(1) 集合論

(a) $|A| = n \Rightarrow |P(A)| = 2^n$

(b) $P(A \cup B) = P(A) \cup P(B): false$
$P(A \cap B) = P(A) \cap P(B): true$

(c) $P(A) \cap \{A\} = \{A\}$ ， $P(A) \cup \{A\} = P(A)$

(d) $n = p_1^{e_1} p_2^{e_2} ......p_k^{e_k}$　　**97 台大電機類**

尤拉公式: $\phi(n) = n\prod_{i=1}^{k}(1 - p_i)$ ，　正因數個數: $(e_1 + 1)(e_2 + 1)...(e_k + 1)$

(e)費馬定理:

$\gcd(m, p) = 1 \Rightarrow m^{p-1} = 1 \pmod{p}$

$\gcd(m, p) = 1 \Rightarrow m^{p} = m \pmod{p}$

$m^{\phi(n)} = 1 \pmod{n}$

$A = \{a, b\}, B = \{1, 2, 3\}$, Cartesian product $A \times B$ =?
$A \times B = \{(a,1),(a,2),(a,3),(b,1),(b,2),(b,3)\}$

$\{\emptyset\} \in \{\emptyset, \{\emptyset\}\}$	$\{\emptyset\}$ *is a element of* $\{\emptyset, \{\emptyset\}\}$
$\{\emptyset\} \subseteq \{\emptyset, \{\emptyset\}\}$	*all element in set* $\{\emptyset\}$ *are in* $\{\emptyset\} \subseteq \{\emptyset, \{\emptyset\}\}$

Ex 1-6　***C.C.L*** 例題

(1) $|P \cup Q| \le |P| + |Q|$　$\because |P| + |Q| = |P \cup Q| + |P \cap Q|$

(2) $|P \cap Q| \le \min\{|P|, |Q|\}$　$\because |P \cap Q| \le |P|$ *and* $|P \cap Q| \le |Q|$

(3) $|P \oplus Q| = |P| + |Q| - 2|P \cap Q|$

(4) $|P - Q| \ge |P| + |Q|$

$S = \{1, 2, 4\}$　(a) $P(S) - S = ?$　(b) $P(S) - \{S\} = ?$

$P(S) = \{\emptyset, \{1\}, \{2\}, \{4\}, \{1,2\}, \{2,4\}, \{1,4\}, \{1,2,4\}\}$

(a) $P(S) - S = \{\emptyset, \{1\}, \{2\}, \{4\}, \{1,2\}, \{2,4\}, \{1,4\}, \{1,2,4\}\} - \{1,2,4\}$
$= \{\emptyset, \{1\}, \{2\}, \{4\}, \{1,2\}, \{2,4\}, \{1,4\}, \{1,2,4\}\} = P(S)$

(b) $P(S) - \{S\} = \{\emptyset, \{1\}, \{2\}, \{4\}, \{1,2\}, \{2,4\}, \{1,4\}, \{1,2,4\}\} - \{\{1,2,4\}\}$
$= \{\emptyset, \{1\}, \{2\}, \{4\}, \{1,2\}, \{2,4\}, \{1,4\}\}$

$n \in Z^+, prove \quad 43 \quad divide \quad 6^{n+2}+7^{2n+1}$
$n=1: 6^{1+2}+7^{2+1}=559 \Rightarrow 43 \mid 559$ 假設 $n=k$ 成立,則 $43 \mid 6^{k+2}+7^{2k+1}$ 考慮 $n=k+1$,則 $6^{(k+1)+2}+7^{2(k+1)+1}=6(6^{k+2})+6(7^{2k+1})+43(7^{2k+1})$ 因爲 $43 \mid 6(6^{k+2}+7^{2k+1})$,而且 $43 \mid 43(7^{2k+1})$ 所以 $43 \mid 6^{(k+1)+2}+7^{2(k+1)+1}$ 由(1)(2)(3) 得證

Ex 1-10

Find all of the possible solutions of 250x+111y=7,where both x and y are integer
$250=111\times 2+28$ $111=28\times 3+27$ $28=27\times 1+1$ $1=28+27\times(-1)$ $=28+(111-28\times 3)\times(-1)=111\times(-1)+28\times 4$ $=111\times(-1)+(250-111\times 2)\times 4=111\times(-9)+250\times 4$ $\therefore 1=111\times(-9)+250\times 4=111\times(-9-250k)+250\times(4+111k), \forall k \in Z$ $7=111\times 7(-9-250k)+250\times 7(4+111k), \forall k \in Z$ $\Rightarrow x=7(4+111k), y=7(-9-250k)$

Ex 1-11 95 東華資工 95 雲科資工

Prove $\sqrt{2}$ is irrational number
suppose $\sqrt{2}$ is rational number $\Rightarrow \exists m,n \ni \sqrt{2}=\frac{m}{n}, \gcd(m,n)=1$ $2=\frac{m^2}{n^2} \Rightarrow m^2=2n^2 \Rightarrow 2 \mid m^2 \Rightarrow 2 \mid m$ $\therefore let \quad m=2k, k \in N$ $\because m^2=2n^2 \Rightarrow (2k)^2=2n^2 \Rightarrow 4k^2=2n^2$ $\Rightarrow 2k^2=n^2 \Rightarrow 2 \mid n^2 \Rightarrow 2 \mid n$ $\therefore \gcd(m,n) \geq 2 \quad contradition \quad to \quad \gcd(m,n)=1$

$\phi(16200)=?$
$16200=2^3 3^4 4^5 \Rightarrow \phi(16200)=16200(1-\frac{1}{2})(1-\frac{1}{3})(1-\frac{1}{4})=4320$

multiplicative inverse 92 ntu

Find the inverse of 4 module 7

$$7 = 4\times1+3$$
$$4 = 3\times1+1$$
$$1 = 4-3 = 4-(7-4) = 2\times4-7$$
$$2\times4 = 1 \pmod{7}$$
$$4^{-1} = 2 \pmod{7}$$
$$inverse\ \ of\ \ 4 \Rightarrow 2+7k, k\in Z$$

Example 1.13

$$\begin{cases} x = 2 & (\bmod\ 3) \\ x = 3 & (\bmod\ 5) \Rightarrow x = ? \\ x = 2 & (\bmod\ 7) \end{cases}$$

Sol:

$$N_1 = 5\times7 = 35, N_2 = 3\times7 = 21, N_3 = 3\times5 = 15$$
$$M_1 = N_1^{-1} = 2 \quad \because 35\times2 = 1 \ (\bmod\ 3)$$
$$M_2 = N_2^{-1} = 1 \quad \because 21\times1 = 1 \ (\bmod\ 5)$$
$$M_3 = N_3^{-1} = 1 \quad \because 15\times1 = 1 \ (\bmod\ 7)$$
$$x = (r_1M_1N_1 + r_2M_2N_2 + r_3M_3N_3) \ \bmod\ n_1n_2n_3$$
$$= (2\times35\times2 + 3\times21\times1 + 2\times15\times1) \ \bmod\ 105$$
$$= 233 \ \bmod\ 105 = 23(\ \bmod\ 105)$$

Example 1.14

$$\begin{cases} x = 5 & (\bmod\ 6) \\ x = 3 & (\bmod\ 10) \Rightarrow x = ? \\ x = 8 & (\bmod\ 15) \end{cases}$$

Sol:

$$\begin{cases} x = 5 \ (\bmod\ 6) \Rightarrow x = 1 \ (\bmod\ 2), x = 2 \ (\bmod\ 3) \\ x = 3 \ (\bmod\ 10) \Rightarrow x = 1 \ (\bmod\ 2), x = 3 \ (\bmod\ 5) \\ x = 8 \ (\bmod\ 15) \Rightarrow x = 2 \ (\bmod\ 3), x = 3 \ (\bmod\ 5) \end{cases}$$

$$\Rightarrow \begin{cases} x = 1 \ (\bmod\ 2) \\ x = 2 \ (\bmod\ 3) \\ x = 3 \ (\bmod\ 5) \end{cases} \Rightarrow \begin{cases} N_1 = 35, N_2 = 10, N_3 = 6, M_1 = 1, M_2 = 1, M_3 = 1 \\ x = (r_1M_1N_1 + r_2M_2N_2 + r_3M_3N_3) \ \bmod\ n_1n_2n_3 \\ = 53(\ \bmod\ 30) = 23(\ \bmod\ 30) \end{cases}$$

CCL exercise 1-5 小黃習題

True or flase $(a) if \quad A \in B, B \subseteq C \Rightarrow A \in C$ $(b) if \quad A \in B, B \subseteq C \Rightarrow A \subseteq C$ $(c) if \quad A \subseteq B, B \in C \Rightarrow A \in C$ $(d) if \quad A \subseteq B, B \in C \Rightarrow A \subseteq C$
(a) True $(b) false, A=\{\varnothing\}, B=\{\{\varnothing\}\}, C=\{\{\varnothing\}\}$ $(c) false, A=\varnothing, B=\{\varnothing\}, C=\{\{\varnothing\}\}$ $(d) false, A=\{\varnothing\}, B=\{\varnothing\}, C=\{\{\varnothing\}\}$

CCL exercise 1-8

What can you say about the sets P and Q if $(a) P \cap Q = P$ $(b) P \cup Q = P$ $(c) P \oplus Q = P$ $(d) P \cap Q = P \cup Q$
$(a) P \subseteq Q$ $(b) Q \subseteq P$ $(c) Q = \varnothing$ $(d) P = Q$

CCL exercise 1-22

True or False $(a) A \cup P(A) = P(A) \qquad (b) A \cap P(A) = A$ $(c) \{A\} \cup P(A) = P(A) \qquad (d) \{A\} \cap P(A) = A$ $(e) A - P(A) = A \qquad (f) P(A) - \{A\} = P(A)$
$(a) A=\{a\}, P(A)=\{\varnothing, \{a\}\}$ $(b) A=\{a\}, P(A)=\{\varnothing, \{a\}\}$ $(c) T \quad (d) F \quad (e) F \quad (f) F$

95 靜宜資管 , 92 政大資料

(a)How many zeros are there at the end of 400 ! (n)How many zeros are there at the end of 100 !
$(a) \left\lfloor \frac{400}{5} \right\rfloor + \left\lfloor \frac{400}{25} \right\rfloor + \left\lfloor \frac{400}{125} \right\rfloor = 80+16+3=99 \quad (b) \left\lfloor \frac{100}{5} \right\rfloor + \left\lfloor \frac{100}{25} \right\rfloor = 20+4=24$

CCL exercise 1-54

Among 1-300,how many of them are not divisible by 3,not by 5,nor by 7? how many of them are divisible by 3,not by 5,nor by 7?
$(a)300-\left(\left\lfloor\frac{300}{3}\right\rfloor+\left\lfloor\frac{300}{5}\right\rfloor+\left\lfloor\frac{300}{7}\right\rfloor-\left\lfloor\frac{300}{15}\right\rfloor-\left\lfloor\frac{300}{35}\right\rfloor-\left\lfloor\frac{300}{21}\right\rfloor+\left\lfloor\frac{300}{105}\right\rfloor\right)$ $=300-(100+60+42-20-8-14+2)=138$ $(b)\left\lfloor\frac{300}{3}\right\rfloor-\left\lfloor\frac{300}{15}\right\rfloor-\left\lfloor\frac{300}{21}\right\rfloor+\left\lfloor\frac{300}{105}\right\rfloor=100-20-14+2=68$

Grimaldi exercise 3-1-8

$A=\{1,2,3,4,5,6,7\}$,determine the number of (a) subsets of A (b)nonempty subsets of A (c)proper subsets of A (a) nonempty proper subsets of A (e)subsets of A contains three element (f) subsets of A contains 1,2 (g) subsets of A contains 5 element include 1,2 (h) subsets of A with an even number of elements (i) subsets of A with an even number of elements
$(a)2^7 \quad (b)127 \quad (-\varnothing) \quad (c)127 \quad (-A) \quad (d)126$ $(e)\binom{7}{3}=5 \quad (f)2^5=32 \quad (g)\binom{5}{3}=5$ $(h)\binom{7}{0}+\binom{7}{2}+\binom{7}{4}+\binom{7}{6}=64$ $(i)\binom{7}{1}+\binom{7}{3}+\binom{7}{5}+\binom{7}{7}=64$

Grimaldi exercise 3-2-5

Determine which of the following statement are true and which are false $(a)Z^+\subseteq Q^+ \quad (b)Z\subseteq Q^+ \quad (c)Q^+\subseteq R \quad (d)R^+\subseteq Q$ $(e)Q^+\cap R^+=Q^+ \quad (f)Z^+\cup R^+=R^+ \quad (g)R^+\cap C=R^+$ $(h)C\cup R=R \quad (i)Q^*\cap Z=Z$
(a)T (b) T (c) T (d)F (e) T (f) T (g) T (h) F (i) F

小黃習題 1-23

Prove $(A-B)-C=(A-C)-(B-C)$

$$(A-C)-(B-C)=(A-C)\cap\overline{(B-C)}=(A\cap\overline{C})\cap\overline{(B\cap\overline{C})}$$
$$=(A\cap\overline{C})\cap(\overline{B}\cup C)=(A\cap\overline{C}\cap\overline{B})\cup(A\cap\overline{C}\cap C)=(A\cap\overline{B}\cap\overline{C})$$
$$=(A\cap\overline{B})\cap\overline{C}=(A-B)\cap\overline{C}=(A-B)-C$$

小黃習題 1-92

What is the last digit in 7^{2004}

$$7^4\equiv 1 \pmod{10} \Rightarrow 7^{2004}=(7^4)^{501}\equiv 1^{501}\equiv 1 \pmod{10}$$

所以 last digit in 7^{2004} be 1

小黃習題 1-94

Solve $13x\equiv 7 \pmod{31}$

$31=13*2+5$

$13=5*2+3$

$5=3*1+2$

$3=2*1+1$

$1=3-2*1$

$1=3-(5-3*1)*1=3*2-5*1$

$1=(13-5*2)*2-5*1=13*2-5*5$

$1=13*2-(31-13*2)*5=13*12+(-31)*5$

$1=13*(12+31k)+(-31-13k)*5$

$7=7*13*(12+31k)+7*(-31-13k)*5, \forall k\in Z$

$x=7*(12+31k), \forall k\in Z$

小黃習題 1-108

$(a)\, find\ \ \phi(715)\qquad (b)\, find\ \ \phi(2431)\qquad (c)\, find\ \ \phi(1763)$

$$(a)715=5*11*13\Rightarrow\quad \phi(715)=715(1-\frac{1}{5})(1-\frac{1}{11})(1-\frac{1}{13})=480$$

$$(b)2431=11*13*17\quad\Rightarrow\quad \phi(2431)=715(1-\frac{1}{11})(1-\frac{1}{13})(1-\frac{1}{17})=1920$$

$$(c)1763\quad 41*43\Rightarrow\quad \phi(1763)=715(1-\frac{1}{41})(1-\frac{1}{43})=1680$$

95 台大電機

which of following statement is false $(a) a \equiv b \pmod{m} \Rightarrow 2a \equiv 2b \pmod{m}$ $(b) a \equiv b \pmod{2m} \Rightarrow a \equiv b \pmod{m}$ $(c) a \equiv b \pmod{m} \Rightarrow 2a \equiv 2b \pmod{2m}$ $(d) a \equiv b \pmod{m^2} \Rightarrow a \equiv b \pmod{m}$ $(e) a \equiv b \pmod{m} \Rightarrow a \equiv b \pmod{2m}$
(e) $(a) b = a + km \Rightarrow 2b = 2a + (2k)m$ $(b) b = a + 2km \Rightarrow b = a + nm, n = 2k$ $(c) b = a + km \Rightarrow 2b = 2a + 2km$ $(d) b = a + km^2 \Rightarrow) b = a + nm, n = mk$

96 中原資工 95 台大電機

$7^{360} \mod 180 = ?$ $(a) 105^{36} \mod 37 = ?$ $(b) 104^{37} \mod 37 = ?$
$\because \gcd(7,180) = 1$, $\phi(180) = 180(1-\frac{1}{2})(1-\frac{1}{3})(1-\frac{1}{5}) = 48$ $\therefore 7^{48} \equiv 1 \pmod{180} \Rightarrow 7^{360} \mod 180 \equiv 7^{24} \mod 180$ $7^4 \equiv 61 \Rightarrow 7^8 \equiv 121, 7^{16} \equiv 61, 7^{24} \equiv 7^8 * 7^{16} = 61*121 = 1$ $(a) \gcd(105,37) = 1$, $105^{37-1} = 1 \pmod{37}$ $(b) \gcd(104,37) = 1$, $104^{37} = 104 \pmod{37}$ p.s. $(1) \gcd(a,p) = 1$, $a^{p-1} = 1 \pmod{p} \Leftrightarrow a^p = a \pmod{p}$ $(2) \gcd(a,n) = 1$, $a^{\phi(n)} = 1 \pmod{n}$

94 中山資工,東華資工,嘉大資工,淡江資管

38 play golf , 21 play tennis , 56 play tennis 8 play golf& tennis , 17 play golf& bridge , 13 play tennis & bridge 5 play golf& tennis& bridge 72 dis not play golf , tennis or tennis How many residents are there in this retirement community ?
[38 +21 +56 −(8+17+13) +5] +72 =154

90 台大資工

$\phi(1000)=?$
$1000=2^3*5^3 \Rightarrow \phi(1000)=1000*(1-\frac{1}{2})*(1-\frac{1}{5})=400$

95 台大電機

Find y in {0,1,2,3,4,5,6,7,8,9} such that $3^{555} \equiv y \pmod{10}$
此題 10 非 p 不能用 費馬小定理做 只能用(c)做 (a) $m \in Z, p: prime \ni \gcd(m,p)=1 \Rightarrow m^{p-1}=1 \ (\bmod\ p)$ (b) $\gcd(m,p)=1 \Rightarrow m^p = m \ (\bmod\ p)$ (c) $m^{\phi(n)}=1 \ (\bmod\ n)$ $\phi(10)=10*(1-\frac{1}{2})*(1-\frac{1}{5})=4 \Rightarrow 3^4 \equiv 1 \pmod{10}$ $\Rightarrow 3^{552} \equiv 1 \pmod{10}$ $\therefore 3^{555} \equiv y \pmod{10} \Rightarrow 3^3 \equiv y \pmod{10} \Rightarrow y \equiv 7 \pmod{10}$

90 台大資工

Find $75^{384} \pmod{97}$
$\because \gcd(75,97)=1 \quad \therefore 75^{96} \equiv 1 \ (\bmod\ 97)$ $\Rightarrow (75^{96})^4 \equiv 1 \ (\bmod\ 97)$

95 靜宜資管

Find $(a) 5^{2003} \bmod 7 \quad (b) 5^{2003} \bmod 11 \quad (c) 5^{2003} \bmod 13$
$(a) \because \gcd(5,7)=1 \quad \therefore 5^6 \equiv 1 \ (\bmod\ 7)$ $5^{2003} \bmod 7 \Rightarrow 5^5 \bmod 7 = 3$ $(b) \because \gcd(5,11)=1 \quad \therefore 5^6 \equiv 1 \ (\bmod\ 11)$ $5^{2003} \bmod 7 \Rightarrow 5^3 \bmod 11 = 1$ $(c) \because \gcd(5,13)=1 \quad \therefore 5^6 \equiv 1 \ (\bmod\ 13)$ $5^{2003} \bmod 7 \Rightarrow 5^{11} \bmod 13 = 8$

90 市教大資工

572^{13} mod 71
$572^{13} \equiv 4^{13} \pmod{71} \equiv 4^3 4^3 4^3 4^3 4 \pmod{71}$ $\equiv (-7)(-7)(-7)(-7)4 \pmod{71} \equiv 9604 \pmod{71} \equiv 19 \pmod{71}$

(2) 關係

$|A| = m, |B| = n \quad |A \times B| = mn$

$|P(A \times B)| = 2^{mn}$ is the cardinality of relation from A to B

$|A| = n$ there are 2^{n^2} Binary relations

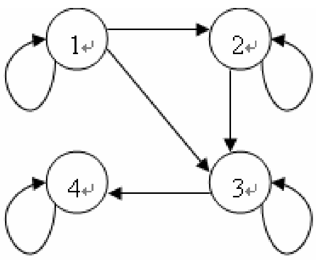
Ex 2-1

Relation $R = \{(1,1),(1,2),(1,4),(2,2),(2,3),(3,3),(3,4),(4,4)\}$ on $\{1,2,3,4\}$

(a)represent this relation by a matrix and a directed graph

(b)find the matrix that represent the relation R^2

$$M_R = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



$$M_{R^2} = (M_R)^2 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Ex 2-2

$A=\{a_1,a_2,a_3\}, B=\{b_1,b_2\}, C=\{c_1,c_2,c_3,c_4\}$

$R_\alpha = \{(a_1,b_1),(a_2,b_1),(a_2,b_2),(a_3,b_2)\}$

$R_\beta = \{(b_1,c_1),(b_1,c_3),(b_2,c_2),(b_2,c_4)\}$

$R_{\alpha\beta} = ?$

$R_{\alpha\beta} = \{(a_1,c_1),(a_1,c_3),(a_2,c_1),(a_2,c_2),(a_2,c_3),(a_2,c_4),(a_3,c_2),(a_3,c_4)\}$

Ex 2-3

$R=\{a,b,c\}, \alpha = \{(a,a),(a,b),(b,b),(b,c),(c,a),(c,c)\}, \alpha : A \to A$

$\alpha^{-1} = ? \overline{\alpha} = ?$

$M_R = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$ inverse→ $M_R{}^T = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ complement→ $\overline{M_R} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

$\alpha^{-1} = \{(a,a),(a,c),(b,a),(b,b),(c,b),(c,c)\}$

$\overline{\alpha} = \{(a,c),(b,a)(c,b)\}$

(a) $\alpha : A \to A$ be $binary$ $relation$

$reflecive : \forall a \in A, aRa$

$irreflecive : \forall a \in A, a \not R a$

$symmetric : \forall a,b \in A, aRb \Rightarrow bRa$

$asymmetric : \forall a,b \in A, aRb \Rightarrow b \not R a$

$antisymmetric : \forall a,b \in A, aRb$ and $bRa \Rightarrow a \not= b$

$transitive : \forall a,b,c \in A, aRb$ and $bRc \Rightarrow aRc$

(b) cardinality of each relation　　96 交大資聯　**97 台大電機類**

reflexive	irreflexive	symmetric	asymmetric	antisymmetric
2^{n^2-n}	2^{n^2-n}	$2^{\frac{n^2+n}{2}}$	$3^{\frac{n^2-n}{2}}$	$2^n 3^{\frac{n^2-n}{2}}$
Reflexive && Symmetric (Compatible relation)		$2^{\frac{n^2-n}{2}}$		

(c) R is binary relation of A

$R : transitive \Leftrightarrow R^2 \subseteq R \Leftrightarrow R^n \subseteq R$

Ex 2-5

Let R_1, R_2 be relation on a set $A = \{a,b,c,d\}$

$R_1 = \{(a,a),(a,b),(a,d)\}$

$R_2 = \{(a,d),(b,c),(b,d),(c,b)\}$

$find$ $R_1R_2, R_2R_1, R_1^{\,2}, R_2^{\,3}$

$$M_{R_1} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, M_{R_2} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, M_{R_1R_2} = M_{R_1}M_{R_2} = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$M_{R_2R_1} = M_{R_2}M_{R_1} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, M_{R_1^2} = (M_{R_1})^2 = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$M_{R_2^3} = (M_{R_2})^3 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{cases} R_1R_2 = \{(a,c),(a,d)\} \\ R_2R_1 = \{(c,d)\} \\ R_1^{\,2} = \{(a,a),(a,b),(a,d)\} \\ R_2^{\,3} = \{(b,c),(b,d),(c,b)\} \end{cases}$$

Ex 2-6

Find a partition of A $A=\{1,2,3,4\}, R=\{(1,1),(2,2),(3,3),(4,4),(1,2),(2,1),(3,4),(4,3)\}$
相異等價類形成 A 的一個分割 相異等價類➔ $\begin{array}{l}[1]=\{1,2\}=[2]\\ [3]=\{3,4\}=[4]\end{array}$ $P=\{[1],[3]\}=\{\{1,2\},\{3,4\}\}$ 形成 A 的一個分割

