THE PIRANI GAUGE FOR THE MEASUREMENT OF SMALL CHANGES OF PRESSURE

BY A. ELLETT AND R. M. ZABEL
PHYSICAL LABORATORY, STATE UNIVERSITY OF IOWA
(Received March 20, 1931)

ABSTRACT

The application of the Pirani gauge to the measurement of small pressure changes is discussed. Both nickel and tungsten wires are used as filaments in the gauge. Nickel wire not only has the greater sensitivity but possesses several other advantages. The theory of the gauge is developed so that it is possible to predict the effect of change in length or diameter of the wire upon the sensitivity of the gauge. The theory also predicts that there is an optimum temperature to which the wire should be heated for maximum sensitivity of the gauge. The observed and computed values of the optimum temperature are compared. In some cases the agreement is as good as can be expected and in the others the discrepancy is easily explained. The maximum sensitivity attained is a galvanometer deflection of 1 mm for a pressure change of air equivalent to 5×10^{-9} mm of mercury.

INTRODUCTION

THE possibility of using the variation in heat conductivity of a gas with pressure as a measure of low pressures was first suggested by Pirani.[1] Recent experimenters[2,3] have shown that such a gauge is capable of responding to very small changes in pressure.

These investigators have been primarily interested in a gauge which will determine the actual pressure existing in a chamber after it has been calibrated by comparison with an absolute gauge such as the McLeod. The aim of this experiment is to develop a gauge with a high sensitivity to very small pressure changes.

THEORY

It is easily shown that the quantity of heat Q conducted by a gas when the mean free path is large compared to the dimensions of the container is

$$Q = nGAtH/6N \tag{1}$$

where n is the number of molecules per cm^3, G the mean molecular velocity, A the area of the heated element, t the temperature difference between the heated element and its surroundings, H the molecular heat at constant volume, and N the number of molecules per gram molecule. Experimental measurements give a value of the heat transfer which is usually less than that predicted by this equation. This is because the molecules striking the heated wire do not attain temperature equilibrium with it. The ratio of the actual amount of heat conducted from the heated element to that computed by

[1] M. Pirani, Verh. d Deutsch. Phys. Ges. **8**, 24, 686 (1906).
[2] A. M. Skellet, J. O. S. A. and R. S. I. **15**, 56 (1927).
[3] L. F. Stanly, Phys. Soc. Proc. **41**, 194 (1929).

Eq. (1) is known as the accommodation coefficient and has been measured for some gases. Soddy and Berry[4] give the values of the actual amount of heat conducted and the values of the accommodation coefficient for gases striking a heated tungsten surface. The values of the accommodation coefficient range from 0.25 in the case of hydrogen to 1.0 in the case of argon and neon. The values of the actual amount of heat conducted by various gases deviate by less than 10 percent from a mean value for a number of the common gases. Hence we may expect that the sensitivity of the Pirani gauge will be nearly the same for this group of gases.

In the application of the above principle to the measurement of pressure changes the temperature change in the wire resulting from a change in the heat conductivity of the gas is usually measured by the change in resistance. This resistance change is in turn measured by a Wheatstone bridge. If the pressure change to be measured is not large the galvanometer deflection is the most accurate method of determining the magnitude of the change.

The bridge potential serves as a source of power to heat the wire. The energy dissipated by a heated wire must be equal to that supplied, hence neglecting conduction to the leads

$$\frac{E^2}{R} = A\gamma(T^4 - T_0^4) + A\alpha p(T - T_0) \tag{2}$$

where E is the potential across the ends of the wire, R the resistance and A the area of the wire, T and T_0 the temperature of the wire and of its surroundings, respectively, p the pressure of the gas and γ and α constants. Since the resistance change corresponding to the pressure changes normally measured is small we may assume that the power supplied to the wire is constant. Then by differentiation of (2),

$$\frac{dT}{dp} = -\frac{\alpha(T - T_0)}{4\gamma T^3 + \alpha p}\cdot$$

At low pressures the amount of heat dissipated from the wire by radiation is large compared to that conducted by gas hence we may neglect αp in comparison to $4\gamma T^3$. Then

$$\frac{dT}{dp} = K\frac{T - T_0}{T^3} \tag{3}$$

where K is a constant.

