Chapter 1

Transformer and Inductor Design Philosophy

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Introduction

The conversion process in power electronics requires the use of transformers, and components which frequently are the heaviest and bulkiest items in the conversion circuits. They also have a significant effect upon the overall performance and efficiency of the system. Accordingly, the design of such transformers has an important influence on overall system weight, on the power conversion efficiency and cost. Because of the interdependence and interaction of parameters, judicious tradeoffs are necessary to achieve design optimization.

Power-Handling Ability

For years, manufacturers have assigned numeric codes to their cores; these codes represent the power-handling ability. This method assigns to each core, a number, which is the product of its window area, (W_a), and core cross-section area, (A_c), and is called, "Area Product," A_p.

These numbers are used by core suppliers to summarize dimensional and electrical properties in their catalogs. They are available for laminations, C-cores, pot cores, powder cores, ferrite toroids, and toroidal tape-wound cores.

The regulation and power-handling ability of a core is related to the core geometry, K_g. Every core has its own inherent, K_g. The core geometry, K_g, is a relatively new for magnetic cores. Manufacturers do not list this coefficient.

Because of their significance, the area product, A_p, and core geometry, K_g, are treated extensively in this book. A great deal of other information is also presented for the convenience of the designer. Much of the material is in tabular form to assist the designer in making the tradeoffs, best-suited for his particular application, in a minimum amount of time.

These relationships can now be used as new tools to simplify and standardize the process of transformer design. They make it possible to design transformers of a lighter weight and smaller volume, or to optimize efficiency without going through a cut and try, design procedure. While developed specifically for aerospace applications, the information has a wider utility, and can be used for the design of non-aerospace transformers, as well.

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Transformer Design

The designer is faced with a set of constraints which must be observed in the design of any transformer. One of these is the output power, P_O, (operating voltage multiplied by maximum current demand), which the secondary winding must be capable of delivering to the load within specified regulation limits. Another relates to minimum efficiency of operation which is dependent upon the maximum power loss, which can be allowed in the transformer. Still another defines the maximum permissible temperature rise for the transformer, when used in a specified temperature environment.

Other constraints relate to volume occupied by the transformer, particularly, in aerospace applications, and weight, since weight minimization is an important goal in the design of space flight electronics. Lastly, cost effectiveness is always an important consideration.

Output power, (P_o), is of greatest interest to the user. To the transformer designer it is the apparent power, (P_t), which is associated with the greater important geometry of the transformer. Assume, for the sake of simplicity, the core of an isolation transformer has only two windings in the window area, (W_a), a primary and a secondary. Also, assume that the window area, (W_a), is divided up in proportion to the power handling capability of the windings, using equal current density. This includes the primary winding handles, P_{in}, and the secondary handles, P_o, to the load. Since the power transformer has to be designed to accommodate the primary, P_{in} and P_o, then:

By definition:

$$P_t = P_{in} + P_o, \quad \text{[watts]}$$

$$P_{in} = \frac{P_o}{\eta}, \quad \text{[watts]}$$

The primary turns can be expressed using Faraday's law:

$$N_p = \frac{V_p\left(10^4\right)}{A_c B_{ac} f K_f}, \quad \text{[turns]}$$

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The winding area of a transformer is fully utilized when:

$$K_u W_a = N_p A_{wp} + N_s A_{ws}$$

By definition the wire area is:

$$A_w = \frac{I}{J}, \quad [\text{cm}^2]$$

Rearranging the equation shows:

$$K_u W_a = N_p \left(\frac{I_p}{J} \right) + N_s \left(\frac{I_s}{J} \right)$$

Now, substitute in Faraday's equation:

$$K_u W_a = \frac{V_p \left(10^4\right)}{A_c B_{ac} f K_f} \left(\frac{I_p}{J} \right) + \frac{V_s \left(10^4\right)}{A_c B_{ac} f K_f} \left(\frac{I_s}{J} \right)$$

Rearranging shows:

$$W_a A_c = \frac{\left[\left(V_p I_p\right) + \left(V_s I_s\right)\right]\left(10^4\right)}{B_{ac} f J K_f K_u}, \quad [\text{cm}^4]$$

The output power, P_o, is:

$$P_o = V_s I_s, \quad [\text{watts}]$$

The input power, P_{in}, is:

$$P_{in} = V_p I_p, \quad [\text{watts}]$$

Then:

$$P_t = P_{in} + P_o, \quad [\text{watts}]$$

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Substitute in P_t:

$$W_a A_c = \frac{P_t(10^4)}{B_{ac} f J K_f K_u}, \quad [\text{cm}^4]$$

By definition, A_p, equals:

$$A_p = W_a A_c, \quad [\text{cm}^4]$$

Then:

$$A_p = \frac{P_t(10^4)}{B_{ac} f J K_f K_u}, \quad [\text{cm}^4]$$

The designer must be concerned with the apparent power, handling capability, P_t, of the transformer core and windings. P_t may vary by a factor, ranging from 2 to 2.828 times the input power, P_{in}, depending upon the type of circuit in which the transformer is used. If the current in the rectifier transformer becomes interrupted, its effective RMS value changes. Thus, transformer size, is not only determined by the load demand, but also, by application, because of the different copper losses incurred, due to the current waveform.

For example, for a load of one watt, compare the power handling capabilities required for each winding, (neglecting transformer and diode losses, so that $P_{in} = P_o$) for the full-wave bridge circuit of Figure 1-1, the full-wave center-tapped secondary circuit of Figure 1-2, and the push-pull, center-tapped full-wave circuit in Figure 1-3, where all the windings have the same number of turns, (N).