Ex 2-7

Prove module relation ($\equiv_n$) is a equivalent relation
$(1)\forall a\in Z, \because n\mid a-a\Rightarrow a\equiv_n a$ $(2)\forall a,b\in Z,$ $\quad if\quad a\equiv_n b$ $\quad \because n\mid a-b\Rightarrow a-b=nk, for\quad some\quad k\quad \in Z$ $\quad \Rightarrow b-a=n(-k),-k\quad \in Z$ $\quad \Rightarrow n\mid b-a$ $\quad \Rightarrow b\equiv_n a$ $(3)\forall a,b,c\in Z,$ $\quad if\quad a\equiv_n b, b\equiv_n c$ $\quad \because n\mid a-b\quad ,\quad n\mid b-c$ $\quad \Rightarrow a-b=nk, b-c=nl, for\quad some\quad k,l\quad \in Z$ $\quad \Rightarrow a-c=(a-b)+(b-c)=nk+nl=n(k+l), k+l\in Z$ $\quad \Rightarrow n\mid a-c$ $\quad \Rightarrow a\equiv_n c$

96 中興資工

A={1,2,3,4,5} How many different equivalence classes (試求相異分割數)
令 P_i 爲 i 個元素之相異分割數 $P_1=\binom{0}{0}P_0=1 \qquad P_2=\binom{1}{0}P_0+\binom{1}{1}P_1=2$ $P_3=\binom{2}{0}P_0+\binom{2}{1}P_1+\binom{2}{2}P_2=5$ $P_4=\binom{3}{0}P_0+\binom{3}{1}P_1+\binom{3}{2}P_2+\binom{3}{3}P_3=15$ $P_5=\binom{4}{0}P_0+\binom{4}{1}P_1+\binom{4}{2}P_2+\binom{4}{3}P_3+\binom{4}{4}P_3=52$

$R=\{(a,b),(b,a),(a,c),(c,d),(d,b),(d,c)\}$ on $\{a,b,c,d\}$ Find (a) reflexive closure(b) symmetric closure(c) transitive closure
$r(R)=R\cup\{(a,a),(b,b),(c,c),(d,d)\}$ $=\{(a,b),(b,a),(a,c),(c,d),(d,b),(d,c),(a,a),(b,b),(c,c),(d,d)\}$ $s(R)=R\cup R^{-1}$ $=\{(a,b),(b,a),(a,c),(c,d),(d,b),(d,c),(c,a),(b,d)\}$ $t(R)=R\cup R^2\cup R^3\cup R^4$ $=\{(a,a),(a,b),(a,c),(a,d),(b,a),(b,b),(b,c),(b,d),$ $(c,a),(c,b),(c,c),(c,d),(d,a),(d,b),(d,c),(d,d)\}$

Ex 2-11

Find the smallest equivalence on the set $\{a,b,c,d,e\}$ contains $R=\{(a,b),(a,c),(d,e)\}$
Method (1): 最小等價類所對應的分割爲 $\{\{a,b,c\},\{d,e\}\}$ 所以 $\{(a,a),(a,b),(a,c),(b,a),(b,b),(b,c),(c,a),(c,b),(c,c),$ $(d,a),(d,b),(e,a),(e,b)\}$ Method (2): $r(R)=\{(a,a),(b,b),(c,c),(d,d),(e,e),(a,b),(a,c),(d,e)\}$ $s(r(R))=\{(a,a),(b,b),(c,c),(d,d),(e,e),$ $(a,b),(b,a),(a,c),(c,a),(d,e),(e,d)\}$ $t(s(r(R)))=R\cup R^2\cup R^3\cup R^4=\begin{bmatrix}1&1&1&0&0\\1&1&1&0&0\\1&1&1&0&0\\0&0&0&1&1\\0&0&0&1&1\end{bmatrix}$

$$a\rightarrow b\equiv \overline{a}\vee b$$

$$\sim\exists x\equiv\forall x\sim$$

$$\sim\forall x\equiv\exists x\sim$$

$finite \quad set \Rightarrow countable \quad set \qquad (\not\Leftarrow)$

$uncountable \quad set \Rightarrow infinite \quad set \qquad (\not\Leftarrow)$

$Z: countable \quad set, infinite \quad set$

$N: countable \quad set, infinite \quad set$

$R: uncountable \quad set, infinite \quad set$

Ex 2-13　95 雲科資工

Prove Z is countable set
$Z \to N \Rightarrow f(a) = \begin{cases} 1, if \quad a = 0 \\ 2a, if \quad a > 0 \\ -2a+1, if \quad a < 0 \end{cases}$

Ex 2-14　96 中興資管 95 東華資工

Prove $N \times N$ is countable set
$N \times N \to N \Rightarrow f(i,j) = \frac{(i+j-1)(i+j-2)}{2} + j$

Ex 2-15

Prove Q is countable set
$let \quad Q = \frac{j}{i}$, $Q(\frac{j}{i}) \to N \Rightarrow f(i,j) = \frac{(i+j-1)(i+j-2)}{2} + j$

Ex 2-16　95 雲科資工

Show real number between (0,1) is uncountable infinite set
假設實數位於(0,1)間是可數的 ,我們是著列出所有的數 $0.a_{11}a_{12}a_{13}....$ $0.a_{21}a_{22}a_{23}....$　　但我們考慮 $b = 0.b_1b_2......$, $b_i = \begin{cases} 1, if \quad r_{ii} = 9 \\ 9 - r_{ii}, if \quad r_{ii} \neq 9 \end{cases}$ $..................$ $0.a_{i1}a_{i2}a_{i3}....$ B 並不在上列中,所以假設矛盾 實數位於(0,1)間是無限不可數的

$\pi_1 = \{\{1,2,3\},\{4\},\{5\}\}, \pi_2 = \{\{1,2\},\{3,4\},\{5\}\}$ be two partition on set $A = \{1,2,3,4,5\}$ find $\pi_1\pi_2, \pi_1 + \pi_2$
$R_{\pi_1} = \{(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3),(4,4),(4,5),(5,4),(5,5)\}$ $R_{\pi_2} = \{(1,1),(1,2),(2,1),(2,2),(3,3),(3,4),(4,3),(4,4),(5,5)\}$ $\because R_{\pi_1} \cap R_{\pi_2} = \{(1,1),(1,2),(2,1),(2,2),(3,3),(4,4),(5,5)$ $\therefore \pi_1\pi_2 = \{\{1,2\},\{3\},\{4\},\{5\}\}$ $\because R_{\pi_1} \cup R_{\pi_2} = \{(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(2,4)$ $(3,1),(3,2),(3,3),(3,4),(4,1),(4,2),(4,3),(4,4),(5,5)\}$ $\therefore \pi_1 + \pi_2 = \{\{1,2,3,4\},\{5\}\}$

cise 4-22

If R reflexive,is R^{-1} reflexive? If R symmetric,is R^{-1} symmetric? If R transitive,is R^{-1} transitive?
Yes , Yes , Yes

C.C.L exercise 4-15

There are 10 distinct element (1)how many relation on A? (a) how many binary relation on A? (b)how many reflective relation on A? (c)how many symmetric relation on A? (d)how many reflective and symmetric relation on A? (e)how many asymmetric relation on A? (f)how many asntisymmetric relation on A?
(1) 2^{n^2} $(a)2^{10}$ $(b)2^{n^2-n} = 2^{10}$ $(c)2^{\frac{n^2+n}{2}} = 2^{55}$ $(d)2^{\frac{n^2-n}{2}} = 2^{45}$ $(e)3^{\frac{n^2-n}{2}} = 3^{45}$ $(f)2^n 3^{\frac{n^2-n}{2}} = 2^{10}3^{45}$

C.C.L example 94 彰師資工 (value :30,45,14)

A chess player wants to prepare for a championship match by playing some practice games in 77 days. she wants to play at least one game a day but no more than 132 games altogether. We show now that no matter how she schedules the game there is a period of consecutive days within which she plays exactly 21 games
a_i 表前 i 天比賽的局數, $i=1,2,...,77$ $1 \le a_1 \le a_2 \le .... \le a_{77} \le 132$, add 21 both side $22 \le a_i + 21 \le 154, i=1,2,...,77$ 考慮 154 個數 $a_1,a_2,....,a_{77},a_1+21,a_2+21,.....,a_{77}+21$ 介於 1 到 153 $a_1,a_2,....,a_{77}$ 中互不相等 ,$a_1+21,a_2+21,.....,a_{77}+21$ 中也互不相等 根據鴿籠原理 必存在 $a_i,a_j,i \ne j \ni a_j = a_i + 21$ ➔第 i+1 天到 第 j 天 比了 21 場

C.C.L exercise 4-48 (C.C.L n=100) 95 清大資工,95 逢甲資工,94 元智資管,93 中華資工

n+1 distinct integers are choosen from 1,2,…,2n,prove there are one of them are divisible by another
Let $X=\{1,2,...,2n\}$, $Y=\{1,3,..,2n-3,2n-1\},\|Y\|=n$ $\forall x \in X, x=2^k y, y \in Y, k \in N$ 此表示式唯一 e.g. $1=2^0 \times 1, 2=2^1 \times 1, 3=2^0 \times 3, 4=2^2 \times 1,...$ 每一個 x 對應惟一 $y \in Y$ 且 $\|Y\|=n$,從 Y 中挑 n+1 個數 必存在 $x_1,x_2 \in X, y \in Y \ni a=2^{x_1}y, b=2^{x_2}y \Rightarrow 2^{x_1}y \mid 2^{x_2}y$

C.C.L exercise 4-52 96 清大資應(丙) 95 高第一資管

Show that among n+1 integers less than or equal to 2n there are two of them that are relatively prime
取 $a_1,a_2,...,a_{n+1}$ 爲任意 n+1 個不大於 2n 的整數 欲証 $a_1,a_2,...,a_{n+1}$ 必有 2 數相鄰 若 $a_1,a_2,...,a_{n+1}$ 皆不相鄰 ,則每 2 數間至少差 2,這 n+1 個數必有 1 數 > 2n ➔矛盾 所以 $a_1,a_2,...,a_{n+1}$ 中必存在 2 數相鄰,且這 2 數 relatively prime

Grimaldi exercise 5-3-2 , 5-3-3

<table>
<tr><td>(1) $f: Z \to Z$,determine whether the function is one-to-one and whether it is onto
(2)
$(a) f(x) = x+7 \quad (b) f(x) = 2x-3 \quad (c) f(x) = -x+5$
$(d) f(x) = x^2 \quad (e) f(x) = x^2 + x \quad (f) f(x) = x^3$</td></tr>
<tr><td>(a)one-to-one , onto
(b)one-to-one ,not onto
(c)one-to-one ,onto
(d)not one-to-one ,not onto
(e) not one-to-one ,not onto
(f) one-to-one ,not onto</td></tr>
</table>

Grimaldi exercise 5-3-4　 97 台大工科類

<table>
<tr><td>Let $A = \{1,2,3,4\}, B = \{1,2,3,4,5,6\}$
(1)
(a)how many function $A \to B$?
(b)how many function $A \to B$ one-to-one ?
(c)how many function $A \to B$ onto ?
(2)
(a)how many function $B \to A$?
(b)how many function $B \to A$ one-to-one ?
(c)how many function $B \to A$ onto ?</td></tr>
<tr><td>$(1)(a)6^4 \quad (b)P_4^6 = 360 \quad (c)0$
$(2)(a)4^6 \quad (b)0 \quad (c) onto(6,4) = \sum_{i=0}^{n} (-1)^i \binom{4}{i} (4-i)^6 = 1560$
$m \to n$
$(a)n^m \quad (b)P_m^n \quad (c) onto(m,n) = \sum_{i=0}^{n} (-1)^i \binom{n}{i} (n-i)^m$</td></tr>
</table>

Grimaldi exercise 5-3-7

Let $A=\{1,2,3,4,5,6,7\}, B=\{v,w,x,y,z\}$ Determine the number of function $f:A\to B$ $(a) f(A)=\{v,x\}\quad (b) \mid f(A)\mid=2\quad (c) \mid f(A)\mid=\{w,x,y\}$ $(d) \mid f(A)\mid=3\quad (e) \mid f(A)\mid=\{v,x,y,z\}\quad (f) \mid f(A)\mid=4$ (g)$\mid A\mid=m\geq n=\mid B\mid, 1\leq k\leq n$,how many function $f:A\to B$ such that $\mid f(A)\mid=k$
$(a) onto(7,2)\quad (b)\binom{5}{2} onto(7,2)\quad (c) onto(7,3)$ $(d)\binom{5}{3} onto(7,3)\quad (e) onto(7,4)\quad (f)\binom{5}{4} onto(7,4)$ $(e)\binom{n}{k} onto(m,k)$

小黃習題 2-15

Let R be relation on A (a) prove if R reflexive ,then R^{-1} reflexive (b) prove if R symmetric ,then R^{-1} symmetric (c) prove if R transitive ,then R^{-1} transitive
$(a)\forall a\in A, \because R: reflexive \Rightarrow (a,a)\in R$ $\Rightarrow (a,a)\in R^{-1}, \therefore R^{-1}: reflexive$ $(b) would\quad prove\quad (a,b)\in R^{-1}\to (b,a)\in R^{-1}$ $\forall a,b\in A, (a,b)\in R^{-1}\Rightarrow (b,a)\in R\Rightarrow (a,b)\in R\Rightarrow (b,a)\in R^{-1}$ $(c) would\quad prove\quad (a,b),(b,c)\in R^{-1}\to (a,c)\in R^{-1}$ $\forall a,b,c\in A, (a,b),(b,c)\in R^{-1}\Rightarrow (b,a),(c,b)\in R\Rightarrow (c,a)\in R\Rightarrow (a,c)\in R^{-1}$

91 台科資工

$A=\{1,2,3,4,5\}, B=\{6,7,8,9\}, C=\{10,11,12,13\}, D=\{\alpha,\beta,\gamma,\delta\}$ $R\subseteq A\times B, S\subseteq B\times C, T=C\times D$ $R=\{(1,7),(4,6),(5,6),(2,8)\}$ $S=\{(6,10),(6,11),(7,10),(8,13)\}$ $T=\{(11,\beta),(10,\beta),(13,\delta),(12,\alpha),(13,\gamma)\}$ $(a) R^{-1}=?\quad ,\quad S^{-1}=?\quad (b) T(SR)=?$
$(a) R^{-1}=\{(7,1),(6,4),(6,5),(8,2)\}, S^{-1}=\{(10,6),(11,6),(10,7),(13,8)\}$ $(b) SR=\{(1,10),(4,10),(4,11),(5,10),(5,11),(2,13)\}$ $T(SR)=\{(1,\beta),(4,\beta),(5,\beta),(2,\gamma),(2,\delta)\}$

93 元智資工 , 92 成大工科 ,91 交大資科 **97 台大電機類**

(a) $R_1 \cap R_2$, reflexive ?, symmetric?, transitive?, (b) $R_1 \cup R_2$, reflexive ?, symmetric?, transitive?,

	$R_1 \cap R_2$	$R_1 \cup R_2$	R_1R_2
reflxive	V	V	V
symmetric	V	V	X
anti symmetric	V	X	X
transitive	V	X	X
asymmetric	V	X	X
equivalent	V	X	X

95 台大資工, 中央資管 ,元智資工,大同資工 94 大同資工,朝陽資工 96 清大資應
91 東華資工 92 朝陽網通

|A|=n ,find the distinct relation R on A (a) no restriction (b) reflexive (c)symmetric (d)antisymmetric (e)compatability (f)total order (g) irreflexive (h) not reflexive (i)asymmetric
$(a)2^{n^2}\quad (b)2^{n^2-n}\quad (c)2^{\frac{n^2+n}{2}}\quad (d)2^n3^{\frac{n^2-n}{2}}\quad (e)2^{\frac{n^2-n}{2}}\quad (f)n!\quad (g)2^{n^2-n}\quad (i)3^{\frac{n^2-n}{2}}$

96 中山資工,95 清大資應

$S=\|n\|$, (a) what is the Maximum value of symmetric relation for $\|R\|$ are there on S ? (b) what is the Maximum value of asymmetric relation $\|R\|$ are there on S ? (c) what is the Maximum value of antisymmetric relation $\|R\|$ are there on S ? (d) what is the Maximum value of reflexive relation $\|R\|$ are there on S ? (e) what is the Maximum value of irreflexive relation $\|R\|$ are there on S ? (f) what is the Maximum value of not reflexive relation $\|R\|$ are there on S ?
關係數 $\|R\|$ 最大為 1 擺滿時有幾個 1 $(a)n^2-n\quad (b)\frac{n^2-n}{2}\quad (c)\frac{n^2+n}{2}\quad (d)2^{n^2}\quad (e)n^2-n\quad (f)2^{n^2}-1$

96 中央資工

$f: A \to A, \forall x, y \in A$ define $x \sim y$ if $f(x) = f(y)$ Prove $\sim$ is an equivalence relation
$(a) \forall x \in A, \because f(x) = f(x), x \sim x$ $(b) \forall x, y \in A, \because if \quad x \sim y \quad f(x) = f(y) \Rightarrow f(y) = f(x), y \sim x$ $(c) \forall x, y, z \in A, \because if \quad x \sim y, y \sim z$ $f(x) = f(y), f(y) = f(z) \Rightarrow f(x) = f(z), x \sim z$

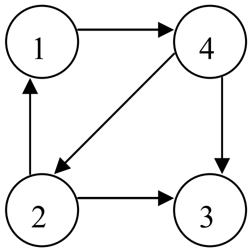
96 中正資工

$R = \{(a,b) \in Z \times Z \mid a^2 - b^2 \quad is \quad divisible \quad by \quad 5\}$ Prove R is an equivalence relation
$(a) \forall a \in Z, \because 5 \mid a^2 - a^2 \Rightarrow (a,a) \in R$ $(b) \forall a, b \in A, \because if \quad (a,b) \in R \Rightarrow 5 \mid a^2 - b^2$ $\Rightarrow \exists k \in Z \ni a^2 - b^2 = 5k \Rightarrow b^2 - a^2 = 5(-k) \Rightarrow 5 \mid b^2 - a^2 \Rightarrow (b,a) \in R$ $(c) \forall a, b, c \in A, \because if \quad (a,b),(b,c) \in R \Rightarrow 5 \mid a^2 - b^2, 5 \mid b^2 - c^2$ $\Rightarrow 5 \mid (a^2 - b^2) + (b^2 - c^2) \Rightarrow 5 \mid a^2 - c^2 \Rightarrow (a,c) \in R$

97 台大電機類

(1) f:one-to-one $\Leftrightarrow f(a) = f(b) \to a = b$ (2) f : onto $\Leftrightarrow \forall b \in B, \exists a \in A \ni f(a) = b$ (3) f,g one-to-one $\Rightarrow$ gf one-to-one (4) f,g onto $\Rightarrow$ gf onto (5) gf one-to-one$\Rightarrow$ f one-to-one (6) gf onto$\Rightarrow$ g onto

92 師大資工

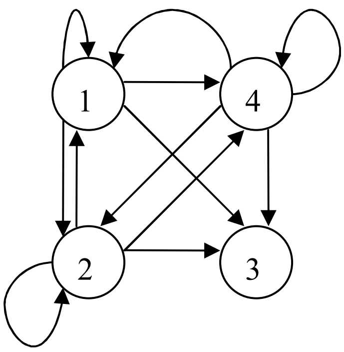


draw the transitive closure of the relation of following digraph

使用 Washall algorithm

$$A_0=\begin{bmatrix}0&0&0&1\\1&0&1&0\\0&0&0&0\\0&1&1&0\end{bmatrix},A_1=\begin{bmatrix}0&0&0&1\\1&0&1&1\\0&0&0&0\\0&1&1&0\end{bmatrix},A_2=\begin{bmatrix}0&0&0&1\\1&0&1&1\\0&0&0&0\\1&1&1&1\end{bmatrix}$$

$$A_3=\begin{bmatrix}0&0&0&1\\1&0&1&1\\0&0&0&0\\1&1&1&1\end{bmatrix},A_4=\begin{bmatrix}1&1&1&1\\1&1&1&1\\0&0&0&0\\1&1&1&1\end{bmatrix}$$



91 中央資管

Prove there are infinite prime in N

$suppose\ \ there\ \ are\ \ finite\ \ primes\ \ in\ \ N$

$p_1,p_2,......p_k$

$pick\ \ E=p_1p_2......p_k+1$

$\because E>p_i,i=1,2,...,k\Rightarrow E\ \ isn`t\ \ prime,but\ \ composite$

$\Rightarrow\exists p_j\ni p_j\mid E,1\le j\le k$

$\Rightarrow p_j\mid E\ \ and\ \ p_j\mid p_1p_2......p_k$

$\Rightarrow p_j\mid(E-p_1p_2......p_k)=p_j\mid 1$

$contradition$

(3)排列,組合

$|A|=m, |B|=n$

$\#\ \ of\ \ function\ \ A\to B?$

$\#\ \ of\ \ 1-1\ \ A\to B?$

$\#\ \ of\ \ onto\ \ A\to B?$ $\quad n^m\ ,\ P_n^m\ ,\ \sum_{i=0}^{n}(-1)^i\binom{n}{i}(n-i)^m$

Prove following are integer (整除)? $(a)\dfrac{(2n)!}{2^n}\quad (b)\dfrac{(n^2)!}{(n!)^n}\quad (c)\dfrac{(k!)!}{(k!)^{(k-1)!}}$
(1)n class objects,every class has 2 identical objects ,total 2n do permutation,ways   (2)n class objects,every class has n identical objects ,total n^2 do permutation,ways   (3) (k-1)! class objects,every class has k identical objects ,total k! do permutation,ways

Ex 3-11

Prove $\binom{n}{r}=\binom{n-1}{r}+\binom{n-1}{r-1}$
$\binom{n-1}{r}+\binom{n-1}{r-1}=\dfrac{(n-1)!}{(n-1-r)!r!}+\dfrac{(n-1)!}{(n-r)!(r-1)!}=\dfrac{(n-r)(n-1)!}{(n-r)!r!}+\dfrac{r(n-1)!}{(n-r)!r!}$ $=\dfrac{(n-r+r)(n-1)!}{(n-r)!r!}=\dfrac{n!}{(n-r)!r!}=\binom{n}{r}$

Ex 3-12

Prove $\sum_{k=0}^{n}\binom{r}{k}\binom{s}{n-k}=\binom{r+s}{n}$
pick n object form r+s do combination method= $\binom{r+s}{n}$ as same as 2 closure r and s, pick k object form r and pick n-k object form s,total   method $\sum_{k=0}^{n}\binom{r}{k}\binom{s}{n-k}$

Example 3.15: (a)n director ,lock open unless k director appear at same time (b)9 director ,lock open unless 4 director appear at same time (c)11 director ,lock open unless 6 director appear at same time
(a)how many key need? $\binom{n}{k-1}$ (b)how many key on every director? $\binom{n-1}{k-1}$ (b)how many key need? $\binom{9}{3}$ (b)how many key on every director? $\binom{8}{3}$ (c)how many key need? $\binom{11}{5}$ (b)how many key on every director? $\binom{10}{5}$

Ex 3-18

(a) nonnegative solution of $x_1+x_2+...+x_7=37, x_1+x_2+x_3=6$ (b) execute times of for i=1 to 24 for j=i to 24 for k=j to 24 for l=k to 24 (c) terms# after expend $(x+y+z)^{10}$
$(a)\binom{8}{6}\binom{34}{31}$ $(b)\binom{27}{4}$ $(c)\binom{12}{10}$ (b) $1 \le l \le k \le j \le i \le 24$ ，等同於 4 個相同球放入 24 個相異箱子,允許空箱

