Stresses in Cylindrical Glass-Metal Seals with Glass Inside

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It was shown previously that, when a cylinder of glass is sealed to the outside of a metal rod, the principal stresses in the glass are of opposite sign, so that tensile stresses cannot be avoided except by a perfect match. In this article the stresses are calculated for a solid glass cylinder sealed to the *inside* of a metal cylinder. It is shown that the stresses are *all of the same sign*, so that a moderate mismatch in thermal expansion, with the metal expansion the greater, is allowable and perhaps desirable. Large differences in expansion should be avoided, because of the shearing stresses at the ends.

MANY modern devices use cylindrical seals, in which a cylinder of glass is sealed to the inside surface of a metal cylinder, either with or without a central wire. These devices include microwave tubes, thyratrons, capacitors, and refrigerators. The stresses in such seals are different from those in glass-wire seals. They can be calculated easily from the equations given in an earlier paper.[1]

CALCULATION OF STRESSES

The form of seal is shown in Fig. 1. Let the components of stress be p_r, p_θ, and p_z, in the radial, tangential, and axial directions, respectively. Only the central portion of the seal will be considered, in which the stresses are independent of z, and in which no shearing stresses exist. The equations of the earlier paper, which refer to stresses in the outer cylinder, are:

Radial stress,

$$p_r = -\left[\frac{E_2\delta}{1+\alpha+\alpha\beta R}\right]\left[\frac{a^2}{b^2}-\frac{a^2}{r^2}\right], \qquad (1)$$

Tangential stress,

$$p_\theta = -\left[\frac{E_2\delta}{1+\alpha+\alpha\beta R}\right]\left[\frac{a^2}{b^2}+\frac{a^2}{r^2}\right], \qquad (2)$$

Axial stress,

$$p_z = -\left[\frac{E_2\delta}{1+\alpha+\alpha\beta R}\right]\left[2\sigma\frac{a^2}{b^2}+\frac{1+\alpha+\alpha\beta R}{1+\beta R}\right], \qquad (3)$$

where

E_1 = elasticity (Young's modulus) of internal cylinder,
E_2 = elasticity (Young's modulus) of external cylinder,
σ = Poisson's ratio, assumed equal for metal and glass,
a, b = radii of internal and external cylinders, respectively,
$R = E_2/E_1$, $\alpha = (a^2/b^2)(1-2\sigma)$, $\beta = b^2/a^2 - 1$,
$\delta = (k_2-k_1)(t-t_0)$,
k_2 and k_1 are the mean contraction coefficients between t_0 and t of the outer cylinder and inner cylinder, respectively,
t is the temperature at which the stress is observed, and
t_0 is the "sealing temperature," i.e., the temperature at which the glass ceases to flow after sealing. δ is, therefore, the difference in total contraction of metal and glass, in cooling from the sealing temperature to the observation temperature (normally room temperature). The stress is defined to be positive when it is tension, negative when compression.

These equations may be modified to represent the stresses of the inner cylinder by making use of three simple relations[2]:

(a) The radial stress is the same for inner and outer cylinders at the boundary $r=a$, and is constant throughout the inner cylinder.

(b) The tangential and radial stresses are equal in the inner cylinder.

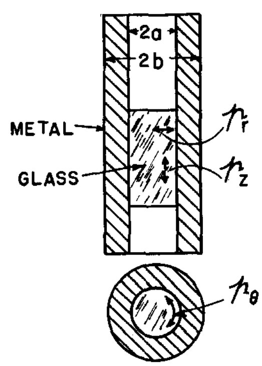

FIG. 1. Cylindrical seal with glass inside. The stress in the central portion can be calculated from expansion data.

[1] A. W. Hull and E. E. Burger, "Glass to metal seals," Physics 5, 387 (1934).

[2] H. Poritsky, "Analysis of thermal stresses in sealed cylinders," Physics 5, 406 (1934).