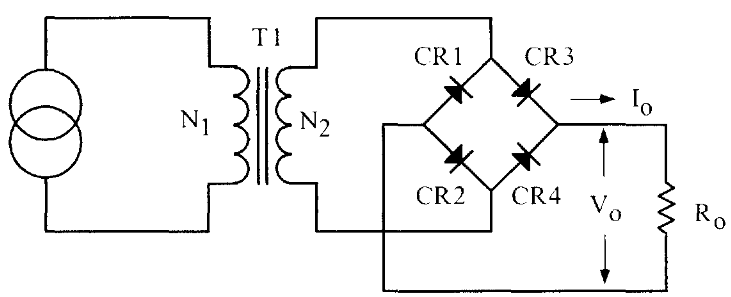

Figure 1-1. Full-Wave Bridge Secondary.

The total apparent power, P_t, for the circuit shown in Figure 1-1 is 2 watts.
This is shown in the following equation:

$$P_t = P_{in} + P_o, \quad [\text{watts}]$$

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$$P_t = 2P_{in}, \quad \text{[watts]}$$

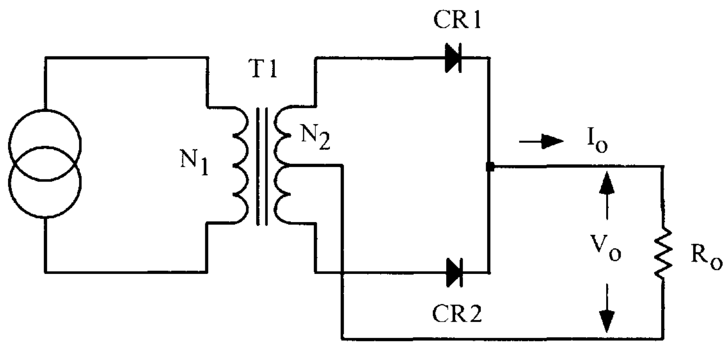

Figure 1-2. Full-Wave, Center-Tapped Secondary.

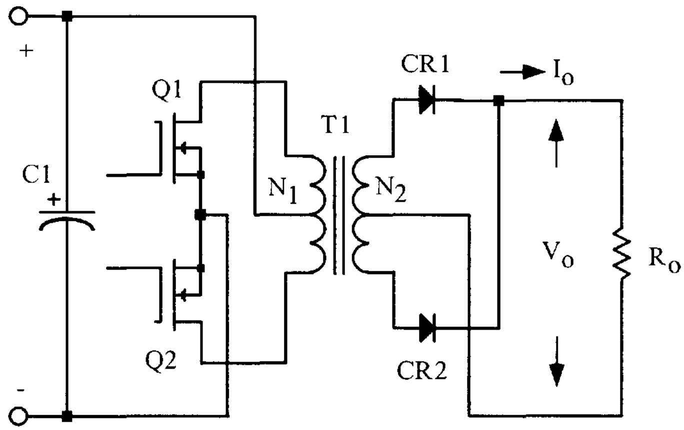

Figure 1-3. Push-Pull Primary, Full-Wave, Center-Tapped Secondary.

The total power, P_t, for the circuit shown in Figure 1-2, increased 20.7%, due to the distorted wave form of the interrupted current flowing in the secondary winding. This is shown in the following equation:

$$P_t = P_{in} + P_o\sqrt{2}, \quad \text{[watts]}$$

$$P_t = P_{in}\left(1+\sqrt{2}\right), \quad \text{[watts]}$$

The total power, P_t, for the circuit is shown in Figure 1-3, which is typical of a dc to dc converter. It increases to 2.828 times P_{in}, because of the interrupted current flowing in both the primary and secondary windings.

$$P_t = P_{in}\sqrt{2} + P_o\sqrt{2}, \quad \text{[watts]}$$

$$P_t = 2P_{in}\sqrt{2}, \quad \text{[watts]}$$

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Transformers with Multiple Outputs

This is an example of how the apparent power, P_t, changes with a multiple output transformers.

Output	Circuit
5 V @ 10A	center tapped V_d = diode drop = 1 V
15 V @ 1A	full wave bridge V_d = diode drop = 2 V

Efficiency = 0.95

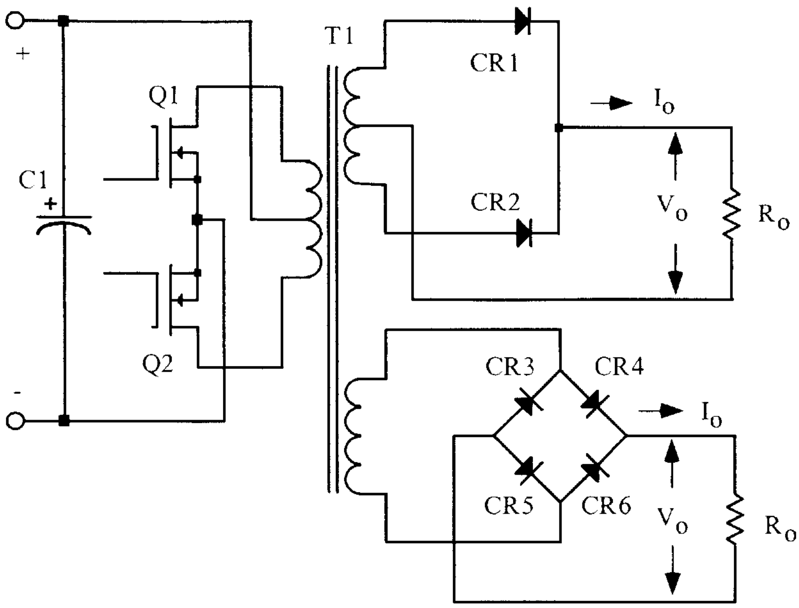

Figure 1-4. Multiple Output Converter.