Obviously there is a relation between T and T_0 for which the temperature change of the wire will be a maximum for a given pressure change. This relation is given by

$$\frac{d}{dT}\left[\frac{(T - T_0)}{T^3}\right] = 0$$

or

$$T = 3T_0/2.$$

[4] F. Soddy and A. Berry, Proc. Roy. Soc. **83**, 254 (1910).

Substituting this value in (3) we obtain,

$$\left[\frac{dT}{dp}\right]_{\max} = \frac{K'}{T_0^2}. \tag{4}$$

Hence the value of the maximum temperature change of the wire for a given pressure change is inversely proportional to the square of the absolute temperature of the surroundings of the wire.

Since the temperature change resulting from a given pressure change is normally small we may assume that

$$R = R_0[1 + \alpha(T - 273)]$$

or

$$dR = R_0 \alpha dT.$$

Substituting this value of dT in (3) we obtain,

$$\frac{dR}{dp} = K'' R_0 \frac{T - T_0}{T^3}. \tag{5}$$

Obviously dR/dp has the same maximum as dT/dp. The maximum galvanometer deflection does not necessarily correspond to the maximum resistance change, however.

The current through the galvanometer in a Wheatstone bridge circuit is given by

$$I_g = \frac{EdR}{4RR_g + 4R^2}$$

if the battery resistance is small and the four arms of the bridge are approximately equal. Substituting for dR from (5),

$$I_g = K''' \frac{R_0 E}{RR_g + R^2} \frac{T - T_0}{T^3} dp. \tag{6}$$

Obviously the galvanometer deflection may be increased by increasing the potential of the bridge. This factor is limited, however, because the wire must be heated to its optimum temperature.

The relation between the potential drop across the wire and the temperature may be obtained from (2). In general the heat conducted by the gas may be neglected in comparison to that dissipated by radiation and T_0^4 in comparison to T^4. Hence

$$E = (\gamma AR)^{1/2} T^2.$$

Substituting this value of E in (6)

$$I_g = K \frac{R_0 (AR)^{1/2}}{RR_g + R^2} \frac{T - T_0}{T} dp. \tag{7}$$

The optimum temperature of the gauge wire for maximum galvanometer deflection may be determined from (7). In the general case this solution is difficult but several special cases which are of interest may be treated.

If we assume that the galvanometer resistance is large compared to the resistance of the arms of the bridge, (7) reduces to

$$I_g = K' \frac{T - T_0}{R^{1/2}T} dp.$$

But

$$R = R_0[1 + \alpha(T - 273) + \beta(T - 273)^2]$$

where the terms of second order must be included if the action of the gauge is to be described over any considerable range. Hence

$$I_g = \frac{K'''(T - T_0)}{[1 + \alpha(T - 273) + \beta(T - 273)^2]^{1/2}T} dp.$$

The optimum temperature of the wire may be determined by

$$\frac{d}{dT}\left[\log \frac{T - T_0}{[1 + \alpha(T - 273) + \beta(T - 273)^2]^{1/2}T}\right] = 0$$

or

$$2\beta T^3 + (\alpha - 546\beta - 4\beta T_0)T^2 - T_0(3\alpha - 1638\beta)T - 2T_0[1 - 273\alpha + (273)^2\beta] = 0. \quad (8)$$

If the galvanometer resistance is small as compared to the resistance in the bridge arms Eq. (7) reduces to

$$I_g = K^* \frac{T - T_0}{R^{3/2}T}$$

and the value of T for maximum I_g is given by

$$6\beta T^3 + (3\alpha - 1638\beta - 8\beta T_0)T^2 - 5T_0(\alpha - 546\beta)T - 2T_0[1 - 273\alpha + (273)^2\beta] = 0. \quad (9)$$

Results

The experimental arrangement used in obtaining the results given below is shown in Fig. 1. The leak was controlled by a stopcock so that it could be opened or closed as desired. This gave a constant pressure change and the corresponding galvanometer deflection could be observed for various bridge potentials and various samples of wire.

The gauge formed one arm of a Wheatstone bridge. The other arms consisted of resistances variable in small steps so that the arms of the bridge could be kept approximately equal. The galvanometer used has a resistance of 17 ohms and a sensitivity of 11.6 mm per microvolt. Resistance was connected in series or parallel with it to obtain the high and low galvanometer resistances used.