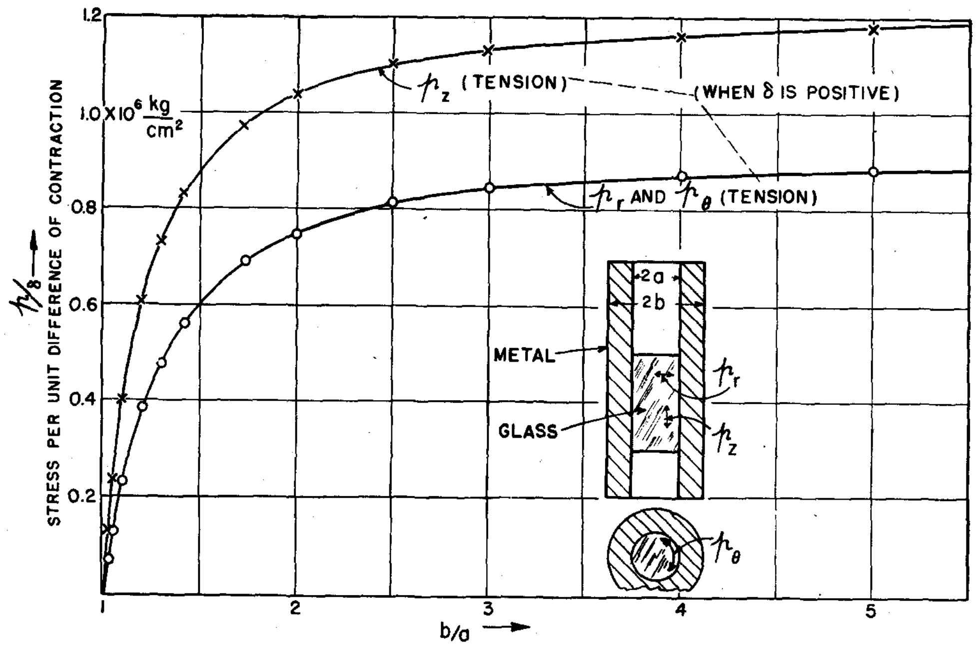

FIG. 2. Stress in internal glass cylinder per unit difference of contraction, δ, between metal and glass ($\delta=\{k_{\text{outer cyl.}}-k_{\text{inner cyl.}}\}\{t-t_0\}$), as function of ratio b/a of metal diameter to glass diameter. When δ is negative, the stresses will be compression.

(c) The total axial stress over any cross section z=constant *of both cylinders* is zero. Hence,

$$p_z(\text{internal}) = -\frac{b^2-a^2}{a^2}p_z(\text{external}).$$

The calculations will be made for an external cylinder of iron, for which $E_2=2.05\times10^6$ kg/cm². This is the metal most likely to be used in thick sections, i.e., with large values of b/a. For Fernico

TABLE I. Stresses in internal glass cylinder per unit difference of contraction δ, between metal and glass, as function of ratio b/a of metal diameter to glass diameter. Positive stresses are tension; negative, compression. When δ is negative, the signs of all stresses are opposite to those given in the table.

b/a	$P_r=P\theta$	P_z
α	+0.91	+1.193
5	+0.882	+1.18
4	+0.872	+1.164
3	+0.845	+1.130
2.5	+ .811	+1.102
2.0	+ .750	+1.040
1.732	+ .691	+0.975
1.414	+ .560	+0.830
1.30	+ .460	+ .735
1.20	+ .388	+ .610
1.10	+ .230	+ .401
1.05	+ .1285	+ .236
1.025	+ .071	+ .1325
1.00	0.000	0.000

($E_2=1.80\times10^6$), with thicknesses ordinarily used ($b/a\leq1.1$), the radial and tangential stresses are about 2 percent greater than those given here, and the axial stress 4 percent greater; while the maximum difference, even for very thick Fernico tubing, does not exceed 10 percent. The elastic constants used for the calculations are therefore:

$$E_1\ (\text{glass}) \quad =0.65\times10^6\ \text{kg/cm}^2,$$
$$E_2\ (\text{metal})=2.05\times10^6\ \text{kg/cm}^2,$$
$$\sigma_1=\sigma_2=0.30.$$

The calculated values for the stresses in the inner cylinder are given in Table I and Fig. 2. For comparison, the stresses in external glass cylinders are shown in Fig. 3.[1]

COMPARISON OF INTERNAL AND EXTERNAL SEALS

A comparison of Figs. 2 and 3 reveals one striking difference. *The stresses in internal seals are all of the same sign.* Thus, when the contraction of the metal is greater than that of the glass (δ negative) all the stresses will be negative, i.e., compressions. This is a desirable condition, because glass is strong in compression. However, excessive differences of contraction should be