The output power seen by the transformer in Figure 1-4 is:

$$P_{o1} = (V_{o1} + V_d)(I_{o1}), \quad \text{[watts]}$$
$$P_{o1} = (5+1)(10), \quad \text{[watts]}$$
$$P_{o1} = 60, \quad \text{[watts]}$$

and:

$$P_{o2} = (V_{o2} + V_d)(I_{o2}), \quad \text{[watts]}$$
$$P_{o2} = (15+2)(1.0), \quad \text{[watts]}$$
$$P_{o2} = 17, \quad \text{[watts]}$$

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Because of the different winding configurations the apparent power, P_t, the transformer will have to be summed to reflect this. When a winding has a center tap and produces a discontinuous current, then, the power in that winding, be it primary or secondary, has to be multiplied by the factor, U. The factor, U, corrects for the rms current in that winding, if the winding has a center tap, then, the factor U is equal to 1.41. If not, the factor, U, is equal to 1.

For an example, summing up the output power of a multiple output transformer, would be:

$$P_{\Sigma} = P_{o1}(U) + P_{o2}(U) + P_{n}(U) + \cdots\cdots$$

then:

$$P_{\Sigma} = P_{o1}(U) + P_{o2}(U), \quad \text{[watts]}$$
$$P_{\Sigma} = 60(1.41) + 17(1), \quad \text{[watts]}$$
$$P_{\Sigma} = 101.6, \quad \text{[watts]}$$

After the secondary has been totaled, then the primary power can be calculated.

$$P_{in} = \frac{P_{o1} + P_{o2}}{\eta}, \quad \text{[watts]}$$
$$P_{in} = \frac{(60) + (17)}{(0.95)}, \quad \text{[watts]}$$
$$P_{in} = 81, \quad \text{[watts]}$$

Then, the apparent power, P_t, equals:

$$P_t = P_{in}(U) + P_{\Sigma}, \quad \text{[watts]}$$
$$P_t = (81)(1.41) + (101.6), \quad \text{[watts]}$$
$$P_t = 215.8, \quad \text{[watts]}$$

Regulation

The minimum size of a transformer is usually determined either by a temperature rise limit, or by allowable voltage regulation, assuming that size and weight are to be minimized. Figure 1-5 shows a circuit diagram of a transformer with one secondary.

Note that α = regulation (%).

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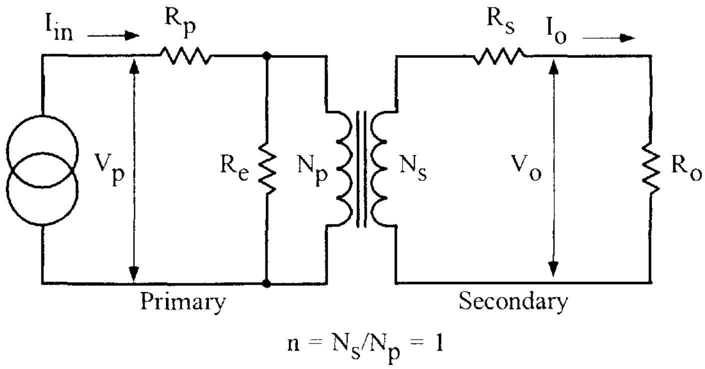

Figure 1-5. Transformer Circuit Diagram.

The assumption is that distributed capacitance in the secondary can be neglected because the frequency and secondary voltage are not excessively high. Also, the winding geometry is designed to limit the leakage inductance to a level, low enough to be neglected under most operating conditions.

Transformer voltage regulation can now be expressed as:

$$\alpha = \frac{V_o(\text{N.L.}) - V_o(\text{F.L.})}{V_o(\text{F.L.})}(100), \quad [\%]$$

in which, V_o(N.L.), is the no load voltage and, V_o(F.L.), is the full load voltage. For the sake of simplicity, assume the transformer in Figure 1-5, is an isolation transformer, with a 1:1 turns ratio, and the core impedance, R_e, is infinite.

If the transformer has a 1:1 turns ratio, and the core impedance is infinite, then:

$$I_{in} = I_o, \quad [\text{amps}]$$

$$R_p = R_s, \quad [\text{ohms}]$$

With equal window areas allocated for the primary and secondary windings, and using the same current density, J,

$$\Delta V_p = I_{in} R_p = \Delta V_s = I_o R_s, \quad [\text{volts}]$$

Regulation is then:

$$\alpha = \frac{\Delta V_p}{V_p}(100) + \frac{\Delta V_s}{V_s}(100), \quad [\%]$$

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Multiply the equation by currents, I:

$$\alpha = \frac{\Delta V_p I_{in}}{V_p I_{in}}(100) + \frac{\Delta V_s I_o}{V_s I_o}(100), \quad [\%]$$

Primary copper loss is:

$$P_p = \Delta V_p I_{in}, \quad [\text{watts}]$$

Secondary copper loss is:

$$P_s = \Delta V_s I_o, \quad [\text{watts}]$$

Total copper loss is:

$$P_{cu} = P_p + P_s, \quad [\text{watts}]$$

Then, the regulation equation can be rewritten to:

$$\alpha = \frac{P_{cu}}{P_o}(100), \quad [\%]$$

Regulation can be expressed as the power lost in the copper. A transformer, with an output power of 100 watts and regulation of 2%, will have a 2 watt loss in the copper:

$$P_{cu} = \frac{P_o \alpha}{100}, \quad [\text{watts}]$$

$$P_{cu} = \frac{(100)(2)}{100}, \quad [\text{watts}]$$

$$P_{cu} = 2, \quad [\text{watts}]$$

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Relationship K_g to Power Transformer Regulation Capability

Transformers

Although most transformers are designed for a given temperature rise, they can also be designed for a given regulation. The regulation and power-handling ability of a core is related to two constants:

$$\alpha = \frac{P_t}{2K_g K_e}, \quad [\%]$$

α = Regulation (%)

The constant, *K_g, is determined by the core geometry, which may be related by the following equations:

$$K_g = \frac{W_a A_c^2 K_u}{\text{MLT}}, \quad [\text{cm}^5]$$

The constant, K_e, is determined by the magnetic and electric operating conditions, which may be related by the following equation:

$$K_e = 0.145 K_f^2 f^2 B_m^2 \left(10^{-4}\right)$$

Where:

K_f = waveform coefficient

4.0 square wave

4.44 sine wave

From the above, it can be seen that factors, such as flux density, frequency of operation, and waveform coefficient, have an influence on the transformer size.

