HKN EE16A MT2 Review

Spring 2016

Topics

Nullspace
Resistance
KVL
KCL
Superposition
Voltage/Current Dividers
Thevenin/Norton
Capacitance
Operational Amplifiers
Power

Null Space

Definition: The set of solutions to the homogeneous equation $Ax = 0$.

The nullspace is a vector subspace

Solve for a null space by solving $Ax = 0$:

1. Reduce A to reduced row-echelon form
2. Identify non-pivot columns -- these are the free variables
3. Rewrite rows of matrix as equations
4. Solve for pivot-column variables in terms of free variables
5. Convert to a span

Null Space - Practice

Find the Null Space of A:

$$A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & 2 \end{bmatrix}$$

Null Space - Practice (Solution)

Find the null space of A

$$A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & 2 \end{bmatrix} \xrightarrow{\text{Row Reduce}} \begin{bmatrix} 1 & -2 & 0 & -1 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Null Space - Practice (Solution)

- We saw that in the row-reduced matrix, there were 3 pivot columns.
- Additionally, we know that there are 5 total "variables"
- Thus, we can say that there are 2 free variables, and obtain a basis for our null space in terms of these free variables!
- The pivot columns occur at x_1, x_3, and x_5
  so we can set $x_2 = r$, $x_4 = s$.

- Let's find our basis in terms of r,s

Null Space - Practice (Solution)

$$\begin{bmatrix} 1 & -2 & 0 & -1 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Solution:

$X_1 = 2r + s$

$X_2 = r$

$X_3 = -2s$

$X_4 = s$

$X_5 = 0$

Null Space of dimension 2

Matrix form:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = r \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} 1 \\ 0 \\ -2 \\ 1 \\ 0 \end{bmatrix}$$

Null Space - Challenge Practice

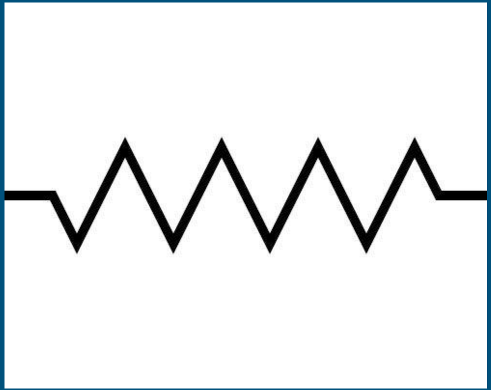
Why are we allowed to row reduce A when calculating Nul(A)?
In other words, row reduction changes A.
How are we guaranteed that A has the same null space as R (A in rref)?

Null Space - Challenge Practice

Why can we row reduce A when calculating Nul(A)?
In other words, row reduction changes A. How are we guaranteed that A has the same null space as R (A in rref)?
R is created by from row operations on A. Thus, R = EA where E is some combination of elementary matrices. If EAx = 0, then Ax = 0.

What is a resistor?

- Circuit element used to dissipate energy ie. lower voltage
- Equation for voltage drop: $\Delta V=IR$
- Unit: Ohm (Ω)
- Good way to think about them is a “bumpy road” that prevents the electrical current from travelling smoothly



Resistivity (ρ)

- How much resistance a material naturally has.
- For example metal will have much lower resistivity than plastic.
- Physical equation for resistance:
  - R is the resistance
  - L is the material's length
  - A is the materials cross section
  - ρ is the constant of resistivity for that material

$$R = \rho \frac{L}{A}$$

Units of resistivity are in Ωm

Quick Resistivity Question

We have a rectangular wire made out of copper whose cross-sectional area is 1e-9 m^2 and whose length is 0.2 m. What is its resistance? (ρ of copper = 1.68 $\times 10^{-8}$ Ωm)

Quick Resistivity Question

We have a rectangular wire made out of copper whose cross-sectional area is 1e-9 m^2 and whose length is 0.2 m. What is its resistance? (ρ of copper = $1.68 \times 10^{-8}\ \Omega m$)

We apply our formula:

$R = \rho L/A = 1.68\times10^{-8}\Omega m(0.2\ m)/1e\text{-}9m^2 = 3.36\ \Omega$

Sanity Check

$$R = \rho \frac{L}{A}$$

What is the resistance of a material when L approaches 0? Infinity?

What is the resistance of a material when A approaches 0? Infinity?

Does this make sense intuitively?

Sanity Check

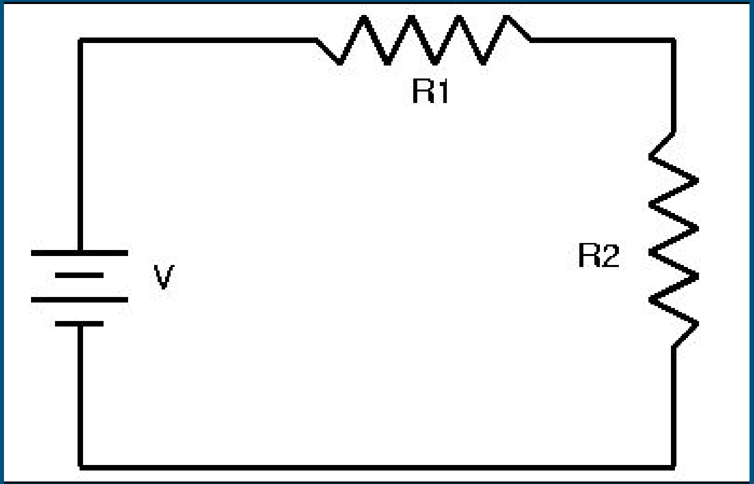
$$R = \rho \frac{L}{A}$$

What happens when we double the length of a resistor?

What happens when we double the area of a resistor?

Resistors in Series

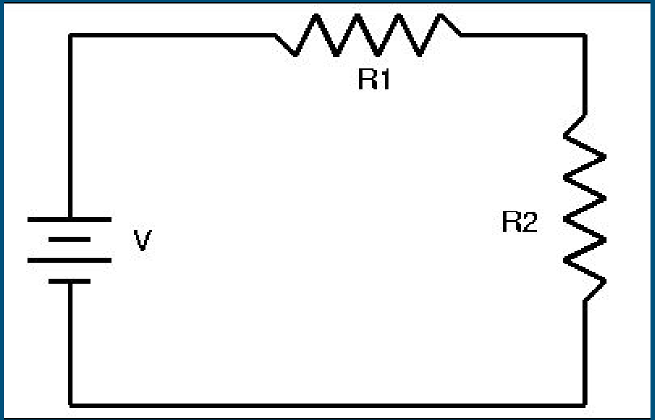
- When they are along a single wire without splitting.
- All of the current through the first resistor must go through the second resistor



Resistors in Series

Want to model as $V_{total} = IR_{total}$ (equivalent total R)

1. $V_1 = IR_1$
2. $V_2 = IR_2$
3. $V_{total} = V_1 + V_2$
4. $V_{total} = IR_1 + IR_2$
5. $V_{total} = I(R_1 + R_2)$
6. $R_{total} = R_1 + R_2$



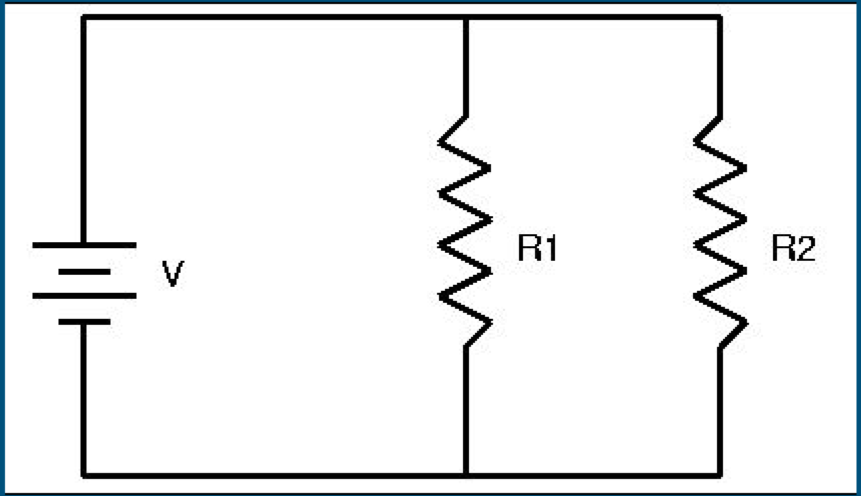
Resistors in Series

TLDR: Just add them

$$R_{total} = \Sigma R_n$$

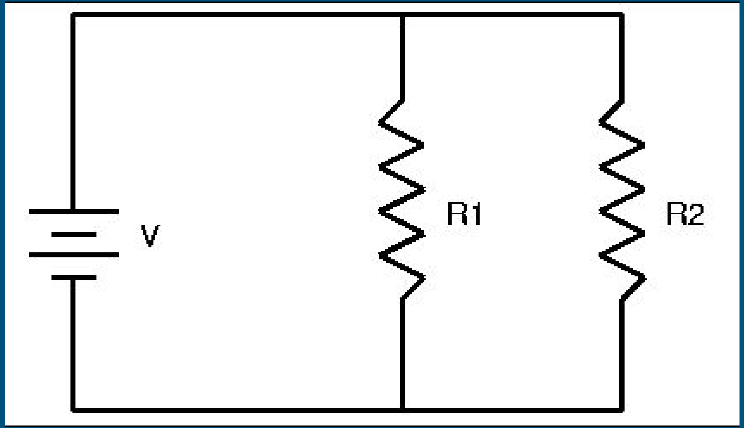
Resistors in Parallel

- When the wire splits into two or more parts
- Voltage across both resistors is the same



Resistors in Parallel

1. $V_1 = V_2 = V_{total}$
2. $V_{total} = I_1R_1$
3. $V_{total} = I_2R_2$
4. $I_{total} = I_1 + I_2$
5. $V_{total}/R_{total} = V_1/R_1 + V_2/R_2$
6. $1/R_{total} = 1/R_1 + 1/R_2$



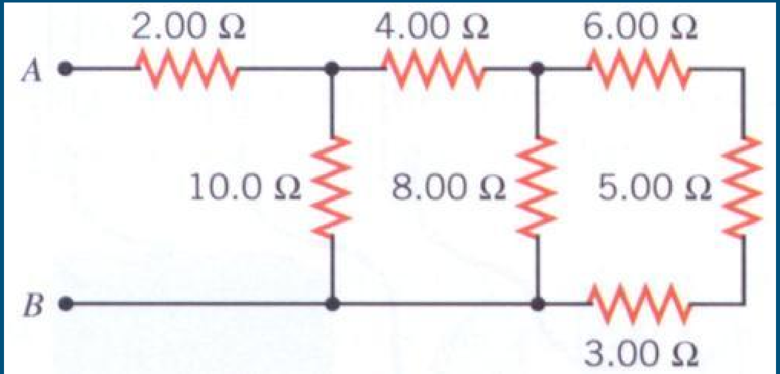
Resistors in Parallel

TLDR:

$$1/R_{total} = \Sigma 1/R_n$$

Quick Resistor Equivalence Problem

Find the equivalent resistance for this circuit

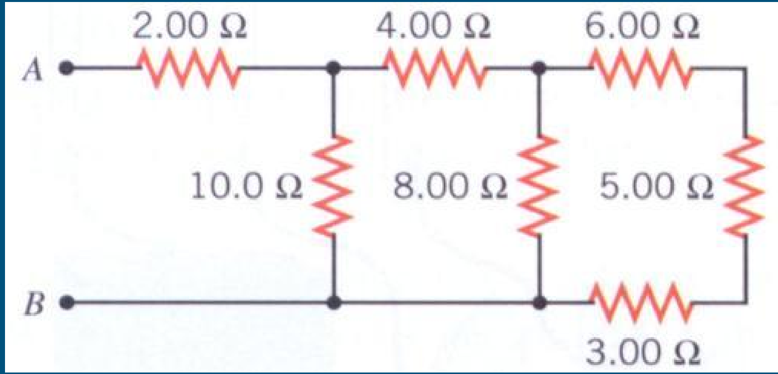


Quick Resistor Equivalence Problem

Add up the resistors all the way on the right: $R_{right} = (6 + 5 + 3)\Omega = 14\Omega$

Now that we have that join with the 8Ω in parallel:

$1/R = ⅛ + 1/14 = 7/56 + 4/56 = 11/56.$ $R = 56/11\Omega$



Then add with the 4Ω since they are in series:

$R = 56/11\Omega + 4\Omega = 100/11\Omega$

Now join with the 10Ω in parallel

$1/R = 1/10 + 11/100 = 21/100.$ $R = 100/21\Omega$

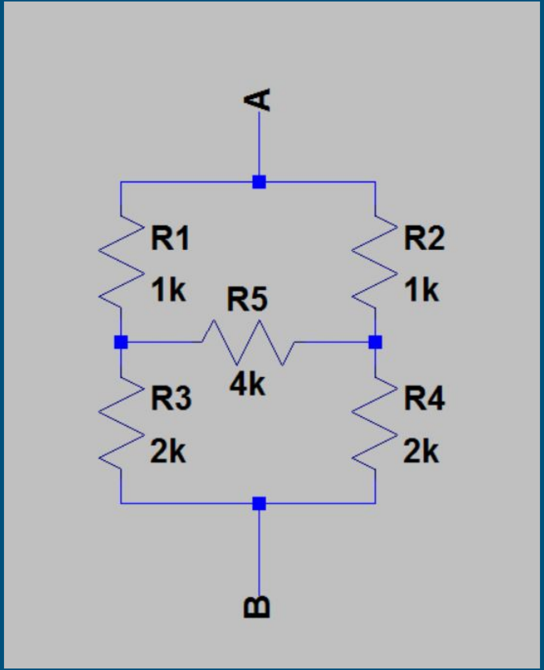
Finally add the 2Ω that are in series with the rest of the circuit: $R_{total} = 2\Omega + 100/21\Omega = 142/21\Omega = 6.762\Omega$

Wheatstone Bridge (Balanced)

How much current flows through the

4k Ohm resistor? Why?