Equation (7) shows that the galvanometer deflection should be directly proportional to the pressure change. This was verified beyond the limits of the McLeod gauge by connecting a small leak to a pump through a stopcock. This leak was adjusted to give several cm deflection of the galvanometer as it was turned on or off. As the pressure in the gauge was increased by the use of another leak the deflection produced by opening or closing the first leak remained constant.

Equation (7) also shows that the sensitivity of the gauge is proportional to the square root of the area of the wire. It is possible to multiply the sensitivity of a nickel wire gauge by two or three merely by flattening the wire.

The effect of the length of the gauge wire upon the sensitivity may be determined from (7). Since A, R, and R_0 are all proportional to the length of the wire it is evident that the sensitivity will be directly proportional to

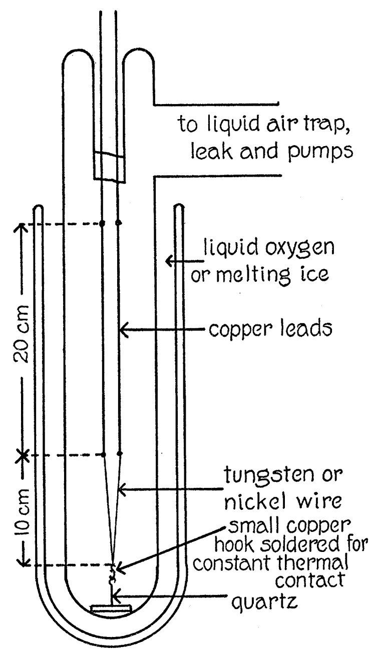

Fig. 1. Construction of the Pirani gauge.

the length of the wire if the galvanometer resistance is large compared to the resistance of the bridge arms. On the other hand, if the galvanometer re-

TABLE I.

Galvanometer resistance	Length of wire (cm)	Maximum deflection
312	20	22.4
	10	12.0
	5	6.3
17	20	23.0
	10	20.1
	5	15.9
3.86	20	24.5
	10	24.8
	5	23.2

sistance is small the sensitivity is independent of the length of the wire. Table I shows the effect of the length of the wire upon the sensitivity of the gauge. In each case the gauge was constructed of nickel wire 0.001 inch in diameter and the walls of the gauge were maintained at the temperature of melting ice.

The variation of the sensitivity of the gauge with diameter of the wire may also be predicted. If the galvanometer resistance is large compared to the resistance of the gauge, the diameter of the wire should be decreased and if the galvanometer resistance is low the diameter of the wire should be increased. Hence for maximum sensitivity the wire should be as long as convenient and the diameter should be adjusted so that its resistance is of the

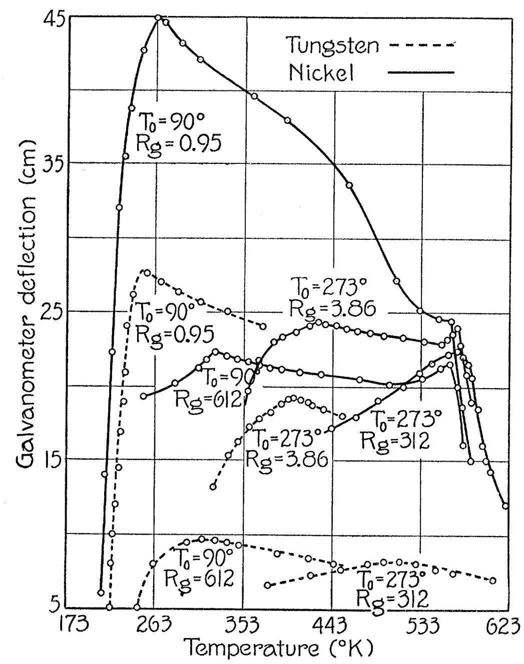

Fig. 2. Curves showing the relation between the temperature of the gauge wire and the sensitivity of the gauge. T_0 is the temperature of the walls of the gauge and R_g the galvanometer resistance.

same order of magnitude as the galvanometer resistance. In general a galvanometer with low resistance is preferable since the corresponding lower resistance in the bridge arms permits the use of lower battery potential which decreases the tendency of the galvanometer to drift.