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*The derivation for these equations is set forth, in detail, by the author in the book, "Transformer and Inductor Design Handbook," Marcel Dekker, Inc., New York, 1988.

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Relationship A_p to Transformer Power Handling Capability

Transformers

According to the newly developed approach, the power handling capability of a core is related to its area product, *A_p, by an equation which may be stated as:

$$A_p = \frac{P_t(10^4)}{K_f K_u B_m J f}, \quad [\text{cm}^4]$$

Where:

K_f = waveform coefficient

4.0 square wave

4.44 sine wave

From the above, it can be seen that factors, such as flux density, frequency of operation, and window utilization factor K_u, defines the maximum space which may be occupied by the copper in the window.

The area product, A_p, of a core is the product of the available window area, W_a, of the core in square centimeters, (cm^2), multiplied by the effective, cross-sectional area, A_c, in square centimeters, (cm^2), which may be stated as:

$$A_p = W_a A_c, \quad [\text{cm}^4]$$

Figures 1-6 through Figure 1-8 show, in outline form, three transformer core types that are typical of those shown in the catalogs of suppliers.

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*The derivation for these equations is set forth in detail by the author in the book "Transformer and Inductor Design Handbook," Marcel Dekker, Inc., New York, 1988.

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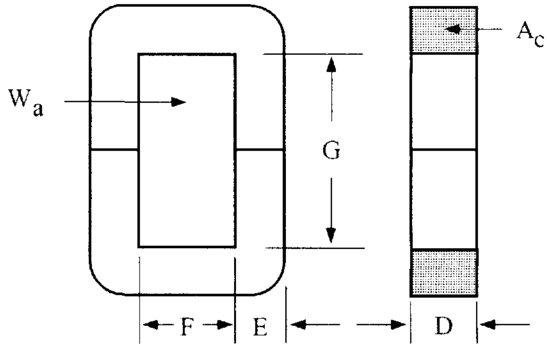

Figure 1-6. C, Core.

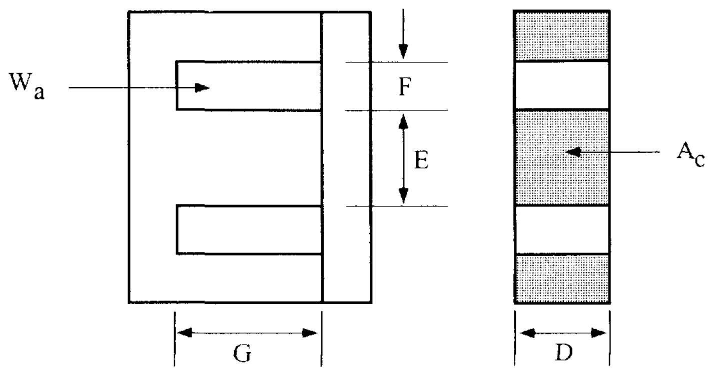

Figure 1-7. EE Core.

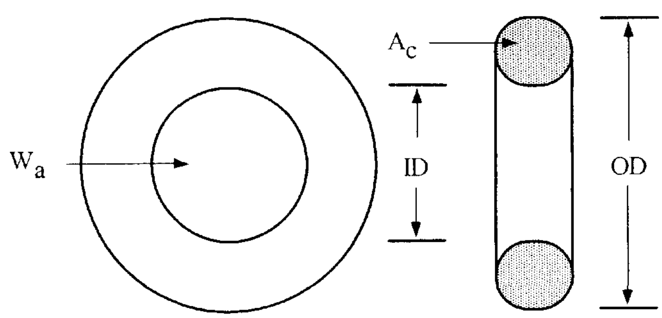

Figure 1-8. Toroidal Core.

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Inductor Design

The designer is faced with a set of constraints which must be observed in the design of any inductor. One of these constraints is copper loss. The winding must be capable of delivering current to the load within specified regulation limits. Another constraint relates to minimum efficiency of operation, which is dependent upon the maximum power loss that can be allowed in the inductor. Still another defines the maximum permissible temperature rise for the inductor, when used in a specified temperature environment. The gapped inductor has three loss components. They are copper loss, P_{cu}, core loss P_{fe}, and gap loss, P_g. Maximum efficiency is reached in an inductor, as in a transformer, when the copper loss, P_{cu}, and the iron loss, P_{fe}, are equal, but only, when the core gap is zero. The loss does not occur in the air gap itself, but is caused by magnetic flux fringing around the gap, and re-entering the core in a direction of high loss. As the air gap increases, the fringing flux increases more and more. Some of the fringing flux strikes the core perpendicular to the lamination, and sets up eddy currents, which cause additional loss. Designing with molypermalloy powder core, the gap loss is minimized because the powder is insulated with a ceramic material, which provides an uniformly distributed air gap. Also designing with ferrites, the gap loss is minimized because ferrite materials have such high resistivity.

Other constraints relate to volume occupied by the inductor, and weight, since weight minimization is an important goal in the design of space flight electronics. Lastly, cost effectiveness is always an important consideration.

Fundamental Conditions in Designing Inductors

The design of a linear reactor depends upon four related factors:

1. Desired inductance, L.
2. Direct current, I_{dc}.
3. Alternating current, ΔI.
4. Power loss and temperature rise, T_r.