(Hint: think about symmetry)



Wheatstone Bridge (Balanced)

How much current flows through the

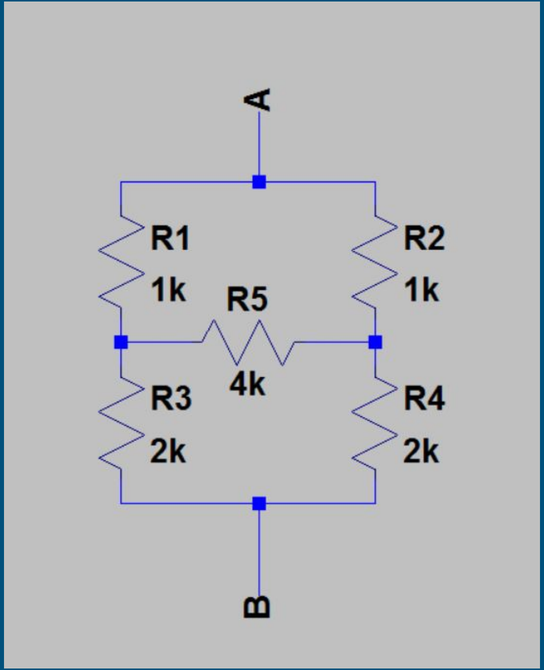
4k Ohm resistor? Why?

(Hint: think about symmetry)

Answer:

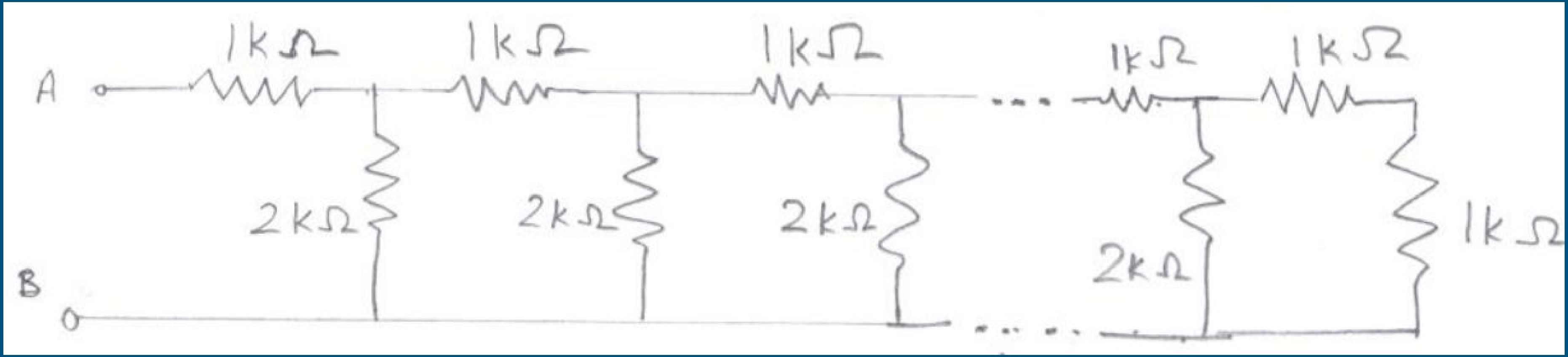
I = 0. By symmetry, no current can

Flow through the 4k Ohm resistor.



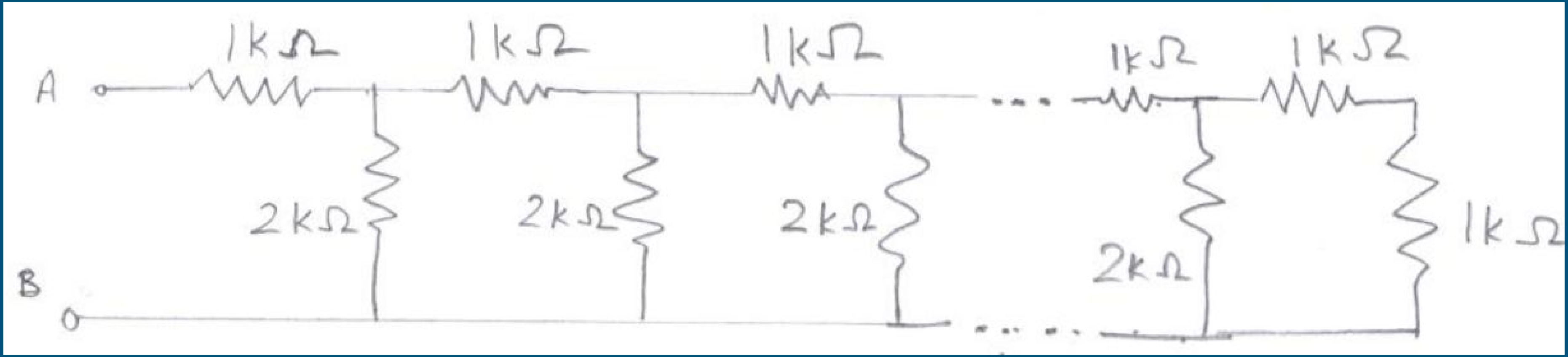
Infinite series

Find the effective resistance between the two nodes.



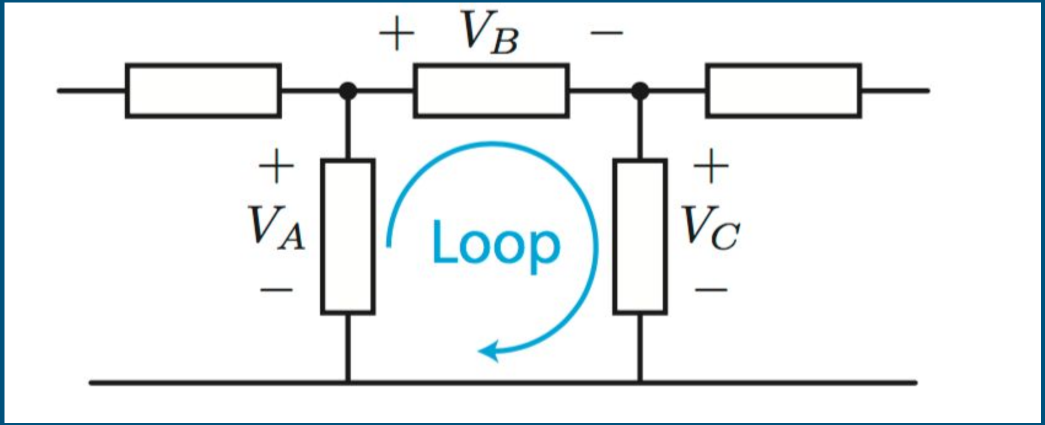
Infinite series

Find the effective resistance between the two nodes. Answer: 2k Ohms



Kirchhoff's Voltage Law

- Net potential around any loop in a circuit is zero
- $\sum V = 0$
- Ex. KVL tells us VA - VB - VC = 0



Kirchhoff’s Current Law (Nodal Analysis)

- Comes from conservation of charge
- $\Sigma I = 0$

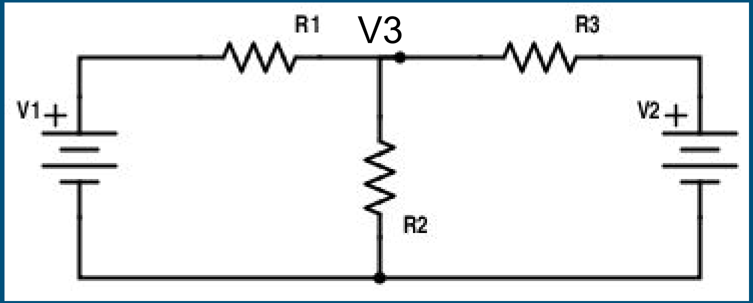
Equivalent restatement:

- Sum of current into a node = Sum of current out of a node

Use KCL to write an equation for each node when solving circuits

Kirchhoff's Current Law (Example)

Solve for V3:



Set the bottom node to be the reference ground, and apply KVL at the top node:

$(V3 - V1)/R1 + V3/R2 + (V3-V2)/R3 = 0$

Superposition

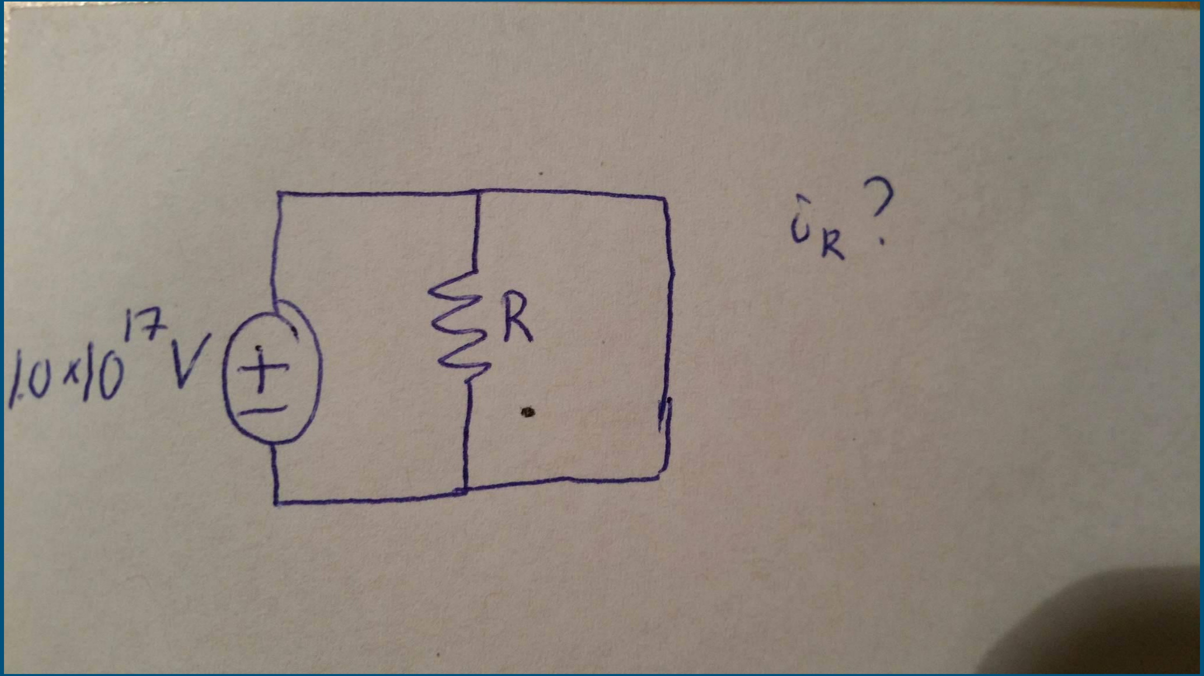
If a circuit has linear elements (like resistors), we can analyze it by considering only one voltage/current source at a time.

To do this, we “zero” out the other sources

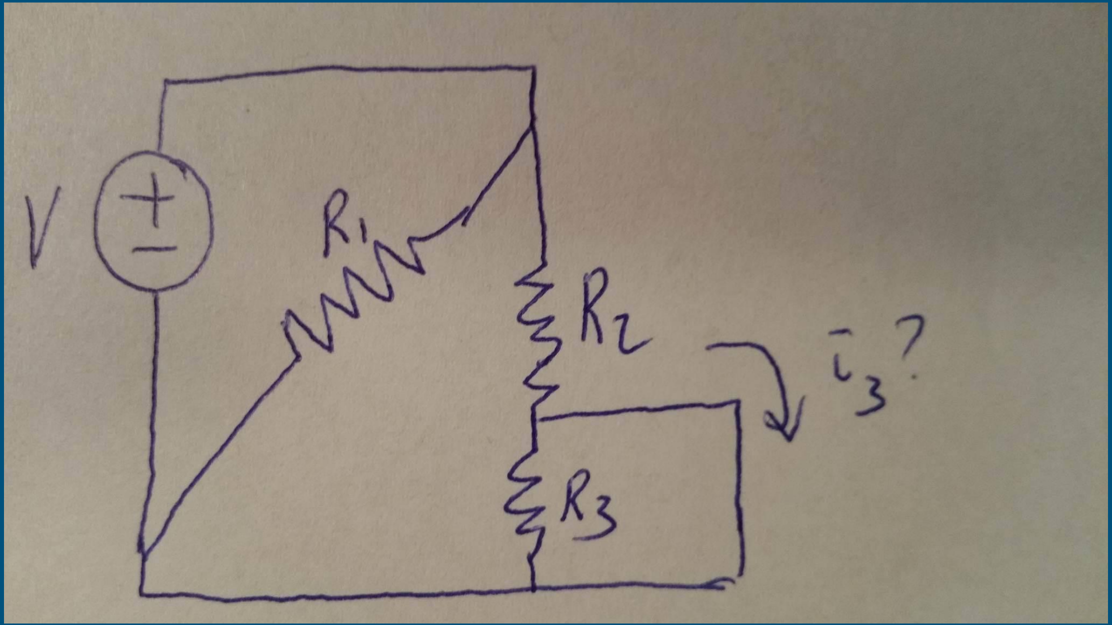
Zeroing out:

- Voltage sources become ideal wires ($V = 0$)
- Current sources become open circuits ($I = 0$)