Ex 3-20

Example 3.19: 87 ncku A urn contains 3 red and 4 black balls, pick out 3 balls,probability Part 1: without replacement (a) all red (b)all black (c)one red ,two black (d)two red ,one black Part 2: with replacement (b) all red (b)all black (c)one red ,two black (d)two red ,one black
$(a)\frac{3}{7}\frac{2}{6}\frac{1}{5}=\frac{1}{35}(b)\frac{4}{7}\frac{3}{6}\frac{2}{5}=\frac{4}{35}(c)3(\frac{3}{7}\frac{4}{6}\frac{3}{5})=\frac{18}{35}(d)3(\frac{3}{7}\frac{2}{6}\frac{4}{5})=\frac{12}{35}$ $(a)(\frac{3}{7})^3(b)(\frac{4}{7})^3(c)(\frac{3}{7})(\frac{4}{7})^2(d)(\frac{3}{7})^2(\frac{4}{7})$

ball (r)	hole (n)	empty allowed	formula
distinct	distinct	allowed	n^r
distinct	distinct	not allowed	$onto(r,n)$ or $n!S_2(r,n)$
identical	distinct	allowed	$\binom{n+r-1}{r}$
identical	distinct	not allowed	$\binom{r-1}{n-1}$
distinct	identical	allowed	$\sum_{i=1}^{\min(r,n)} S_2(r,i)$
distinct	identical	not allowed	$S_2(r,n)$
identical	identical	allowed	partition
identical	identical	not allowed	partition

97 清大資工

object (n)	repeat allowed (r)	do	formula
distinct	allowed	combination	$\binom{n+r-1}{r}$
distinct	not allowed	combination	$\binom{n}{r}$
distinct	allowed	permutation	n^r
distinct	not allowed	permutation	P_n^m

C.C.L example 小黃習題 3-21

Among 11 senators to select a committee of 5 member (a) no restriction,how many ways ? (b) 5 member include A,how many ways ? (c) 5 member not include A,how many ways ? (d) 5 member include A and B,how many ways ? (e) 5 member include A without B,how many ways ? (f) 5 member at least A or B include,how many ways ?
$(a)\binom{11}{5}$ $(b)\binom{10}{4}$ $(c)\binom{10}{5}$ $(d)\binom{9}{3}$ $(e)\binom{9}{4}$ $(f)2\binom{9}{4}+\binom{9}{3}$

C.C.L example

When three dice are rolled,the number of different outcomes is ?
$\binom{6+3-1}{3}=56$

CCL exercise 3-5

Five boys and five girls are to be seated in a row ,In how many ways can they seated if (a) All boys must be seated in the five leftmost seats? (b) No two boys can be seated together? (c) John and Mary must be seated together
$(a)5!5! \quad (b)2*5!5! \quad (c)2*9!$

CCL exercise 3-6

(a) In how many way can 10 boys and 5 girls stand in a line so that no two girls are next to each other ? (b) Repeat part (a) if they must stand around a circle?
$(a)10!*P(11,5) \quad (b)9!*P(10,5)$

CCL exercise 3-7

In how many ways can the letters in the English alphabet be arranged so that there are exactly seven letters between the letter a and b?
$P(24,7)*2*18!$ 先挑出 a,b ➔2! ,剩 24 個 letters 挑 7 個出來排列於 a,b 之間 剩 17 個字母 與 (a 7letters b) 總共 18 個 element 排列

CCL exercise 3-8

(a) In how many ways can the letters a,a,a,a,a,b,c,d,e be permuted such that no two a`s are adjacent (b) Repeated (a) if no two of b,c,d,e be adjacent
$(a)4! \quad (b)P(6,4)$ (a) a _ a _ a _ a _ a 4 position (b) _ a _ a _ a _ a _ a _ 6 position

CCL exercise 3-9　91 清大通訊,90 東吳資科

(a) in how many ways can the letters in the word MISSISSIPPI be arranged? (b) in how many ways can the they be arranged if the two P`s must be separated?
$(a)\dfrac{11!}{4!4!2!} \quad (b)\dfrac{11!}{4!4!2!}-\dfrac{10!}{4!4!}$

CCL exercise 3-11　小黃習題 3-60

(a) Suppose the repeated are not permitted .How many four-digit number can be formed from the six digits 1,2,3,5,7,8 ? (b) How many numbers in part (a) are less than 4000? (c) How many numbers in part (a) are even? (d) How many numbers in part (a) are odd? (e) How many numbers in part (a) are multiple of 5? (f) How many numbers in part (a) contain both the digit 3 and the digit 5?
$(a)P_4^6=360 \quad (b)3*5*4*3=180 \quad (c)2*5*4*3=120$ $(d)4*5*4*3=240 \quad (e)P_3^5=60 \quad (f)4*3*4*3=144$

CCL exercise 3-12

There are 15 “true or false” question in the examination.In how many different ways can a student do the examination if he or she can also choose not to answer some of the questions?
3^{15}

CCL exercise 3-15　小黃習題 3-17

(a) in how many ways can we make up the pattern with 0`s and 1`s ? (b) how many of these patterns are not symmetrical with respect to the vertical axis?	X XXXXX X X X
$(a)2^9 \quad (b)2^9-2^7$	

CCL exercise 3-20

A student must answer 8 out of 10 questions in an examination (a)how many choices does the student have? (b)how many choices does she have if she must answer the first three question? (c)how many choices does she have if she must answer at least four of the first five question?
$(a)\binom{10}{8}$ $(b)\binom{7}{5}$ $(c)\binom{5}{5}\binom{5}{3}+\binom{5}{4}\binom{5}{4}$

CCL exercise 3-24

(a) fifteen basketball players are to be drafted by the three personal teams in Boston,Chicago ,and New York such that each team will draft five players.In how many ways can this be done? (b) fifteen basketball players are to be divided into three teams of five players each. In how many ways can this be done ?
$(a)\binom{15}{5}\binom{10}{5}\binom{5}{5}$ $(b)\dfrac{\binom{15}{5}\binom{10}{5}\binom{5}{5}}{3!}$

CCL exercise 3-27

(a) In how many ways can two numbers be selected from the integer 1,2,….,100 so that their sum is even number ? (b) Use a combinatiorial argument to show that $C(2n,2)=2(n,2)+n^2$
(a) 從 50 個偶數中選出 2 個 或從 50 個奇數中選出 2 個 盒接爲偶 $2\binom{50}{2}$ (b) 從 2n 個數中選出 2 個方法數 等同於 偶數中選出 2 個 + 奇數中選出 2 個 + 偶數中選出 1 個且奇數中選出 1 個

CCL exercise 3-32

How many n-digit decimal numbers have their digits in nondecreasing order? (Note that the first digit of an n-digit number must not be 0)
$\binom{9+n-1}{n}=\binom{8}{n}$

CCL exercise 3-37

(a) Show that $\binom{2n+2}{n+1}=\binom{2n}{n+1}+2\binom{2n}{n}+\binom{2n}{n-1}$ (b) Give a combinatorial interpretation to the equality in part (a)
$\binom{2n+2}{n+1}=\binom{2n+1}{n+1}+\binom{2n+1}{n}$ (a) $=\binom{2n}{n+1}+\binom{2n}{n}+\binom{2n}{n}+\binom{2n}{n-1}$ $=\binom{2n}{n+1}+2\binom{2n}{n}+\binom{2n}{n-1}$ (b) 從 2n 隻普通狗+小白+小黑 挑出 n+1 條狗 方法數   = 從 2n 隻普通狗挑出 n+1 條狗 或 從 2n 隻普通狗挑出 n 條狗和小白   或 從 2n 隻普通狗挑出 n 條狗和小黑 或 從 2n 隻普通狗挑出 n-1 條狗和小白, 小黑

CCL exercise 3-40

Out of a large number of pennies,nickels,dimes ,and quarters, in how many ways can five coins be selected?
視爲不全相異物組合 $x_1+x_2+x_3+x_4=5, x_i \geq 0$ integer solution $\binom{4+5-1}{5}=\binom{8}{5}=56$

Grimaldi exercise 1-1-21 小黃習題 3-3 95朝陽資工,95台大資工,90大同

(a) How many arrangements are there of all the letters in SOCIOLOGICAL?   (b) In how many of the arrangements in part (a) are A and G adjacent?   (c) In how many of the arrangements in part (a) are all the vowels adjacent?
(a) $\frac{12!}{3!2!2!2!}$ (b) $2!*\frac{11!}{3!2!2!2!}$ (c) $\frac{7!}{2!2!}*\frac{6!}{2!3!}$

Grimaldi exercise 1-1-26 小黃習題 3-8

(a)How many different paths in the xy-plane are there from (0, 0) to (7, 7) if a path proceeds one step at a time by going either one space to the right (R) or one space upward (U)?   (b)How many such paths are there from (2, 7) to (9, 14)?   (c)Can any general statement be made that incorporates these two results?
(a) $\frac{14!}{7!7!}$, (b) $\frac{14!}{7!7!}$ (c) (a,b) to (a+m,b+n) ➔ $\frac{(m+n)!}{m!n!}$

Grimaldi exercise 1-3- 小黃習題 3-37

How many arrangements of the letters in MISSISSIPPI have no consecutive S's?
_M_I_I_ I_P_P_I_ MIIIPPI 排法 $\frac{7!}{4!2!}$ S 從 8 個 blank 挑 4 個 ➔ $\binom{8}{4}$ $\binom{8}{4}\frac{7!}{4!2!}$

Grimaldi exercise 1-3-23

Determine the coefficient of x^9y^3 in the expansions of   (a) $(x+y)^{12}$ (b) $(x+2y)^{12}$ (c) $(2x-3y)^{12}$
$(a)\binom{12}{9}$ $(b)2^3\binom{12}{9}$ $(c)2^9(-3)^3\binom{12}{9}$

Grimaldi exercise 1-3-25 小黃習題 3-41

Determine the coefficient of $(a)xyz^2 \ \ in \ \ (x+y+z)^4$ $(b)xyz^2 \ \ in \ \ (w+x+y+z)^4$ $(c)xyz^2 \ \ in \ \ (2x-y-z)^4$ $(d)xyz^{-2} \ \ in \ \ (x-2y+3z^{-1})^4$ $(e)w^3x^2yz^2 \ \ in \ \ (2w-x+3y-2z)^8$
$(a)\binom{4}{1,1,2}=\frac{4!}{1!1!2!}$ $(b)\binom{4}{0,1,1,2}=\frac{4!}{1!1!2!}$ $(c)2(-1)(-1)^2\binom{4}{1,1,2}=2(-1)(-1)^2\frac{4!}{1!1!2!}$ $(d)(-2)(3)^2\binom{4}{1,1,2}=(-2)(3)^2\frac{4!}{1!1!2!}$ $(e)\binom{8}{3,2,1,2}2^3(-1)^2(3)(-2)^2=\frac{8!}{3!2!1!2!}2^3(-1)^2(3)(-2)^2$

Grimaldi exercise 1-3-27　94朝陽資工　93朝陽網通

Determine the sum of all the coefficients in the expansions of $(a)(x+y)^3 \quad (b)(x+y)^{10} \quad (c)(x+y+z)^{10} \quad (d)(w+x+y+z)^5$ $(e)(2s-3t+5u+6v-11w+3x+2y)^{10}$
$(a)2^3 \quad (b)2^{10} \quad (c)3^{10} \quad (d)4^5 \quad (e)4^{10}$

Grimaldi example 1.35

How many such solutions are there to the inequality $x_1+x_2+x_3+x_4+x_5+x_6<10, x_i \ge 0, i=1\sim 6$
$x_1+x_2+x_3+x_4+x_5+x_6+x_7=9, x_i \ge 0, i=1\sim 7$ $\Rightarrow \binom{7+9-1}{9}=\binom{15}{9}$

Grimaldi exercise 1-4-14

(a) Find the coefficient of v^2w^4xz in the expansion of $(3v+2w+x+y+z)^8$ (b) How many distinct terms arise in the expansion in part (a)?
$(a)\binom{8}{2,4,1,0,1}3^2 2^4 \quad (b)\binom{5+8-1}{8}=\binom{12}{8}$ distinct terms in $(x_1+x_2+....+x_n)^r$ be $(n+r-1,r)$

Grimaldi exercise 1-5-3

(a) In how many ways can one travel in the xy-plane from (0, 0) to (3, 3) using the moves R: (x, y)→(x + 1, y) and U: (x, y)→(x, y + 1), if the path taken may touch but never fall **below the line y=x?** In how many ways from (0, 0) to (4, 4)? (b) Generalize the results in part (a). (c) What can one say about the first and last moves of the paths in parts (a) and (b)?
$(a)\frac{1}{3+1}\binom{2*3}{3}=5 \quad , \quad \frac{1}{4+1}\binom{2*4}{4}=14$ $(b)\frac{1}{n+1}\binom{2n}{n} \quad from \quad (0,0) \quad to \quad (n,n) \quad without \quad throught \quad y=x$

小黃習題 3-1　92中正理工

In how many ways can the symbols a,b,c,d,e,e,e,e,e be arranged so that no e is adjacent to another ?
a,b,c,d 插入 5 個 e 之間的 4 個空格　➔4!

小黃習題 3-5

(a) in how many ways can 10 boys and 5 girls stand in a line so that two girls are next to each other? (b) Repeat part (a) if they stand around a circle
$(a)10!P_5^{11} \quad (b)9!P_5^{10}$

小黃習題 3-6

(a) in how many ways can the letters a,b,c,d,e,f be arranged so that the letter b is always to the immediate left of the letter e ? (b) Repeat part (a) if the letter b is always to the left of the letter e ?
(a) be　綁在一起　(b)b 在 e 左邊佔一半, b 在 e 右邊佔一半 (b) $(a)5! \quad (b)\frac{6!}{2}$

小黃習題 3-10

How many ways can a particle move in the xy-plane from the orgin to the point (7,4) (a) if $R:(x,y)\to(x+1,y),U:(x,y)\to(x,y+1)$ allow ? (b) repeat (a) if $D:(x,y)\to(x+1,y+1)$ also allow ?
$(a)\frac{11!}{7!4!} \quad (b)\sum_{k=0}^{4}\frac{(11-k)!}{k!(7-k)!(4-k)!}$ 假設 移動 k 個 D 則需要移動 7-k 個 R , 4-k 個 D ,且 k 由 0~4 都有可能發生

小黃習題 3-11　,94高第一電通, 94雲科資工, 94雲科資管

How many ways can a particle move in the xy-plane from the (-1,5) to the point (-5,-2) $W:(x,y)\to(x-1,y),S:(x,y)\to(x,y-1),SW:(x,y)\to(x-1,y-1)$
假設 移動 k 個 SW 則需要移動 4-k 個 R , 7-k 個 D ,且 k 由 0~4 都有可能發生 $\sum_{k=0}^{4}\frac{(11-k)!}{k!(4-k)!(7-k)!}$

小黃習題 3-45

Find $2\binom{n}{0}+\binom{n}{1}+2\binom{n}{2}+\binom{n}{3}+...+2\binom{n}{n-2}+\binom{n}{n-1}+2\binom{n}{n}=?$
Let $A=\binom{n}{0}+\binom{n}{2}+...+\binom{n}{n-2}+\binom{n}{n}, B=\binom{n}{1}+\binom{n}{3}+...+\binom{n}{n-3}+\binom{n}{n-1}$ $\because (1+x)^n=\sum_{r=0}^{n}\binom{n}{r}x^r, x=1\Rightarrow 2^n=\sum_{r=0}^{n}\binom{n}{r}=A+B, x=-11\Rightarrow 0=\sum_{r=0}^{n}\binom{n}{r}=A-B$ $\therefore A=B=2^{n-1}$ $\Rightarrow 2\binom{n}{0}+\binom{n}{1}+2\binom{n}{2}+\binom{n}{3}+...+2\binom{n}{n-2}+\binom{n}{n-1}+2\binom{n}{n}=2A+B=2^n+2^{n-1}$

小黃習題 3-

Simplify the following expression $3\binom{n}{0}+3^2\binom{n}{1}+3^3\binom{n}{2}...+3^n\binom{n}{2}$
$\because (1+x)^n=\sum_{r=0}^{n}\binom{n}{r}x^r, when \quad x=3$ $\sum_{r=0}^{n}\binom{n}{r}3^r=3\binom{n}{0}+3^2\binom{n}{1}+3^3\binom{n}{2}...+3^n\binom{n}{2}=4^n$

小黃習題 3-55

How many ways are there (a) to distribute 6 distinct blue balls and 4 distinct red balls into five distinct box (b) to distribute 6 distinct blue balls and 4 identical red balls into five distinct box
$(a)5^6 5^4 \quad (b)5^6\binom{5+4-1}{4}=5^6\binom{8}{4}$

93清大資工

(a)Suppose the repeated are not permitted .How many four-digit number can be formed from the six digits 1,3,5,6,8,9 ? (b)How many numbers in part (a) are less than 4000? (c)How many numbers in part (a) contain both the digit 1 and the digit 9?
$(a)6*5*4*3=360 \quad (b)2*5*4*3=120 \quad (c)\binom{4}{2}4!=144$

小黃習題 3-66　96高第一資管

$n \in Z$ show that $\binom{n}{0}+\binom{n}{2}+\binom{n}{4}+....=\binom{n}{1}+\binom{n}{3}+\binom{n}{5}+....$
$\because (1+x)^n=\sum_{r=0}^{n}\binom{n}{r}x^r, when \quad x=-1, \binom{n}{0}-\binom{n}{1}+\binom{n}{2}-\binom{n}{3}+\binom{n}{4}-\binom{n}{5}+...+(-1)^n\binom{n}{n}$ $\Rightarrow \binom{n}{0}+\binom{n}{2}+\binom{n}{4}+....=\binom{n}{1}+\binom{n}{3}+\binom{n}{5}+....$

小黃習題 3-67　95宜蘭資工

Prove $\binom{n}{1}+2\binom{n}{2}+3\binom{n}{3}+....+n\binom{n}{n}=n2^{n-1}$
$\because (1+x)^n=\sum_{r=0}^{n}\binom{n}{r}x^r, different \quad both \quad side$ $n(1+x)^{n-1}=\sum_{r=1}^{n}r\binom{n}{r}x^{r-1} \quad , when \quad x=1 \Rightarrow \quad n2^{n-1}=\binom{n}{1}+2\binom{n}{2}+3\binom{n}{3}+....+n\binom{n}{n}$

小黃習題 3-84

Find the number of all onto function from a set with 5 element to a set with 3 element
$onto(m,n)=n!S_2(m,n)=\sum_{i=0}^{n}(-1)^i\binom{n}{i}(n-i)^m$ $3!S_2(5,3)=\sum_{i=0}^{3}(-1)^i\binom{3}{i}(3-i)^5$

小黃習題 3-85

How many ways are there assign five jobs to four different employees if each employee is assigned at least one job ?
$onto(5,4)=\sum_{i=0}^{n}(-1)^i\binom{n}{i}(n-i)^m=\sum_{i=0}^{4}(-1)^i\binom{4}{i}(4-i)^5=240$

96 靜宜資工 95 台大資工　考到爆

How many n bit palindrome combine by {1,2,....,k} ?
$k^{\frac{n+1}{2}}$ or $k^{\left\lfloor \frac{n}{2} \right\rfloor}$

91 東吳資料 ,93 淡江資工

(a)How many three-element subset of {1,2,...,11,12} has no consecutive integer? (b) How many seven-element subset of {1,2,...,49,50} has no consecutive integer?
(a) 由 S 中取 3 個不連續整數 a,b,c， $1\le a<b<c\le 12$ $x_1=a-1, x_2=b-a, x_3=c-b, x_4=12-c$ $x_1+x_2+x_3+x_4=11, x_1,x_4\ge 0, x_2x_3\ge 2$ $x_1+x_2+x_3+x_4=7, x_i\ge 0, 1\le i\le 4$ $\binom{10}{7}$ (b) 由 S 中取 7 個不連續整數 $y_1\sim y_7$ ， $1\le y_1<y_2<....<y_7\le 50$ $x_1=y_1-1, x_2=y_2-y_1, ..., x_8=50-y_7$ $x_1+...+x_8=49, x_1,x_8\ge 0, x_2\sim x_7\ge 2$ $x_1+...+x_8=37, x_i\ge 0, 1\le i\le 8$ $\binom{44}{7}$

93 政大資管

How many ways to put 8 different balls into 5 different boxes such that no box allowed to be empty ?
$onto(m,n)=\sum_{i=0}^{n}(-1)^i\binom{n}{i}(n-i)^m$ $onto(8,5)=\sum_{i=0}^{5}(-1)^i\binom{5}{i}(5-i)^8=5^8-\binom{5}{1}4^8+\binom{5}{2}3^8-\binom{5}{3}2^8+\binom{5}{4}1^8$

93 政大資管

How many ways to put 8 identical balls into 5 different boxes such that no box allowed to be empty ?
$x_1+x_2+x_3+x_4+x_5=8, x_i\ge 1 \cong x_1+x_2+x_3+x_4+x_5=3, x_i\ge 0$ $\binom{5+3-1}{3}=\binom{7}{3}$

96 輔大資工

$A=\{1,2,...,10\}, B=\{1,2,...,7\}$

(a) how many function $f:A\to B$ such that $|f(A)|=7$?

(b) how many function $f:A\to B$ such that $|f(A)|=4$?

(c) how many function $|f(A)|\le 4$?

$$(a)onto(10,7)=\sum_{i=0}^{7}(-1)^i\binom{7}{i}(7-i)^{10} \quad (b)\binom{7}{4}onto(10,4)$$

$$(c)\binom{7}{1}onto(10,1)+\binom{7}{2}onto(10,2)+\binom{7}{3}onto(10,3)+\binom{7}{4}onto(10,4)$$

96 暨大資工

$A=\{1,2,3,4\}, B=\{2,5\}, C=\{3,4,7\}$

(a) the relation of relation from A to B

(b) how many function $f:A\to B$ is one-to-one

(c) how many function $f:A\to B$ is onto

(d) how many function $f:A\to C$ such that $|f(A)|=2$

$$(a)2^{4*2}=2^8 \quad (b)0 \quad (c)onto(4,2)=\sum_{i=0}^{2}(-1)^i\binom{2}{i}(2-i)^4 \quad (d)\binom{3}{2}onto(4,2)$$

95 海大電機

(a) how many ways to put 7 different balls into 4 different box such that no box allow empty?

