule ["main","finish"]
4 inter-rule conflicts:
5 ["main","state"] _write > _read
```

```
Rule ["main", "gcd", "swap"]
6
        inter-rule conflicts:
7
           ["main", "gcd", "x"] _write > _read
8
           ["main", "gcd", "x"] _write > _read
9
           ["main", "gcd", "y"] _write > _read
10
           ["main", "gcd", "y"] _write > _read
11
           ["main", "gcd", "y"] _write > _read
12
           ["main", "gcd", "busy"] _write > _read
13
      Rule ["main", "gcd", "subtract"]
14
        inter-rule conflicts:
15
           ["main", "gcd", "x"] _write > _read
16
           ["main", "gcd", "y"] _write > _read
17
           ["main", "gcd", "y"] _write > _read
18
           ["main", "gcd", "busy"] _write > _read
19
```

This shows that the first rule in the schedule, main.init, fired. The second rule in the schedule, main.finish did not fire: it had an inter-rule conflict, where it was trying to execute the method main.state.\_read, whereas rule main.init has already fired and executed main.state.\_write, and \_read cannot follow \_write in a clock. And so on, for the remaining rules in the schedule, main.gcd.swap and main.gcd.subtract.

When main.init fires, it invokes the main.gcd.start(24,16) method which stores 24 and 16 into registers main.gcd.x and main.gcd.y, respectively. Continuing through clock 5, we see:

```
.... Clock 1
2
                                                             So: x = 16, y = 24
      Rule ["main", "gcd", "swap"] fired
3
    .... Clock 2
4
5
                                                             So: x = 16, y = 8
      Rule ["main", "gcd", "subtract"] fired
6
    .... Clock 3
7
8
      Rule ["main", "gcd", "swap"] fired
                                                             So: x = 8, y = 16
9
10
    .... Clock 4
11
12
      Rule ["main", "gcd", "subtract"] fired
                                                             So: x = 8, y = 8
13
    .... Clock 5
14
15
      . . .
                                                             So: x = 8, y = 0
      Rule ["main", "gcd", "subtract"] fired
16
```

At this point, rule main.finish is enabled: its rule condition is true since main.state=1, and the condition of the method it calls, main.gcd.getResult, is true because main.gcd.busy=1 and main.gcd.y=0. Indeed, if we continue to clock 6 we see:

```
1 .... Clock 6 ...
```

```
Rule ["main", "finish"] fired

SDISPLAY: The GCD is

DISPLAY: 8

...
```

and we see the outputs from the rule's \$display statements.

In clock 7, we see that no rules fire (they are all Unavail), so the simulation prints the final system state and stops. You can see that the main.state register has value 2. the main.gcd.x register has value 8, the main.gcd.y register has value 0, and the main.gcd.busy register has value 0.

```
.... Clock 7
      Rule ["main", "init"] unavail
2
      Rule ["main", "finish"] unavail
3
      Rule ["main", "gcd", "swap"] unavail
4
      Rule ["main", "gcd", "subtract"] unavail
5
    No rule fired on cycle 7
6
    ---- final system:
      0 ["main", "state"]
                               PMIReg 2
      1 ["main", "gcd", "x"]
                                 PMIReg 8
      2 ["main", "gcd", "y"]
                                 PMIReg 0
10
      3 ["main", "gcd", "busy"]
                                    PMIReg 0
11
      4 ["main", "gcd"] UMI ...
12
      5 ["main"]UMI ...
13
```

## 8.2 PipelineFIFO

The GCD example is nice, but it does not have any interesting concurrency since only one rule condition is enabled at any time (this is why the particular choice of its schedule does not matter either). We now look at a more interesting example, a 2-stage pipeline between which is a "PipelineFIFO". The code is in the file Progs/PipelineFIFO.kbsv.

The first module in the file is mkPipelineFIFO:

```
module mkPipelineFIFO;
1
        let full = mkCReg (2, 0);
2
        let data = mkCReg (2, 0);
3
4
      rules
5
        -- no rules
6
      methods
8
        method A enq (x) if (full._read1 () == 0);
9
           data._write1 (x);
10
           full._write1 (1)
11
        endmethod
^{12}
```

