＊＊作答前請先核對試題、答案卷(試卷)與准考證之所組別與考科是否相符！

線性代數 共50分

1. (a) Let matrix A = [a_1, a_2, a_3], where a_1, a_2, a_3 are vectors in R^3. If $4a_1-3a_2+2a_3=0$, find det(A). (2%)

(b) Given the Adjoint of matrix A denoted as adj(A), find det(A), A, det($3A^{-1}A^T$).

Where

adj(A) =

2	1	0
4	3	2
-2	-1	2

(6%)

(c) For the linear system Ax= b (6%)

(i) Find the rank of A and a basis for the column space of A.

(ii) Find a basis for the null space N(A)? What is the dimension of N(A)?

Where

A=

0	1	1	3	4
1	-2	1	1	2
1	2	5	13	5
-1	3	0	2	-2

2. For this problem, just give the answer, no need to show your computation.

(a) Let L be the linear transformation of the reflection about the line $ax+by=0$, from R^2 to R^2, where $a^2+b^2\neq 0$.
Find the matrix representation of L with respect to the standard basis, find the dimension of the kernel space, and find a basis of the range space. (6%)

(b) Let L be the linear transformation from R^3 to R^3 having the following matrix representation with respect to the standard basis;

$$\begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}. \qquad \text{Find } L(L(L(v))),\text{ where } v=\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Find the dimension of the kernel space, and find a basis of the range space of L. (4%)

(c) Let A_1 be the matrix representation of a linear transformation L from R^n to R^n with respect to the basis B_1. Let B_2 be another basis of R^n, and let P be the transition matrix corresponding to the change of basis from B_1 to B_2. What is the matrix representation of L with respect to the basis B_2? Express it in terms of A_1 and P. (2%)

3. Given an m by n matrix $A=[\, a_1, a_2, \ldots, a_n \,]$, where $a_i \bullet a_j = 0$, $i \neq j$, $1 \leqq i, j \leqq n$.
   (a) What can you say regarding the properties of $A^T A$? (at least 3 statements to be made for full score of 4 points) (4%)
   (b) Let m=n=3 and f(x)=Ax. Give two specific examples of A and explain the kinds of geometric operations thus involved respectively. (8%)

4. Let $A = [a_{ij}]$ be an nxn matrix with eigenvalues $\lambda_1, \ldots, \lambda_n$. Show that
   $$\sum_{j=1}^{n} (\lambda_j - a_{jj}) = 0.$$ (6%)

5. Compute cos(A) for $A = \begin{bmatrix} -2 & -6 \\ 1 & 3 \end{bmatrix}$. (6%)

〈接下頁〉

國立交通大學 96 學年度碩士班考試入學試題

科目：線性代數與離散數學(1002)　　考試日期：96 年 3 月 17 日 第 2 節

系所班別：資訊學院聯招　　組別：資訊聯招　　

**作答前請先核對試題、答案卷(試卷)與准考證之所組別與考科是否相符！

離散數學 50分

6. For each sub-problem, you must clearly indicate in the first line of your answer whether you want to prove or you want to disprove. No points will be given if you do both.

(a) (4 points) Let P(x,y) be a propositional function.

Prove or disprove the following:

$\forall y \exists x P(x,y) \rightarrow \exists x \forall y P(x,y)$ is always true for all interpretations.

(b) (4 points) Let P(x,y) be a propositional function.

Prove or disprove the following:

$\exists x \forall y P(x,y) \rightarrow \forall y \exists x P(x,y)$ is always true for all interpretations.

(c) (4.5 points) Let f be a function from the set A to the set B, $f: A \rightarrow B$. Given any subset $A' \subseteq A$, we define $f(A') = \{ f(a) \in B \mid a \in A' \}$. Therefore, $f(A')$ is a subset of B. Given any subset $B' \subseteq B$, we define $ⓕ(B') = \{ a \in A \mid f(a) \in B' \}$. Therefore, $ⓕ(B')$ is a subset of A.

Prove or disprove the following:

For any subset $A' \subseteq A$, we always have $A' \subseteq ⓕ(f(A'))$.

7. For each problem, you first write down your answer. Then write down your explanation.

(a) (4 points) How many bit strings of length 10 contains either five consecutive 1s or five consecutive 0s?

(b) (4 points) We toss a coin 5 times. There are 2^5 possible outcomes. How many of them contain no two consecutive heads?

(c) (4.5 points) How many different dice are there? Two dice are considered identical if they become exactly the same after proper rotations and flips.

8. (12.5 points, 2.5 points for each) For each of the following (a)-(e), if the statement is true (always), write TRUE. Otherwise write FALSE. If the statement is correct, briefly state why. If the statement is wrong, briefly explain why. Your justification is worth more points (60%) than your true-or-false designation (40%).

(a) There exists a simple graph with 6 vertices, whose degrees are

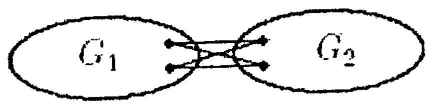
2, 2, 2, 3, 4, 4.

(b) There exists a simple graph with 6 vertices, whose degrees are

0, 1, 2, 3, 4, 5

(c) There exists a simple graph with degrees 1, 2, 2, 3.

(d) A graph containing an Eulerian circuit is called an Eulerian graph. If G_1 and G_2 are Eulerian graphs, and we add the following edges between them, the resulting graph is Eulerian.



(e) Let T be a minimum spanning tree of G. Then, for any pair of vertices s and t, the shortest path from s to t in G is the path from s to t in T.

9. A binary relation R on a set S is a subset of S^2.

(a) (7 points, 1 point for each) Assume $|S|=n$, i.e. the cardinality of S is n. Then,

i. How many symmetric relations are there on S?

ii. How many antisymmetric relations are there on S?

iii. How many asymmetric relations are there on S?

iv. How many irreflexive relations are there on S?

v. How many reflexive and symmetric relations are there on S?

vi. How many relations neither reflexive nor irreflexive are there on S?

vii. How many equivalent relations are there on S?

(b) (1 point) Let $R1$ be the reflexive closure of R. Please fill the blank $R1=\{(a,b)\in S^2:$ ______________________ $\}$.

(c) (1 point) Let $R2$ be the symmetric closure of R. Please fill the blank $R2=\{(a,b)\in S^2:$ ______________________ $\}$.

(d) (3.5 points) Prove that the transitive closure of $R \cup R1 \cup R2$ is an equivalent relation.