With these requirements established, the designer must determine the maximum values for, B_{dc}, and for, B_{ac}, which will not produce magnetic saturation. The designer must make tradeoffs, which will yield the highest inductance for a given volume. The core material, which is chosen, dictates the maximum flux density which can be tolerated for a given design. The basic equation for maximum flux is:

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$$B_{\max} = \frac{0.4\pi N\left(I_{dc} + \frac{\Delta I}{2}\right)\left(10^{-4}\right)}{l_g + \frac{\text{MPL}}{\mu_m}}, \quad \text{[tesla]}$$

The inductance of an iron-core inductor, carrying dc and having an air gap, may be expressed as:

$$L = \frac{0.4\pi N^2 A_c\left(10^{-8}\right)}{l_g + \frac{\text{MPL}}{\mu_m}}, \quad \text{[henrys]}$$

Inductance is dependent on the effective length of the magnetic path, which is the sum of the air gap length, (l_g), and the ratio of the core mean length to relative permeability, (l_m/μ_r).

When the core air gap, (l_g), is larger, compared to relative permeability, (l_m/μ_r), because of the high relative permeability (μ_r), variations in (μ_r) do not substantially effect the total effective magnetic path length, or the inductance. The inductance equation, then reduces to:

$$L = \frac{0.4\pi N^2 A_c\left(10^{-8}\right)}{l_g}, \quad \text{[henrys]}$$

Final determination of the air gap size requires consideration of the effect of fringing flux, which is a function of gap dimension, the shape of the pole faces, and the shape, size and location of the winding. Its net effect is the shorting of the air gap.

Fringing Flux

Fringing flux decreases the total reluctance of the magnetic path and, therefore, increases the inductance by a factor, F, to a value greater than that calculated from inductance equation. Fringing flux is a larger percentage of the total for larger gaps. The fringing flux factor is:

$$F = 1 + \frac{l_g}{\sqrt{A_c}} \ln \frac{2G}{l_g}$$

Inductance, L, computed, does not include the effect of fringing flux. The value of inductance, L, corrected for fringing flux is:

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$$L = \frac{0.4\pi N^2 A_c F\left(10^{-8}\right)}{l_g + \dfrac{\text{MPL}}{\mu_\text{m}}}, \quad \text{[henrys]}$$

Distribution of fringing flux is also affected by another aspect of core geometry, the proximity of coil turns to the core, and whether, there are turns on both legs.

$$B_\text{max} = \frac{0.4\pi NF\left(I_{dc} + \dfrac{\Delta I}{2}\right)\left(10^{-4}\right)}{l_g + \dfrac{\text{MPL}}{\mu_\text{m}}}, \quad \text{[tesla]}$$

The fringing flux is around the gap and re-entering the core in a direction of high loss as shown in Figure 1-9.

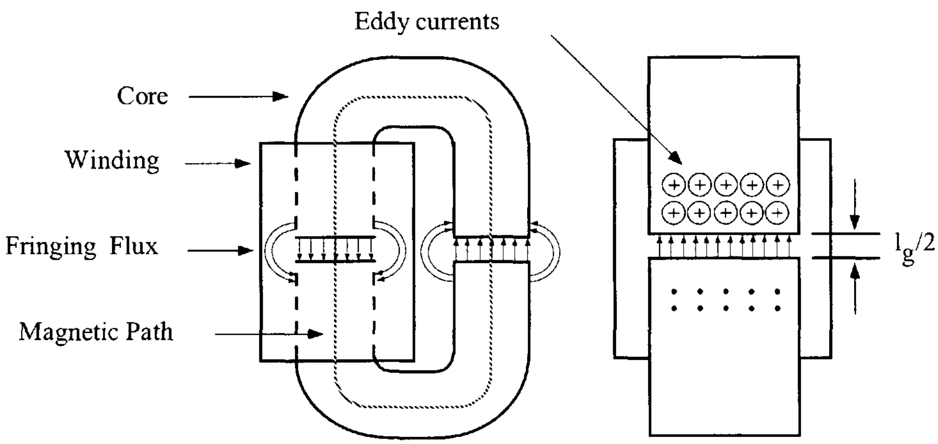

Figure 1-9. Fringing Flux Around the Gap of an Inductor Design with a C Core.

Effective permeability may be calculated from the following expression:

$$\mu_e = \frac{\mu_m}{1 + \mu_m\left(\dfrac{l_g}{\text{MPL}}\right)}$$

After establishing the required inductance and the dc bias current which will be encountered, dimensions can be determined. This requires consideration of the energy handling capability, which is controlled by the size of the inductor.

The energy handling capability of a core is:

$$\text{Energy} = \frac{LI^2}{2}, \quad \text{[watt-second]}$$

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Toroidal Powder Core Selection

The design of an inductor, also, frequently involves consideration of the effect of its magnetic field on other devices near where it is placed. This is especially true in the design of high-current inductors for converters and switching regulators used in spacecraft, which may also employ sensitive magnetic field detectors. For this type of design problem, it is frequently imperative that a toroidal core be used. The magnetic flux in a powder toroid, (core), can be contained inside the core more readily, than in a lamination, or C type core, as the winding covers the core along the whole magnetic path length.

The author has developed a simplified method of designing optimum, dc carrying inductors with powder cores. This method allows the correct core permeability to be determined, without relying on trial and error.

With these requirements established, the designer must determine the maximum values for, B_{dc}, and for B_{ac}, which will not produce magnetic saturation, and must make tradeoffs that will yield the highest inductance for a given volume. The chosen core permeability dictates the maximum dc flux density, which can be tolerated for a given design. Powder cores come in a range of permeability. Figure 1-10 shows how the required permeability changes with dc bias. As the magnetizing force (dc bias) is increased, the permeability will start to drop. When the permeability has reached 90% of its initial value, as shown in Figure 1-10, the permeability falls off, relatively fast.