```
13
        method V notEmpty ();
14
           full._read0 ()
15
        endmethod
16
17
        method V first () if (full._read0 () == 1);
18
           data._read0 ()
19
        endmethod
20
21
        method A deq () if (full._read0 () == 1);
22
           full._write0 (0)
23
        endmethod
24
    endmodule
25
```

This implements a one-element FIFO. The full register is 1 if the FIFO contains a datum, and 0 if it does not. The data register holds the datum itself. The module contains no rules, just method definitions. The enq method may be called when full=0, i.e., the FIFO is empty, in which case it stores the datum x in the data register and stores 1 in full. The notEmpty method just returns the full value. The first and deq methods may be invoked when full=1. The first method returns the datum in the FIFO, and the deq method writes 0 full, i.e., it marks the FIFO as empty.

If full were an ordinary register, we could never invoke enq and deq in the same cycle. Both methods execute full.\_read() and write full.\_write(). Whether we scheduled (a rule that invoked) enq before (a rule that invoked) deq or vice versa, we would run into an inter-rule conflict because \_read() must precede \_write in a clock.

Thus, we use Concurrent Registers (CRegs) for full and data. The methods on the output side of the FIFO, notEmpty, first and deq, use \_read0 and \_write0. The method on the input side of the FIFO, enq, uses \_read1 and \_write1. The interRuleConflictTable permits \_write0 and \_read0 to precede \_write1 and \_read1 in a clock. Thus, provided the output side methods are scheduled before the input side method, they can execute in the same clock. Conceptually, we can dequeue an element from the FIFO, thus emptying it, and enqueue a new element into the FIFO, thus re-filling it, all within the same clock.

The main test program is shown here:

```
module main;
1
        let x = mkReg(0);
2
        let f = mkPipelineFIFO ();
3
4
      rules
5
        rule feed;
6
          f.enq(x._read());
7
          x.\_write (x.\_read() + 1)
8
        endrule
9
10
        rule drain (f.notEmpty ());
11
```

```
$\frac{12}{3}$ $\display ("RESULT");
$\frac{13}{4}$ $\frac{1}{3}$ $\frac
```

The feed rule repeatedly enqueues x into the FIFO and increments x. The drain rule repeatedly dequeues an item from the FIFO and displays it. At the bottom of the file, we show two candidate schedules (one of them is commented out):

```
-- Sched 1 (pipelining happens)
    schedule
      [ main, drain ]
3
      [ main, feed ]
4
5
    /*
6
    -- Sched 2 (pipelining does not happen, rules alternate)
7
    schedule
8
      [ main, feed ]
      [ main, drain ]
10
    */
11
```

## 8.2.1 Executing the semantics with schedule 1

We can execute the program; the transcript using Sched 1 is captured in log\_PipelineFIFO. Looking at the first 3 clocks, we see:

```
.... Clock 0
1
      Rule ["main", "drain"] unavail
2
      Rule ["main", "feed"] fired
3
    .... Clock 1
4
      Rule ["main", "drain"] fired
5
    $DISPLAY: RESULT
6
    $DISPLAY: 0
7
      Rule ["main", "feed"] fired
8
    .... Clock 2
9
      Rule ["main", "drain"] fired
10
    $DISPLAY: RESULT
11
    $DISPLAY: 1
12
      Rule ["main", "feed"] fired
13
    .... Clock 3
14
      Rule ["main", "drain"] fired
15
    $DISPLAY: RESULT
16
    $DISPLAY: 2
      Rule ["main", "feed"] fired
```

In clock 0, rule drain did not fire, since the FIFO is initially empty, and so the notEmpty, first and deq are not enabled. Rule feed fired, since the enq method is enabled when the FIFO is empty.

In each subsequent clock, now that the FIFO is no longer empty, we see that both rules fire, as expected. First drain fires and displays the result; this leaves the FIFO empty, enabling the feed rule, which also fires. The displayed outputs are 0, 1, 2, ... as expected.

The system stops after 100 clocks, as requested on the command line. The final output is 99, as expected, and the final state can also be seen:

```
.... Clock 100
      Rule ["main", "drain"] fired
    $DISPLAY: RESULT
3
    $DISPLAY: 99
4
      Rule ["main", "feed"] fired
5
    Cycle limit reached: 100
6
    ---- final system:
7
      0 ["main", "x"]
                          PMIReg 101
8
      1 ["main", "f", "full"]
                                  PMICReg 2 1
9
      2 ["main", "f", "data"]
                                  PMICReg 2 100
10
      3 ["main","f"]UMI ...
11
      4 ["main"]UMI ...
12
```

## 8.2.2 Executing the semantics with schedule 2

If we instead execute the program with the other schedule, Sched 2, we get a different execution. Its transcript is in file log\_PipelineFIFO\_Sched2. The execution trace begins at clock 0:

In clock 0, the FIFO is empty, so rule feed is enabled and fires, ultimately using methods full.\_read1(), full.\_write1() and data.\_write1(). Then, rule drain cannot fire, since it ultimately invokes full.\_read0(), full.\_write0() and data.\_read0(), and these cannot follow the already-invoked full.\_read1(), full.\_write1() and data.\_write1() (inter-rule conflict). Moving on to clock 1:

```
1 .... Clock 1
2 Rule ["main", "feed"] unavail did not fire
3 Rule ["main", "drain"] fired
```

```
$DISPLAY: RESULT 5 $DISPLAY: 0
```

In clock 1, rule feed cannot fire because the FIFO is full and so the enq method is disabled. Rule drain can fire; it empties the FIFO, and displays the dequeued item.

```
1 .... Clock 2
2 Rule ["main", "feed"] fired
3 Rule ["main", "drain"] did not fire
4 ...
```

In clock 2, rule feed is once again enabled since the FIFO is empty, but this again prevents rule drain from firing, just like in clock 0. In subsequent clocks we see this pattern repeating, with either rule feed or rule drain firing.

```
.... Clock 3
      Rule ["main", "feed"] unavail
                                            did not fire
2
      Rule ["main", "drain"] fired
3
    $DISPLAY: RESULT
4
    $DISPLAY: 1
5
    .... Clock 4
6
      Rule ["main", "feed"] fired
      Rule ["main","drain"]
                                            did not fire
         . . .
9
    .... Clock 5
10
      Rule ["main", "feed"] unavail
11
      Rule ["main", "drain"] fired
12
    $DISPLAY: RESULT
13
    $DISPLAY: 2
14
    .... Clock 6
15
      Rule ["main","feed"] fired
16
      Rule ["main","drain"]
                                            did not fire
17
18
         . . .
```

After we reach the requested cycle limit (100), simulation stops. The final output is 49 and the final state of register x is 51 (these were 99 and 101, respectively, with Sched 1).

```
1 .... Clock 99
2 Rule ["main", "feed"] unavail
3 Rule ["main", "drain"] fired
4 $DISPLAY: RESULT
5 $DISPLAY: 49
6 .... Clock 100
7 Rule ["main", "feed"] fired
```