Warm Up. Find current through R

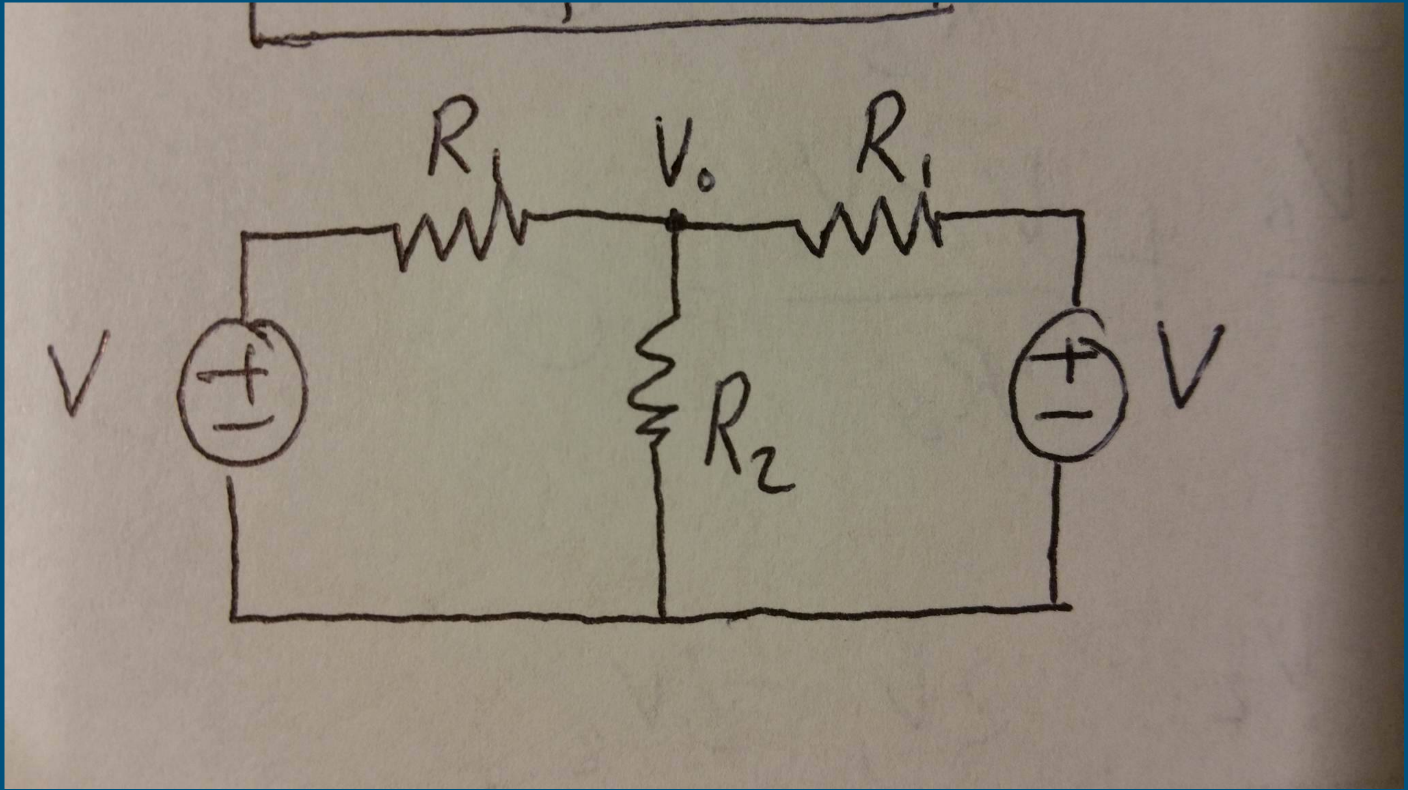


Find i3



Practice

Find V0



Voltage Divider

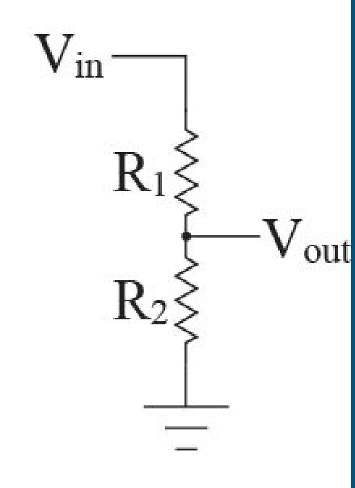
Super useful, a way to lower the voltage by a desired amount by using resistors.

$V_{out} = V_{in}R_2/(R_1+R_2)$

Intuitively thinking: If R_1 is much greater, then most of the voltage is lost across R_1, so $V_2 = V_{out}$ is small

Voltage across each resistor is proportional to resistance

Can also be used to find voltage across top resistor



Voltage Divider

If we want to halve our input voltage how can we use a voltage divider to do so?

Voltage Divider

If we want to halve our input voltage how can we use a voltage divider to do so?

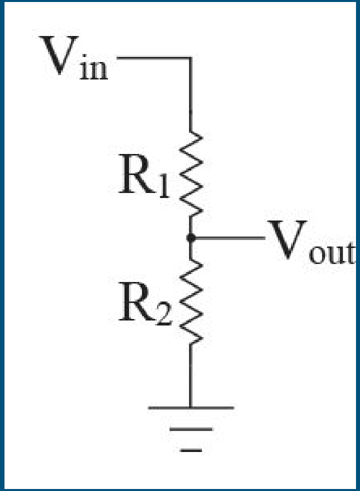
Well if we look at the equation it makes sense.

$V_{out} = V_{in}R_2/(R_1+R_2)$

We simply take $R_1 = R_2$ and then the fraction

$R_2 / (R_1 + R_2) = ½$

So we halve our voltage.



Current Divider

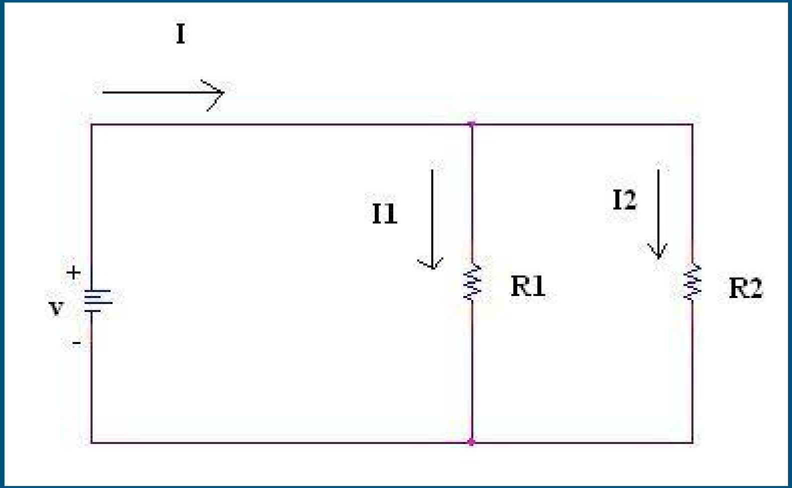
Similar to the Voltage divider, a way to get a fraction of the input current based on the resistors of our circuit.

Useful to solve problems.

$I_1 = R_2/(R_1 + R_2)\ I_{total}$

Intuitively: The more resistance in our R_2 value means that

there will be more current going through the first part (I_1)



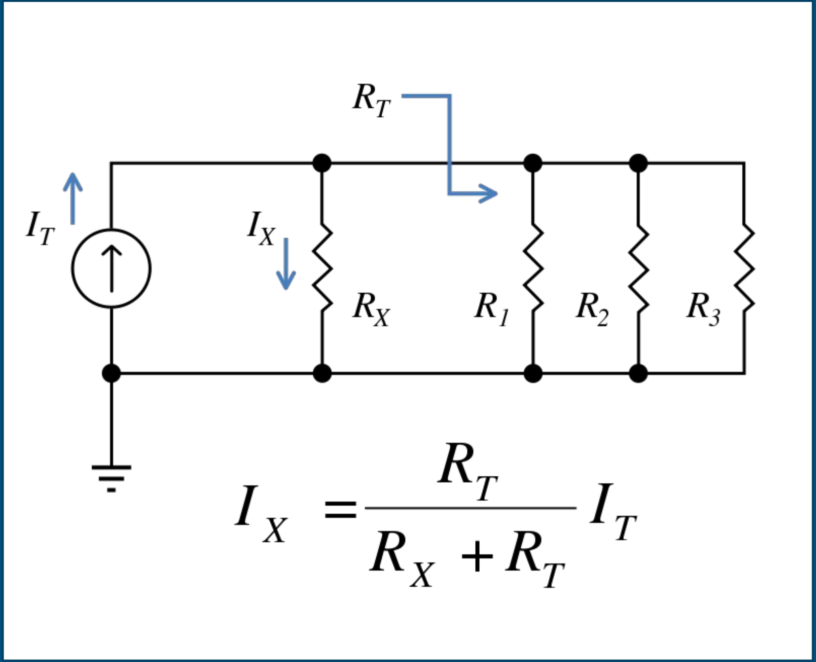
Current Divider

Note that in our formula

$I_X = R_T/(R_X + R_T)\ I_T$

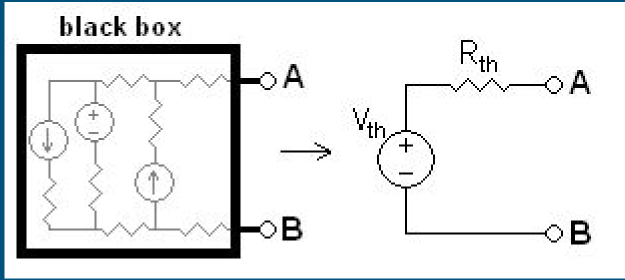
The R_T corresponds to the equivalent resistance of the rest of the circuit.

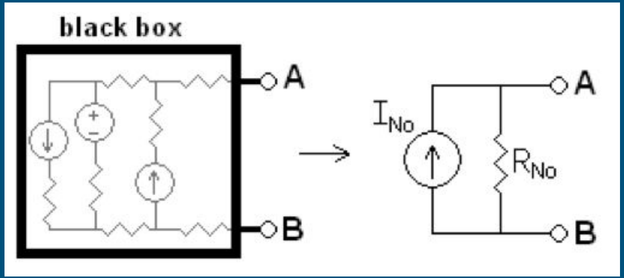
(Not including R_X)



Thévenin and Norton Equivalent Circuits

Theory: Any linear electrical network with voltage and current sources and only resistances can be replaced by an equivalent voltage source V_{th} in series connection with an equivalent resistance R_{th}, or an equivalent current source I_{no} in parallel with an equivalent resistance R_{no}.





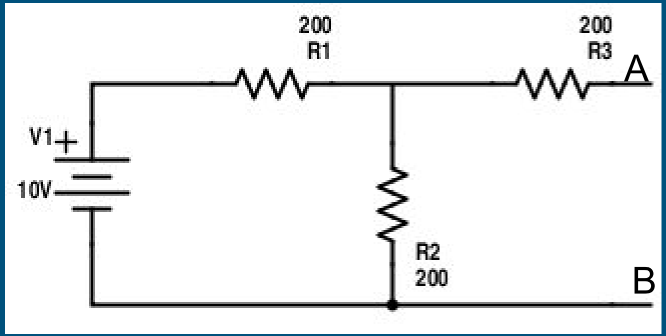
Thévenin and Norton Equivalent Circuits

How to solve Thévenin/Norton:

1. Find Thevenin voltage, V_{Th}, by treating the output terminals as an open circuit
2. Find Norton current, I_x, by treating the output terminals as a short circuit
3. Lastly, solve for Thevenin resistance: $R = V_{Th} / I_x$

Thévenin/Norton Equivalent (example)

A. Find the Thévenin voltage and resistance between nodes A and B
B. Find the Norton current and resistance between nodes A and B



Thévenin/Norton Equivalent (solution)

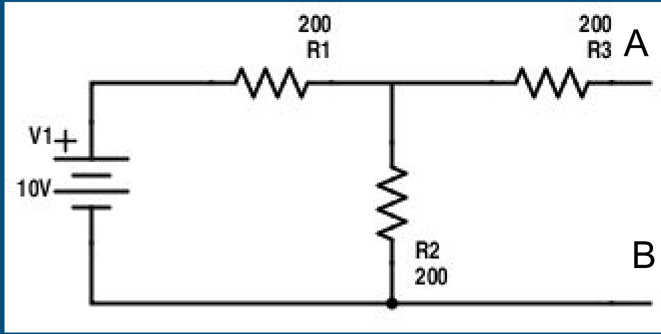
$V_{Th} = V_1 * (R_2 / (R_1 + R_2)) = 5V$ (Voltage Divider)

$I_{total} = 10V / (200 \parallel 200 + 200)\ \Omega = 10V / 300\ \Omega = 1/30\ A$

$I_{No} = I_{total} * (R_2 / (R_2 + R_3)) = (1/30\ A) * (200/400) = 1/60\ A$

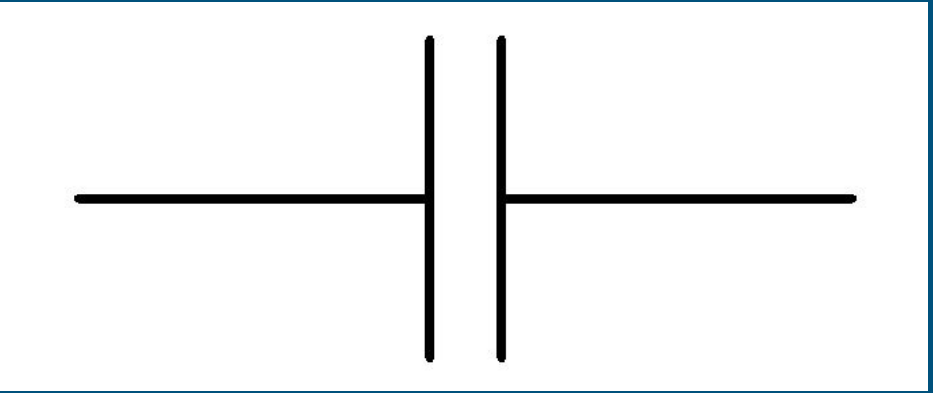
$R_{Th} = R_{No} = V_{Th} / I_{No} = 300\ \Omega$

A. $V_{Th} = 5V$, $R_{Th} = 300\ \Omega$
B. $I_{No} = 1/60\ A$, $R_{No} = 300\ \Omega$



Capacitance

- What is a capacitor?
- Generally two plates that store charge, with non-conductive material between plates
- Unit is the Farad (F)



Capacitance

Three equations to know:

$C = Q/V$ (How much charge per voltage in the capacitor)