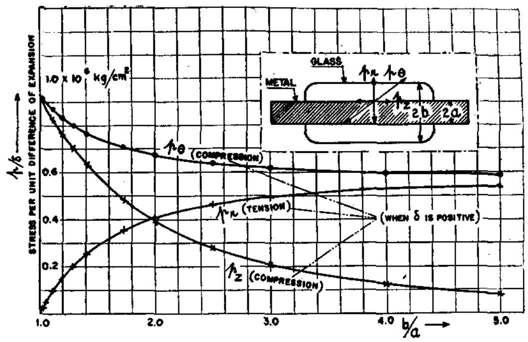

FIG. 3. Stress in cylindrical seal at surface of wire ($r=a$) per unit difference of contraction, δ, between metal and glass, ($\delta=\{k_{\text{outer cyl.}}-k_{\text{inner cyl.}}\}\{t-t_0\}$) as function of ratio b/a of glass diameter to wire diameter. When δ is negative, the signs of stresses are the reverse of those indicated, *viz.* tension in place of compression, and *vice versa.*

avoided, since the shearing stresses at the ends are proportional to the difference in contraction.

With external seals the situation is different. Here the two largest stresses, namely, the radial and the tangential stresses, are of opposite sign, so that there is no safe condition except that of zero stress. For example, a metal which contracts more than the glass will give an axial stress which is a compression, as observed with the quartz wedge testing equipment[3]; and this is sometimes mistaken for a desirable mismatch. Actually, as may be seen in Fig. 3, the tangential stress also is a compression, but the radial stress is tension, tending to break the bond between metal and glass. This bond is the weakest part of the seal; hence this type of stress should be avoided.

INTERNAL SEAL WITH CENTRAL WIRE

When a central wire is added to the internal seal (Fig. 4), one has a combination of the two cases considered above, namely an internal and an external seal. For the central wire seal, in which the glass is external, the only safe condition is a close match to the glass. The external metal cylinder, however, may be mismatched to yield a mild compression of the glass. The values given in Table I and Fig. 2 will apply to this case, with small error, provided the central wire is well matched to the glass.

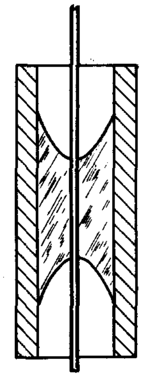

FIG. 4. Cylindrical seal with central wire.

[3] A. W. Hull and E. E. Burger, Rev. Sci. Inst. **7**, 98 (1936).

Particle Size Determination from X-Ray Line Broadening

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The x-ray line broadening method of determining particle size was compared with direct measurement on electron micrographs. By controlled heating of the carbonate, magnesium oxide particles were prepared from 50 to 1000A in diameter. Particle size calculated from x-ray data taken on a Geiger counter spectrometer agreed to ±10 percent with the microscope measurements. Mechanical mixtures of two different sizes were examined by the x-ray method, but the particle sizes could not be determined unless the two maxima of the distribution curve were completely resolved.

INTRODUCTION

THE broadening of x-ray diffraction lines is one of the most accurate indirect methods of determining particle size for crystallites smaller than 1000A diameter. The theory relating line broadening to particle size was developed by Scherrer,[1] von Laue,[2] and Bragg.[3] It was considered in more detail by Warren,[4] Patterson,[5]

[1] Scherrer, "Bestimmung der Grosse und der inneren Struktur von Kolloidteilchen mittels Rontgenstrahlen," Nach. Gesell. Wiss. Göttingen, Zitzungsber., July 26, 1918 in R. Zsigmondy, *Kolloidchemie* (Otto Spamer, Leipzig, 1920), third edition.

[2] Von Laue, Zeits. f. Krist. **64**, 115 (1926).

[3] W. Bragg, *The Crystalline State* (G. Bell and Sons, London, 1933).

[4] B. E. Warren, "X-ray diffraction study of carbon blacks," J. Chem. Phys. **2**, 551 (1934).

[5] A. L. Patterson, "The diffraction of x-rays by small crystalline particles," Phys. Rev. **56**, 972 (1939).