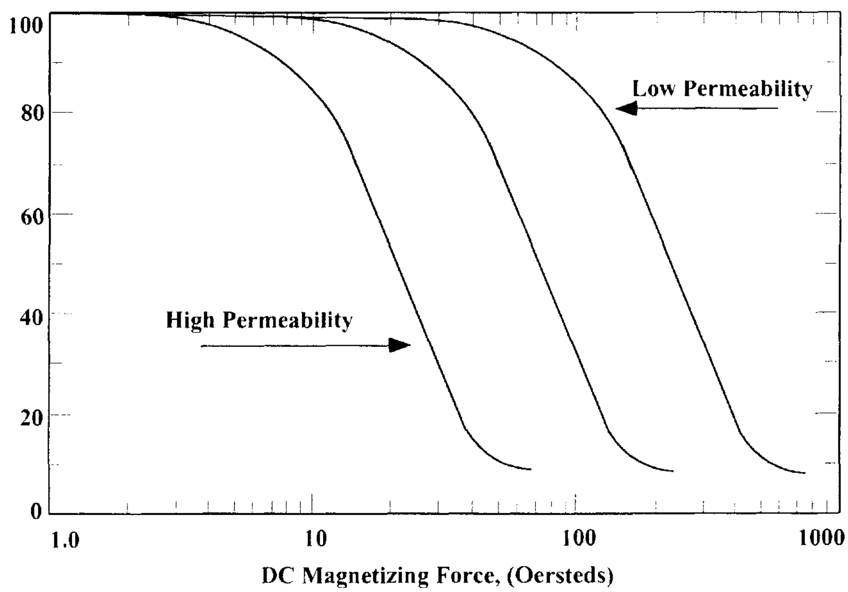

Figure 1-10. Typical Permeability Versus dc Bias for Powder Cores.

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If an inductance is to be constant with an increasing direct current, there must be a negligible drop in inductance over the operating current range. Then the maximum, H, is an indication of a core's capability. The magnetization force, H, is in oersteds:

$$H = \frac{0.4\pi NI}{\text{MPL}}, \quad \text{[oersteds]}$$

Inductance decreases with increasing flux density and magnetizing force for various materials of different values of permeability. The selection of the correct permeability for a given design is made after solving for the energy handling capability:

$$\Delta\mu = \frac{B_m(\text{MPL})(10^4)}{0.4\pi W_a J K_u}$$

It should be remembered that maximum flux density depends upon, $I_{dc} + \Delta I/2$, in the manner shown in Figure 1-11. Different powder cores materials, with different permeability, will operate with a high or lower dc bias. Table 1-1 shows the different types of powder cores that are offered to the design engineer.

$$B_{\max} = \frac{0.4\pi N\mu_r\left(I_{dc} + \frac{\Delta I}{2}\right)(10^{-4})}{\text{MPL}}, \quad \text{[tesla]}$$

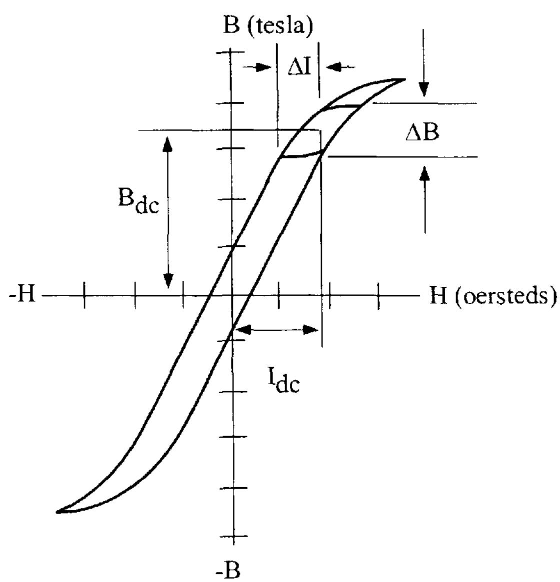

Figure 1-11. Flux Density Versus $I_{dc} + \Delta I$.

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Table 1-1

Powder Cores					
Materials		MPP	Hugh Flux	Kool Mu	Iron Powder
Initial Permeability	μ_i	14 - 550	14 - 160	60 - 125	10 - 100
Flux Density	B_m	0.7T	1.4T	1.0T	1.03 - 1.4T
Density	δ	8.5	8	6.15	6.5 - 7

Relationship K_g to Power Inductor Regulation Capability

Inductors

Inductors, like transformers, are designed for a given temperature rise. They can also be designed for a given regulation. The regulation and energy handling ability of a core is related to two constants:

$$\alpha = \frac{(\text{Energy})^2}{K_g K_e}, \quad [\%]$$

$$\alpha = \text{Regulation } (\%)$$

The constant, *K_g, is determined by the core geometry:

$$K_g = \frac{W_a A_c^2 K_u}{\text{MLT}}, \quad [\text{cm}^5]$$

The constant, K_e, is determined by the magnetic and electric operating conditions:

$$K_e = 0.145 P_o B_m^2 \left(10^{-4}\right)$$

Where:

$$P_o = \text{output power}$$

$$B_{max} = B_{dc} + \frac{\Delta B}{2}, \quad [\text{tesla}]$$

From the above, it can be seen that flux density is the predominant factor, governing the size.

*The derivation for these equations are set forth in detail by the author in the book, "Transformer and Inductor Design Handbook," Marcel Dekker, Inc., New York, 1988.

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Relationship A_p to Inductor Energy Handling Capability

Inductors

According to the newly developed approach, the energy handling capability of a core is related to its area product, *A_p, by an equation, which may be stated as follows:

$$A_p = \frac{2(\text{Energy})(10^4)}{K_u B_m J}, \quad [\text{cm}^4]$$

From the above, it can be seen that factors, such as flux density, B_{max}, window utilization factor, K_u, (which defines the maximum space, which may be occupied by the copper in the window), and the current density, J. All have an influence on the inductor area product.

The area product A_p of a core is the product of the available window area, W_a, of the core in square centimeters, (cm^2), multiplied by the effective, cross-sectional area, A_c, in square centimeters, (cm^2), which may be stated as:

$$A_p = W_a A_c, \quad [\text{cm}^4]$$

Figures 1-6 through 1-8 show, in outline form, three transformer core types that are typical of those shown in the catalogs of the suppliers.