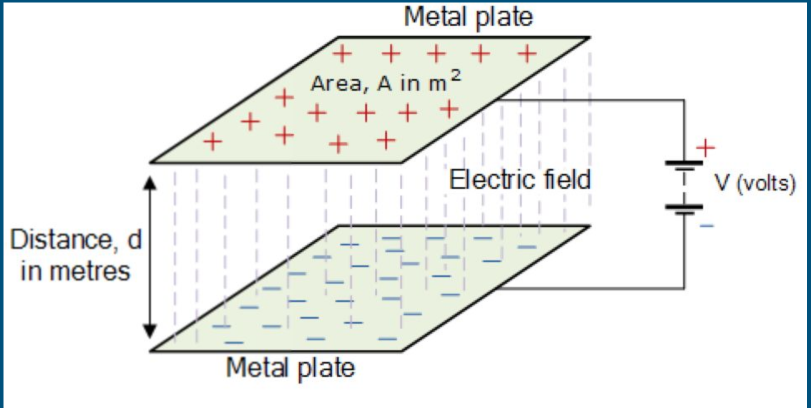
$C = \varepsilon A/d$

$E = \frac{1}{2} CV^2$ (Amount of energy stored in the capacitor)

Physical Meaning of Capacitance

$C = \varepsilon A/d$

- C: capacitance
- A: Area of capacitor (one plate)
- d: distance between plates
- ε: "permittivity", a constant depending on the material in the space between the two plates



Capacitors in Parallel

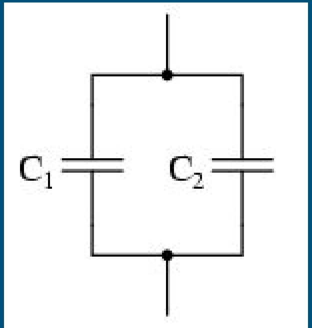
We know that the two capacitors must be at the same voltage but not necessarily have the same charge. So:

$C_{total} = Q/V$

$= (Q_1 + Q_2)/V$

$= Q_1/V + Q_2/V$

$C_{total} = C_1 + C_2$



Capacitors in Parallel

TLDR: Just add them

$$C_{total} = \Sigma C_n$$

Capacitors in Series

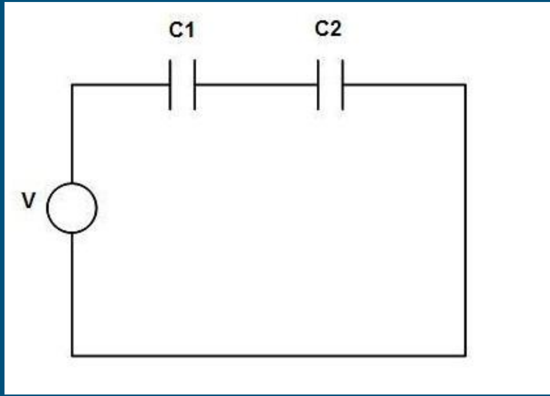
We know that both C_1 and C_2 have the **same charge Q** stored in them since the current going through each of the capacitors must leave through the other. On the other hand, the voltages sum to the total voltage

Knowing this:

$1/C_{total} = V_{total}/Q = (V_1 + V_2)/Q$

$= V_1/Q + V_2/Q$

$1/C_{total} = 1/C_1 + 1/C_2$



Capacitors

TLDR:

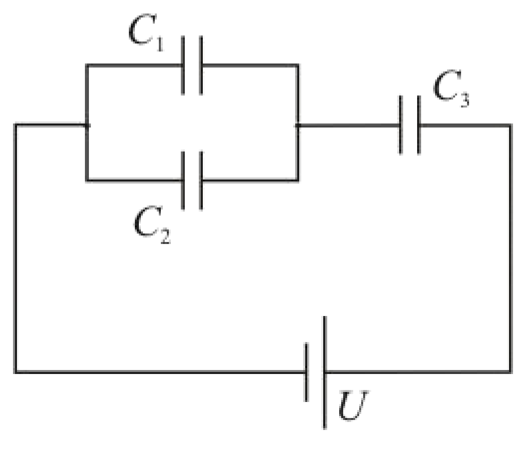
In series: $1/C_{total} = 1/C_1 + 1/C_2$

In parallel: $C_{total} = C_1 + C_2$

Opposite of resistors!

Quick Capacitor Problem

Find the total capacitance in this circuit.



Quick Capacitor Problem

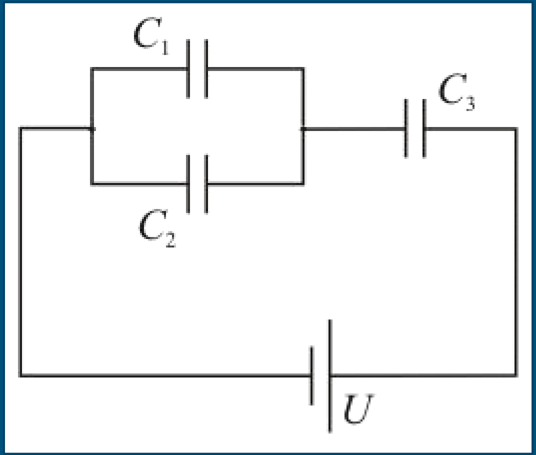
Find the total capacitance in this circuit.

The parallel portion becomes: $C_{par} = C_1 + C_2$

Then we add the series:

$1/C_{total} = 1/C_{par} + 1/C_3 = (C_{par} + C_3)/(C_{par}\,C_3)$

$C_{total} = (C_{par}\,C_3)/(C_{par} + C_3)$



Charging a Capacitor

- When a capacitor is supplied with current, it charges up.
- Since V = Q/C the voltage across the capacitor increases with time.
- Similarly when the capacitor discharges it loses charge and as a result voltage.
- Charge on capacitor after t seconds under constant current is simply I*t. (current * time)

Charge Sharing

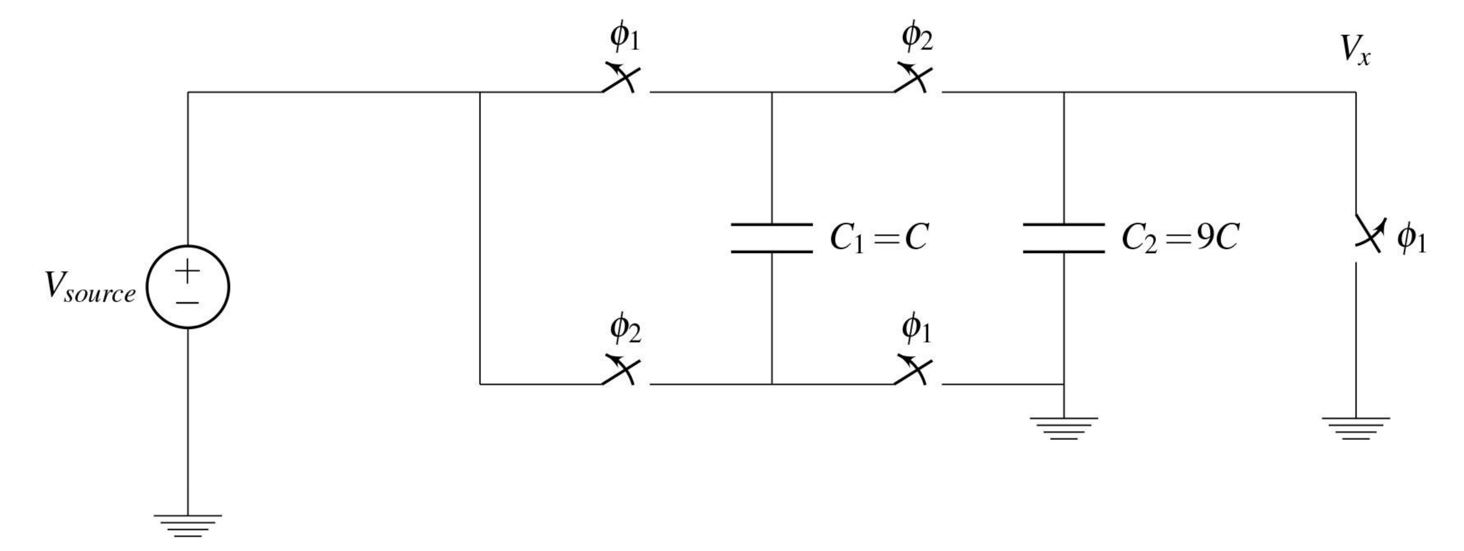
Capacitors can be first charged up, then reconfigured into a different circuit, usually via switches

States to Analyze:

1. Initial state -- after charging up
2. Final state -- after charges redistribute in new configuration

Charge Sharing Problem

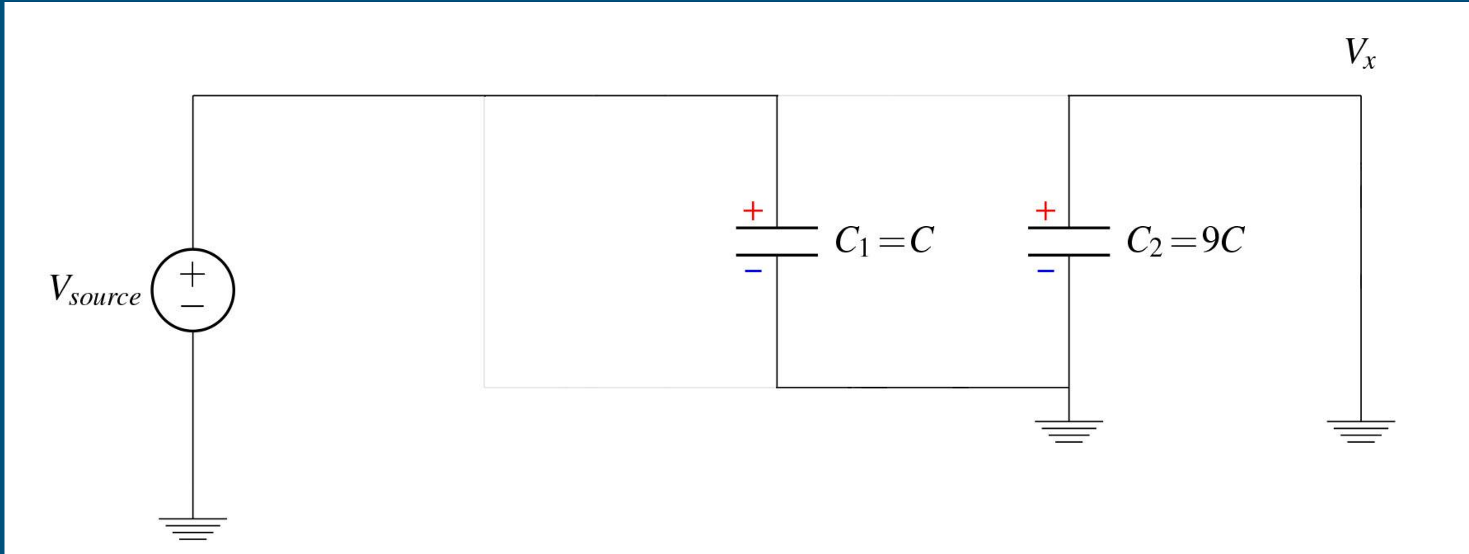
During phase 1, all φ_1 switches are <u>closed</u>, and all φ_2 switches are <u>open</u>.
During phase 2, all φ_1 switches are <u>open</u>, and all φ_2 switches are <u>closed</u>.
What is V_x during phase 2?



Charge Sharing Problem: Phase 1

$Q_1 = C_1V_1 = CV_S$

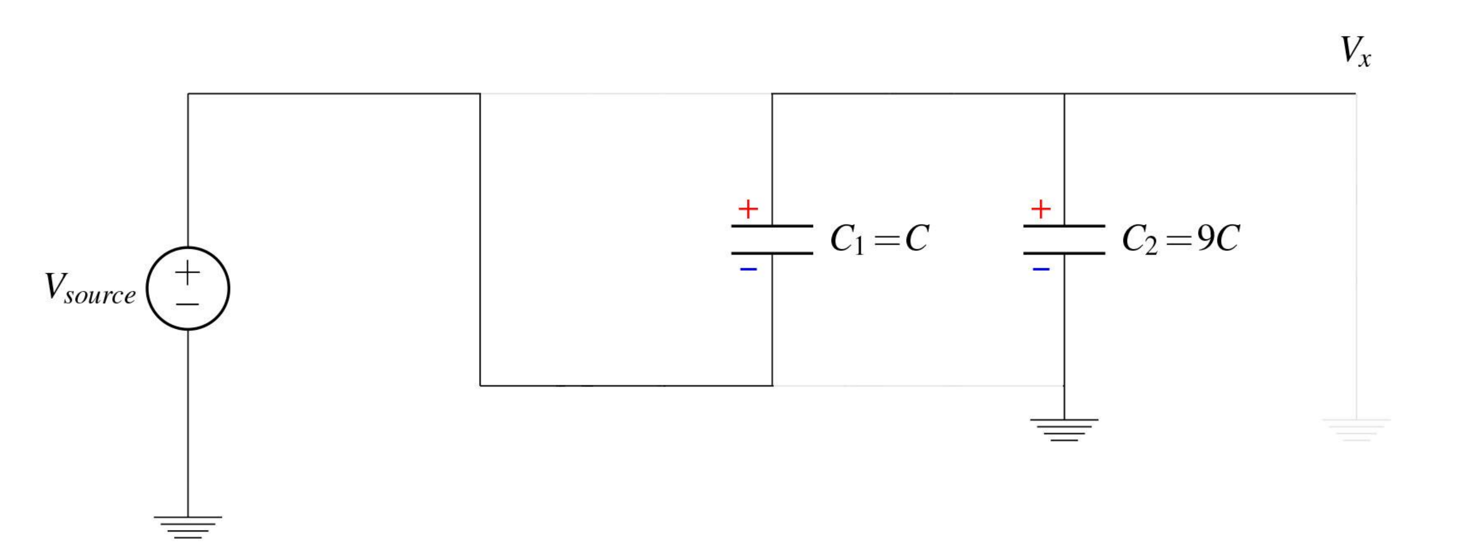
$Q_2 = C_2V_2 = (9C)(0) = 0$



Charge Sharing Problem: Phase 2

$V_1 = V_X - V_S \quad \rightarrow \quad Q_1 = C_1(V_X - V_S) = C(V_X - V_S)$

$V_2 = V_X \quad \rightarrow \quad Q_2 = C_2V_X = 9CV_X$



Charge Sharing Problem: Solution!