```
Rule ["main","drain"]
8
        inter-rule conflicts:
9
10
    Cycle limit reached: 100
11
    ---- final system:
12
      0 ["main","x"]
                          PMIReg 51
13
      1 ["main", "f", "full"]
                                  PMICReg 2 1
14
      2 ["main","f","data"]
                                  PMICReg 2 50
15
      3 ["main", "f"] UMI ...
16
      4 ["main"]UMI ...
17
```

In summary, observe that both schedules provide functionally correct execution: we enqueue and dequeue correct values. In this functional sense, neither schedule is "correct" or "incorrect".

However, Sched 1 admits more concurrency than Sched 2, because its sequence is "better aligned" with the inter-rule ordering constraints on the methods of the data and full CRegs. In fact, the bsc compiler will automatically pick Sched 1 using this criterion, i.e., it tries to maximize concurrency.

## 8.3 BypassFIFO

The next example is the "dual" of the PipelineFIFO and is called the BypassFIFO. The code is in the file Progs/BypassFIFO.kbsv. We leave it as an exercise for the reader, with just the following observations.

In the PipelineFIFO, the output-side methods (notEmpty, first and deq) of the FIFO used port 0 methods (\_read0 and \_write0) of the full and data Concurrent Registers, and the input-side method (enq) used port 1 methods (\_read1 and \_write1). This allowed concurrent input and output on the FIFO, provided the output side was scheduled before the input side.

In the BypassFIFO, we do exactly the opposite: the input-side method (enq) uses port 0 methods (\_read0 and \_write0), and the output-side methods (notEmpty, first and deq) use port 1 methods (\_read1 and \_write1) of the full and data Concurrent Registers. This allows concurrent input and output on the FIFO, provided the input side is scheduled before the output side.

The main test program and the two candidate schedules are identical to those in PipelineFIFO.kbsv. The transcript using Sched 1 is shown in file log\_BypassFIFO\_Sched1, and using Sched 2 is shown in file log\_BypassFIFO. As with the PipelineFIFO, both show functionally correct execution, just different levels of concurrency. With Sched 2, both rules can fire on every clock; with Sched 1 they alternate.

## 8.4 PipeFwdBwd

Our final example combines the last two and is representative of structures we find, for example, in CPU pipelines. In a CPU pipeline, the main data flow is in the "forward" direction through the pipe, and the pipeline stages are separated by PiplineFIFOs. Thus, in each clock, rules corresponding to all stages can fire concurrently, allowing the entire pipeline to advance in each clock. However, within each clock, semantically the last stage executes first, then the previous one, then its previous one, and so on until the first stage.

Most CPUs also also have a "reverse" data flow, where later stages may provide updated register values or branch-redirection information back to early stages. For consistent rule ordering with the forward direction, therefore, we use BypassFIFOs in the reverse direction.

Our example is in the file PipeFwdBwd.kbsv. It also has two stages, using a PipelineFIFO in the forward direction from rule feed to rule drain, and a BypassFIFO in the reverse direction from rule drain to rule feed.

We leave it as an exercise to the reader to run PipeFwdBwd.kbsv with each of the two schedules, and to explain their respective behaviors. For convenience, log\_PipeFwdBwd is a transcript of execution with Sched 1, and log\_PipeFwdBwd\_Sched2 is a transcript of execution with Sched 2.

## 9 Conclusion and closing observations

We have described the dynamic execution semantics of BSV programs formally by providing a Haskell program directly implementing the semantics.

We also described a process of static elaboration, but that is not central here; any alternative mechanism is fine that results in a collection of module instances, where each module is either primitive or is user-defined, where user-defined modules have rules and methods that can invoke methods of primitives or other modules.

## 9.1 Relationship to RTL and Chisel: Atomic transactions

In the literature and training materials on BSV, we often refer to rules as *atomic transactions*. Atomicity is trivial in the semantics described in this document because we define *serial* execution of rules according to a schedule, and so there is no question of interleaving of rule actions.

A central issue encountered in concurrent programming is that atomic operations are non-modular, or non-compositional [10]. In other words, a system of modules, each of which implements (locally) atomic operations does not guarantee that system-level actions are atomic. The classic example is a pair of shared bank account modules with deposit and withdraw methods, each of which are guaranteed atomic. Two concurrent processes that attempt a logical transfer operation by withdrawing from one account and depositing into the other may nevertheless see inconsistent results due to bad interleavings of these methods.

This is the reason why user-level atomic transactions cannot be implemented correctly merely with libraries—the compiler and/or run-time system must have visibility over *all* actions of a user-defined atomic transaction, and this set may span many modules/functions/procedures (cf. "Transactional Memory" [22]).