The relation between the sensitivity of the gauge and the temperature of the wire is shown in Fig. 2. The tungsten and nickel wires used in obtaining these curves were 20 cm long and 0.001 inch in diameter. The temperature was determined by the known relation between the temperature and resistance. For tungsten the values $\alpha=5.24\times10^{-3}$ and $\beta=0.7\times10^{-6}$ were used in the well-known resistance formula. The temperature of the nickel wire was read from the curve shown in Fig. 3 which was determined for the wire used in this experiment.

It will be noted that all the temperature sensitivity curves for nickel wire (Fig. 2) have either a primary or a secondary maximum at approximately 570°* after which the sensitivity decreases very rapidly. This is obviously due to the rapid increase in the slope of the temperature resistance curve and the sudden break at 615°. The observed temperature at which this break occurs should be less when the wire is used as a gauge because the ends of the gauge wire are cooled by conduction of heat to the leads. This view is substantiated by the fact that the apparent temperature at which the maximum occurs decreases when a shorter wire is used.

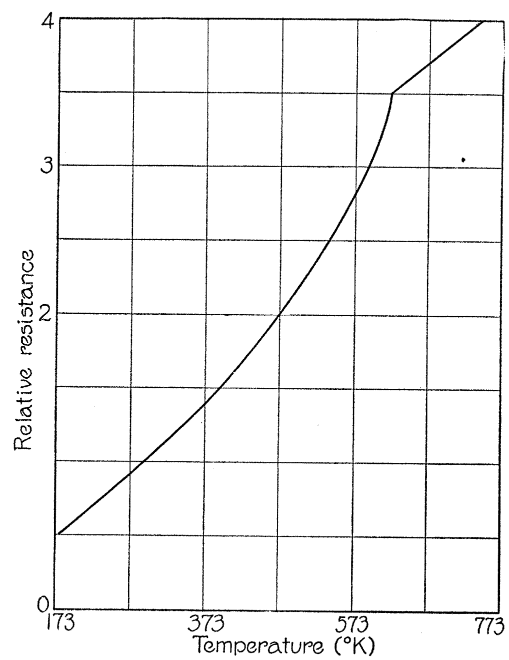

Fig. 3. Temperature resistance curve for nickel wire.

In Table II, T_0 is the temperature of the gauge walls, R_g is the galvanometer resistance, R is the resistance of the wire at its optimum temperature, E is the bridge potential necessary to produce this temperature, T(obs.) and D the optimum temperature and the galvanometer deflection at that temperature, respectively, as determined from Fig. 2, and T (comp.) the optimum temperature computed from Eqs. (8) and (9). In obtaining the values of T (comp.) the proper equation is chosen depending upon the value of the galvanometer resistance, and the values of T_0, α, and β are substituted after which the equation may be solved for T. For tungsten the values of α and β previously given were used and for nickel the same constants were determined from Fig. 3 to fit the curve approximately in the region in which the observed temperature was known to lie. It will be noted that the computed temperatures when $T_0=90°$ are much lower than the observed. This is partially due to heat conduction along the leads and partially to the decrease in the constant γ appearing in Eq. (2) or, in other words, a decrease in the efficiency of radiation at low temperatures. In fact with a potential of only 0.0003 volt applied to the bridge the lowest temperature attained by the nickel filament was 209° and by the tungsten 213°.

The secondary maxima in the temperature sensitivity curves of nickel wire are not predicted by the equations but they probably would be if the variation in slope of the temperature resistance curve were taken into account and the equation of the curve were expressed more accurately.

TABLE II

	T_0	R_g	R	E	T Comp.	T Obs.	D
Nickel	90	612	41.9	0.5	248	313	22.3
	90	0.95	30.6	0.3	136	261	45.0
	273	312	99.0	2.9	533	563	22.4
	273	3.86	61.8	1.1	429	393	24.5
Tungsten	90	612	23.3	0.5	195	311	9.6
	90	0.95	17.0	0.4	120	254	27.6
	273	312	46.5	3.0	715	513	8.2
	273	3.86	33.5	1.3	415	401	19.3

* All temperatures given on the Kelvin scale.

Table II shows that the sensitivity attained by the use of nickel wire is in all cases greater than that for tungsten. Nickel wire has the additional advantage that it may be flattened more easily than tungsten which greatly increases the sensitivity as the previous discussion has shown. Tungsten wire has the disadvantage that the slightest vibration will cause its resistance to change sufficiently to keep the galvanometer moving back and forth over several cm.