Phase 1:

$Q_1 = CV_S$

$Q_2 = 0$

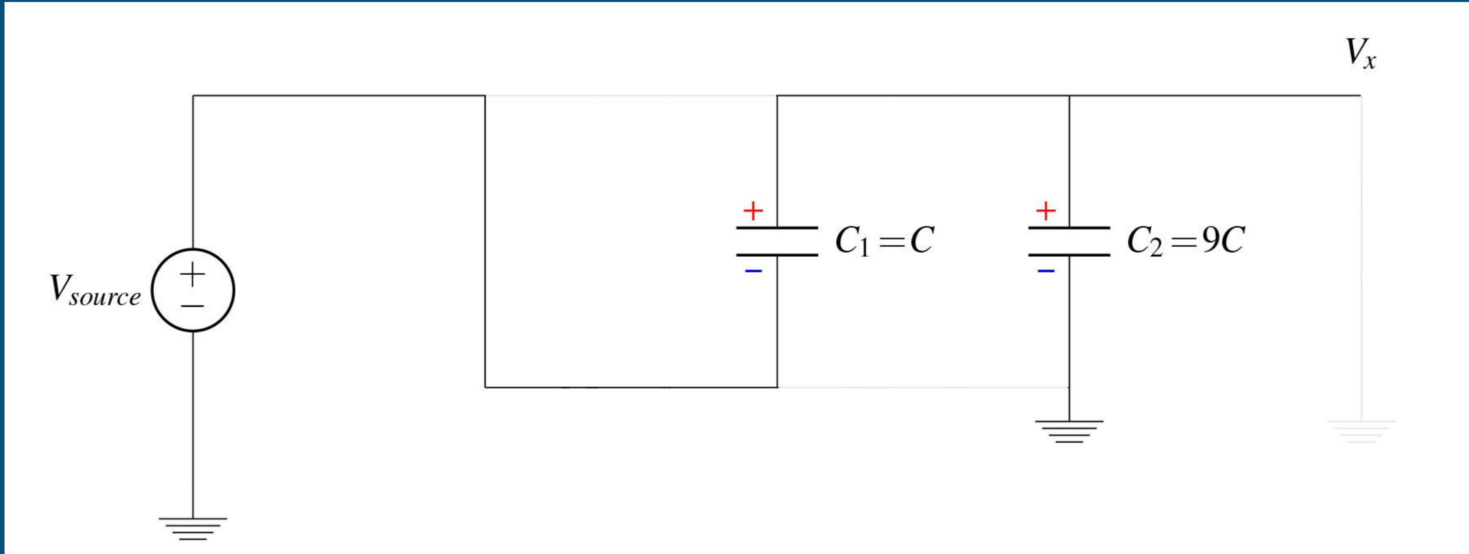
Phase 2:

$Q_1 = C(V_X - V_S)$

$Q_2 = 9CV_X$

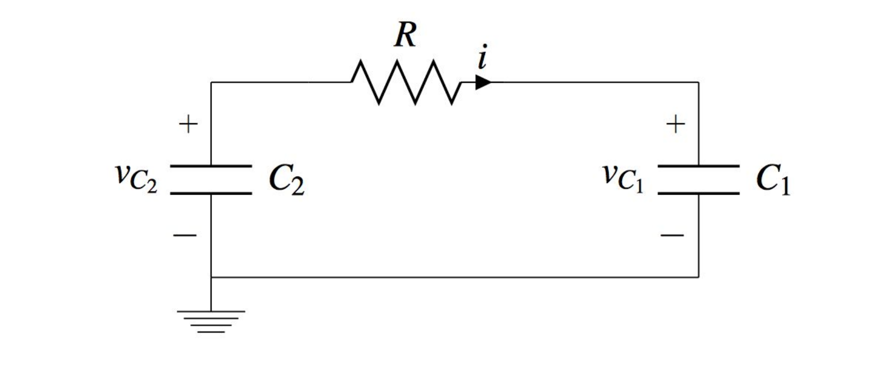
$$CV_S + 0 = C(V_X - V_S) + 9CV_X$$

$$CV_S = CV_X - CV_S + 9CV_X \quad \rightarrow \quad 2CV_S = 10CV_X \quad \rightarrow \quad \boxed{V_X = V_S / 5}$$



Another Charge Sharing Problem

Suppose C2 has a charge Q2 and C1 has a charge Q1. Find the charge on C1 at steady state.



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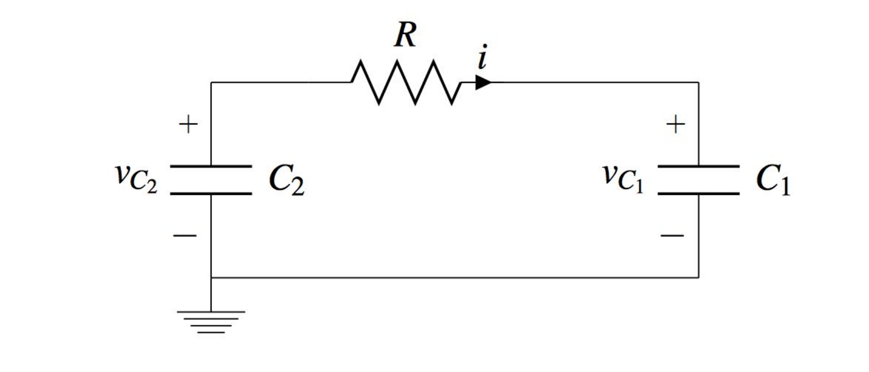


Answer: The resistor can't remove charge!

So Q1final = C1(Q1+Q2)/(C1+C2)

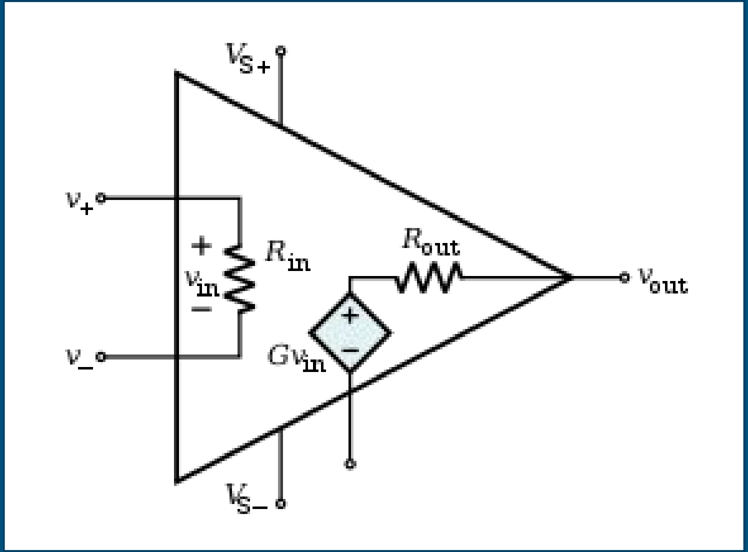
Op Amps

Very useful! Here's what one looks like:

Important facts:

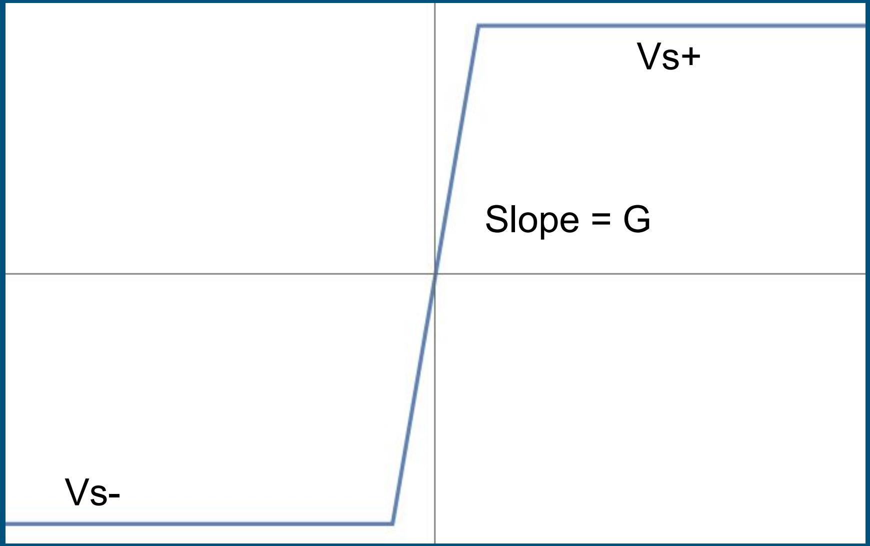
- Vout = G(V+ - V-)
- Vs+ ≥ Vout ≥ Vs-

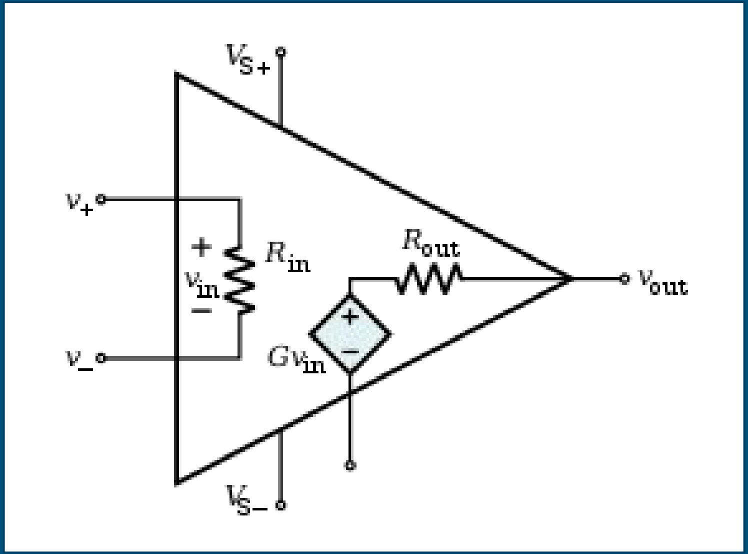
The last fact says that the output voltage "clips" if the input voltage difference is too large



Op Amps

Graph of the output voltage:





Golden Rules

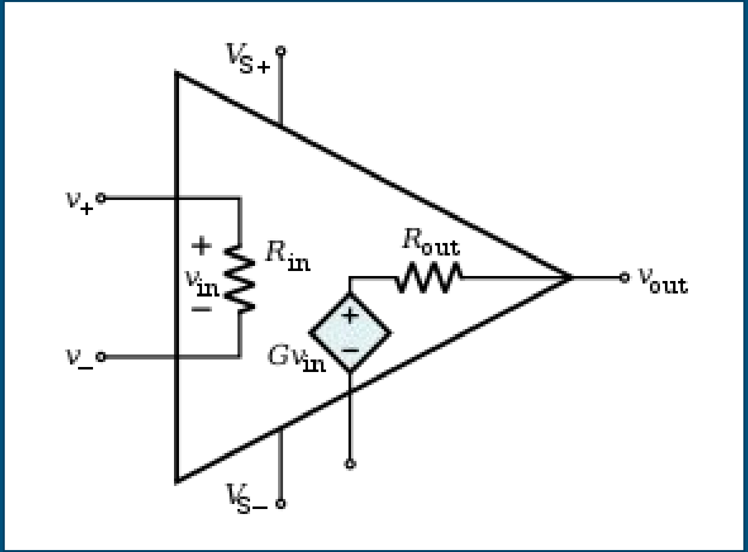
Ideal op-amps: take G → ∞, Rin → ∞, Rout →0

For all ideal op amps:

- Input terminals draw no current: I- = I+ = 0A

For all ideal op amps in negative feedback:

- No voltage difference between the two input terminals: V- = V+

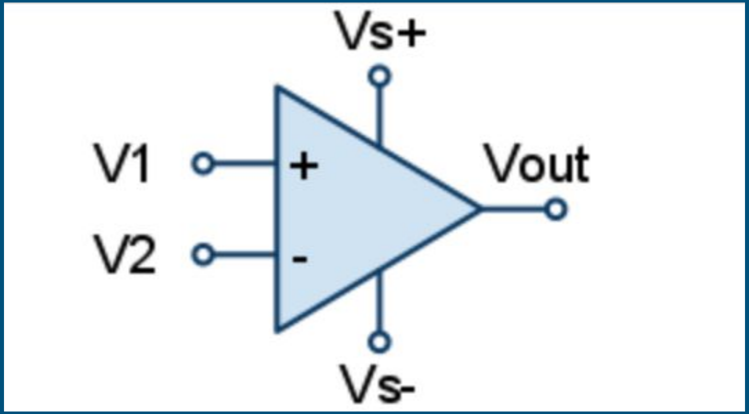


Common Op-amp Configurations

What is Vout when:

V1 > V2?

V2 > V1?