(b) how many ways to put 7 identical balls into 4 different box such that no box allow empty?

$$(a)onto(7,4)=\sum_{i=0}^{4}(-1)^i\binom{4}{i}(4-i)^7$$

$$(b)x_1+x_2+x_3+x_4=7, x_i\ge 1 \cong x_1+x_2+x_3+x_4=3, x_i\ge 0$$

$$\binom{4+3-1}{3}=\binom{6}{3}$$

97 政大資料

How many terms in the expansion of $(w+x+y+z)^{12}$
$(w+x+y+z)^{12}=\sum\binom{12}{n_1\quad n_2\quad n_3\quad n_3}w^{n_1}x^{n_2}y^{n_3}z^{n_4}$ 展開後的項數相當於解 $n_1+n_2+n_3+n_4=12\Rightarrow\binom{15}{12}$

97 暨大資工

A={1,2,3,4,5} B={u,v,w,x,y,z} How many function A➔B satisfy |f(A)|=2
$\binom{6}{2}onto(5,2)$

97 北教大資工

20 張卡片,放入 12 個信封(編號 1~20) (1)20 張卡片,都不一樣,允許信封爲空,方法數 (2)20 張卡片,都不一樣,信封全不爲空,方法數 (3) 20 張卡片,都一樣,允許信封爲空,方法數 (4) 20 張卡片,都一樣,信封全不爲空,方法數
$(1)12^{20}\quad(2)onto(20,12)\quad(3)\binom{31}{20}\quad(4)\binom{19}{8}$

97 高第一電通

5 digit 10 進位遞增(減)數 有幾個 (13578,23689) ? 5 digit 10 進位非遞增(減)數 有幾個 (13578,23689) ?
$(1)\binom{10}{5}$ (2)$0\le x_1\le\ldots\le x_5\le 9$,令 $y_1=x_1-0,y_2=x_2-x_1,\ldots,y_6=9-x_5$ $y_1+\ldots+y_6=9,x_i\ge 0\Rightarrow\binom{14}{9}$

97 淡江資管

$f: R \to R^2, f(x) = (x^2, x^3), g: R^2 \to R, g(x,y) = 2x + y$ $f: one-to-one?\quad f: onto?$ $g: one-to-one?\quad g: onto?$
(1) $f(x) = f(y) \Rightarrow (x^2, x^3) = (y^2, y^3) \Rightarrow \begin{cases} x^2 = y^2 \\ x^3 = y^3 \end{cases} \Rightarrow x = y$ 所以 one-to-one 不存在 $x \in R \ni f(x) = (1,2)$ 所以 不 onto (2) $g(1,1) = 3 = g(0,3)$ 所以 不 one-to-one $\forall y \in R,$ 取 $\forall y \in R, (0,y) \in R^2 \ni f(0,y) = y$ 所以 g 爲 onto

95 台大工科

How many arrangement of the letters in ESOESETALLAHA have no adjacent E`s
$\binom{11}{3}\dfrac{10!}{3!2!1!1!2!1!}$

95 高雄電機

(0,0) ➔ (6,4) only (x,y)➔(x+1,y) or (x,y)➔(x,y+1) U may never exceed the number of R
$\dfrac{10!}{6!4!} - \dfrac{10!}{7!3!}$

96 輔大資工

A={1,2,3….,10} , B={1,2,3,….,7} (a) how many function $f: A \to B$ satisfy $\lvert f(A) \rvert = 7$ (b) how many function $f: A \to B$ satisfy $\lvert f(A) \rvert = 4$ (c) how many function $f: A \to B$ satisfy $\lvert f(A) \rvert \le 4$
$(a) onto(10,4)\quad (b)\binom{7}{4} onto(10,4)\quad (c)\sum_{i=1}^{4}\binom{7}{i} onto(10,i)$

91 台大資工

Strling`s formula for n!
$n! = \left(\dfrac{n}{e}\right)^n \sqrt{2\pi n}$

91 東吳資科

|A|=m |B|=n (a) what is the number of relation from A to B? (b) what is the number of function from A to B? (c) what is the number of 1-1 from A to B? (d) what is the number of onto from A to B?
$(a)2^{mn} \quad (b)n^{m} \quad (c)P_{m}^{n} \quad (d)\sum_{i=0}^{n}(-1)^{i}\binom{n}{i}(n-i)^{m}$

90 師大資教

$A=\{1,2,3,4\}, B=\{a,b,c,d,e,f\}$ (a) # function A➔B? (b)# 1-1 A➔B? (c)#onto A➔B? (d) #onto B➔A?
$(a)6^{4} \quad (b)P_{4}^{6}=360 \quad (c)0$ $(d)\sum_{i=0}^{n}(-1)^{i}\binom{n}{i}(n-i)^{m}=\sum_{i=0}^{n}(-1)^{i}\binom{4}{i}(4-i)^{6}=1560$

93 東華資工

5 boy and five girl attend a concert together (a)how many ways can they sit together no boy sit together? (b)how many ways can they sit together no boy and no girl sit together? (c)how many ways can they sit if all boy sit together? (d) how many ways can they sit if a boy sit at each end? (e) how many ways can they sit if a boy and a girl refuse to sit next to each other?
$(a)5!6! \quad (b)2(5!\times 5!) \quad (c)6!5! \quad (d)40\times 8! \quad (e)10!-9!2$

(4)生成函數

4.1 find a generating function for the sequence $3+4^n$
$A(x)=\sum_{n=0}^{\infty}(3+4^n)x^n=3\sum_{n=0}^{\infty}x^n+\sum_{n=0}^{\infty}(4x)^n=\frac{3}{1-x}+\frac{1}{1-4x}=\frac{4-13x}{(1-x)(1-4x)}$

4.2

(1) $1\binom{n}{1}+2\binom{n}{2}+\ldots+n\binom{n}{n}=?$ 92 ntu,nctu,cnu

(2) $1^2\binom{n}{1}+2^2\binom{n}{2}+\ldots+n^2\binom{n}{n}=?$

(3) $2^0\binom{n}{0}+2^1\binom{n}{1}+\ldots+2^n\binom{n}{n}=?$

(4) $k^0\binom{n}{0}+k^1\binom{n}{1}+\ldots+k^n\binom{n}{n}=?$

(5) $3\binom{n}{1}+3^2\binom{n}{2}+\ldots+3^n\binom{n}{n}=?$

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(1) $(1+x)^n=\sum_{i=0}^{n}\binom{n}{i}x^i$ *different both side* $\Rightarrow n(1+x)^{n-1}=\sum_{i=0}^{n}i\binom{n}{i}x^{i-1}\Rightarrow n(2)^{n-1}$

(2) $(1+x)^n=\sum_{i=0}^{n}\binom{n}{i}x^i$ *different both side* $\Rightarrow n(1+x)^{n-1}=\sum_{i=0}^{n}i\binom{n}{i}x^{i-1}$

multiple x both side $\Rightarrow nx(1+x)^{n-1}=\sum_{i=0}^{n}i\binom{n}{i}x^i$ *different both side* $\Rightarrow$

$(n(1+x)^{n-1}+(n-1)nx(1+x)^{n-2})=\sum_{i=0}^{n}i^2\binom{n}{i}x^{i-1}\Rightarrow n(2)^{n-1}+(n-1)n(2)^{n-2}$

$(3)(1+x)^n=3^n$

$(4)(1+x)^n=(k+1)^n$

(5) $trap!!!!!\Rightarrow(3+1)^n-3^0\binom{n}{0}=4^n-1$

$(a)\binom{-n}{r}=(-1)^r\binom{n+r-1}{r}$ $\quad\quad (b)(1-x)^{-n}=\sum_{r=0}^{n}\binom{n+r-1}{r}x^r$

小黃習題 4-15 (a)94 中山資工 (b)95 靜宜資管 (c)88 逢甲資工

4.3 find coefficient of (a) x^5 in $(1-2x)^{-7}$ (b) x^2 in $(1-3x)^{-4}$ (c) x^2 in $(1-2x)^{-6}$
(a) $2^5\binom{7+5-1}{5}=2^5\binom{11}{5}$ (b) $\binom{4+2-1}{2}(3)^2=90$ (c) $\binom{6+2-1}{2}(2)^2=84$

94 元智資工 93 清大資應,師大資工,朝陽資工

4.4 find coefficient of x^{15} in $(x^2+x^3+x^4+...)^4$ **93 nthu**
$(x^2+x^3+x^4+...)^4=(x^2)^4(1+x+x^2+...)^4$ $x^8(1+x+x^2+...)^4=x^8(1-x)^{-4}=x^8\sum_{r=0}^{\infty}\binom{4+r-1}{r}x^r$ $coefficient\ \ of\ \ x^{15}\ \Rightarrow\binom{4+7-1}{7}=\binom{10}{7}=120$

Grimaldi exercise for (a) 95 彰師資工 96 朝陽資工 94 中山資工

find coefficient of (a) x^6 in $\frac{1}{(x-3)(x-2)^2}$ (b) x^5 in $\frac{1}{(x-2)(x-3)}$
$let\ \ \frac{1}{(x-3)(x-2)^2}=\frac{A}{x-3}+\frac{B}{x-2}+\frac{C}{(x-2)^2}\Rightarrow\frac{1}{(x-3)(x-2)^2}=\frac{1}{x-3}-\frac{1}{x-2}-\frac{1}{(x-2)^2}$ $=-\frac{1}{3}\left(\frac{1}{1-\frac{x}{3}}\right)+\frac{1}{2}\left(\frac{1}{1-\frac{x}{2}}\right)-\frac{1}{4}\left(\frac{1}{(1-\frac{x}{2})^2}\right)=-\frac{1}{3}\sum_{r=0}^{\infty}\left(\frac{x}{3}\right)^r+\frac{1}{2}\sum_{r=0}^{\infty}\left(\frac{x}{2}\right)^r-\frac{1}{4}\sum_{r=0}^{\infty}\binom{2+r-1}{r}\left(\frac{x}{2}\right)$ x^6 的係數為 $-\frac{1}{3}\left(\frac{1}{3}\right)^6+\frac{1}{2}\left(\frac{1}{2}\right)^6-\frac{1}{4}\binom{2+6-1}{6}\left(\frac{1}{2}\right)^6=-\left(\frac{1}{3}\right)^7+\left(\frac{1}{2}\right)^7-\frac{7}{4}\left(\frac{1}{2}\right)^6$ (b) $\frac{1}{(x-2)(x-3)}=-\frac{1}{x-2}+\frac{1}{x-3}=\frac{1}{2}\left(\frac{1}{1-\frac{x}{2}}\right)-\frac{1}{3}\left(\frac{1}{1-\frac{x}{3}}\right)=\frac{1}{2}\left(\sum_{r=0}^{\infty}(\frac{x}{2})^r\right)-\frac{1}{3}\left(\sum_{r=0}^{\infty}(\frac{x}{3})^r\right)$ x^5 的係數為 $\left(\frac{1}{2}\right)^6-\left(\frac{1}{3}\right)^6$

4.7 find coefficient of x^{16} in $(1+x^4+x^8)^{10}$
$cofficient\ \ of\ \ x^{16}\ \ in\ \ (\frac{1-x^{12}}{1-x^4})^{10}=(1-x^{12})^{10}(1-x^4)^{-10}$ $=\sum_{i=0}^{10}\binom{10}{i}(x^{12})^i\sum_{r=0}^{\infty}\binom{10+r-1}{r}(x^4)^r$ $\binom{10}{0}\binom{10+4-1}{4}-\binom{10}{1}\binom{10+1-1}{1}=715-100=615$

小黃習題 4-8　　93 師大資教,87 台科資工

4.8 find coefficient of x^{83} in $(x^5+x^8+x^{11}+x^{14}+x^{17})^{10}$
$(x^5+x^8+x^{11}+x^{14}+x^{17})^{10}=(x^5)^{10}(1+x^3+x^6+x^9+x^{12})^{10}=(x^5)^{10}(\frac{1-x^{15}}{1-x^3})^{10}$ $x^{50}(\frac{1-x^{15}}{1-x^3})^{10}=x^{50}\sum_{i=0}^{10}\binom{10}{i}(-x^{15})^i\sum_{r=0}^{\infty}\binom{10+r-1}{r}(x^3)^r$ $\Rightarrow\binom{10}{0}\binom{10+11-1}{11}-\binom{10}{1}\binom{10+6-1}{6}+\binom{10}{2}\binom{10+1-1}{1}$ $=\binom{20}{11}-10\binom{15}{6}+450$

4.9 find positive integer solution of $x_1+x_2+x_3+x_4+x_5=45$ $x_i\ \ must\ \ divisible\ \ by\ \ 3\ \ ,i=1,2,3,4,5$
$x^{15}(1+x^3+x^6+.....)^5=x^{15}\left(1-x^3\right)^{-5}=x^{15}\sum_{r=0}^{\infty}\binom{5+r-1}{r}(x^3)^r\Rightarrow\binom{5+10-1}{10}=\binom{14}{10}$

CCL exercsie 9-40

(a) show that $\binom{r}{0}^2+\binom{r}{1}^2+\binom{r}{2}^2+....+\binom{r}{i}^2+....+\binom{r}{r}^2=\binom{2r}{r}$ (b) show the generating function for a where $a_r=\binom{2r}{r}$ is $(1-4z)^{-\frac{1}{2}}$
(a) $\binom{r}{0}^2+\binom{r}{1}^2+\binom{r}{2}^2+....+\binom{r}{i}^2+....+\binom{r}{r}^2$ is the constant term of the product $(1+z)^r(1+\frac{1}{z})^r=(1+z)^r(z+1)^r z^{-r}=(1+z)^{2r}z^{-r}$ Therefore it is the coefficient of z^r in $(1+z)^{2r}$ which is $\binom{2r}{r}$

4.11 4-ary n sequence(every digit may 0,1,2,3)

(a) how many sequence conatins even '0'?

(b) how many sequence conatins even '0' and even '1'?

$$(a)\quad 0 \Rightarrow \frac{e^x+e^{-x}}{2} \quad , \quad 1,2,3 \Rightarrow e^x$$

$$\therefore (\frac{e^x+e^{-x}}{2})e^x e^x e^x = \frac{1}{2}(e^{4x}+e^{2x}) = \frac{1}{2}(\sum_{i=0}^{\infty}\frac{(4x)^r}{r!}+\sum_{i=0}^{\infty}\frac{(2x)^r}{r!}) = \frac{1}{2}(4^n+2^n)$$

$$(b)\quad 0,1 \Rightarrow \frac{e^x+e^{-x}}{2} \quad , \quad 2,3 \Rightarrow e^x$$

$$\therefore (\frac{e^x+e^{-x}}{2})^2 e^x e^x = \frac{1}{2}(e^{4x}+2e^{2x}+1)$$

$$= \frac{1}{2}(\sum_{i=0}^{\infty}\frac{(4x)^r}{r!}+2\sum_{i=0}^{\infty}\frac{(2x)^r}{r!}+1) = \frac{1}{2}(4^n+2^{n+1}+1)$$

$$\Rightarrow cofficient \quad of \quad \frac{x^n}{n!} = \frac{1}{2}(4^n+2^{n+1})$$

4.12 20-digit ternary (0,1,2)

(a) how many sequence conatins even '1'?

(b) how many sequence conatins even '1' and even '2'?

$$(a)\quad 1 \Rightarrow \frac{e^x+e^{-x}}{2} \quad , \quad 0,2 \Rightarrow e^x$$

$$\therefore (\frac{e^x+e^{-x}}{2})e^x e^x = \frac{1}{2}(e^{3x}+1) = \frac{1}{2}(\sum_{i=0}^{\infty}\frac{(3x)^r}{r!}+1) = \frac{1}{2}(3^n+1), n=20 \Rightarrow \frac{1}{2}(3^{20}+1)$$

$$(b)\quad 1,2 \Rightarrow \frac{e^x+e^{-x}}{2} \quad , \quad 0 \Rightarrow e^x$$

$$\therefore (\frac{e^x+e^{-x}}{2})^2 e^x = \frac{1}{4}(e^{3x}+2e^x+e^{-x})$$

$$= \frac{1}{4}(\sum_{i=0}^{\infty}\frac{(3x)^r}{r!}+2\sum_{i=0}^{\infty}\frac{(x)^r}{r!}+\sum_{i=0}^{\infty}(-1)^r\frac{(x)^r}{r!}) = \frac{1}{4}(3^n+2+(-1)^n), n=20 \Rightarrow \frac{1}{4}(3^{20}+3)$$

CCL exercsie 9-36

Determine the sum $\binom{n}{1}+2\binom{n}{2}+....+i\binom{n}{i}+....+n\binom{n}{n}$
$\binom{n}{0}+\binom{n}{1}z+\binom{n}{2}z^2+....+\binom{n}{i}z^i+....+\binom{n}{n}z^n=(1+z)^n$ Different both side $\binom{n}{1}1+\binom{n}{2}2z+....+\binom{n}{i}iz^{i-1}+....+\binom{n}{n}nz^{n-1}=n(1+z)^{n-1}$ Put z=1 ➔ $\binom{n}{1}+2\binom{n}{2}+....+i\binom{n}{i}+....+n\binom{n}{n}=n2^{n-1}$

CCL exercsie 9-

Determine the sum $\binom{n}{0}+2\binom{n}{1}+2^2\binom{n}{2}+....+2^n\binom{n}{n}$
$\sum_{i=0}^{n}2^i\binom{n}{i}z^i=(1+2z)^n \quad set \quad z=1\Rightarrow\sum_{i=0}^{n}2^i\binom{n}{i}=3^n$

CCL exercsie 9-38

Determine the sum $\binom{n}{0}\binom{m}{k}+\binom{n}{1}\binom{m}{k-1}+....+\binom{n}{k}\binom{m}{0}$
$(1+z)^n(1+z)^m=(1+z)^{n+m}\Rightarrow sum=\binom{n+m}{k}$

CCL exercsie 9-39

Determine the sum $\binom{2n}{2}+\binom{2n-1}{2-1}+....+\binom{2n-i}{n-i}+....+\binom{n}{0}$
$\binom{2n+1}{n}$

Grimaldi example 9.14

In how many ways can a police captain distribute 24 rifle shells to four police officers so that each officer gets at least three shells ,but not more than eight ?
coefficient of x^{24} in $f(x)=(x^3+x^4+....+x^8)^4$ same as coefficient of x^{12} in $f(x)=(1+x+x^2+x^3+x^4+x^5)^4$ coefficient of x^{12} in $\frac{(1-x^6)^4}{(1-x)^4}=(1-x^6)^4(1-x)^{-4}$ $(1-4x^6+6x^{12}-4x^{18}+x^{24})(1-x)^{-4}$ $\binom{12+4-1}{4}-4\binom{6+4-1}{4}+6\binom{4}{4}=\binom{15}{4}-4\binom{9}{4}+6=125$

Grimaldi exercise 9-2-1

Find generating functions for the following sequence (for example: 0,1,3,9,27,... ➔ $\frac{x}{1\text{-}3x}$) $(a)\binom{8}{0},\binom{8}{1},\binom{8}{2},.....,\binom{8}{8}$ $(b)\binom{8}{1},2\binom{8}{2},3\binom{8}{3},.....,8\binom{8}{8}$ $(c)1,-1,1,-1,.....$ $(d)0,0,0,6,-6,6,-6,....$ $(e)1,0,1,0,......$ $(f)0,0,1,a,a^2,a^3,......a\neq 0$
$(a)(1+x)^8$ $(b)8(1+x)^7$ $(c)(1+x)^{-1}$ $(d)\frac{6x^3}{1+x}$ $(e)(1-x^2)^{-1}$ $(f)\frac{x^2}{1-ax}$

Grimaldi exercise 9-2-2 小黃習題 4-4

Determine the sequence generated by each of the following generating function $(a)f(x)=(2x-3)^3$ $(b)\frac{x^4}{1-x}$ $(c)\frac{x^3}{(1-x^2)}$ $(d)\frac{1}{1+3x}$ $(e)\frac{1}{3-x}$ $(f)\frac{1}{1-x}+3x^7-11$
$(a)\binom{8}{0},\binom{8}{1},\binom{8}{2},.....,\binom{8}{8}$ $(b)-27,54,-36,8,0,0,.....$ $(c)0,0,0,0,1,1,1,1,.....$ $(d)x^3(1+x^2+x^4+x^6+....)=x^3+x^5+x^7+x^9....$ $(e)1,-3,3^2,-3^3,....$ $(f)\frac{1}{3},\left(\frac{1}{3}\right)^2,\left(\frac{1}{3}\right)^3,....$

Grimaldi exercise 9-2-5　　木易習題

(a) find the coefficient of x^7 in $(1+x+x^2+....)^{15}$ (b) find the coefficient of x^7 in $(1+x+x^2+....)^n, n\in Z^+$
$(a)\binom{-15}{7}(-1)^7=(-1)^7\binom{15+7-1}{7}(-1)^7=\binom{21}{7}$ $(b)\binom{-n}{7}(-1)^7=(-1)^7\binom{n+7-1}{7}(-1)^7=\binom{n+6}{7}$

Grimaldi exercise 9-2-6　小黃習題 4-5　木易習題　**97 成大資工**

find the coefficient of x^{50} in $(x^7+x^8+x^9+....)^6$
$\because (x^7+x^8+x^9+....)^6=x^{42}(1+x+x^2+....)^6=x^{42}\left(1-x\right)^{-6}$ $=x^{42}\sum_{r=0}^{\infty}\binom{6+r-1}{r}(x)^r$ $cofficient\ \ of\ \ x^{45}\ \ in\ \ (x^7+x^8+x^9+....)^6=\binom{6+8-1}{8}=\binom{13}{8}$

Grimaldi exercise 9-2-7　小黃習題 4-6　木易習題

find the coefficient of x^{20} in $(x^2+x^3+x^4+x^5+x^6)^5$
$x^{10}\left(\frac{1-x^5}{1-x}\right)^5=x^{10}(1-x^5)^5(1-x)^{-5}=x^{10}\sum_{i=0}^{5}\binom{5}{i}(-x^5)^i\sum_{r=0}^{\infty}\binom{5+r-1}{r}(x)^r$ $cofficient\ \ of\ \ x^{20}\ =\binom{5+10-1}{10}-5\binom{5+5-1}{5}+10=\binom{14}{10}-5\binom{9}{5}+10$

Grimaldi exercise 9-2- 小黃習題 4-9 95 師大資工 木易習題

find the coefficient of x^{15} in $(a)x^3(1-2x)^{10}$ $(b)\frac{x^3-5x}{(1-x)^3}$ $(c)\frac{(1+x)^4}{(1-x)^4}$
$(a)0$ $(b)\binom{3+12-1}{12}-5\binom{3+14-1}{14}=\binom{14}{12}-5\binom{16}{14}$ $(c)\frac{(1+x)^4}{(1-x)^4}=(1+4x+6x^2+4x^3+x^4)(1-x)^{-4}$ $\binom{4+15-1}{15}+4\binom{4+14-1}{14}+6\binom{4+13-1}{13}+4\binom{4+12-1}{12}+\binom{4+11-1}{11}$ $=\binom{18}{15}+4\binom{17}{14}+6\binom{16}{13}+4\binom{15}{12}+\binom{14}{11}$

Grimaldi example 9.26

In how many ways can four of the letters in ENGINE be arrange ?
G and I appear one time ➔generating function= $(1+\frac{x}{1!})$ E and N appear two time ➔generating function= $(1+\frac{x}{1!}+\frac{x^2}{2!})$ Total = $(1+\frac{x}{1!})^2(1+\frac{x}{1!}+\frac{x^2}{2!})^2$ 取 4 個字母排列方法數爲 $\frac{x^4}{4!}$ 之係數 $(1+x)^2(1+x+\frac{1}{2}x^2)^2=1+4x+7x^2+\frac{17}{4}x^4+\frac{3}{2}x^5+\frac{1}{4}x^6$ $\frac{x^4}{4!}$ 之係數= $4!*\frac{17}{4}x^4=102$

Grimaldi example 9-4-6

find the exponent generating function for (1) HAWAII (2)MISSISSIPPI (3)ISOMORPHISM
$(1)(1+x)^2(1+x+\frac{x^2}{2!})^2$ $(2)(1+x)(1+x+\frac{x^2}{2!})(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!})^2$ $(3)(1+x)^3(1+x+\frac{x^2}{2!})^4$

Grimaldi example 9.28

A ship carries 48 flags,12 each of the color red, white, blue and black. Twelve of these flags are placed on a vertical pole in order to communicate a signal to other ships how many of these signal use an even number of blue flags and an odd number of black flags
$f(x)=(1+x+\frac{x^2}{2!}+...)(1+\frac{x^2}{2!}+\frac{x^4}{4!}+...)(x+\frac{x^3}{3!}+\frac{x^5}{5!}+...)$ $f(x)=\left(e^x\right)^2\left(\frac{e^x+e^{-x}}{2}\right)\left(\frac{e^x-e^{-x}}{2}\right)=\frac{1}{4}e^{2x}(e^{2x}-e^{-2x})=\frac{1}{4}(e^{4x}-1)$ $=\frac{1}{4}\left(\sum_{i=0}^{\infty}\frac{(4x)^i}{i!}-1\right)=\frac{1}{4}\sum_{i=0}^{\infty}\frac{(4x)^i}{i!}$ $the\ \ cofficient\ \ of\ \ \frac{x^{12}}{12!}\ \ be\ \ \frac{1}{4}(4)^{12}=4^{11}$

