※ 請使用 2B 鉛筆於答案卡上作答

多重選擇題，共20題，每題5分。
（每題五個選項，其中至少有一個是正確的答案。各題之選項獨立計分，每答對一個選項，得題分之五分之一；該題不作答者，得零分。每答錯一個選項時，倒扣題分的五分之一，每題倒扣至零分為止。）

1. Consider the following algorithm.

Input: a sequence $\{b_0, b_1, b_2, ..., b_{n-1}\}$ of boolean elements, all initially false.

1. For each $i = 2$ to $n - 1$, set b_i = true.
2. Let $p = 2$.
3. Repeat steps 4-5 while $p^2 < n$.
4. For each $j = 2$ to $(n - 1)/p$, set $b_{j \times p}$ = false.
5. Increment p until b_p is true.

Which of the following statement(s) is (are) true?

A. The first for loop (line 1) iterates $n - 2$ times.
B. The second outside loop (line 3) also iterates $n - 2$ times.
C. The complexity of this algorithm is $\Theta(n^3)$.
D. The complexity of this algorithm is $\Theta(n^2)$.
E. At the end, b_i is true if and only if i is prime.

2. Which of the following statement(s) is (are) true?

A. $o(g(n)) \supset O(g(n)) - \Theta(g(n))$
B. $\omega(g(n)) \subseteq \Omega(g(n)) - \Theta(g(n))$
C. $f(n) \in \Theta(g(n))$ if and only if $f(n) \in O(g(n))$ and $f(n) \in \Omega(g(n))$
D. $f(n) \in O(g(n))$ and $h(n) \in \Omega(g(n))$ imply $f(n) \in O(h(n))$
E. $f(n) \in \Omega(g(n))$ and $g(n) \in \Omega(h(n))$ imply $f(n) \in \Omega(h(n))$

3. Which of the following statement(s) is (are) true?

A. The infix expression of the postfix expression "x 4 + y 3 - / z +" is "$(x+4)/(y-3) + z$."
B. With postfix notation, there is no need for parentheses because there is no ambiguity.
C. Evaluating the postfix expression "7 2 + 5 8 4 / - *" returns 27.
D. If we use a stack to evaluate the postfix expression in C from left to right, a total of 8 pop operations are performed.
E. Continued. A total of 5 push operations are performed.

4. Which of the following statement(s) is (are) true?

A. Arrays are inefficient for implementing dynamic sorted lists because the insert and delete operations require moving half the elements, on average.
B. Linked lists are efficient for implementing dynamic sorted lists and they also provide direct access.
C. A stack implements the last-in-first-out (LIFO) protocol; a queue implements the first-in-first-out (FIFO) protocol.
D. A queue is also a LILO (last-in-last-out) structure.
E. One can implement a stack with only two queues. The run-time complexity for the add() and remove() methods are both $\Theta(n^2)$.

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5. Which of the following statement(s) is (are) true?
   A. With closed addressing, the hash table's storage cells are defined as multi-record buckets, usually implemented as linked lists.
   B. With open addressing, a special algorithm is used to locate an open (empty) cell to store the colliding record. This is also called separate chaining.
   C. Rehashing is a way to overcome primary clustering, which is when records begin to accumulate in long strings of adjacent positions instead of being uniformly distributed throughout the table.
   D. Quadratic probing and double hashing reduce primary clustering.
   E. Linear probing suffer from secondary clustering, which occurs when different keys hash to the same index, but quadratic probing does not.

6. Consider the following code segment. (Assume that x and n are both positive and won't cause overflow.)

```
long foo(long x, long n){
    long f;
    if(n%2 == 0) f = 1 else f = x;
    if(n < 2) return f;
    return f*foo(x*x,n/2);
}
```

   Which of the following statement(s) is (are) ture?
   A. foo(x,n) returns x^n.
   B. foo(x,n) returns n^x.
   C. This recursive method runs in $\Theta(\log n)$ time.
   D. This recursive method runs in $\Theta(\log x)$ time.
   E. This recursive method runs in $\Theta(n \log n)$ time.