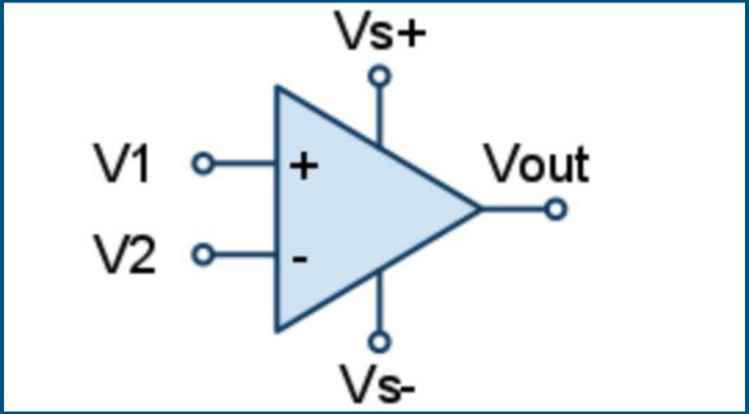
Common Op-amp Configurations

What is Vout when:

V1 > V2? Answer: Vs+

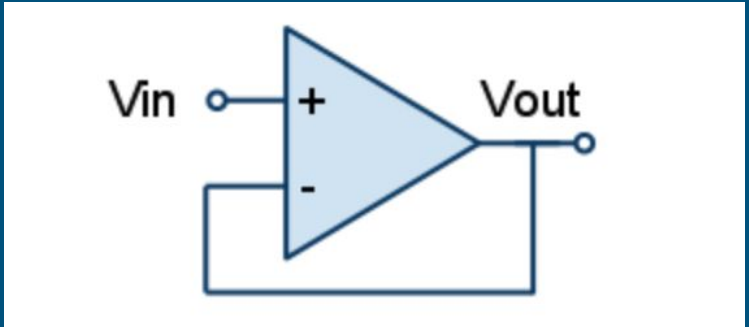
V2 > V1? Answer: Vs-

Reason: For ideal op-amps, the gain is really large (infinite). If there is a difference between V1 and V2, Vout will clip.



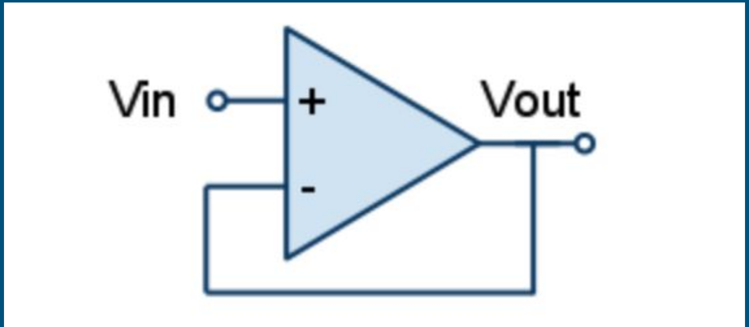
Common Op-amp Configurations

Use the golden rules to find Vout.



Common Op-amp Configurations

Use the golden rules to find Vout.



The op-amp is in negative feedback, so V+ = V-

Vin = V+, and Vout = V-

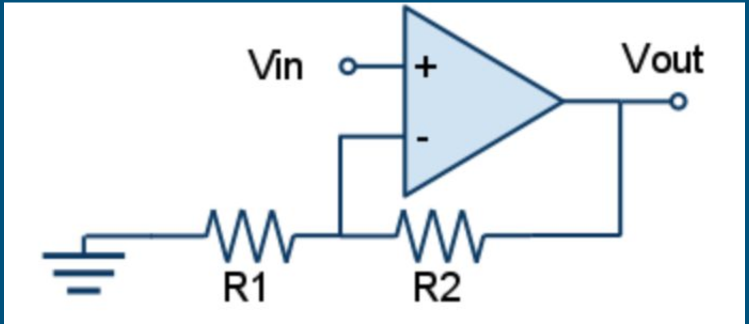
So Vout = Vin.

This configuration is called a voltage buffer.

Common Op-amp Configurations

Noninverting amplifier:

Compute Vout.



Common Op-amp Configurations

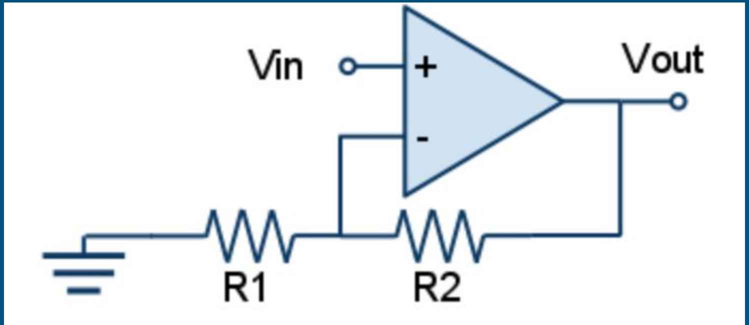
Noninverting amplifier:

Compute Vout.

Apply golden rules: Vin = V+ = V-

Use voltage divider: V- = Vout R1/(R1+R2)

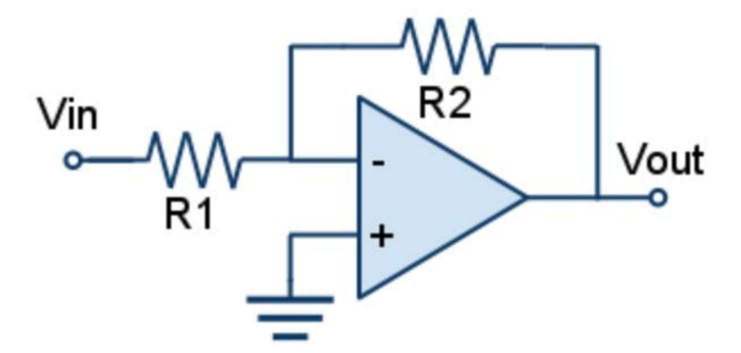
Combine and simplify: Vout = Vin (1 + R2/R1)



Common Op-amp Configurations

Inverting amplifier:

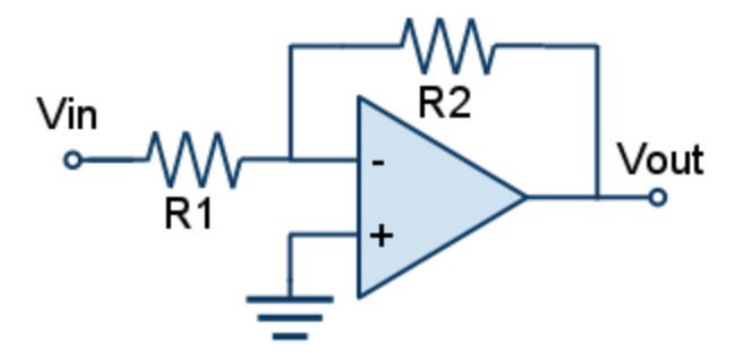
Compute Vout.



Common Op-amp Configurations

Inverting amplifier:

Compute Vout.

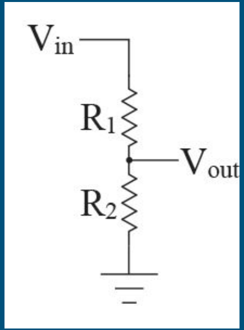


Apply KCL at the negative terminal:
$Vin/R1 + Vout/R2 = 0$

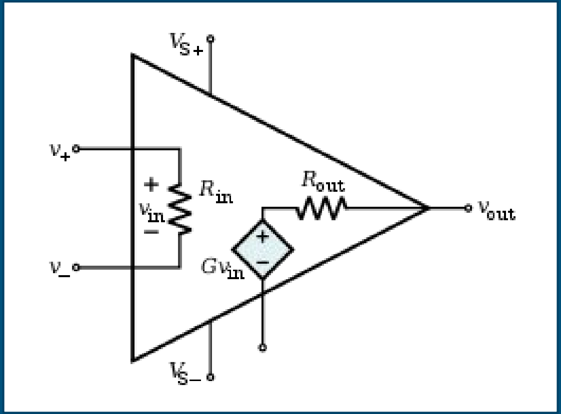
So $Vout = Vin(-R2/R1)$

Op-Amps vs Voltage Dividers

With voltage dividers: Vout depends on the load.

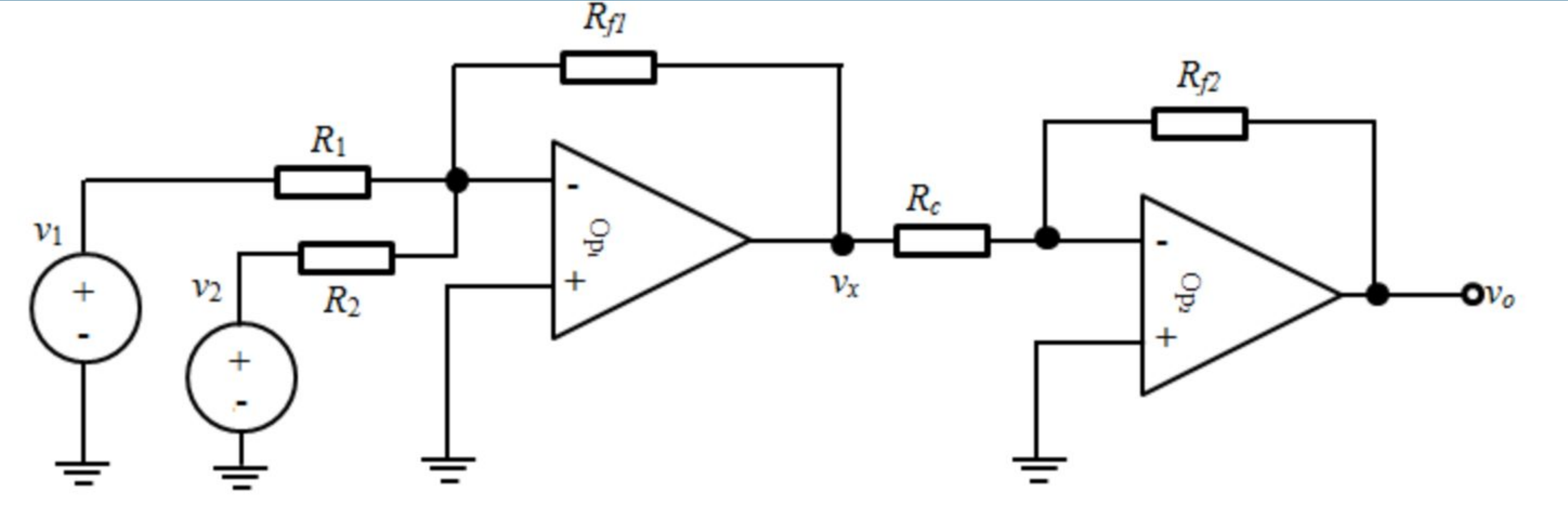


With op-amps: Vout is INDEPENDENT of the load. This is really useful.



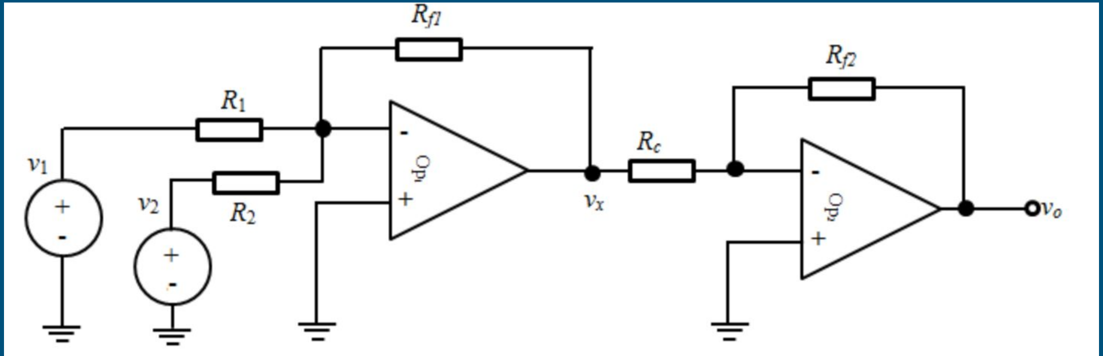
Op-Amps vs Voltage Dividers

So we can do this...



Op-Amps vs Voltage Dividers

This circuit looks complicated, but is actually simple to analyze.

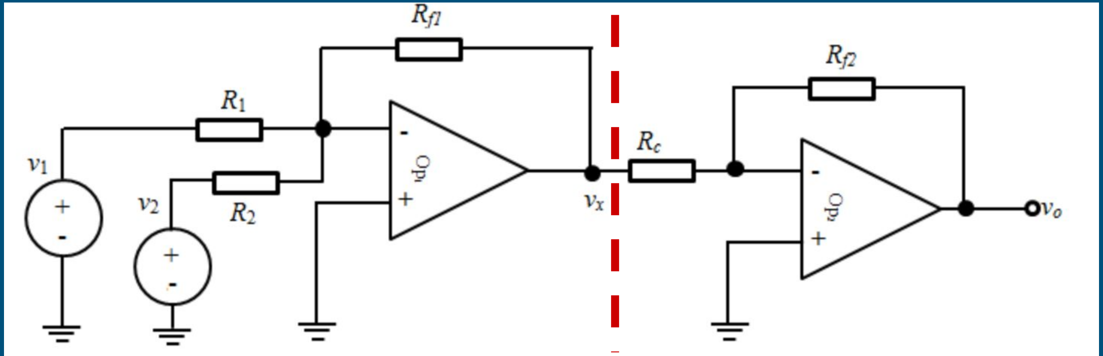


Op-Amps vs Voltage Dividers

This circuit looks complicated, but is actually simple to analyze.

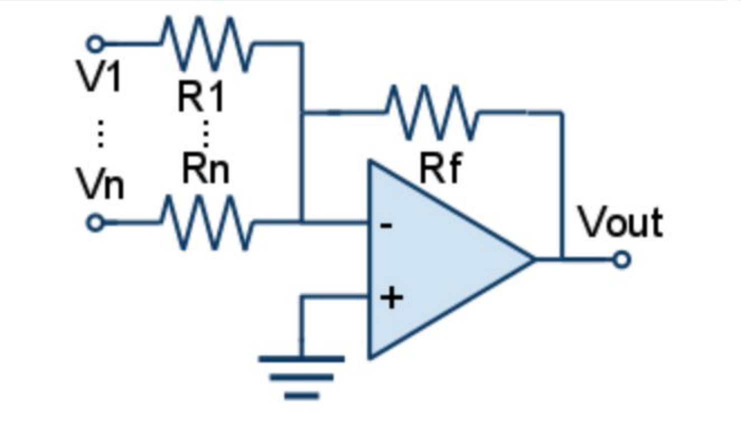
Split the circuit into two blocks, and analyze them individually.

Then combine to get the final result.



Op-Amp Practice

Compute Vout.