*The derivation for these equations are set forth in detail by the author in the book, "Transformer and Inductor Design Handbook," Marcel Dekker, Inc., New York, 1988.

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Transformer Losses

Transformer efficiency, regulation, and temperature rise are all interrelated. Not all of the input power to the transformer is delivered to the load. The difference between the input power and output power is converted into heat. This power loss can be broken down into two components: core loss and copper loss. The core loss is a fixed loss, and the copper loss is a variable loss that is related to the current demand of the load. Copper loss increases by the square of the current and is also termed quadratic loss. Maximum efficiency is achieved when the fixed loss is equal to the quadratic loss at the rated load. Transformer regulation is the copper loss, P_{cu}, divided by the output power, P_o:

$$P_{\Sigma} = P_{cu} + P_{fe}, \quad \text{[watts]}$$

Inductor Losses

The losses in an inductor are made up of three components: (1) copper loss, P_{cu}; (2) iron loss, P_{fe}; and (3) gap loss, P_g. The copper loss and iron loss have been previously discussed. Gap loss is independent of core material thickness and permeability. Maximum efficiency is reached in an inductor, as in a transformer, when the copper loss, P_{cu}, and the iron loss, P_{fe}, are equal, but only when the core gap is zero. The loss does not occur in the air gap itself, but is caused by magnetic flux fringing around the gap and re-entering the core in a direction of high loss. As the air gap increases, the flux across the gap fringes more and more. Some of the fringing flux strikes the core, perpendicular to the strip or tape, and sets up eddy currents, which cause additional losses in the core. If the gap dimension gets too large the fringing flux will strike the copper winding and produce eddy currents, generating heat, just like an induction heater. The fringing flux will jump the gap and produce eddy currents, in both the core and winding, as shown in Figure 1-12.

$$P_{\Sigma} = P_{cu} + P_{fe} + P_g, \quad \text{[watts]}$$

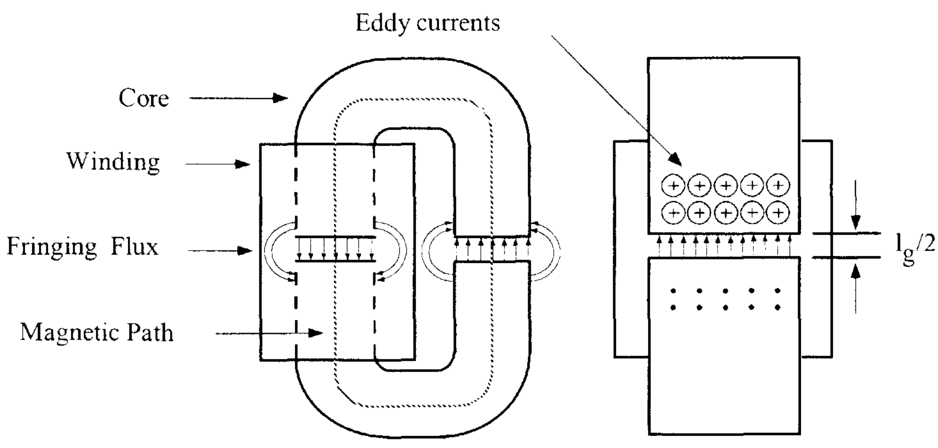

Figure 1-12. Fringing Flux Around the Gap of an Inductor Design with a C Core.

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Eddy Current Losses, Skin Effect

As the frequency increases, there are additional losses that occur in the winding, due to eddy currents induced in the conductors by the magnetic fields within the winding. Skin effect is caused by eddy currents, induced in a wire by the magnetic field of the current carried by the wire itself. Skin depth is defined as the distance below the surface, where the current density has fallen to, 1/e, or 37 percent of its value at the surface. Skin depth of copper at 20°C is:

$$\varepsilon = \frac{6.62}{\sqrt{f}}, \quad [\text{cm}]$$

Skin effect is illustrated in Figure 1-13. The required wire size is a number 17, magnet wire. The operating frequency is 100 kHz. The skin depth is 0.0209 centimeters. A number 17 magnet wire, operating at 100 kHz, will yield an unused area of 0.00422 cm^2, as shown in Figure 1-13. If you take the skin depth ε, and, assume it to be the radius of the wire, then, you can calculate the minimum wire area. Take this area and match it with the closest, AWG. Then, take the area of the AWG, and divide it into the required area, and that will be the number of strands.

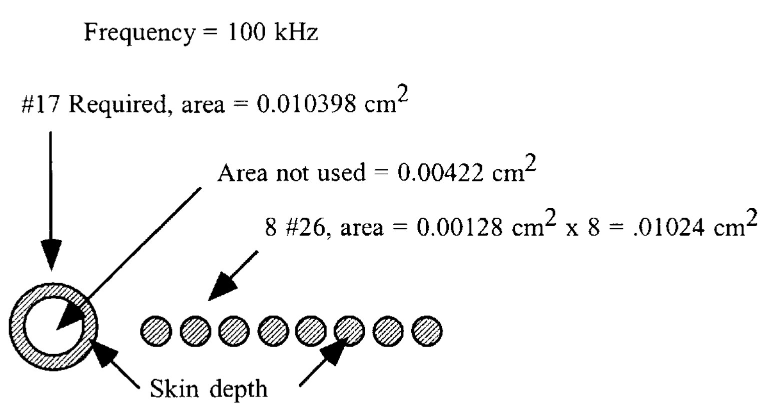

Figure 1-13. Skin Depth Illustration.