Immersing the walls of the gauge in liquid oxygen approximately multiplies the sensitivity of the gauge by two when the galvanometer resistance is high and by seven when the galvanometer resistance is low. This difference in the sensitivity ratio is to be expected for Table II shows that the resistance of the bridge arms at optimum temperature is much lower when $T_0=90°$ than when $T_0=273°$. Equation (7) and the preceding discussion show that this decrease in resistance will decrease the sensitivity when the galvanometer resistance is high but not when the galvanometer resistance is low.

Many factors enter into the computation of the relative sensitivity of the gauge when the walls are at the temperature of melting ice and at the temperature of liquid oxygen. For example the relation between the pressure and the heat conductivity of the gas will change both because more molecules must be present at liquid oxygen temperatures to exert a given pressure and because the mean molecular velocity is smaller. Also, as previously suggested, the radiation constant (γ in Eq. (2)) probably decreases at the lower temperature and it is reasonable to expect that the value of the accommodation coefficient depends upon the temperature. The effect of heat conduction from the filament to the leads upon the sensitivity of the gauge has been neglected so that it is impossible to predict the change in sensitivity which will result from the reversal in the direction of this flow. Such a reversal actually occurs in some cases for when the walls of the gauge are immersed in liquid oxygen the temperature of the filament is less than the temperature of the leads. Due to the impossibility of computing the magnitude of the latter effect and the difficulty in computing some of those previously given no attempt is made to predict the increase in sensitivity when the walls of the gauge are reduced to the temperature of liquid oxygen.

Reducing the walls of the gauge to the temperature of liquid oxygen makes the reaction of the gauge very slow. For example with the gauges at the temperature of melting ice the galvanometer would reach equilibrium about 10 seconds after the stopcock was closed. At liquid oxygen temperature the time required to attain equilibrium was often several minutes. This time lag may be accounted for to some extent by the fact that there must be three times as many molecules in the gauge and their velocity must be much slower when the walls of the gauge are at the temperature of liquid oyxgen so that it should take longer for the gauge to pump out. This factor cannot, however, account for the great difference observed in the time lag in the two cases and neither can it account for the observed fact that the time lag of the gauge decreases rapidly with an increase in temperature of the filament since the area of the filament is so small in comparison to the area of the walls that it can have no appreciable effect upon the mean velocity of the molecules. The observed dependence of the time lag upon the temperature of the wire suggests that it may to some extent be due to the adsorption of a layer of gas by the filament.

It is possible to obtain a galvanometer deflection of 1 mm for a pressure change of air equivalent to $5 \pm 1 \times 10^{-9}$ mm of mercury by forming the gauge from a well-flattened piece of nickel wire 25 cm long and originally 0.0015 inch in diameter and using the galvanometer specified above at maximum sensitivity.

Under similar conditions the sensitivity of the gauge for hydrogen is $4 \pm 1 \times 10^{-9}$ mm of mercury per mm galvanometer deflection.

The sensitivity and characteristics of different gauges will vary due to variations in the original size and the degree of flatness of the wire.

Conclusion

In order to measure pressure changes of less than 10^{-7} mm of mercury it is obvious that the zero position of the galvanometer must be steady. If the ordinary precautions necessary in connecting a Wheatstone bridge of high sensitivity are observed, if the bridge potential is constant, and if the walls of the gauge are maintained at a constant temperature such as melting ice or liquid oxygen, the zero drift of the gauge becomes negligible provided that the pressure changes take place over a short period of time.

The operation of the gauge is not satisfactory when the walls are exposed to the variations of room temperature. A compensating gauge greatly improves the stability of the system but is not as satisfactory as one gauge immersed in a constant temperature bath.

The stability of the gauge may be improved by keeping the bridge potential low. This may be accomplished by proper choice of the galvanometer as previously described and by operating the gauge below its optimum temperature. The extent to which the temperature should be reduced below its optimum value depends upon the difficulty arising from zero drift and upon the rate at which the sensitivity decreases with decrease in temperature. In some cases it is possible to reduce the bridge sensitivity to one half* of its value at the optimum temperature while the gauge sensitivity is reduced only 15 percent. In such cases the decrease in bridge potential would probably be an advantage.

* That is $\Delta R/R$ for unit galvanometer deflection is doubled.