This explains the fundamental power of BSV over traditional RTL languages like Verilog [14], SystemVerilog [17] and VHDL [15]. The semantics of those languages is based entirely on clocks and classical clocked digital logic. There is no concept of user-defined atomic transactions like rules in BSV.

The more recent language Chisel [5] is a much higher-level language than those Verilog, SystemVerilog or VHDL, but its semantics is also fundamentally clock based, just like RTL. Chisel has a syntactic "when" construct that at first glance look like rule and method conditions in BSV, but

they have nothing to do with atomic transactions; they are just a syntactic means of prioritizing updates to a state element, *locally to a syntatic scope* (in particular, they are not modular or compositional).

## 9.2 Relationship to HLS (High Level Synthesis)

The last 10 years has seen the emergence of a number of commercial tools for hardware design under the category of "High Level Synthesis", or HLS for short [8] (we'll refer to this as "classical HLS"). The input design language for these tools is an existing language, typically C or C++ (or SystemC, which is just a C++ library [16]). This is paradoxical, since C and C++ are, formally, completely sequential languages, from which classical HLS tries to generate hardware which, quintessentially, has massive, fine-grained parallelism. Classical HLS tools achieve this with "automatic parallelization", which builds on several decades of research into automatic parallelization of software for vector computers and SIMD computers. The central activity is to analyze for-loops over dense rectangular arrays and to recognize when iterations can be performed safely in parallel, from which they can be mapped into parallel hardware. As a consequence, classical HLS has the same strengths and limitations as classical automatic parallelization; a deeper discussion of these limitations may be found in [9].

It should be evident that BSV and BSV compilation has almost nothing in common with classical HLS. The BSV programmer (unlike the C/C++ programmer) thinks in parallel, and explicitly designs and implements parallel algorithms. The "high-level synthesis" in BSV and the bsc compiler refers to a quite different set of properties and activities:

- its extremely powerful Haskell-like type system,
- its extremely powerful Haskell-like static-elaboration,
- its automatic analysis of rules to produce highly concurrent rule schedules,
- and its generation of efficient hardware from this analysis to execute rules concurrently.

Unlike classical HLS, where hardware quality drops off sharply (and may even becomes infeasible) as one moves out of the "sweet spot" of simple loops on dense rectangular arrays, BSV has universal applicability to all kinds of clocked digital hardware and is, in that sense, a genuinely full replacement for RTL.

## 9.3 Relationship of this formal semantics to the full BSV language

Ideally, formal semantics should be defined directly on the source language used by the programmer. If this can be done, and the semantic definition is "simple", then the everyday practicing programmer can directly use it as a mental model while creating, debugging and analyzing actual programs. Everyday practicing programmers find it difficult to usefully exploit formal semantics that is defined on a kernel language that is too far removed from the source language, or that involve esoteric source-to-source transformations.

We have attempted to approach this ideal in this semantics. The language defined in Sec. 4 is admittedly much smaller and much simplified compared to full BSV. But the principal difference

is in types and type-checking, and in a few syntatic facilities like interface definitions and staticelaboration loops. These are quite orthogonal to dynamic semantics and do not affect the mental model held by the everyday practicing programmer. We have also omitted anything related to BSV packages, which again are just about namespace management and separate compilation, and quite orthogonal to dynamic semantics.

Thus, our kernel language captures the essence of BSV for purposes of explaining dynamic semantics. This semantics is in fact the mental model one uses while doing BSV programming.

## 9.4 Relationship of this formal semantics to the bsc compiler

This formal semantics is purely a dynamic semantics explained directly on the abstract syntax trees of source code—it takes no position on how to find schedules (rule sequences), nor on compilation to actual hardware. As a result, this semantics is more liberal than what can actually be achieved by any compiler (which, by definition, is working statically).

The *bsc* compiler only produces a *static* schedule—a single fixed schedule that is constant across clocks. Further, these schedules are always permutations of the rule-instance set, i.e., each rule-instance appears in the schedule exactly once. The dynamic semantics has no such restriction.

As described earlier in Sec. 3.7, this semantics takes no position on schedules. A schedule is a parameter to this semantics, and may therefore be produced by an oracle, even one that dynamically selects the next rule at the last moment. There is no such thing in this semantics as a "wrong" schedule; different schedules merely admit more or less concurrency (because they'll encounter less or more conflicts, respectively).