一個由 n 個數字組成的數列,其中每個數字可為 0,1,2,3 即四元 n 序列中 (a) 含偶數個 0 (b)含偶數個 0 與偶數個 1
$(a)\left(\frac{e^x+e^{-x}}{2}\right)\left(e^x\right)^3=\frac{1}{2}(e^{4x}+e^{2x})=\frac{1}{2}(\sum_{n=0}^{\infty}\frac{(4x)^n}{n!}+\sum_{n=0}^{\infty}\frac{(2x)^n}{n!})$ $=\frac{1}{2}(4^n+2^n)\sum_{n=0}^{\infty}\frac{x^n}{n!}\Rightarrow cofficient\ \ of\ \ \frac{x^n}{n!}\ \ be\ \ \frac{1}{2}(4^n+2^n)$ $(b)\left(\frac{e^x+e^{-x}}{2}\right)^2\left(e^x\right)^2=\frac{1}{4}(e^{4x}+2e^{2x}+1)=\frac{1}{4}(\sum_{n=0}^{\infty}\frac{(4x)^n}{n!}+2\sum_{n=0}^{\infty}\frac{(2x)^n}{n!})$ $=\frac{1}{4}(4^n+2*2^n)\sum_{n=0}^{\infty}\frac{x^n}{n!}\Rightarrow cofficient\ \ of\ \ \frac{x^n}{n!}\ \ be\ \ \frac{1}{4}(4^n+2^{n+1})$

20-digit ternary (0,1,2) sequence (a)an even number of 1 (b) an even number of 1 and even number of 2
$(a)\left(\frac{e^x+e^{-x}}{2}\right)\left(e^x\right)^2=\frac{1}{2}(e^{3x}+e^x)=\frac{1}{2}(\sum_{i=0}^{\infty}\frac{(3x)^i}{i!}+\sum_{i=0}^{\infty}\frac{(x)^i}{i!})$ $=\frac{1}{2}\sum_{i=0}^{\infty}(3^i+1)\frac{(x)^i}{i!}\Rightarrow cofficient\ \ of\ \ \frac{x^{20}}{20!}\ \ be\ \ \frac{1}{2}(3^{20}+1)$ $(b)\left(\frac{e^x+e^{-x}}{2}\right)^2\left(e^x\right)=\frac{1}{4}(e^{3x}+2e^x+e^{-x})=\frac{1}{4}(\sum_{i=0}^{\infty}\frac{(3x)^i}{i!}+2\sum_{i=0}^{\infty}\frac{(x)^i}{i!}+(-1)^i\sum_{i=0}^{\infty}\frac{(x)^i}{i!})$ $=\frac{1}{4}\sum_{i=0}^{\infty}(3^i+2+(-1)^i)\frac{(x)^i}{i!}\Rightarrow cofficient\ \ of\ \ \frac{x^{20}}{20!}\ \ be\ \ \frac{1}{4}(3^{20}+3)$

n-digit binary sequence (a) even number of 0 and even number of 1 (b) even number of 0 and odd number of 1 (b) odd number of 0
$(a)\left(\frac{e^x+e^{-x}}{2}\right)^2=\frac{1}{4}(e^{2x}+2+e^{-2x})=\frac{1}{4}(\sum_{i=0}^{\infty}\frac{(2x)^i}{i!}+(-2)^i\sum_{i=0}^{\infty}\frac{(x)^i}{i!})$ $=\frac{1}{4}\sum_{i=0}^{\infty}(2^i+(-2)^i)\frac{(x)^i}{i!}\Rightarrow cofficient\ \ of\ \ \frac{x^n}{n!}\ \ be\ \ \frac{1}{4}(2^n+(-2)^n)$ $(b)\left(\frac{e^x+e^{-x}}{2}\right)\left(\frac{e^x-e^{-x}}{2}\right)=\frac{1}{4}(e^{2x}-e^{-2x})=\frac{1}{4}(\sum_{i=0}^{\infty}\frac{(2x)^i}{i!}-(-2)^i\sum_{i=0}^{\infty}\frac{(x)^i}{i!})$ $=\frac{1}{4}\sum_{i=0}^{\infty}(2^i-(-2)^i)\frac{(x)^i}{i!}\Rightarrow cofficient\ \ of\ \ \frac{x^n}{n!}\ \ be\ \ \frac{1}{4}(2^n-(-2)^n)$ $(c)\left(\frac{e^x-e^{-x}}{2}\right)e^x=\frac{1}{2}(e^{2x}-1)=\frac{1}{2}(\sum_{i=0}^{\infty}\frac{(2x)^i}{i!})$ $\Rightarrow cofficient\ \ of\ \ \frac{x^n}{n!}\ \ be\ \ \frac{1}{2}(2^n)=2^{n-1}$

How many r-digit quaternary sequences (0,1,2,3) have at least one 1, one 2 ,one 3
$e^x\left(e^x-1\right)^3=e^{4x}-3e^{3x}+3e^{2x}-e^x$ $=\sum_{i=0}^{\infty}\frac{(4x)^i}{i!}-3\sum_{i=0}^{\infty}\frac{(3x)^i}{i!}+3\sum_{i=0}^{\infty}\frac{(2x)^i}{i!}-\sum_{i=0}^{\infty}\frac{(x)^i}{i!}$ $=\sum_{i=0}^{\infty}(4^i-3^{i+1}+3*2^i-1)\frac{(x)^i}{i!}$ $\Rightarrow cofficient\ \ of\ \ \frac{x^n}{n!}\ \ be\ \ 4^n-3^{n+1}+3*2^n-1$

小黃習題 4-14

Find the number of positive integer x that exist where $x \le 9999999$ and sum of the digits in x equals 31
對應 gernerating function $A(x)=(1+x+...+x^9)^7$,欲求 x^{31} 的係數 $(1+x+...+x^9)^7=\left(1-x^{10}\right)^7\left(1-x\right)^{-7}=\left(\sum_{i=0}^{7}\binom{7}{i}(-x^{10})^i\right)\left(\sum_{r=0}^{\infty}\binom{7+r-1}{r}x^r\right)$ $=(1-7x^{10}+21x^{20}-35x^{30}+....)\sum_{r=0}^{\infty}\binom{6+r}{r}x^r$ x^{31} 的係數爲 $\binom{31+6}{31}-7\binom{21+6}{21}+21\binom{11+6}{11}-35\binom{1+6}{1}$

小黃習題 4-21

How many integer solution to $x_1+x_2+x_3+x_4+x_5=21, 0\le x_i\le 7, i=1\sim 5$
對應 gernerating function $A(x)=(1+x+...+x^7)^5$,欲求 x^{21} 的係數 $(1+x+...+x^9)^7=\left(1-x^8\right)^5\left(1-x\right)^{-5}=\left(\sum_{i=0}^{5}\binom{5}{i}(-x^8)^i\right)\left(\sum_{r=0}^{\infty}\binom{5+r-1}{r}x^r\right)$ $=(1-5x^8+10x^{16}+....)\sum_{r=0}^{\infty}\binom{r+4}{r}x^r$ x^{21} 的係數爲 $\binom{21+4}{21}-5\binom{13+4}{13}+10\binom{5+4}{5}=\binom{25}{21}-5\binom{17}{13}+10\binom{9}{5}$

小黃習題 4-26

Find the number of numbers which are between 1000 and 9999 ,and sum of digit is 12
$x_1+x_2+x_3+x_4=12, 1\le x_1\le 9, 0\le x_2,x_3,x_4\le 9$ 對應 gernerating function $A(x)=(x+...+x^9)(1+x+...+x^9)^3$ 欲求 x^{12} 的係數 $A(x)=(x+...+x^9)(1+x+...+x^9)^3$ $=x\left(\frac{1-x^9}{1-x}\right)\left(\frac{1-x^{10}}{1-x}\right)^3=x(1-x^9)(1-x^{10})^3(1-x)^{-4}$ $=(x-x^{10})(1-3x^{10}+3x^{20}-x^{30})\sum_{r=0}^{\infty}\binom{4+r-1}{r}x^r$ $=(x-x^{10}-3x^{11}+3x^{20}+.....)\sum_{r=0}^{\infty}\binom{4+r-1}{r}x^r$ x^{12} 的係數爲 $\binom{3+12-1}{12}-\binom{3+2-1}{2}-3\binom{3+1-1}{1}=\binom{14}{12}-\binom{4}{2}-3\binom{3}{1}$

93 政大資科

find the coefficient of x^{32} in $(1+x^5+x^9)^{10}$
$cofficient\ \ of\ \ x^{32}\ \ in\ \ (1+x^5+x^9)^{10}=\sum_{r=0}^{\infty}\binom{10}{a\ \ b\ \ c}(1)^a(x^5)^b(x^9)^c$ $only\ \ when\ \ \sum_{r=0}^{\infty}\binom{10}{6\ \ 1\ \ 3}(1)^6(x^5)^1(x^9)^3\Rightarrow\frac{10!}{6!3!}=\frac{10\times9\times8\times7}{1\times2\times3}=840$

91 師大資教

(a) Find the coefficient of x^2yz^2 in expansion $\left(\frac{x}{2}+y-3z\right)^5$ (b) How many distinct terms in expansion $\left(\frac{x}{2}+y-3z\right)^5$ (c) Sum of the coefficient in expansion $\left(\frac{x}{2}+y-3z\right)^5$
$(a)\left(\frac{1}{2}\right)^2 1^1(-3)^2\binom{5}{2\ \ 1\ \ 2}=\left(\frac{1}{2}\right)^2 1^1(-3)^2\frac{5!}{2!1!2!}=\frac{135}{2}$ $(b)\binom{3+5-1}{5}=\binom{7}{5}=21$ $(c)\left(\frac{1}{2}+1-3\right)^5=-\left(\frac{3}{2}\right)^5$

91 逢甲資工

Integer solution of $x_1+x_2+x_3+x_4=19,-5\le x_i\le10$
$(1+x+..+x^{15})^4=\left(\frac{1-x^{16}}{1-x}\right)^4=\left(1-x^{16}\right)^4\left(1-x\right)^{-4}$ $=\left(1-4x^{16}+6x^{32}+...\right)\left(1-x\right)^{-4}=\left(1-4x^{16}+6x^{32}+...\right)\sum_{i=0}^{\infty}\binom{4+r-1}{r}$ $cofficient\ \ of\ \ x^{39}\ \ upper=\binom{4+39-1}{39}-4\binom{4+23-1}{23}+6\binom{4+7-1}{7}$ $=\binom{42}{39}-4\binom{26}{23}+6\binom{10}{7}$

92 中央資管

RNA chain is a string on the alphabet {A,C,G,U} (a) ways of a length 6 RNA chain consisting of 3 C`s and 3 A`s (b) ways of a length k RNA chain with even number of U
$(a)\dfrac{6!}{3!3!}$ $(b)e^x e^x e^x\left(\dfrac{e^x+e^{-x}}{2}\right)=\dfrac{1}{2}e^{4x}+\dfrac{1}{2}e^{2x}$ $A(x)=\dfrac{1}{2}e^{4x}+\dfrac{1}{2}e^{2x}=\dfrac{1}{2}\left(\sum_{r=0}^{\infty}\dfrac{(4x)^r}{r!}+\sum_{r=0}^{\infty}\dfrac{(2x)^r}{r!}\right)=\dfrac{1}{2}\sum_{r=0}^{\infty}(4^r+2^r)\dfrac{x^r}{r!}$ $cofficient\ \ of\ \ \dfrac{x^k}{k!}\Rightarrow\dfrac{1}{2}(4^k+2^k)$

92 台科資工

Use generating function to solve $a_{n+2}-5a_{n+1}+6a_n=0, n\ge 0, a_0=5, a_1=13$
$\sum_{n=2}^{\infty}a_n x^n-5\sum_{n=2}^{\infty}a_{n-1}x^n+6\sum_{n=2}^{\infty}a_{n-2}x^n=0$ $A(x)-a_0-a_1x-5x(A(x)-a_0)+6x^2A(x)=\dfrac{2x^2}{1-x}$ $A(x)(1-5x+6x^2)=5-12x$ $\Rightarrow A(x)=\dfrac{5-12x}{1-5x+6x^2}=\dfrac{5-12x}{(1-2x)(1-3x)}$ $\Rightarrow A(x)=\dfrac{2}{(1-2x)}+\dfrac{3}{(1-3x)}=2\sum_{n=0}^{\infty}(2x)^n+3\sum_{n=0}^{\infty}(3x)^n$ $a_n=(2)^{n+1}+(3)^{n+1}$

94 海大資工 ，長庚資工

Use generating function to solve $a_n=8a_{n-1}+10^{n-1}, a_1=9$
$\sum_{n=1}^{\infty}a_n x^n=8\sum_{n=1}^{\infty}a_{n-1}x^n+\sum_{n=1}^{\infty}10^{n-1}x^n$ $A(x)-a_0=8xA(x)+\dfrac{x}{1-10x}\Rightarrow A(x)-1=8xA(x)+\dfrac{x}{1-10x}$ $\Rightarrow A(x)=\dfrac{1-9x}{(1-8x)(1-10x)}\Rightarrow A(x)=\dfrac{\frac{1}{2}}{1-8x}+\dfrac{\frac{1}{2}}{1-10x}$ $\Rightarrow A(x)=\dfrac{1}{2}\sum_{n=0}^{\infty}(8x)^n+\dfrac{1}{2}\sum_{n=0}^{\infty}(10x)^n\Rightarrow a_n=\dfrac{1}{2}\left(8^n+10^n\right)$

93 清大通訊

Use generating function to solve $a_{n+1}=2a_n, a_0=\frac{1}{2}$
$a_{n+1}=2a_n, a_0=\frac{1}{2}, let \quad A(x)=\sum_{n=0}^{\infty}a_n x^n$ $a_{n+1}=2a_n \Rightarrow \sum_{n=1}^{\infty}a_n x^n - 2\sum_{n=1}^{\infty}a_{n-1}x^n = 0$ $A(x)-a_0-2xA(x)=0$ $A(x)(1-2x)=\frac{1}{2} \Rightarrow A(x)=\frac{\frac{1}{2}}{(1-2x)} \Rightarrow A(x)=\frac{1}{2}\sum_{n=0}^{\infty}(2x)^n$ $\Rightarrow a_n=\frac{1}{2}2^n=2^{n-1}$

93 元智資管

Use generating function to solve $a_n=5a_{n-1}-6a_{n-2}, a_0=1, a_1=-2$
$\sum_{n=2}^{\infty}a_n x^n = 5\sum_{n=2}^{\infty}a_{n-1}x^n - 6\sum_{n=2}^{\infty}a_{n-2}x^n$ $A(x)-a_0-a_1x=5x(A(x)-a_0)-6x^2A(x)$ $A(x)-1+2x=5xA(x)-5x-6x^2A(x)$ $-1+7x=-A(x)+5xA(x)-6x^2A(x)$ $A(x)=\frac{1-7x}{6x^2-5x+1}=\frac{-5}{2x-1}+\frac{4}{3x-1}=\frac{5}{1-2x}+\frac{-4}{1-3x}$ $=5\sum_{n=0}^{\infty}(2x)^n-4\sum_{n=0}^{\infty}(3x)^n \Rightarrow a_n=5(2)^n-4(3)^n$

97 成大電機

N digit from {0,1,2,3,4} in each of which the total number of 0`s and 1`s is even
(1)0 與 1 皆 even $\left(\frac{e^x+e^{-x}}{2}\right)^2(e^x)^3=\frac{1}{4}(e^{5x}+2e^{3x}+e^x)=\frac{1}{4}\left(\sum_{n=0}^{\infty}\frac{(5x)^n}{n!}+2\sum_{n=0}^{\infty}\frac{(3x)^n}{n!}+\sum_{n=0}^{\infty}\frac{(x)^n}{n!}\right)$ $=\frac{1}{4}\left(5^n+2*3^n+1\right)\sum_{n=0}^{\infty}\frac{(x)^n}{n!}$ $\left(\frac{e^x-e^{-x}}{2}\right)^2(e^x)^3=\frac{1}{4}(e^{5x}-2e^{3x}+e^x)=\frac{1}{4}\left(\sum_{n=0}^{\infty}\frac{(5x)^n}{n!}-2\sum_{n=0}^{\infty}\frac{(3x)^n}{n!}+\sum_{n=0}^{\infty}\frac{(x)^n}{n!}\right)$ $=\frac{1}{4}\left(5^n-2*3^n+1\right)\sum_{n=0}^{\infty}\frac{(x)^n}{n!}$ $\frac{1}{4}\left(5^n+2*3^n+1\right)+\frac{1}{4}\left(5^n-2*3^n+1\right)=\frac{1}{2}\left(5^n+1\right)$

92 台大資工

$\sum_{i=1}^{n} i\binom{n}{i}$
$(1+x)^n=\sum_{i=0}^{n}\binom{n}{i}x^i, different\ \ both\ \ side\ \ n(1+x)^{n-1}=\sum_{i=1}^{n}i\binom{n}{i}x^{i-1}$ $x=1\ \ put\ \ \Rightarrow n(1+1)^{n-1}=\sum_{i=1}^{n}i\binom{n}{i}\Rightarrow n2^{n-1}=\sum_{i=1}^{n}i\binom{n}{i}$

(5)遞迴函數

5.1 solve the recurrence equation $a_n - 7a_{n-1} + 10a_{n-2} = 4n - 5, a_0 = 2, a_1 = 4$
$let\ \ a_n^{(h)} = c_1 2^n + c_2 5^n, a_n^{(p)} = d_1 n + d_2$ $a_n = a_n^{(h)} + a_n^{(p)} = c_1 2^n + c_2 5^n + n + 2 \Rightarrow a_n = -\frac{1}{3}2^n + \frac{1}{3}5^n + n + 2$

5.2 solve the recurrence equation $a_n - 2a_{n-1} + a_{n-2} = 2, a_0 = 1, a_1 = 1$
$let\ \ a_n^{(h)} = c_1 n + c_2\ \ ,\ \ a_n^{(p)} = d_1 n^2$ $a_n = a_n^{(h)} + a_n^{(p)} = c_1 n + c_2 + n^2 \Rightarrow a_n = n^2 - n + 1, n \geq 0$

5.3 solve the recurrence equation $a_n - 4a_{n-1} + 3a_{n-2} = 2 \times 3^n, a_0 = 2, a_1 = 13$
$let\ \ a_n^{(h)} = c_1 + c_2 \times 3^n, let\ \ a_n^{(p)} = dn3^n$ $a_n = a_n^{(h)} + a_n^{(p)} = 1 + 3^n + 3n3^n$

5.4 solve the recurrence equation $a_n = 5a_{n-1} - 6a_{n-2} + 4n, a_0 = 9, a_1 = 14$
$let\ \ a_n^{(h)} = c_1 \times 2^n + c_2 \times 3^n, let\ \ a_n^{(p)} = d_1 n + d_2$ $a_n = a_n^{(h)} + a_n^{(p)} = 2^n + 3^n + 2n + 7$

5.5 solve the recurrence equation $a_n - 4a_{n-1} + 4a_{n-2} = 3 \times 2^n, a_0 = 1, a_1 = 7$
$let\ \ a_n^{(h)} = (c_1 + c_2 n) \times 2^n, let\ \ a_n^{(p)} = dn^2 2^n$ $a_n = a_n^{(h)} + a_n^{(p)} = (1 + n + \frac{3}{2}n^2)2$

5.6 solve the recurrence equation $a_{n+2} - 8a_{n+1} + 15a_n = 6 \times 3^n + 10 \times 5^n, a_0 = 2, a_1 = 10$
$let\ \ a_n^{(h)} = c_1 \times 3^n + c_2 \times 5^n, let\ \ a_n^{(p)} = d_1 n \times 3^n + d_2 n \times 5^n$ $a_n = a_n^{(h)} + a_n^{(p)} = (1 - n)3^n + (1 + n)5^n$

4 special recurrence equation

5.7	solve the recurrence equation $a_{n+2}{}^2 - 5a_{n+1}{}^2 + 6a_n{}^2 = 7n, a_0 = a_1 = 1$

$$let \quad b_n = a_n{}^2 \Rightarrow b_{n+2} - 5b_{n+1} + 6b_n = 7n, b_0 = b_1 = 1$$

$$\Rightarrow b_n = \left(-5\times 2^n + \frac{3}{4}3^n + \frac{7}{2}n + \frac{21}{4}\right) \Rightarrow a_n = \pm\left(-5\times 2^n + \frac{3}{4}3^n + \frac{7}{2}n + \frac{21}{4}\right)^2$$

5.8	solve the recurrence equation $\begin{cases} a_{n+1} = 3a_n - 2b_n \\ b_{n+1} = 2a_n + 8b_n \end{cases} \quad a_0 = b_0 = 1$

$$let \begin{cases} a_{n+1} = 3a_n - 2b_n \text{............}(1) \\ b_{n+1} = 2a_n + 8b_n \text{............}(2) \end{cases}$$

$$from \quad (2) \quad a_n = \frac{1}{2}b_{n+1} - 4b_n \quad put \quad (1)$$

$$\frac{1}{2}b_{n+2} - 4b_{n+1} = \frac{3}{2}b_{n+1} - 12b_n - 2b_n \Rightarrow \frac{1}{2}b_{n+2} - \frac{11}{2}b_{n+1} + 14b_n = 0$$

$$\Rightarrow b_{n+2} - 11b_{n+1} + 28b_n = 0, let \quad b_n = c_1 4^n + c_2 7^n$$

$$\begin{cases} b_{n+1} = 2a_n + 8b_n \\ a_0 = b_0 = 1 \end{cases} \Rightarrow b_1 = 10 \Rightarrow b_n = -4^n + 2\times 7^n$$

$$b_n = -4^n + 2\times 7^n \quad put \quad (2)$$

$$-4^{n+1} + 2\times 7^{n+1} = 2a_n + 8(-4^n + 2\times 7^n)$$

$$a_n = 2\times 4^n - 7^n$$

5.9	solve the recurrence equation use generating function $a_{n+1} = 2a_n, a_0 = \frac{1}{2}$

$$\sum_{n=1}^{\infty} a_n x^n = 2\sum_{n=1}^{\infty} a_{n-1}x^n \Rightarrow A(x) - a_0 = 2xA(x) \Rightarrow A(x)(1-2x) = a_0 = \frac{1}{2}$$

$$\Rightarrow A(x) = \frac{1}{2(1-2x)} = \frac{1}{2}\sum_{n=0}^{\infty}(2x)^n \Rightarrow a_n = \frac{1}{2}(2)^n$$

Grimaldi example 10.12

We have a 2*n chessboard ,we wish to cover such chessboard using 2*1 dominoes and 1*2 dominoes
$b_n = b_{n-1} + b_{n-2}, b_1 = 1, b_2 = 2$ $b_n = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^{n+1} - \left(\frac{1-\sqrt{5}}{2}\right)^{n+1}$

Grimaldi exercise 10.2.15 93 台科資管

solve the recurrence equation $a_{n+2} = a_{n+1}a_n, a_0 = 1, a_1 = 2$
Let $T_n = \log a_n$ $a_{n+2} = a_{n+1}a_n, a_0 = 1, a_1 = 2 \Rightarrow T_n = T_{n-1} + T_{n-2}, T_0 = 0, T_1 = 1$ $a_n = 2^{F_n}$

Grimaldi example 10.26

solve the recurrence equation $a_n - 3a_{n-1} = 5(7)^n, a_0 = 2$
$a_n^{(h)} = c(3)^n, a_n^{(p)} = d(7)^n$ $d(7)^n - 3d(7)^{n-1} = 5(7)^n \Rightarrow d = \frac{35}{4}$ $a_n = a_n^{(h)} + a_n^{(p)} = 2(3)^n + \frac{35}{4}(7)^n$

Grimaldi example 10.27

solve the recurrence equation $a_n - 3a_{n-1} = 5(3)^n, a_0 = 2$
$a_n^{(h)} = c(3)^n, a_n^{(p)} = dn(3)^n$ $dn(3)^n - 3d(n-1)(3)^{n-1} = 5(3)^n \Rightarrow d = 5$ $a_n = a_n^{(h)} + a_n^{(p)} = (5n+2)(3)^n$