Eddy Current Losses, Proximity Effect

Proximity effect is caused by eddy currents induced in a wire, due to the alternating magnetic field of other conductors in the vicinity. Proximity effect is more serious than skin effect because skin effect can be overcome by going to a smaller diameter wire. In the proximity effect, eddy currents, caused by adjacent layers, increase exponentially in amplitude, as the number of layers increases. Proximity effect, skin effect and high frequency together will cause the transformer, with multiple layers, to have losses that are excessive, due to current crowding from the skin effect. With each additional layer, the I^2R losses in that layer, increase by the square of the current of the previous layer. Selecting the correct AWG, as well as the winding geometry, is very important in keeping the losses down. The proximity effect can be reduced, significantly, by interleaving the primary and secondary, as shown in Figures 1-14 and Figure 1-15.

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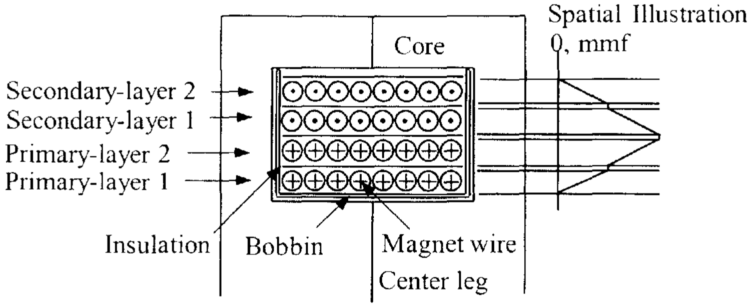

Figure 1-14. Primary and Secondary are Separated.

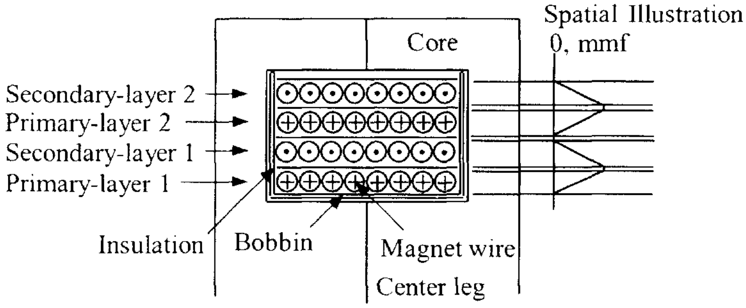

Figure 1-15. Primary and Secondary are Interleaved.

Temperature Rise and Surface Area

The heat, generated by the core loss, copper loss, gap loss, and the losses due to the skin effect and proximity effect, produces a temperature rise, which must be controlled to prevent damage to, or the failure of the windings by the breakdown of the insulation at elevated temperatures. This heat is dissipated from the exposed surfaces of the transformer or inductor by a combination of radiation and convection. Therefore, the dissipation is dependent upon the total, exposed surface area of the core and windings.

Temperature rise in a transformer winding cannot be predicted with complete precision, despite the fact that many techniques are described in the literature for its calculation. One, reasonably accurate method for open core and winding construction is a homogeneous method. It is also based upon the assumption that the core and winding losses may be lumped together as:

Transformer:

$$P_{\Sigma} = P_{cu} + P_{fe}, \quad \text{[watts]}$$

Inductor:

$$P_{\Sigma} = P_{cu} + P_{fe} + P_{g}, \quad \text{[watts]}$$

Also, The assumption is made that the thermal energy is dissipated uniformly throughout the surface area of the core and winding assembly. The effective surface area, A_t, required to dissipate heat, (expressed as watts dissipated per unit area), is:

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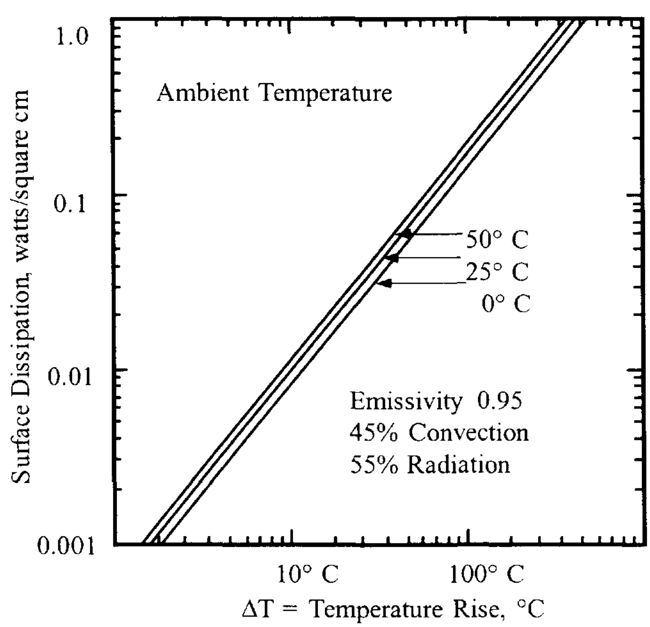
$$A_t = \frac{P_\Sigma}{\psi}, \quad [\mathrm{cm}^2]$$

"ψ" is the power density of the average power, dissipated per unit area from the surface of the transformer and, P_Σ, is the total power lost or dissipated.

The temperature rise that can be expected for various levels of power loss is shown in Figure 1-16. It is based on data obtained from Blume, (1938), for heat transfer, effected by a combination of 55% radiation and 45% convection, from surfaces having an emissivity of 0.95, in an ambient temperature of 25°C, at sea level. Power loss, (heat dissipation), is expressed in watts per square centimeter of the total surface area. Heat dissipation, by convection from the upper side of a horizontal flat surface, is on the order of 15-20% more than from a vertical surface. Heat dissipation, from the underside of a horizontal flat surface, depends upon surface area and conductivity. Below are two, prominently used power loss factors, expressed in watts per square centimeter of the total surface area.

$$\psi = 0.03 W/\mathrm{cm}^2 @ 25°\mathrm{C\ rise}$$
$$\psi = 0.07 W/\mathrm{cm}^2 @ 25°\mathrm{C\ rise}$$



$$T_r = 450\,(\psi)^{(0.826)}$$

Figure 1-16. Temperature Rise Versus Surface Dissipation.

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