The *bsc* compiler chooses a schedule by static analysis of the rules in a program. In other words, it is statically predicting something about dynamic execution (rule and method conditions, rule conflicts and hardware conflicts). This semantics has exact information, but the compiler can only have estimated or approximate information.

Thus, there are rule firings that may be permitted in this dynamic semantics that will be prohibited in code compiled by bsc, because bsc can only use conservative approximations (e.g., bsc may see a potential conflict where the dynamic semantics knows there is none).

Thus, the semantics described here is more liberal than what is produced today by bsc, and covers possible future improvements in bsc schedule analysis and compilation.

## References

- [1] A. Akhiani, D.Doligez, P.Harter, L.Lamport, J.Scheid, M.Tuttle, and Y. Yu. Cache Coherence Verification with TLA+. In *Proc. World Congress on Formal Methods in the Development of Computing Systems-Volume II*, pages 1871–1872, September 20-24 1999.
- [2] Arvind and X. Shen. Using term rewriting systems to design and verify processors. *IEEE Micro*, 19(3):36–46, 1998.

- [3] L. Augustsson, J. Schwartz, and R. S. Nikhil. Bluespec Language Definition. Technical report, Sandburst Corp., 2001. Earlier version of BSV, now called "Bluespec Classic".
- [4] F. Baader and T. Nipkow. Term Rewriting and All That. Cambridge University Press, 1998.ISBN 0 521 45520 0.
- [5] J. Bachrach, H. Vo, B. Richards, Y. Lee, A. Waterman, R. Avizienis, J. Wawrzynek, and K. Asanovic. Chisel: Constructing Hardware in a Scala Embedded Language. In *Proc. Design Automation Conference (DAC)*, 2012. https://chisel.eecs.berkeley.edu/.
- [6] Bluespec, Inc. Bluespec<sup>TM</sup> SystemVerilog Version Reference Guide, 2015. (BSV). Originally published 2003, with extensive subsequent revisions.
- [7] K. Chandy and J. Misra. Parallel Program Design: A Foundation. Addison Wesley, 1988.
- [8] P. Coussy, D. Gajski, M. Meredith, and A. Takach. An Introduction to High-Level Synthesis. *IEEE Design and Test of Computers*, pages 8–17, July/August 2009.
- [9] S. A. Edwards. The Challenge of Hardware Synthesis from C-like Languages. In *Proc. Design Automation and Test in Europe (DATE), Munich, Germany, March* 2005.
- [10] T. Harris, S. Marlow, S. P. Jones, and M. Herlihy. Composable Memory Transactions. In ACM Conf. on Principles and Practice of Parallel Programming (PPoPP'05), 2005.
- [11] C. Hoe, James and Arvind. Operation-Centric Hardware Description and Synthesis. *IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems (TCAD)*, 23(9), September 2004.
- [12] J. C. Hoe. Operation-Centric Hardware Description and Synthesis. PhD thesis, MIT, June 2000.
- [13] J. C. Hoe and Arvind. Synthesis of Operation-Centric Hardware Descriptions. In *IEEE/ACM Intl. Conf. on Computer Aided Design (ICCAD)*, pages 511–518, 2000.
- [14] IEEE. IEEE Standard Verilog (R) Hardware Description Language, March 2001. IEEE Std 1364-2001.
- [15] IEEE. IEEE Standard VHDL Language Reference Manual, IEEE Std 1076-1993, 2002.
- [16] IEEE. IEEE Standard for Standard SystemC Language Reference Manual, January 9 2012. IEEE Std 1666-2011.
- [17] IEEE. IEEE Standard for System Verilog—Unified Hardware Design, Specification and Verification Language, 21 February 2013. IEEE Std 1800-2012, 1800-2005.
- [18] R. Joshi, L. Lamport, J. Matthews, S. Tasiran, M. Tuttle, and Y. Yu. Checking Cachecoherence Protocols with TLA+. Formal Methods in System Design, 22(2):125–131, March 2003.
- [19] J. Klop. Term Rewriting Systems, volume 2, pages 1–116. Oxford University Press, 1992.
- [20] L. Lamport. Specifying Systems: The TLA+ Language and Tools for Hardware and Software Engineers. Addison-Wesley Professional (Pearson Education), 2002.

- [21] B. W. Lampson. Atomic Transactions. In B. W. Lampson, M. Paul, and H. J. Siegert, editors, Distributed Systems - Architecture and Implementation, An Advanced Course, volume 105 of Lecture Notes in Computer Science, pages 246–265. Springer Verlag, 1981.
- [22] J. Larus and C. Kozyrakis. Transactional Memory. Communications of the ACM, 51(7):80–88, July 2008.
- [23] C. Metayer, J.-R. Abrial, and L. Voisin. The Event-B Language, May 31 2005. http://rodin.cs.ncl.ac.uk/deliverables.htm.
- [24] R. S. Nikhil. Bluespec: a general-purpose approach to High-Level Synthesis based on parallel atomic transactions, pages 129–146. Springer, June 2008. Chapter 8. Each chapter is from a different tool vendor or research group in the field.
- [25] R. S. Nikhil. Abstraction in Hardware System Design. Communications of the ACM, 54(10):36–44, October 2011.
- [26] R. S. Nikhil. Types, Functional Programming and Atomic Transactions in Hardware Design. In In search of elegance in the theory and practice of computation Essays dedicated to Peter Buneman (Festschrift), Lecture Notes in Computer Science, LNCS 8000, pages 418–431. Springer-Verlag, Berlin, Heidelberg, 2013.
- [27] R. S. Nikhil and K. R. Czeck. *BSV by Example*. CreateSpace, December 2010. ISBN-10: 1456418467; ISBN-13: 978-1456418465; avail: Amazon.com.
- [28] S. Peyton Jones. *Haskell 98 Language and Libraries: the Revised Report*. Cambridge University Press, 2003. haskell.org.
- [29] J. Stoy, X. Shen, and Arvind. Proofs of Correctness of Cache-Coherence Protocols. In *FME2001: Formal Methods for Increasing Software Productivity*, Berlin, Germany, March 2001. Springer Verlag LNCS 2021.
- [30] Terese. Term Rewriting Systems. Cambridge University Press, 2003. Cambridge Tracts in Theoretical Computer Science 55.