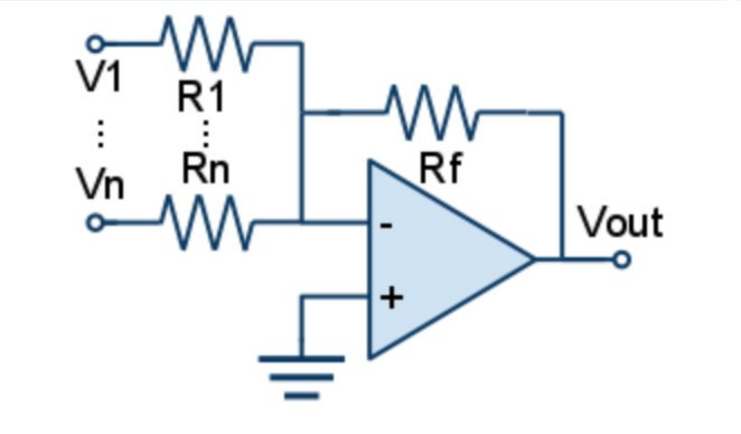
Op-Amp Practice

Compute Vout.

Solution: This is similar to the inverting amplifier. KCL at the negative terminal gives: V1/R1 + … + Vn/Rn + Vout/Rf = 0

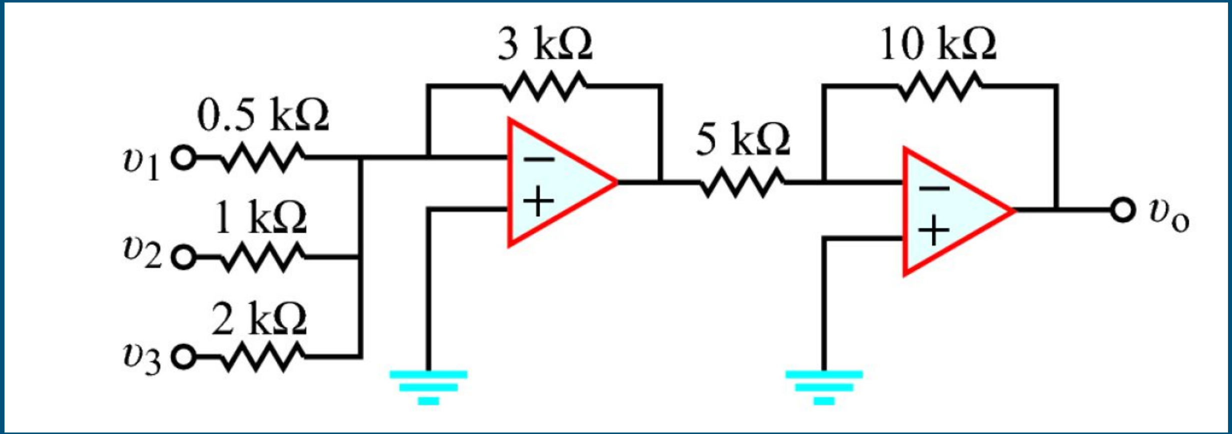
So, Vout = -Rf(V1/R1 + … + Vn/Rn)

If there is only one voltage source, then this just reduces to the inverting amplifier.



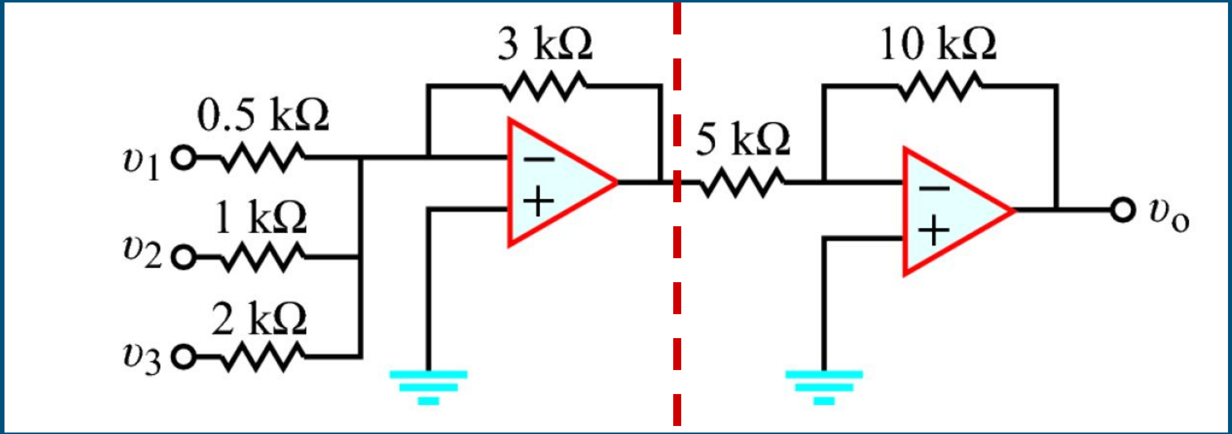
Op-Amp Practice

Compute Vout.



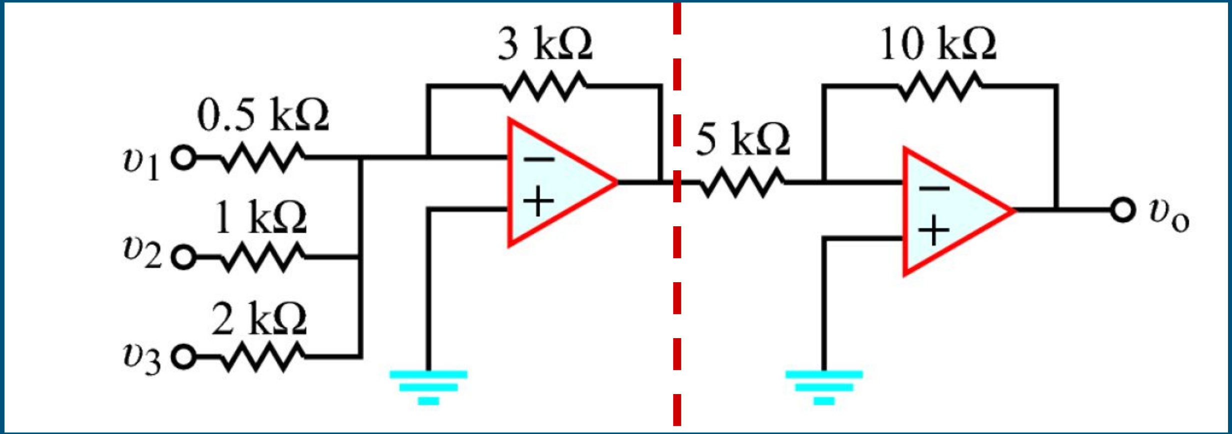
Op-Amp Practice

First, divide the circuit into blocks.



Op-Amp Practice

Vout of the first block is -3(2 V1 + V2 + V3/2)



Vout of the second block is -2Vin

So Vo = (-2)(-3 (2 V1 + V2 + V3/2)) = 12 V1 + 6 V2 + 3 V3

Op-Amp Summary

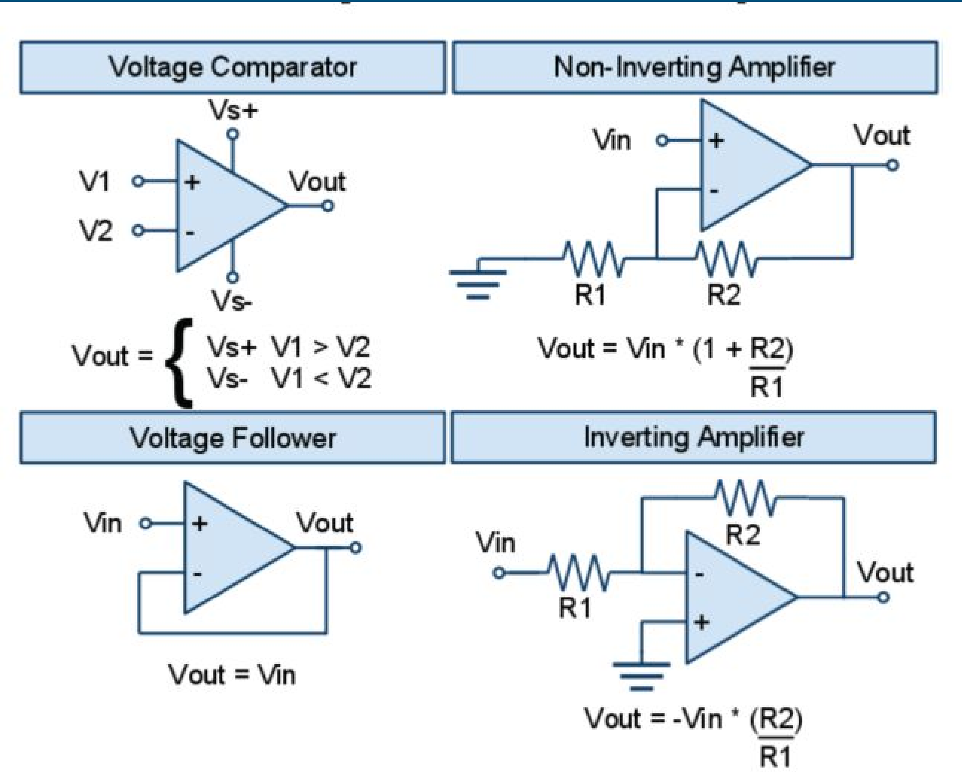
In general, Vout = G(V+ - V-).

Golden rules:

- I+ = I- = 0
- V+ = V- (for negative feedback only)

Vout is independent of the load resistance, so we can analyze circuits in blocks.

Op-Amp Summary

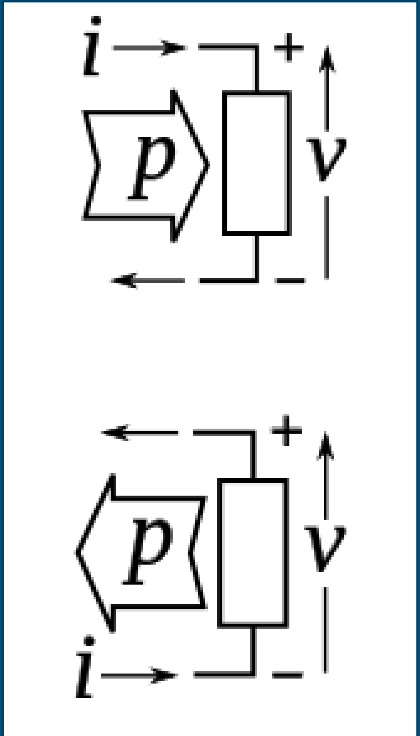


Power (definition)

- Amount of energy supplied/dissipated per unit time:
  - Units: Watts (Joules/second)
  - General Equation: $P = IV$
    - For resistors, $P = IV = (V^2)/R = (I^2)R$

Power Conventions

- We use the “passive sign convention”
  - Elements consuming power have current entering the higher voltage node (top diagram)
  - Elements supplying power have current exiting the higher voltage node (bottom diagram)



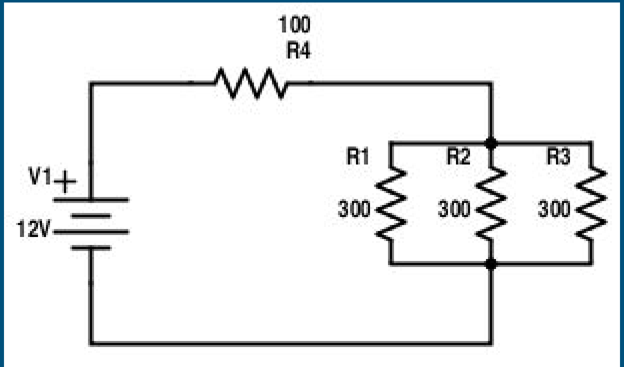
Power

- A circuit element either SUPPLIES power, or CONSUMES power
  - Voltage/Current sources SUPPLY power
  - Resistors always CONSUME power
    - Convert electrical energy to heat
  - Capacitors can either supply or consume power
    - Consumes energy when charging the capacitor
    - Supplies energy when discharging the capacitor

Power (practice problem 1)

Question:

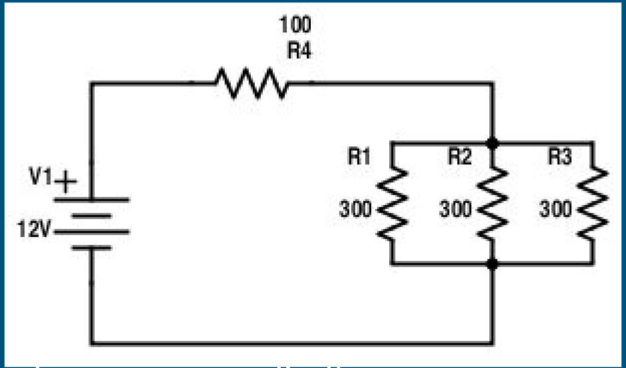
A. Find the total power consumed by the resistors
B. Find the power supplied by the battery



Power (problem 1 solution)

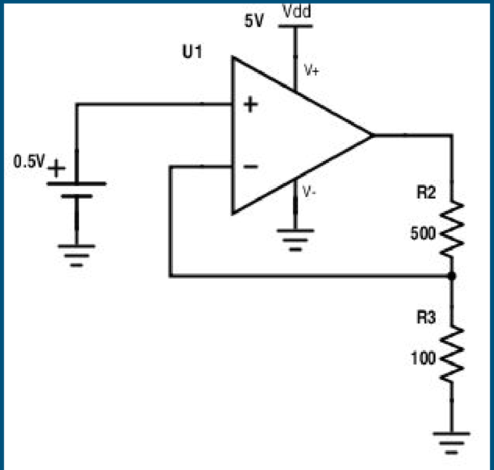
A. Power dissipated by resistor
   a. Equivalent Resistance
      i. Parallel network: 1/R = 1/300 + 1/300 + 1/300 = 1/100. R = 100
      ii. Total resistance = 100 + 100 = 200
   b. Total current
      i. 12/200 = 0.06 A = 60 mA
   c. Power
      i. P = (I^2)(R) = (60 mA)^2 * 200 Ohms
      ii. **P = 0.72 W**

B. Power given by battery
   a. By conservation of energy, total power = 0, (power consumed = power supplied)
   b. P = 0.72W



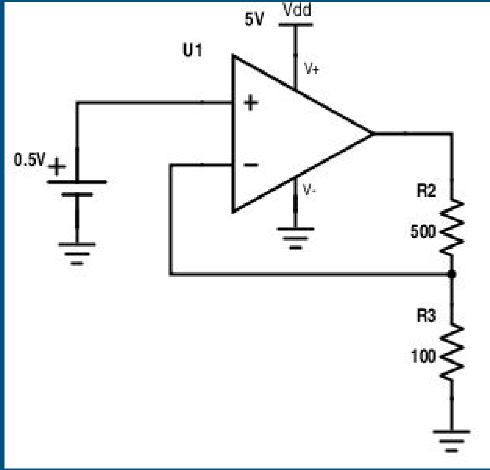
Power (practice problem 2)

A. Find the current going through the resistors
B. Find the power dissipated through the resistors
C. Find the power consumed by the op-amp given that it is ideal, and 100% efficient
D. Find the power consumed by the op-amp given that it is ideal, and 75% efficient.