Grimaldi example 10.38

Use generating function solve the recurrence equation $a_n - 3a_{n-1} = n, a_0 = 1$
$\sum_{n=1}^{\infty} a_n x^n - 3\sum_{n=1}^{\infty} a_{n-1} x^n = \sum_{n=1}^{\infty} nx^n$ $\Rightarrow (f(x)-1) - 3xf(x) = \frac{x}{(1-x)^2} \Rightarrow f(x) = \frac{1}{1-3x} + \frac{x}{(1-x)^2(1-3x)}$ $\frac{x}{(1-x)^2(1-3x)} = \frac{A}{1-x} + \frac{B}{(1-x)^2} + \frac{C}{1-3x} = \frac{-\frac{1}{4}}{1-x} + \frac{-\frac{1}{2}}{(1-x)^2} + \frac{\frac{3}{4}}{1-3x}$ $f(x) = \frac{1}{1-3x} - \frac{1}{4}\left(\frac{1}{1-x}\right) - \frac{1}{2}\left(\frac{1}{(1-x)^2}\right) + \frac{3}{4}\left(\frac{1}{1-3x}\right)$ $f(x) = -\frac{1}{4}\left(\frac{1}{1-x}\right) - \frac{1}{2}\left(\frac{1}{(1-x)^2}\right) + \frac{7}{4}\left(\frac{1}{1-3x}\right)$ $a_n = \frac{7}{4}(3)^n - \frac{1}{2}n - \frac{3}{4}$

Grimaldi example 10.39

Use generating function solve the recurrence equation $a_{n+2} - 5a_{n+1} + 6a_n = 2, a_0 = 3, a_1 = 7$
$\sum_{n=0}^{\infty} a_{n+2} x^{n+2} - 5\sum_{n=0}^{\infty} a_{n+1} x^{n+2} - 6\sum_{n=0}^{\infty} a_n x^{n+2} = 2\sum_{n=0}^{\infty} x^{n+2}$ $\Rightarrow \sum_{n=0}^{\infty} a_{n+2} x^{n+2} - 5x\sum_{n=0}^{\infty} a_{n+1} x^{n+1} - 6x^2\sum_{n=0}^{\infty} a_n x^n = 2x^2\sum_{n=0}^{\infty} x^n$ $let \quad f(x) = \sum_{n=0}^{\infty} a_n x^n \quad be \quad G.F. \quad of \quad solution$ $(f(x) - a_0 - a_1 x) - 5x(f(x) - a_0) + 6x^2 f(x) = \frac{2x^2}{1-x}$ $\Rightarrow (f(x) - 3 - 7x) - 5x(f(x) - 3) + 6x^2 f(x) = \frac{2x^2}{1-x}$ $\Rightarrow f(x) = \frac{3 - 11x + 10x^2}{(1-5x+6x^2)(1-x)} = \frac{(3-5x)(1-2x)}{(1-3x)(1-2x)(1-x)} = \frac{(3-5x)}{(1-3x)(1-x)}$ $f(x) = \frac{2}{1-3x} + \frac{1}{1-x} = 2\sum_{n=0}^{\infty}(3x)^n + \sum_{n=0}^{\infty} x^n \Rightarrow a_n = 2(3)^n + 1$

Grimaldi exercise 10.3.6　94 中山資工

solve the recurrence equation $a_{n+2}-6a_{n+1}+9a_n=3(2)^n+7(3)^n, a_0=1, a_1=4$
$a_n^{(h)}=(c_1n+c_2)(3)^n, a_n^{(p)}=d_1(2)^n+d_2n^2(3)^n$ $a_n=a_n^{(h)}+a_n^{(p)}=(\frac{17}{18}n-2)(3)^n+3(2)^n+\frac{7}{18}n^2(3)^n$

96 中興網媒　96 靜宜資工

Find a recurrence relation for the number of binary strings of length n that do not contains three consecutive 0`s
When n=1　………0 和 …………1 都可以　→ $a_1=2$ When n=2　…….00 ,….01 ,…….10 ,…….10 都可以 → $a_2=4$ When n=3　只有 …..000 不行　→ $a_3=7$ $\begin{cases} a_n=a_{n-1}+a_{n-2}+a_{n-3}, n\ge 4 \\ a_1=2, a_2=4, a_3=7 \end{cases}$

solve the recurrence equation $(a) a_{n+2}+a_n=0, a_0=0, a_1=3$ $(b) a_{n+2}+4a_n=0, a_0=1, a_1=1$ $(c) a_n+2a_{n-1}+2a_{n-2}=0, a_0=1, a_1=3$
$(a)\alpha^2+1=0\Rightarrow\alpha=\pm i$ $a_n=B_1\cos\frac{n\pi}{2}+B_2\sin\frac{n\pi}{2}$ $\begin{cases} a_0=B_1=0 \\ a_0=B_2=3 \end{cases}\Rightarrow a_n=3\sin\frac{n\pi}{2}$ $(b)\alpha^2+4=0\Rightarrow\alpha=\pm 2i$ $a_n=2^n\left(B_1\cos\frac{n\pi}{2}+B_2\sin\frac{n\pi}{2}\right)$ $\begin{cases} a_0=B_1=1 \\ a_0=2B_2=1 \end{cases}\Rightarrow a_n=2^n\left(\cos\frac{n\pi}{2}+\frac{1}{2}\sin\frac{n\pi}{2}\right)$ $(c)\alpha^2+2\alpha+2=0\Rightarrow\alpha=-1\pm i$ $a_n=\left(\sqrt{2}\right)^n\left(B_1\cos\frac{3n\pi}{4}+B_2\sin\frac{3n\pi}{4}\right)$ $\begin{cases} a_0=B_1=1 \\ a_0=\sqrt{2}(-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}B_2)=3 \end{cases}\Rightarrow a_n=2^n\left(\cos\frac{3n\pi}{4}+4\sin\frac{3n\pi}{4}\right)$

Find $\begin{vmatrix} 6 & 3 & 0 & & & & \\ 3 & 6 & 3 & & O & & \\ 0 & 3 & 6 & & & & \\ & & & \ddots & & & \\ & & & & 6 & 3 & 0 \\ & O & & & 3 & 6 & 3 \\ & & & & 0 & 3 & 6 \end{vmatrix}$
$a_1 = \|6\| = 6, a_2 = \begin{vmatrix} 6 & 3 \\ 3 & 6 \end{vmatrix} = 27 \Rightarrow \begin{cases} a_n = 6a_{n-1} - 9a_{n-2} \\ a_1 = 1, a_2 = 27 \end{cases}$ $\Rightarrow a_n = (1+n)3^n$

2 元 n 序列 ,不含連續個 0
When n=1 ………0 和 …………1 都可以 ➔ $a_1 = 2$ When n=2 ….01 ,……10 ,…….10 都可以 ➔ $a_2 = 3$ $\begin{cases} a_n = a_{n-1} + a_{n-2} \\ a_1 = 2, a_2 = 3 \end{cases}$ ➔ $a_n = F_{n+2} = \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^{n+2} - \left(\frac{1-\sqrt{5}}{2}\right)^{n+2}\right)$

S={1,2,3,……,n} ,求 S 子集中不含連續整數的個數
When S={1} $\varnothing, \{1\}$ 都可以 ➔ $a_1 = 2$ When S={1,2} $\varnothing, \{1\}, \{2\}$ 都可以 ➔ $a_2 = 3$ $\begin{cases} a_n = a_{n-1} + a_{n-2} \\ a_1 = 2, a_2 = 3 \end{cases}$ ➔ $a_n = F_{n+2} = \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^{n+2} - \left(\frac{1-\sqrt{5}}{2}\right)^{n+2}\right)$

In how many ways can a 1*n rectangular board be tiled using 1*1 and 1*2 pieces
When n=1 1*1 都可以 ➔ $a_1 = 1$ When n=2 two 1*1 , one 1*2 ➔ $a_2 = 2$ $\begin{cases} a_n = a_{n-1} + a_{n-2} \\ a_1 = 1, a_2 = 2 \end{cases}$ ➔ $a_n = F_{n+1} = \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^{n+1} - \left(\frac{1-\sqrt{5}}{2}\right)^{n+1}\right)$

小黃習題　5-47

Use generating fuction to solve　recurrence equation

$(a)a_{n+1}-a_n=3^n, n\ge 0, a_0=1$

$(b)a_n-3a_{n-1}=5^{n-1}, n\ge 1, a_0=1$

$(a)let \quad A(x)=\sum_{n=0}^{\infty}a_n x^n$

$a_{n+1}-a_n=3^n \Rightarrow \sum_{n=1}^{\infty}a_n x^n-\sum_{n=1}^{\infty}a_{n-1}x^n=\sum_{n=1}^{\infty}3^{n-1}x^n=x\sum_{n=1}^{\infty}(3x)^{n-1}$

$\Rightarrow A(x)-a_0-xA(x)=\frac{x}{1-3x}\Rightarrow A(x)-1-xA(x)=\frac{x}{1-3x}$

$\Rightarrow (1-x)A(x)=\frac{x}{1-3x}+1\Rightarrow A(x)=\frac{x}{(1-3x)(1-x)}+\frac{1}{1-x}$

$=\frac{1/2}{1-3x}+\frac{1/2}{1-x}=\frac{1}{2}\sum_{n=0}^{\infty}x^n+\frac{1}{2}\sum_{n=0}^{\infty}(3x)^n=\frac{1}{2}\sum_{n=0}^{\infty}(1+3)^n\Rightarrow a_n=\frac{1}{2}(1+3)^n, n\ge 0$

$(b)let \quad A(x)=\sum_{n=0}^{\infty}a_n x^n$

$a_{n+1}-3a_n=5^{n-1}\Rightarrow \sum_{n=1}^{\infty}a_n x^n-3\sum_{n=1}^{\infty}a_{n-1}x^n=\sum_{n=1}^{\infty}5^{n-1}x^n=x\sum_{n=1}^{\infty}(5x)^{n-1}$

$\Rightarrow A(x)-a_0-3xA(x)=\frac{x}{1-5x}\Rightarrow A(x)-1-3xA(x)=\frac{x}{1-5x}$

$\Rightarrow A(x)=\frac{1}{1-3x}+\frac{x}{(1-3x)(1-5x)}=\frac{1}{1-3x}+\frac{-1/2}{1-3x}+\frac{1/2}{1-5x}$

$=\frac{1/2}{1-3x}+\frac{1/2}{1-5x}=\frac{1}{2}\sum_{n=0}^{\infty}(3x)^n+\frac{1}{2}\sum_{n=0}^{\infty}(5x)^n\Rightarrow a_n=\frac{1}{2}(3^n+5^n), n\ge 0$

小黃習題　5-47

Use generating fuction to solve　recurrence equation

$(d)a_{n+2}-3a_{n+1}+2a_n=0, n\ge 0, a_0=1, a_1=6$

$(e)a_{n+2}-2a_{n+1}+a_n=2^n, n\ge 0, a_0=1, a_1=2$

$(f)a_n=5a_{n-1}-6a_{n-2}, n\ge 2, a_0=1, a_1=-2$

$(d)a_{n+2}-3a_{n+1}+2a_n=0$

$$\sum_{n=2}^{\infty}a_nx^n-3\sum_{n=2}^{\infty}a_{n-1}x^n+2\sum_{n=2}^{\infty}a_{n-2}x^n=0$$

$$A(x)-a_0-a_1x-3x\left(A(x)-a_0\right)+2x^2A(x)=0$$

$$\Rightarrow A(x)-1-6x-3x\left(A(x)-1\right)+2x^2A(x)=0$$

$$\Rightarrow A(x)(1-3x+2x^2)=1+3x$$

$$\Rightarrow A(x)=\frac{1+3x}{1-3x+2x^2}=\frac{5}{1-2x}+\frac{(-4)}{1-x}=5\sum_{n=0}^{\infty}(2x)^n-4\sum_{n=0}^{\infty}(x)^n$$

$$\Rightarrow a_n=5(2^n)-4, n\ge 0$$

$(e)a_{n+2}-2a_{n+1}+a_n=2^n$

$$\sum_{n=2}^{\infty}a_nx^n-2\sum_{n=2}^{\infty}a_{n-1}x^n+\sum_{n=2}^{\infty}a_{n-2}x^n=\sum_{n=2}^{\infty}2^{n-2}x^n$$

$$A(x)-a_0-a_1x-2x\left(A(x)-a_0\right)+x^2A(x)=x^2\sum_{n=2}^{\infty}(2x)^{n-2}$$

$$\Rightarrow A(x)-1-2x-2x\left(A(x)-1\right)+x^2A(x)=\frac{x^2}{1-2x}$$

$$\Rightarrow A(x)(x^2-2x+1)=1+\frac{x^2}{1-2x}$$

$$\Rightarrow A(x)=\frac{x^2}{(1-2x)(1-x)^2}+\frac{1}{(1-x)^2}=\frac{1-2x+x^2}{(1-2x)(1-x)^2}=\frac{1}{1-2x}=\sum_{n=0}^{\infty}(2x)^n$$

$$\Rightarrow a_n=2^n, n\ge 0$$

$(f)a_n=5a_{n-1}-6a_{n-2}$

$$\sum_{n=2}^{\infty}a_nx^n-5\sum_{n=2}^{\infty}a_{n-1}x^n+6\sum_{n=2}^{\infty}a_{n-2}x^n=0$$

$$A(x)-a_0-a_1x-5x\left(A(x)-a_0\right)+6x^2A(x)=0$$

$$\Rightarrow A(x)=\frac{1-7x}{6x^2-5x+1}=\frac{1-7x}{(1-2x)(1-3x)}=\frac{5}{1-2x}+\frac{(-4)}{1-3x}=5\sum_{n=0}^{\infty}(2x)^n-4\sum_{n=0}^{\infty}(3x)^n$$

$$\Rightarrow a_n=5(2^n)+4(3^n), n\ge 0$$

97 清大資應,95 朝陽資工,92 健康資管,89 朝陽資管

Solve the recurrence relation $\sqrt{a_n}=\sqrt{a_{n-1}}+2\sqrt{a_{n-2}}, n\ge 2, a_0=1, a_1=1$
$let\ \ b_n=\sqrt{a_n}\Rightarrow b_n=b_{n-1}+2b_{n-2}, b_0=1, b_1=1$ 特徵方程式 $\alpha^2-\alpha-2=0\Rightarrow(\alpha-2)(\alpha+1)=0$ $\alpha^2-\alpha-2=0\Rightarrow(\alpha-2)(\alpha+1)=0$ $b_n=c_1 2^n+c_2(-1)^n, \because b_0=1, b_1=1\Rightarrow\begin{cases}c_1+c_2=1\\2c_1-c_2=1\end{cases}\Rightarrow b_n=\frac{2}{3}2^n+\frac{1}{3}(-1)^n$ $\Rightarrow a_n=b_n^{\ 2}=\left[\frac{2}{3}2^n+\frac{1}{3}(-1)^n\right]^2$

97 清大資應

Find the number of positive integer x that exist where $x\le 999999$ and sum of the digits in x equals 20
對應 gernerating function $A(x)=(1+x+...+x^9)^6$,欲求 x^{20} 的係數 $(1+x+...+x^9)^6=\left(1-x^{10}\right)^6\left(1-x\right)^{-6}=\left(\sum_{i=0}^{6}\binom{6}{i}(-x^{10})^i\right)\left(\sum_{r=0}^{\infty}\binom{6+r-1}{r}x^r\right)$ x^{20} 的係數爲 $\binom{25}{20}-\binom{6}{1}\binom{15}{10}+\binom{6}{2}\binom{5}{0}$

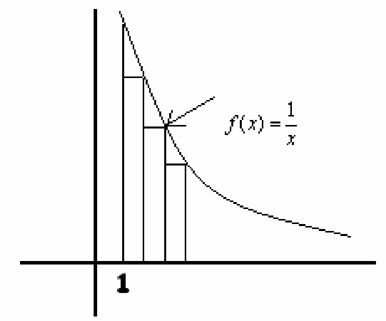
97 中山資工 96 暨南資工

The general solution for $a_{n+2}+b_1a_{n+1}+b_2a_n=b_3n+b_4$ be $c_1 2^n+c_2 3^n+n-7$ Find $b_i, 1\le i\le 4$
$(\alpha-2)(\alpha-3)=\alpha^2-5\alpha+6\Rightarrow a_{n+2}-5a_{n+1}+6a_n=b_3n+b_4$ $a_n^{(p)}=n-7\quad put\quad upper$ $(n+2)-7-5\left[(n+1)-7\right]+6(n-7)=b_3n+b_4$ $2n-17=b_3n+b_4$ $a_{n+2}-5a_{n+1}+6a_n=2n-17$

97 宜蘭電子　96 清大資應　96 中正資工 95 中興資工

Bit string of length n do not have two consecutive 0s (a) list the recursion formula (b) how many such string when length be 8
(a) 第 n 個數字爲 0 ,則第 n-1 個數字必需爲 1 ,前面 n-2 個數字不含連續 0 有 a_{n-2} 種 第 n 個數字爲 1 ,前面 n-1 個數字不含連續 0 有 a_{n-1} 種 當只有一個數字 有 2 種 0,1 當有兩個數字 有 3 種 01,10,11 $a_n = a_{n-1} + a_{n-2}, a_1 = 2, a_2 = 3$ (b) $a_8 = 55$ (費氏數左移)

93 輔大電子

Prove the big -O of $H_n = 1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n}$ be O(logn)
$\because \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n} < \int_1^n \frac{1}{x} dx = \ln n$ $\therefore H_n - 1 < \ln n \Rightarrow H_n < \ln n + 1 \Rightarrow O(H_n) = O(\ln n + 1) = O(\log n)$ 

Prove distinct binary tree has $\frac{1}{n+1}\binom{2n}{n}$
prove n node , 1 be root reminder n-1 partition into left and right subtree,suppose left has k,right=n-1-k left subtrees has b_k knids, right subtrees has b_{n-1-k} so a tree has $b_k \times b_{n-1-k}$ kinds we derived the following function $\begin{cases} b_n = b_0 \times b_{n-1} + b_1 \times b_{n-2} + \ldots b_{n-1} \times b_0 \\ 1, n=0 \end{cases}$ $let \quad B(x) = b_0 + b_1 x + b_2 x^2 + \ldots = \sum_{n=0}^{\infty} b_n x^n$ ➔ $b_0 \times b_{n-1} + b_1 \times b_{n-2} + \ldots b_{n-1} \times b_0 = \sum_{n=0}^{\infty} b_n x^n - b_0 = xB(x)^2$ ➔ $B(x) - 1 = xB(x)^2 \Rightarrow xB(x)^2 - B(x) + 1 = 0 \Rightarrow B(x) = \frac{1-\sqrt{1-4x}}{2x}$ $B(x) = \frac{1-\sqrt{1-4x}}{2x} = \sum_{n=0}^{\infty} \binom{\frac{1}{2}}{n} (-4x)^n = \sum_{n=0}^{\infty} \frac{\frac{1}{2}(\frac{1}{2}-1)\ldots(\frac{1}{2}-n+1)}{n!} (-1)^n 4^n (x)^n$ $= -\sum_{n=0}^{\infty} \frac{1}{(2n-1)} \binom{2n}{n} (x)^n$ answer must >0 so $B(x) = \frac{1-\sqrt{1-4x}}{2x} = \frac{1+\sum_{n=0}^{\infty} \frac{1}{(2n-1)} \binom{2n}{n} (x)^n}{2x} = \frac{1}{2} \sum_{n=0}^{\infty} \frac{1}{(2n-1)} \binom{2n}{n} (x)^{n-1}$ $\frac{1}{2} \frac{1}{2(n+1)-1} \binom{2(n+1)}{(n+1)} = \frac{1}{n+1} \binom{2n}{n}$

(6)**圖論**

Terminology

(a) Strong Connected Graph 95中央企管

G=(V, E) 爲有向圖 $\forall x, y \in V, x \neq y$ x到y存在一條路徑,y到x也存在一條路徑

(b) cut point(artivulation point) v

把點v由G中移除成爲 $G_1 = (V - \{v\}, E_1)$,則圖形G將不在連接

(c) cut set

把邊集合 E_1 從 G 中移除成爲 $G_1 = (V, E - \{E_1\})$,則圖形 G 將不在連接

當 cut set 中只有一條邊稱:bridge

(d) biconnected graph

無向圖 G=(V, E) 不存在 cut point

(e) bipartite

$G = (V, E)$ be undirected graph

$$let\ \ V = V_1 \cup V_2, V_1 \cap V_2 = \varnothing, \forall \{x, y\} \in E$$
$$x \in V_1, y \in V_2 \ \ or \ \ x \in V_2, y \in V_1$$

(f)Eulerian path(circuit)

一個 travse 每個邊剛好一次的 path

(g)Hamiltonian Path (circuit)

一個 travse 每個點剛好一次的 path

(h) Planar

一個圖形畫在平面上,所有邊交會的地方只在點的地方

(i)Kuratowski`s theorem

A graph is nonplanar if and only if contains a subgraph that is homeomorphic to either K_5 or $K_{3,3}$

	Euler trail	Euler circuit
無向圖	奇數 degree 爲 0 或 2 個	全部點 degree 爲偶數
有向圖	**|in-degree - out-degree |=1**	**in-degree=out-degree**

C.C.L exercise 5.8 **Havel-Hakimi theorem** 小黃習題 6-7

(a)show (4,3,2,2,1) is graphical

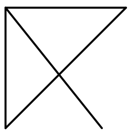
(b)show (3,3,3,1) is graphical

(c)without loss of generality , suppose $d_1 \ge d_2 \ge ... \ge d_n$

Show that $(d_1, d_2, ..., d_n)$ is graphical iff $(d_2 - 1, d_3 - 1, ..., d_{d_1} - 1, d_{d_1+1} - 1, d_{d_1+2} ..., d_n)$ is graphical

(d)use the result in part(c) to determine whether (5,5,3,3,2,2,2) is graphical

(a)



(b) (3,3,3,1) ➔ (2,2,0)

since(2,2,0) **isn`t graphical** so is (3,3,3,1)

Havel-Hakimi theorem

(d)(5,5,3,3,2,2,2) ➔(4,2,2,2,1,1)➔(1,1,1,1,0)

since (1,1,1,1,0) graphical so is (5,5,3,3,2,2,2)

	K_n	$K_{m,n}$	Q_n	$Grid_{m,n}$
|vertices|	n	m+n	2^n	$(m+1)(n+1)$
|edge|	C(n,2)	mn	$n2^{n-1}$	$m(n+1)+n(m+1)$
H path	-	$\|m-n\| \le 1$	-	-
H circuit	$n \ge 3$	$m = n \ge 2$	$n \ge 2$	m,n are all not even
E path	**m is odd**	**(a)m,n is even** **(b)m=1,n=1** **(c) n=2,m:odd**	n is even	
E circuit	$n \ge 3$ **:odd**	**m,n is even**	n is even	m=n=1
Spanning tree#	n^{n-2}	$m^{n-1}n^{n-1}$	$2^{5*2^{n-2}-n-1}$	
Planar	$n < 5$	$m < 3, n < 3$	$n < 4$	-
Chromatic num $\chi()$	n	2	2	2

distinct hamiltonian path = $\frac{n!}{2}$

distinct hamiltonian crcuit= $\frac{(n-1)!}{2}$

disjoint hamiltonian path= $\left\lfloor \frac{n}{2} \right\rfloor$

dis joint hamiltonian crcuit= $\left\lfloor \frac{n-1}{2} \right\rfloor$

$\deg(x)+\deg(y) \geq n-1, \forall x, y \in V, x \neq y$ ➔G has a hamiltonian path

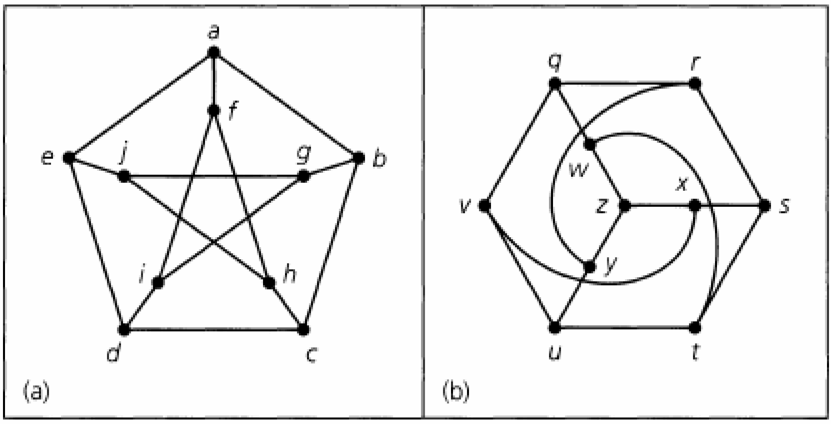
$\deg(x)+\deg(y) \geq n, \forall x, y \in V, x \neq y$ ➔G has a hamiltonian crcuit

$\deg(v) \geq \frac{n-1}{2}, \forall v \in V$ ➔G has a hamiltonian path

$\deg(v) \geq \frac{n}{2}, \forall v \in V$ ➔G has a hamiltonian crcuit

Grimaldi example 11.8 94 中山資工 92 台科資工

Whether or not these graphs are isomorphism



One finds that the correspondence given by

$a \rightarrow q$	$c \rightarrow u$	$e \rightarrow r$	$g \rightarrow x$	$i \rightarrow z$
$b \rightarrow v$	$d \rightarrow y$	$f \rightarrow w$	$h \rightarrow t$	$j \rightarrow s$

Grimaldi exercise 11.2.12

(a) Let G be an undirected graph with n vertices.If G is isomorphism to its own complement $\overline{G}$,how many edge must G have ?

(b) if G is a self-complementary graph on n vertices,where $n > 1$,prove that **n=4k** or **n=4k+1** for some $k \in Z^+$

$(a) let \quad G = (V, E_1), \overline{G} = (V, E_2)$

$$\begin{cases} |E_1| + |E_2| = \binom{n}{2} \\ |E_1| = |E_2| \end{cases} \Rightarrow |E_1| = |E_2| = \frac{n(n-1)}{4}$$

$(b) let \quad G = (V, E_1), \overline{G} = (V, E_2)$

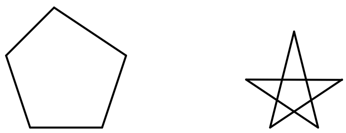
$$\begin{cases} |E_1| + |E_2| = \binom{n}{2} \\ |E_1| = |E_2| \end{cases} \Rightarrow |E_1| = |E_2| = \frac{n(n-1)}{4}$$

$\because |E_1| \in N \Rightarrow n = 4k \quad or \quad n = 4k + 1$

Grimaldi exercise 11.2.13

Let G be a cycle on n vertex . Prove $G \ \ is \ \ self-complementary \Leftrightarrow n = 5$

$(\Leftarrow)$



$(\Rightarrow)$

Because G be a cycle on n vertex

$let \quad G = (V, E_1), \overline{G} = (V, E_2)$

$$|V| = |E_1| = |E_2| = n, |E_1| + |E_2| = \frac{n(n-1)}{2}$$

$$2n = \frac{n(n-1)}{2} \Rightarrow 4n = n^2 - n \Rightarrow n^2 - 5n = 0 \Rightarrow n = 5$$

Grimaldi exercise 11.2.16

(a) how many subgraph $H=(V,E)$ of K_6 satisfy $|V|=3$?

(b) how many subgraph $H=(V,E)$ of K_6 satisfy $|V|=4$?

(c) how many subgraph does K_6 have ?

(d) how many subgraph does K_n have $,n\geq 3$?

$$(a)\binom{6}{3}2^{\binom{3}{2}} \quad (b)\binom{6}{4}2^{\binom{4}{2}} \quad (c)\sum_{k=1}^{6}\binom{6}{k}2^{\binom{k}{2}} \quad (d)\sum_{k=1}^{n}\binom{n}{k}2^{\binom{k}{2}}$$

Grimaldi exercise 11.3.8

(a) find the number of edge in Q_8

(b) maximum distance of one such pairs of vertices of Q_8

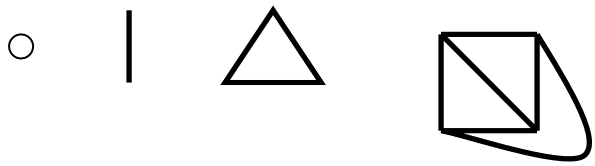
(c) find the length of longest path in Q_8

$(a)|E| \quad of \quad Q_n = n2^{n-1} \Rightarrow 8*2^7$

$(b)8 \quad (c)2^8-1=255$

$(a) prove \quad K_1, K_2, K_3, K_4 \quad is \quad planar$

$(b) prove \quad K_{1,1}, K_{2,2} \quad is \quad planar$



96 彰師資工

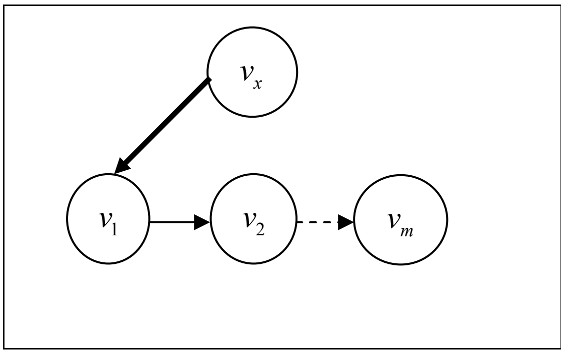
G=(V,E) be loop-free connected planar graph, |V|=v,|E|=e>2 ,|Region|=r (1)Prove $(1)\frac{3}{2}r \le e \le 3v-6$ (2)Prove when G contains no triangular $(2)e \le 2v-4$ (3)Prove when cycle of G unless 5 edges (counterexample to Petersen graph)
(1) let n 為所有區域所圍成邊的總和 ,(含重覆邊) 每個 region 至少含 3 個邊 ➔ $3r \le n$ 每個邊至少在 2 個 region 邊界上➔ $n \le 2e$ $\Rightarrow 3r \le n \le 2e \Rightarrow r \le \frac{2}{3}e$ 因為 G 為連通平面圖滿足 **v-e+r=2** $2 = v-e+r \le v-e+\frac{2}{3}e \Rightarrow 2 \le v-\frac{1}{3}e \Rightarrow e \le 3v-6$ (2) 因為 G 不含三角形 $\Rightarrow 4r \le n \le 2e \Rightarrow r \le \frac{1}{2}e$ 因為 G 為連通平面圖滿足 **v-e+r=2** $2 = v-e+r \le v-e+\frac{1}{2}e \Rightarrow 2 \le v-\frac{1}{2}e \Rightarrow e \le 2v-4$ (3) $5r \le n \le 2e \Rightarrow r \le \frac{2}{5}e$ $2 = v-e+r \le v-e+\frac{2}{5}e \Rightarrow 2 \le v-\frac{3}{5}e \Rightarrow 3e \le 5v-10$ So Petersen graph is nonplanar (v=10,e=15) $3*15 \not\le 5*10-10$

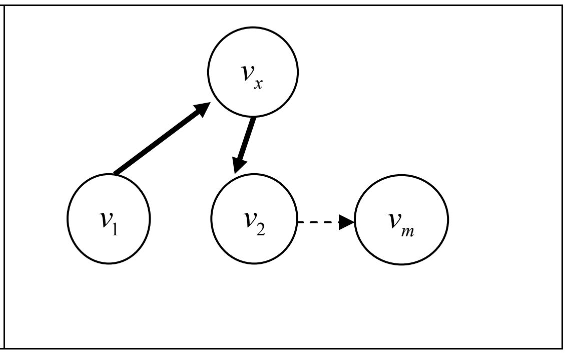
Grimaldi example 11.29

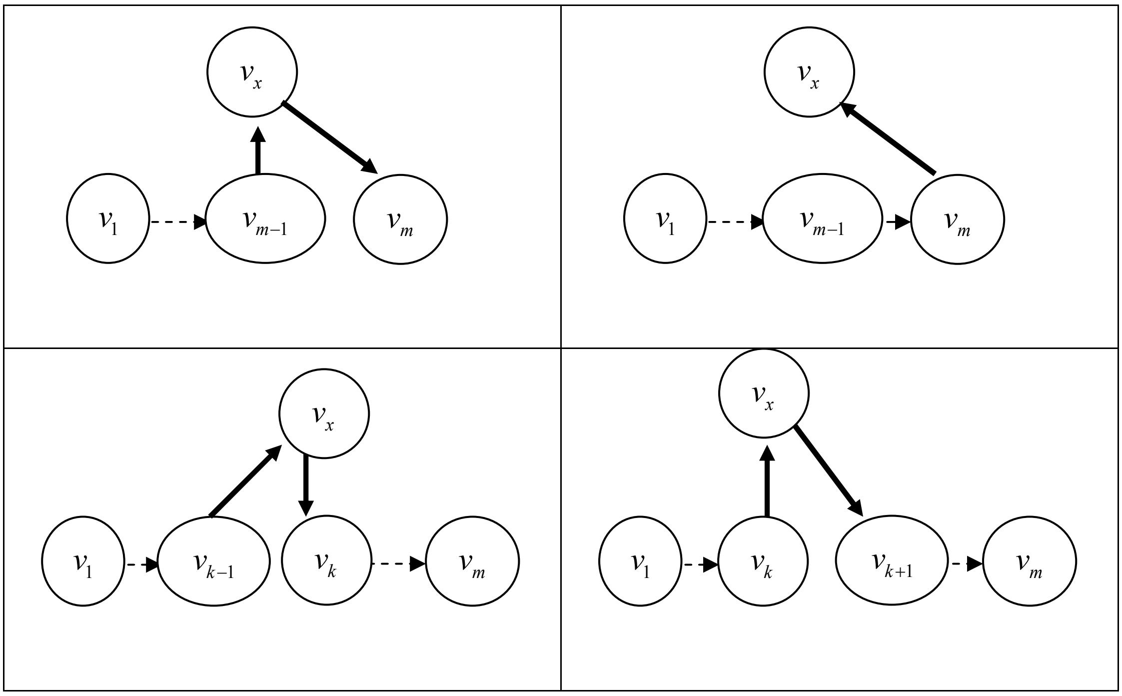
At professor ,17 students have lunch together each day at a circular table. They are trying to get to know one another better, so they make an effort to sit next to two different colleagues each afternoon.For how many afternoons can they do this ? How can they arrange themselves on these occasions?
K_n has $\frac{n-1}{2}$ Hamilton cycles ➔ $\left\lfloor \frac{n-1}{2} \right\rfloor = \left\lfloor \frac{17-1}{2} \right\rfloor = 8$

Grimaldi theorem 11.7

K_n^* be a complete directed graph,prove K_n^* always has Hamilton path
In G include directed path $\{v_1,v_2,...,v_m\}$ and m-1 edges Now add v_x into the path Situation (1) 邊在 $\{v_x,v_1\}$ or $\{v_x,v_1\}$ 上 (a)若邊在 $\{v_x,v_1\}$ 上 ,將原路徑加上 $\{v_x,v_1\}$ 得到新路徑 $\{v_x,v_1,v_2,...,v_m\}$ (b)若邊在 $\{v_1,v_x\}$ 以 $\{v_1,v_x\}$, $\{v_x,v_2\}$ deplace $\{v_1,v_2\}$,新路徑 $\{v_1,v_x,v_2,...,v_m\}$ Situation (2) 邊在 $\{v_x,v_m\}$ or $\{v_m,v_x\}$ 上 (a)若邊在 $\{v_x,v_m\}$ 上, $\{v_{m-1},v_x\}\{v_x,v_m\}$ replace $\{v_{m-1},v_m\}$,新路徑 $\{v_1,...v_{m-1},v_x,v_m\}$ (b)若邊在 $\{v_m,v_x\}$ 上,得到新路徑 $\{v_1,v_2,...,v_m,v_x\}$ Situation (3) 邊在 $\{v_x,v_k\}$ or $\{v_k,v_x\}$ $2\le k\le m-1$ (a)若邊在 $\{v_x,v_k\}$ 上, $\{v_{k-1},v_x\},\{v_x,v_k\}$ replace $\{v_{k-1},v_k\}$,新路徑 $\{v_1,...,v_{k-1},v_x,v_k,...,v_m\}$ (b)若邊在 $\{v_k,v_x\}$ 上, $\{v_k,v_x\},\{v_x,v_{k+1}\}$ replace $\{v_k,v_{k+1}\}$,新路徑 $\{v_1,...,v_k,v_x,v_{k+1},...,v_m\}$







independence number $\beta(G)$

(a)The independence number $\beta(G)$ of a graph is the cardinality of the largest independent set.

(b)A set of vertices of *G* is **independent** (獨立的) if **no two of them are adjacent**

(c) 應用: **(1)** 化學藥品存放，避免交互作用。

(2) 水族館設計，會互相捕食的魚不能放同一個水族箱。

clique number $\omega(G)$ **97 台大電機**

A clique in a graph ***G*** is a maximal complete subgraph.

domination number $\sigma(G)$

(a) **domination number** $\sigma(G)$ of ***G*** is the minimal cardinality of a dominating set of ***G***.

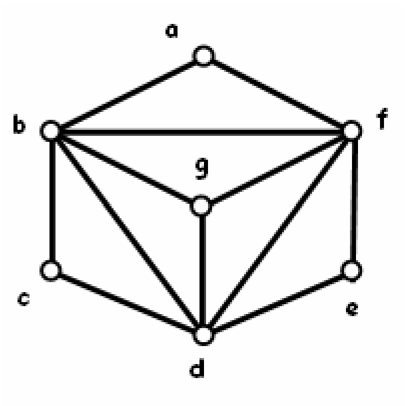
(b) dominating set ***S*** is a minimal dominating set if no proper subset of ***S*** is also a dominating set.

Let G=(V,E) **domination number is the smallest number of** $|V_1|$

$\ni V_1 \cup V_2 = V$ **,which** $b \in V_2$ **are adjacent of** $a \in V_1$

Vertex cover number

The smallest number of subset of vertices of G=(V,E),and all edge spanning be this set is E , vertex cover number =|the vertices of this subset|



(a)independence number $\beta(G)$ =? (b) clique number $\omega(G)$=?

(c) domination number $\sigma(G)$ (d) Vertex cover number

$\beta(G)=4$, $\omega(G)=4$, $\sigma(G)=2$ Vertex cover number=3

(a){a,c,e,g} which are all belong to V and two of then are not **adjacent**

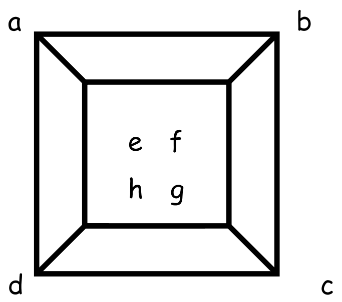
(b)pick V={b,d,f,g} if forms a K_4

(c) {b,e} or {f,c} be **smallest** dominating set

Because when {b,e} ,b`s adjacent ➔{a,f,g,s,c} , add e covers all certices of G

When {f,c} ,f`s adjacent ➔{a,b,g,d,e} , add c covers all certices of G

(d){b,f,d} b span {ba,bf,bg,bd,bc} ,f span {fa,fg,fd,fe} ,d span {dc,dg,de}



(a)independence number $\beta(G)$ =? (b) clique number $\omega(G)$=?

(c) domination number $\sigma(G)$ (d) Vertex cover number

$\beta(G)=4$, $\omega(G)=2$, $\sigma(G)=2$ Vertex cover number=4

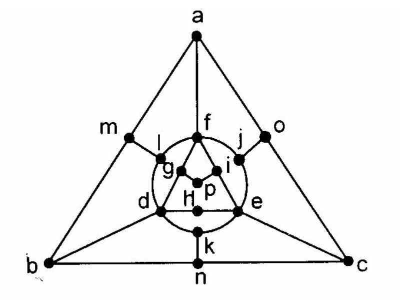
(a){a,c,f,h} or (b,d,e,g) which are all belong to V and two of then are not **adjacent**

(b)pick each edge of G form a K_2

(c) {a,g} be **smallest** dominating set

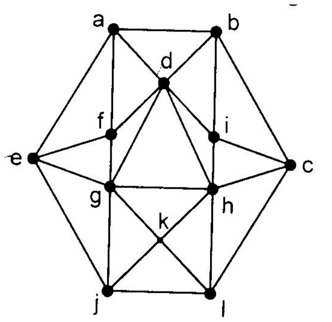
a`s adjacent ➔{b,d,e} , g`s adjacent ➔{c,f,h}

(d){a,c,f,h} or {b,d,e,g}

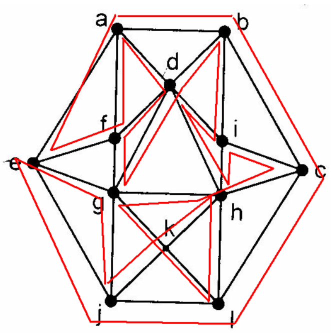
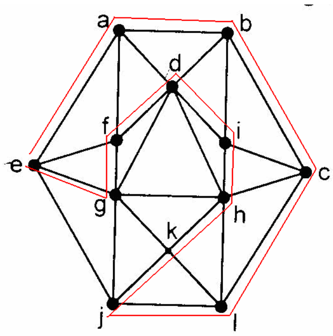


Do following has Euler path? Explain why

Odd degree vertice >3 No Euler path and Euler circuit

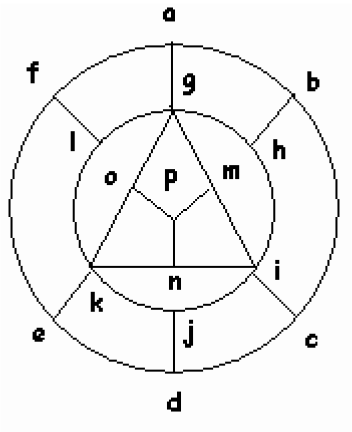


Do this graph has Euler circuit? Hamiltonian cycle?

 Euler circuit	 Hamiltonian cycle

小黃習題 6-01

Let $G=(V,E)$ be an undirected graph with $\mid V\mid=v,\mid E\mid=e$,and no loop , prove that $2e\le v^2-v$
$the\ \ maximum\ \ edge\ \ of\ \ graph \Rightarrow K_n$ $\mid E\mid=e=\dfrac{v^2-v}{2}\Rightarrow so\ \ 2e\le v^2-v$



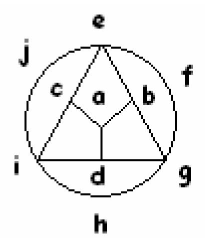
show this graph has no Hamiltonian path

點 g,點 k,點 i ,degree 皆為 5,因此各有 3 邊不會出現於 Hamiltonian path 中

點 b,點 d,點 f, 點 p ,degree 皆為 3, 因此各有 1 邊不會出現於 Hamiltonian path 中

27-3*3-4*1=14

本圖 16 個點,要造出 Hamiltonian path 至少要 15 個邊



Is these graph has Hamiltonian cycle ? why?

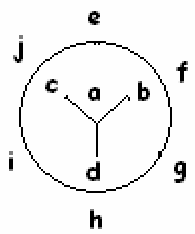
因為對於點 j 來說 邊 ij, 邊 je 為必存在 Hamiltonian cycle 中

因為對於點 f 來說 邊 fe, 邊 fj 為必存在 Hamiltonian cycle 中

因為對於點 g 來說 邊 gf, 邊 gh 為必存在 Hamiltonian cycle 中

對於點 e, 點 g, 點 i , 邊 ce, 邊 be, 邊 gb, 邊 gd, 邊 ic, 邊 id

去掉後 圖形如右

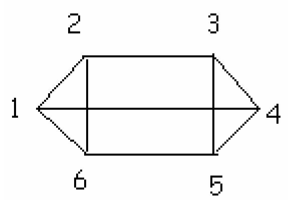


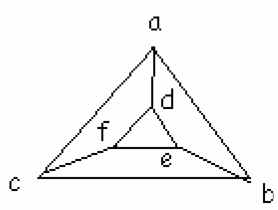
小黃習題 6-18

(a)show that the sum of the in-degree over all vertices is equal to the sum of the out-degree over all vertices in any directed graph (b)show that the sum of the squares of the in-degree over all vertices is equal to the sum of the squares of out-degree over all vertices in any directed complete graph
(a) $G=(V,E)$,對每個邊而言,會使某個點 indegree 累加 1,使某個點 out degree 累加 1 所以 $\sum_{v\in V} id(v)=\sum_{v\in V} od(v)$ (a) 由(a)知 $\sum_{v\in V} id(v)-od(v)=0$,又 directed complete graph $id(v)+od(v)=n-1$ $0=(n-1)0=(n-1)\sum_{v\in V}\left[id(v)-od(v)\right]=\sum_{v\in V}(n-1)\left[id(v)-od(v)\right]$ $=\sum_{v\in V}\left[id(v)+od(v)\right]\left[id(v)-od(v)\right]=\sum_{v\in V} id(v)^2-od(v)^2$ $\Rightarrow \sum_{v\in V} id(v)^2=\sum_{v\in V} od(v)^2$

小黃習題 6-23

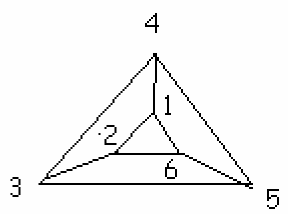
Prove or disprove that the following graphs are isomorphic



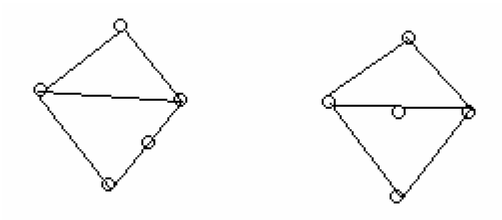


則其中一種對應關係爲 f(4)=a , f(3)=c ,f(5)=b ,f(1)=d , f(2)=f, f(6)=e

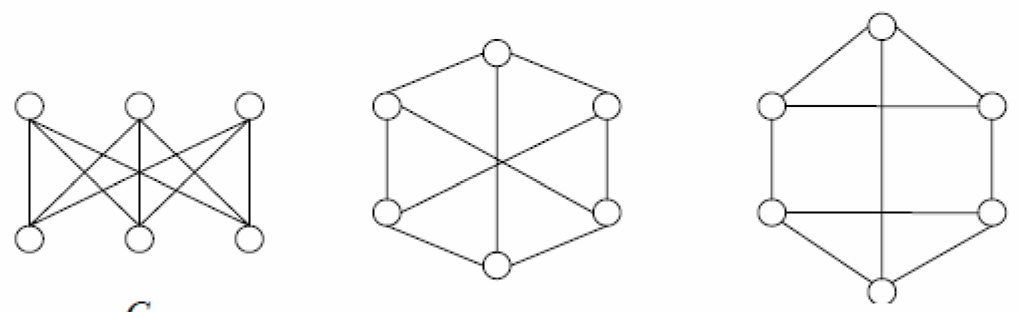
重繪如下



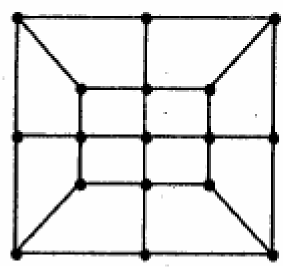
小黃習題 6-24

Prove or disprove that the following graphs are isomorphic 
Not isomorphic,左邊有一長爲 3 的 cycle ,右邊沒有

小黃習題 6-26

Prove or disprove that the following graphs are isomorphic  G_1 G_2 G_3
G_1, G_2 爲 $K_{3,3}$ 同構 ,G_3 有 1 個 K_3 subgraph , $K_{3,3}$ 不含 K_3 subgraph G_1, G_2 與 G_3 不同構

小黃習題 6-59

Show the following graph has no Hamiltonian cycle 	此 graph 爲 2-colorable 所以爲 bipartite ,bipartite2 個 disjoint set vertices 個數不一樣 所以不存在 Hamiltonian cycle

小黃習題 6-70　95 長庚資管

Let $G=(V,E)$ be an undirected connected loop-free graph.Suppose futher that G is planar and determines 53 regions.If,for some planar embedding of G,each region at least five edges in the boundary,prove that $\|V\|\geq 82$
令 N 為所有 region 的邊數總和 每個 region 至少由 5 個邊構成，且每個邊至少 2 個 region 相鄰 $5*53\leq N\leq 2\|E\|\Rightarrow\|E\|\geq 132.5\Rightarrow\|E\|\geq 133$ 由 Euler formula $\|V\|-\|E\|+\|Region\|=2\Rightarrow\|V\|-\|E\|+53=2$ $\Rightarrow\|V\|=\|E\|-51,\because\|E\|\geq 133,\therefore\|V\|\leq 82$

小黃習題 6-71

How many regions would there be in a plane graph with 10 vertices each of degree 3
$\sum_{v\in V} degree=2\|E\|=3*10\Rightarrow\|E\|=15$ $\|V\|-\|E\|+\|Region\|=2\Rightarrow 10-15+\|Region\|=2\Rightarrow\|Region\|=7$

小黃習題 6-72

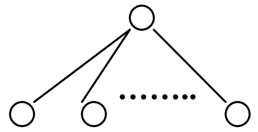
If planar graph with n vertices all of degree 4 has 10 regions,determine n
$\sum_{v\in V} degree=2\|E\|=4*n\Rightarrow\|E\|=2n$ $\|V\|-\|E\|+\|Region\|=2\Rightarrow n-2n+10=2\Rightarrow\|Region\|=8$

小黃習題 6-72

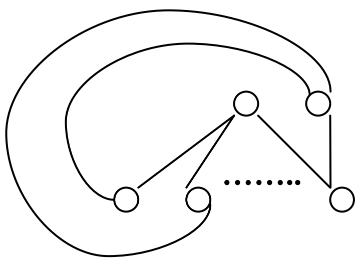
For what value of m and n is $K_{m,n}$ palanar ?