Power (practice problem 2 solution)

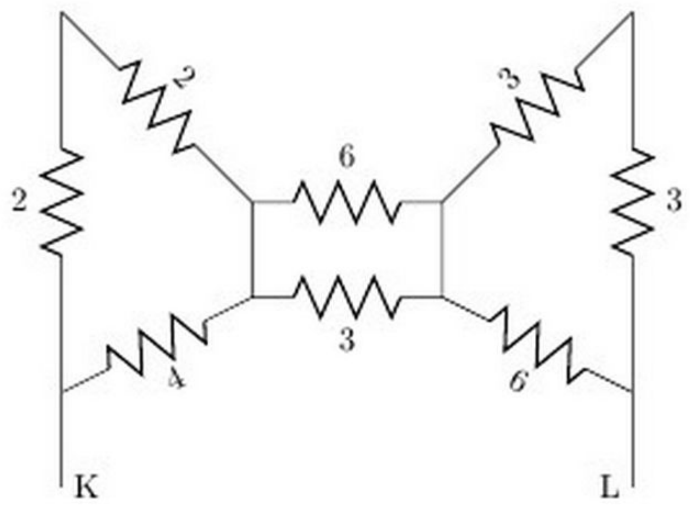
A. This is a non-inverting amplifier with resistor ratio of 5
   a. Gain = (1 + R2/R3) = (1 + 500/100) = 6
   b. Output Voltage = 0.5V * 6 = 3V
   c. 3V / 600Ohms = 5mA
B. P = IV
   a. 5mA * 3V = 15mW
C. Where does the opamp get its power?
   a. The power supply! Takes power from supply and puts it at the output
   b. 100% efficiency = all power from vdd gets transferred to output
   c. 0W consumed by opamp
D. What if it’s 75% efficient?
   a. op-amp converts 75% of power supply power into output power
   b. 15mW output power
   c. (15mW / 0.75) = 20mW supplied by Vdd
   d. 20mW - 15mW = 5mW consumed by op-amp



More Practice Problems

Parallel or Series?

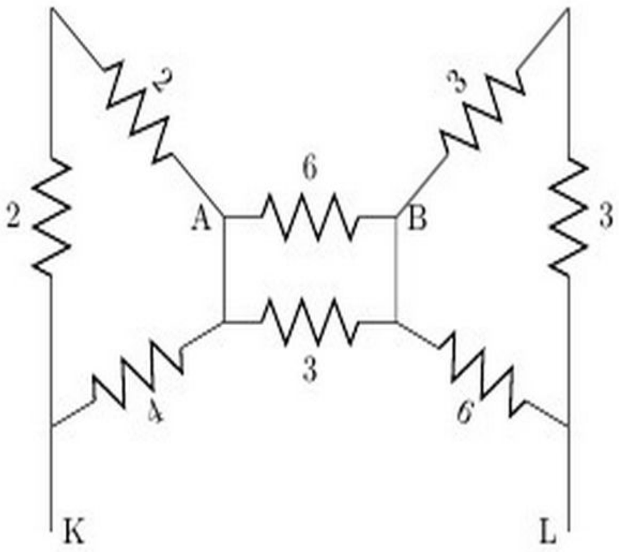
1. Compute the equivalent resistance of the following resistor network between the nodes K and L



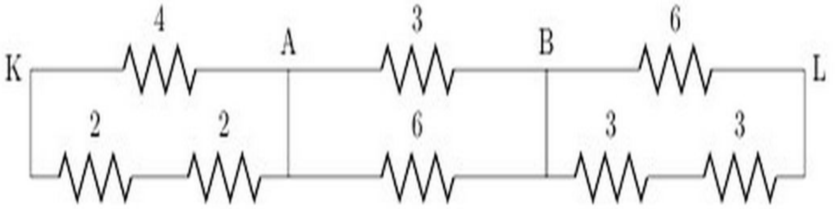
Hint: Try labeling the nodes!!!

Parallel or Series?

Let's label some additional nodes to make the calculation easier:



We can reduce the circuit to the following:



We can further reduce the circuit to the following:

This gives us an equivalent resistance of 7 ohms.

Supplying or Dissipating?

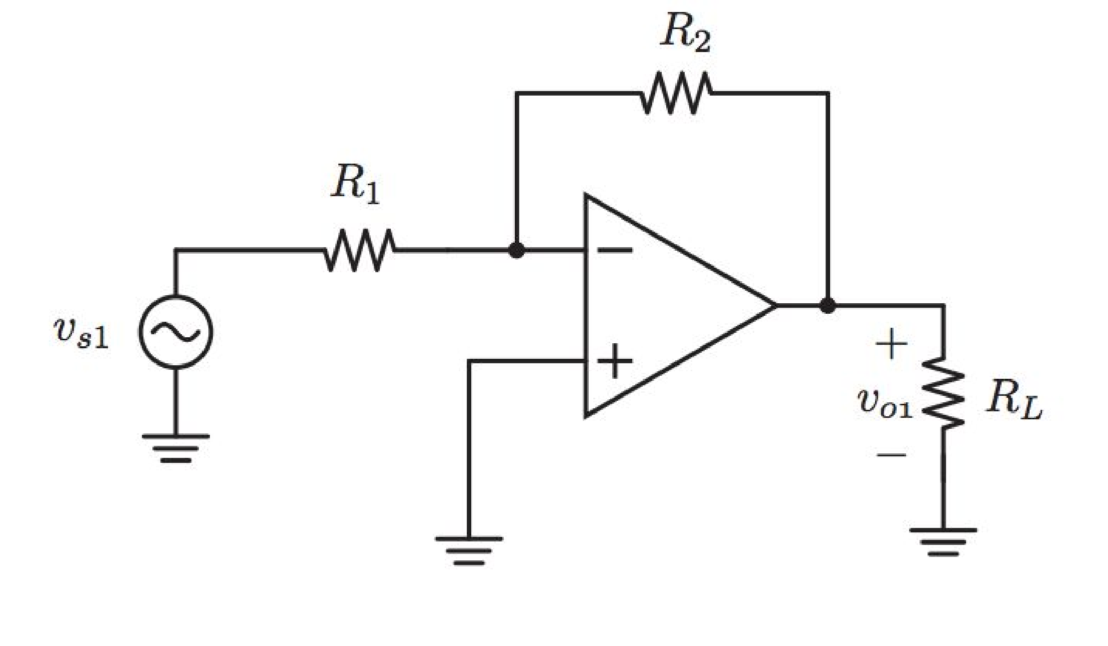
RECAP:

$P = I*V = V^2/R = I^2*R$

P < 0 : supplying

P > O : dissipating

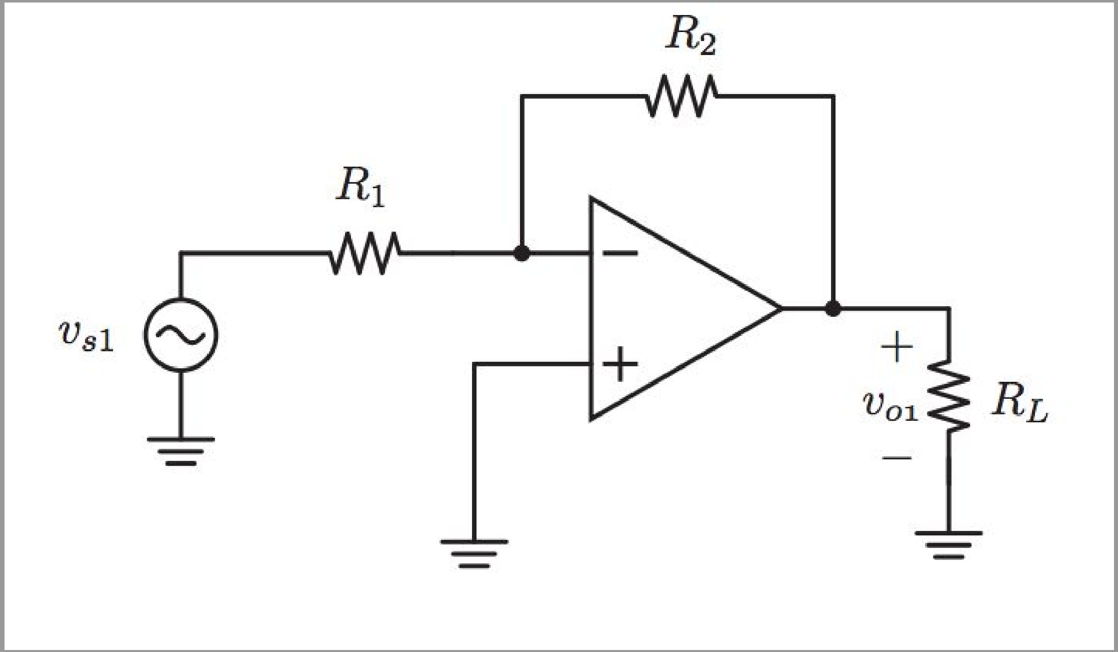
What is the power dissipated/generated by the independent source?



Supplying or Dissipating?

Vs1 is supplying

P = $Vs1^2/R1$

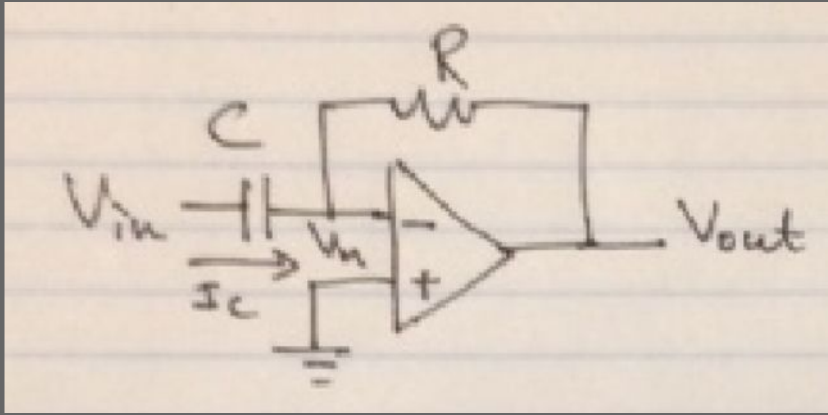


Out of scope

We won’t be presenting the slides following slides. They’re out of scope/not useful.

Calculus in op-amps!

Find Vout as a function of Vin

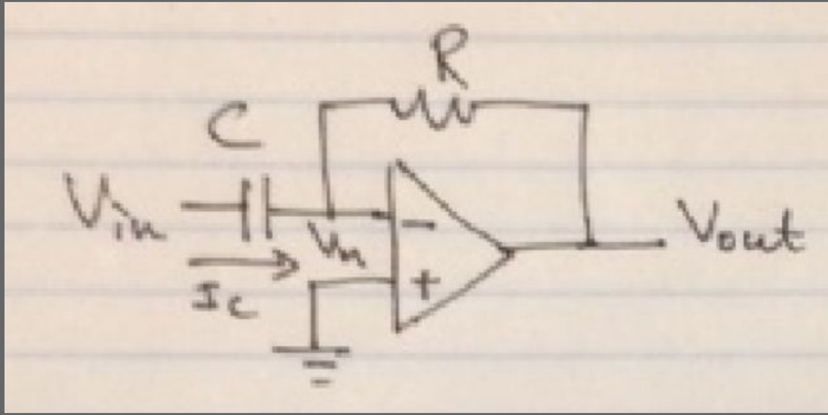


Solution

Virtual Ground! -ve Feedback!

Ic = C*(dVin/dt) = -Vout/R

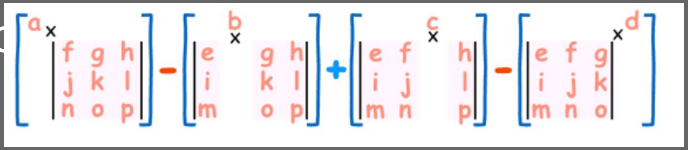
=> Vout = -RC*(dVin/dt)



Determinants

- Determinants
    - Can only be calculated for a square matrix
    - The determinant is ZERO when attempting to calculate the eigenvalue
        - $\det(A-\lambda I) = 0$
    - 2x2 matrix: Determinant is ad - bc
    - Higher dimension matrices: Multiply each value in a column or row by the determinant of the matrix nc

      remember to multiply in a + - ch



        - Sample 4x4 Matrix determinant

Eigenvectors

- $Av = \lambda v$ where v is the eigenvector, λ is the eigenvalue, I is the identity matrix
- $Av - \lambda v = 0$
- $(A - \lambda I)\, v = 0$

Sample problem:

Find the eigenvalues and eigenvectors of:

A = [[2,7], [-1,-6]]

Spoiler: Highlight to reveal