不失一般性假設 $m\leq n$

m=1 時



m=2 時

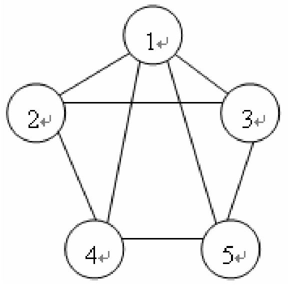


所以當 $m\geq 3,n\geq 3\Leftrightarrow K_{m,n}$ 為 nonplanar

91 清大通訊

Describe the necessary condition for isomorphic $G_1=(V_1,E_1),G_2=(V_2,E_2)$
$(a)\|V_1\|=\|V_2\|$ $(b)\|E_1\|=\|E_2\|$ (c)*degree sequence same* (d)*subgraph same* (e)2對應點距離相同 (f)2對應點連結性相同

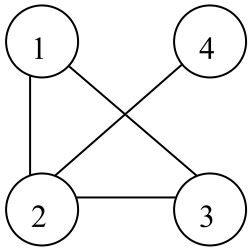
91　中正通訊

How many minimal spanning tree of 
$M=\begin{bmatrix}4&-1&-1&-1&-1\\-1&3&-1&-1&0\\-1&-1&3&0&-1\\-1&-1&0&3&-1\\-1&0&-1&-1&3\end{bmatrix}, cof(M_{1,1})=\begin{vmatrix}3&-1&-1&0\\-1&3&0&-1\\-1&0&3&-1\\0&-1&-1&3\end{vmatrix}$

91 東吳資料

(a) Find the distinct Hamiltonian circuit in K_n (b) Find the disjoint Hamiltonian circuit in K_n (c)Find the distinct Hamiltonian circuit in K_7 (d)Find the disjoint Hamiltonian circuit in K_7
$(a)\frac{(n-1)!}{2}\quad (b)\frac{(n-1)}{2}\quad (c)\frac{6!}{2}\quad (d)\frac{6}{2}=3$

92 師大資工

 find the number of walks of length 4 from vertex 3 to vertex 4
$A^4 = \begin{bmatrix} 7 & 6 & 6 & 4 \\ 6 & 11 & 6 & 2 \\ 6 & 6 & 7 & 4 \\ 4 & 2 & 4 & 3 \end{bmatrix}$

92 中正資工

How many MST of K_4 ?
$\#MST\ \ of\ \ K_n = n^{n-2} \Rightarrow K_4 = 4^{4-2} = 16$

92 中正通訊

How many MST of $K_{3,3}$?
$\#MST\ \ of\ \ K_{m,n} = m^{n-1}n^{m-1} \Rightarrow K_{3,3} = 3^{3-1}3^{3-1} = 81$

94 元智資工

Grid (a) number of vertices of $G_{m,n}$ (b)number of edges of $G_{m,n}$ (c)condition for Euler circuit exist (d)condition for Hamilton circuit exist (d) chromatic number of $G_{m,n}$
$(a)(m+1)(n+1)$ $(b)n(m+1)+m(n+1)$ $(c)only\ \ G_{1,1}$ $(d)m,n\ \ not\ \ all\ \ even$ $(e)2$

木易例題

find graph information in adjacent matrix If A, A^2, A^3 is given
(a) 相鄰矩陣第 i 列和=第 i 行和 爲 點 i 之 degree (b) 相鄰矩陣 A^n 中的 a_{ij} 爲 點 i 到點 j 長度爲 n 之相異 walk 個數 (c) 相鄰矩陣 A^2 中的 a_{ii} 爲 點 i 之 degree (d) 相鄰矩陣 A^3 中的 $\frac{a_{ii}}{2}$ 表示包含點 i 的三角形總數 (e) 相鄰矩陣 A^3 中的 $\frac{\sum a_{ii}}{6}$ 爲 G 之所含三角形總數

木易例題

Suppose G is a graph with vertex a,b,c,d,e,f $\begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix}$	(a) the sum of edge in G (b) the degree od each vertex (c) the number of loops (d) the length of longest path in G (e) th number of components in G (f) the distance from a to c
(a) 1`s sum of upper(lower) triangle=9 (b) a,b,c,d,e,f➔2,4,2,3,4,3 (c) {a,b,d} {b,d,e} {b,e,f} {f,c,e} {a,b,d,e} {b,e,f,c} {a,d,e,f,b} {b,d,e,c,f} {b,f,e,d} {a,b,f,c,e,d} (d)6 (e)1 (f)3	

木易例題

*condition of Hamiltonian graph

(a)if G is a complete graph K_n ,then G has a Hamiltonian circuit if $n \geq 3$

(b) if G is a hypercube graph Q_n ,then G has Hamiltonian circuit if $n \neq 1$

(C) if G is a wheel graph W_n always has Hamiltonian circuit length→ $n+1$

(d) if G is a hypercube graph Q_n ,then G has Hamiltonian path if $|m-n| \leq 1$

(e) if G is a hypercube graph Q_n ,then G has Hamiltonian circuit if $m = n \geq 2$

(f)undirected complete graph contain a Hamiltonian path and circuit

(g) directed complete graph K_n^* graph contains a Hamiltonian path

(h)if deg(x)+deg(y) ≥n-1,for all $x, y \in V, x \neq y$ then G has a Hamiltonian path

(i) if deg(x)+deg(y) ≥n,for all $x, y \in V, x \neq y$ then G has a Hamiltonian circuit

(j) if deg(v) $\geq \frac{n-1}{2}$ for all $v \in V$ then G has a Hamiltonian path

(k) if deg(v) $\geq \frac{n}{2}$ for all $v \in V$ then G has a Hamiltonian circuit

木易例題

* Hamiltonian graph

(a) complete graph K_n has $\frac{n!}{2}$ distinct Hamiltonian path

(b) complete graph K_n has $\frac{(n-1)!}{2}$ distinct Hamiltonian circuit

(c) wheel graph W_n , has n distinct Hamiltonian circuit

(d) bipartite graph $K_{m,n}$, has $\frac{n!(n-1)!}{2}$ distinct Hamiltonian circuit

bipartite graph $K_{m,n}$, has $(n!)^2$ distinct Hamiltonian path

(e) K_n has $\left\lfloor \frac{n-1}{2} \right\rfloor$ disjoint Hamiltonian circuit

(f) K_n has $\left\lfloor \frac{n}{2} \right\rfloor$ disjoint Hamiltonian path

97 中山資工

How many subgraph in K_6
$\sum_{i=1}^{6}\binom{6}{i}2^{\binom{i}{2}}$

97 清大資工 97 竹教大資工 95 中央資管

Prove that every planar graph has a node of degree at most 5
用矛盾證法: $G=(V,E), \forall v\in V, \deg(v)\geq 6$ 因爲 G 爲 connected 所以 $\lvert E\rvert \leq 3\lvert V\rvert -6$ 且 $6\lvert V\rvert = 2\lvert E\rvert$ 由上兩式 $6\lvert V\rvert \leq 6\lvert V\rvert -12$ 矛盾 所以必 $\exists v\in V, \deg(v)\leq 5$

97 彰師資工 95 師大資工

Chromatic number of $(1)K_n\quad(2)K_{m,n}\quad(3)C_n\quad(4)Q_n\quad(5)W_n$
$(1)K_n:n\quad(2)K_{m,n}:2\quad(3)C_n:\begin{cases}odd:3\\even:2\end{cases}\quad(4)Q_n:2\quad(5)W_n\begin{cases}odd:4\\even:3\end{cases}$

96 清大通訊 , 95 清大通訊

Nonisomorphic unrooted tree with 4 node? 5 node? 6 node?
Nonisomorphic rooted tree with 4 node? 5 node? 6 node?

vertex	4	5	6
Unroot tree	2	3	6
Root tree	4	9	20

Grimaldi exercise 11.3.25　96 交大生資

(a) find the maximum length of a trail in $(1)K_6 \quad (2)K_8 \quad (3)K_{10} \quad (4)K_{2n}, n \in Z^+$ (b) find the maximum length of a circuit in $(1)K_6 \quad (2)K_8 \quad (3)K_{10} \quad (4)K_{2n}, n \in Z^+$
(a) $(1)\binom{6}{2}-\frac{1}{2}(6-2)=13 \quad (2)\binom{8}{2}-\frac{1}{2}(8-2)=25$ $(3)\binom{10}{2}-\frac{1}{2}(10-2)=41 \quad (4)\binom{2n}{2}-\frac{1}{2}(n-2)=2n^2-2n+1$ (b) $(1)\binom{6}{2}-\frac{1}{2}(6)=12 \quad (2)\binom{8}{2}-\frac{1}{2}(8)=24$ $(3)\binom{10}{2}-\frac{1}{2}(10)=40 \quad (4)\binom{2n}{2}-\frac{1}{2}(n)=2n(n-1)$

95 高科資管

Prove Q_n has Hamilton cycle
$G_1=\{0,1\}, G_2=\{00,01,11,10\}$ $G_3=\{000,001,011,010,110,111,101,100\}$ G_n 爲 n bit 之 Gray code 每一 element 差一個 bit 即在 Q_n 有相連 G_n 即爲一組 Hamilton cycle

92 nsysu,ntnu

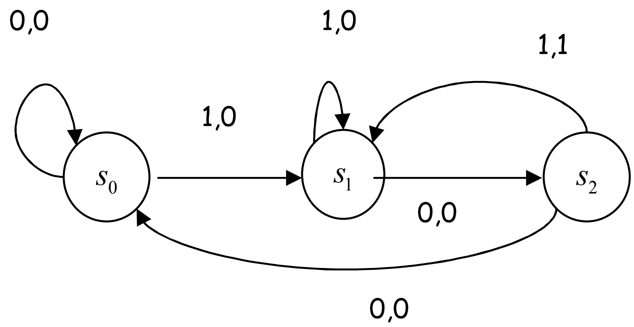
(a) K_n distinct Hamiltonian path (b) K_n distinct Hamiltonian circuit (c) K_n disjoint Hamiltonian path (d) K_n disjoint Hamiltonian circuit (e) $K_{m,n}$ distinct Hamiltonian path (f) $K_{m,n}$ distinct Hamiltonian circuit
$(a)\frac{n!}{2} \quad (b)\frac{(n-1)!}{2} \quad (c)\left\lfloor\frac{n-1}{2}\right\rfloor \quad (d)\left\lfloor\frac{n}{2}\right\rfloor \quad (e)\frac{n!(n-1)!}{2} \quad (f)(n!)^2$

(7)有限狀態機

Mealy machine

	state		output	
	0	1	0	1
s_0	s_0	s_1	0	0
s_1	s_2	s_1	0	0
s_2	s_0	s_1	0	1

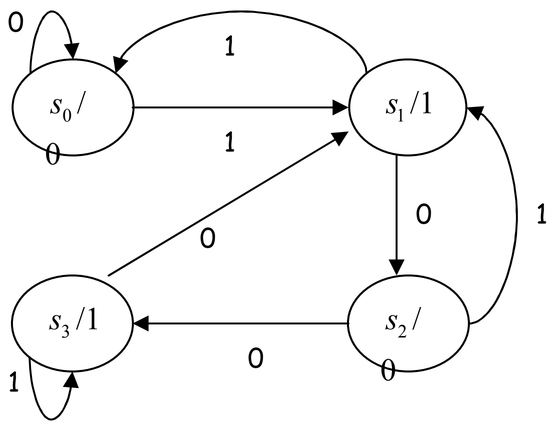
鍵號爲現在 Input,Output

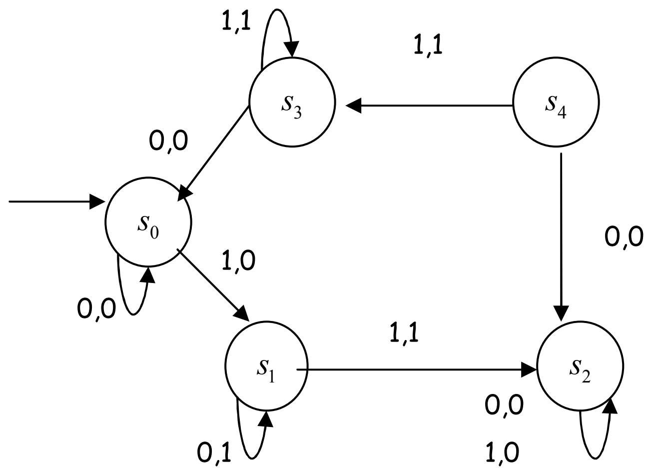


(1) Moore machine

	input		output
state	0	1	
s_0	s_0	s_1	0
s_1	s_2	s_0	1
s_2	s_3	s_1	0
s_3	s_1	s_3	1

鍵號爲現在 Input , output 看下一個 state 內容





(a) find the state table

(b) in which state input string 10010 produces the output 10000 ?

(a)

	state		output	
	0	1	0	1
s_0	s_0	s_1	0	0
s_1	s_1	s_2	1	1
s_2	s_2	s_2	0	0
s_3	s_0	s_3	0	1
s_4	s_2	s_3	0	1

(c) input 1 ➔ output 1 possible s_1 or s_3 or s_4

s_1 : input 10010 ➔ output 10000

s_3 : input 10010 ➔ output 10001

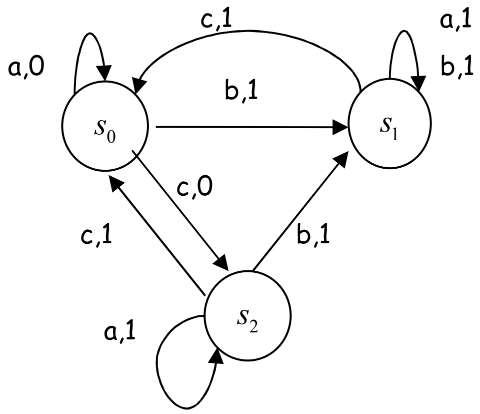
s_4 : input 10010 ➔ output 10001

s_1 be answer

(a)Draw the transition diagram of following

	a	b	c	a	b	c
s_0	s_0	s_1	s_2	0	1	0
s_1	s_1	s_1	s_0	1	1	1
s_2	s_2	s_1	s_0	1	1	1

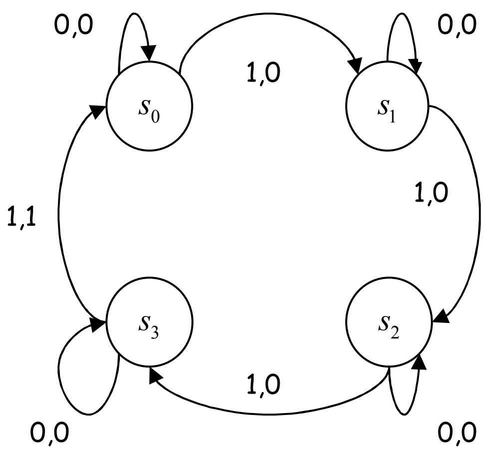
(b)input string "aabbcc" output string ?



input	a	a	b	b	c	c
State	s_0	s_0	s_1	s_1	s_0	s_2
output	0	0	1	1	1	0

95 成大資工

(d) find the state transition table



	X		Y	
	0	1	0	1
s_0	s_0	s_1	0	0
s_1	s_1	s_2	0	0
s_2	s_2	s_3	0	0
s_3	s_3	s_0	0	1

finite state machine 狀態化簡

	input		output
state	0	1	
A	B	F	0
B	A	F	0
C	G	A	0
D	H	B	0
E	A	G	0
F	H	C	1
G	A	D	1
H	A	C	1

Answer

令

$P_2 = \{\{A,B\},\{C,D\},\{E\},\{F\},\{G,H\}\}$

$\Rightarrow P_2 = \{\alpha,\beta,\gamma,\delta,\varepsilon\}$

	input		output
state	0	1	
α	α	δ	0
β	ε	α	0
γ	α	ε	0
δ	ε	β	1
ε	α	β	1

P_0 爲先把 output 0 分一類,1 分一類 ➔ $P_0 = \{\{A,B,C,D,E\},\{F,G,H\}\}$

(A,0)➔B,(B,0)➔A B,A 皆在 P_0 **中的** $\{A,B,C,D,E\}$ **中**

(A,1)➔F,(B,1)➔F F 皆在 P_0 **中的** $\{F,G,H\}$

(A,0)➔B,(C,0)➔G B,G 不在 P_0 中同一區塊

(A,0)➔B,(D,0)➔H B,H 不在 P_0 中同一區塊

(A,0)➔B,(E,0)➔A B,A 皆在 P_0 **中的** $\{A,B,C,D,E\}$ **中**

(A,1)➔F,(E,1)➔G F,G 皆在 P_0 **中的** $\{F,G,H\}$

{A,B,E}

(C,0)➔G, (D,0)=H **G,H 皆在** P_0 **中的** $\{F,G,H\}$

(C,1)➔A, (D,1)=B **B,A 皆在** P_0 **中的** $\{A,B,C,D,E\}$ **中**

{C,D}

(F,0)➔H, (G,0)=A ,A,H 不在 P_0 中同一區塊

(G,0)➔A, (H,0)=A **A 皆在** P_0 **中的** $\{A,B,C,D,E\}$

(G,1)➔D, (H,1)=C **D,C 皆在** P_0 **中的** $\{A,B,C,D,E\}$

所以 $P_1 = \{\{A,B,E\},\{C,D\},\{F\},\{G,H\}\}$

(A,0)➔B,(B,0)➔A B,A 皆在 P_1 **中的** $\{A,B,E\}$ **中**

(A,1)➔F,(B,1)➔F F 皆在 P_1 **中的** $\{F\}$

(A,1)➔F,(E,1)➔G F, G 不在 P_1 中同一區塊

(C,0)➔G, (D,0)=H G,H 皆在 P_1 **中的** $\{G,H\}$

(C,1)➔A, (D,1)=B B,A 皆在 P_1 **中的** $\{A,B,E\}$ **中**

(G,0)➔A, (H,0)=A A 皆在 P_1 **中的** $\{A,B,E\}$

(G,1)➔D, (H,1)=C D,C 皆在 P_0 **中的** $\{C,D\}$

所以 $P_2 = \{\{A,B\},\{C,D\},\{E\},\{F\},\{G,H\}\}$

95 台科資工,94 逢甲資工,93 中山資工

(a)Determine the minimal machine for the table

(b)find a minimal string distinguish s_4 and s_6

	X		Y	
	0	1	0	1
s_1	s_7	s_6	1	0
s_2	s_7	s_7	0	0
s_3	s_7	s_2	1	0
s_4	s_2	s_3	0	0
s_5	s_3	s_7	0	0
s_6	s_4	s_1	0	0
s_7	s_3	s_5	1	0
s_8	s_7	s_3	0	0

$P_1=\{\{s_1,s_3,s_7\},\{s_2,s_4,s_5,s_6,s_8\}\}$

$(s_1,0),(s_3,0)\ \ in\ \ \{s_1,s_3,s_7\},(s_1,1),(s_3,1)\ \ in\ \ \{s_2,s_4,s_5,s_6,s_8\}$

$(s_1,0),(s_7,0)\ \ in\ \ \{s_1,s_3,s_7\},(s_1,1),(s_7,1)\ \ in\ \ \{s_2,s_4,s_5,s_6,s_8\}$

$(s_2,0),(s_4,0)\ \ not\ \ in\ \ same\ \ subset\ \ of\ \ P_1$

$(s_2,0),(s_5,0)\ \ in\ \ \{s_1,s_3,s_7\},(s_2,1),(s_5,1)\ \ in\ \ \{s_1,s_3,s_7\}$

$(s_2,0),(s_6,0)\ \ not\ \ in\ \ same\ \ subset\ \ of\ \ P_1$

$(s_2,0),(s_8,0)\ \ in\ \ \{s_1,s_3,s_7\},(s_2,1),(s_8,1)\ \ in\ \ \{s_1,s_3,s_7\}$

$(s_4,0),(s_6,0)\ \ in\ \ \{s_2,s_4,s_5,s_6,s_8\},(s_4,1),(s_6,1)\ \ in\ \ \{s_1,s_3,s_7\}$

$P_2=\{\{s_1,s_3,s_7\},\{s_2,s_5,s_8\},\{s_4,s_6\}\}$

$(s_1,1),(s_3,1)\ \ not\ \ in\ \ same\ \ subset\ \ of\ \ P_2$

$(s_1,1),(s_7,1)\ \ not\ \ in\ \ same\ \ subset\ \ of\ \ P_2$

$(s_3,0),(s_7,0)\ \ in\ \ \{s_1,s_3,s_7\},(s_3,1),(s_7,1)\ \ in\ \ \{s_2,s_5,s_8\}$

$(s_4,0),(s_6,0)\ \ not\ \ in\ \ same\ \ subset\ \ of\ \ P_2$

$P_3=\{\{s_1\},\{s_3,s_7\},\{s_4\},\{s_2,s_5,s_8\},\{s_6\}\}$

let $P_3=\{\{s_1\},\{s_3,s_7\},\{s_4\},\{s_2,s_5,s_8\},\{s_6\}\}=\{\alpha,\beta,\gamma,\delta,\varepsilon\}$

	0	1	0	1
α	β	ε	1	0
β	β	δ	1	0
γ	δ	β	0	0
δ	β	β	0	0
ε	γ	α	0	0

P_1 中 s_4 , s_6 在同一個 subset

P_2 中 s_4 , s_6 在同一個 subset

P_3 中 s_4 , s_6 在不同 **subset**

所以 **000** 爲 **1** 分別 s_4 , s_6 的 **string**

94 雲科資工, 92 雲科資工,91 師大資教

Determine the minimal machine for the table

	X		Y	
	0	1	0	1
s_1	s_6	s_3	0	0
s_2	s_3	s_1	0	0
s_3	s_2	s_4	0	0
s_4	s_7	s_4	0	0
s_5	s_6	s_7	0	0
s_6	s_5	s_2	1	0
s_7	s_4	s_1	0	0

$P_1 = \{\{s_1, s_2, s_3, s_4, s_5, s_7\}, \{s_6\}\}$

$P_2 = \{\{s_1, s_5\}, \{s_2, s_3, s_4, s_7\}, \{s_6\}\}$

$P_3 = \{\{s_1, s_5\}, \{s_2, s_7\}, \{s_3, s_4\}, \{s_6\}\}$

$P_4 = \{\{s_1\}, \{s_5\}, \{s_2, s_7\}, \{s_3, s_4\}, \{s_6\}\}$

let $P_4 = \{\{s_1\}, \{s_5\}, \{s_2, s_7\}, \{s_3, s_4\}, \{s_6\}\} = \{\alpha, \beta, \gamma, \delta, \varepsilon\}$

	0	1	0	1
α	ε	δ	0	0
β	ε	γ	0	0
γ	δ	α	0	0
δ	γ	δ	0	0
ε	β	γ	1	0