7. The following code segment is a recursive version of the Fibonacci number.

```
long f(long n){
    if(n < 1) return 0;
    if(n < 3) return 1;
    return f(n-1) + f(n-2);
}
```

   Which of the following statement(s) is (are) true?
   A. $f(10) = 55$
   B. $f(8)$ generates 18 recursive calls.
   C. The number of recursive calls grows exponentially with n.
   D. $f(0) + f(1) + f(2) + \ldots + f(n) = f(n+2) - 1$
   E. One reason that the recursive Fibonacci function is very inefficient is that the same value is computed many times.

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Graph Notations for Questions 8—20

In questions 8 to 20, a graph is define as $G = (V, E)$, where

$V = \{1,2,3,\ldots v\}$,

and

$E \subseteq \{(u,v) \mid \forall\, u,v \in V, u \neq v\}$.

Note that graphs here do not contain self-circular edges, *e.g.*, (u,u), nor multiple edges between two nodes, *i.e.*, at most one entry (u,v) in E for each distinct u and v.

8. Which of the following statement(s) about "tree and graph" is (are) true?
A. Any undirected graph without cycle is a tree.
B. Any undirected graph with n nodes and $(n-1)$ edges, where $n \geq 1$, is a tree.
C. Any undirected graph where every node connects to at least one other node is a tree.
D. Any connected undirected graph is a tree.
E. None of the above.

9. Which of the following statement(s) about "binary search tree (BST)" is (are) true?
A. In a BST, every subtree is also a BST.
B. In a binary tree, if both the left and right subtrees of the root are BSTs, this tree is a BST.
C. Deleting a node from a BST with n nodes takes at most $\Theta(\log n)$ time.
D. In-order traversal of a BST visits all nodes in exactly ascended order.
E. None of the above.

10. Which of the following statement(s) about "traversal on BST" is (are) true?
A. Given the visiting sequence of pre-order traversal, the corresponding BST is unique.
B. Given the visiting sequence of in-order traversal, the corresponding BST is unique.
C. Given both visiting sequences of pre-order traversal and post-order traversal, the corresponding BST is unique.
D. Given both visiting sequences of pre-order traversal and in-order traversal, the corresponding BST is unique.
E. Given both visiting sequences of in-order traversal and post-order traversal, the corresponding BST is unique.

11. Which of the following statement(s) about "AVL tree" is (are) true?
A. In an AVL tree, every subtree is also an AVL tree.
B. In a binary tree, if both the left and right subtrees of the root are AVL trees, this tree is an AVL tree.
C. The height of an AVL tree with n nodes is at most $\Theta(\log n)$.
D. After inserting a new node to an AVL tree, at most two rotations are needed to re-balance the tree.
E. None of the above.

12. Define $n(h)$ as the number of nodes of an AVL tree of height h, and define $n(0) = 1$ (Height of the AVL tree with only 1 node is 0). Which of the following statement(s) is (are) true?
A. $5 \leq n(2) \leq 7$.
B. $\max(n(h)) = 2\times\max(n(h\text{-}1)) + 1$.
C. $\min(n(h)) = 2\times\min(n(h\text{-}1)) + 1$.
D. $\max(n(h)) = \Theta(2^h)$.
E. $\min(n(h)) = \Theta(h)$.

13. Which of the following statement(s) about "min heap" is (are) true?
   A. In a min heap, every subtree is also a min heap.
   B. In the worst case, initializing a min heap with n elements takes $\Theta(\log n)$ time.
   C. In the average case, initializing a min heap with n elements takes $\Theta(\log n)$ time.
   D. In the worst case, popping an element from a min heap with n elements takes $\Theta(\log n)$ time.
   E. None of the above.

14. Initially, each set contains one unique element. Which of the following statement(s) about "the union-and-find problem" is (are) true?
   A. To have a set with n elements, at least $(n\text{-}1)$ unions are required.
   B. In the worst case, finding an element in a set of size n takes $\Theta(\log n)$ time
   C. If we always make the shorter tree as a subtree of the taller during union, the height of the tree representing a set with n elements is at most $\Theta(\log n)$.
   D. u unions and f finds can be done in $\Theta(u+f)$ time.
   E. None of the above.

15. Which of the following statement(s) about "red-black trees" is (are) true?
   A. In a red-black tree, every subtree is also a red-black tree.
   B. In a red-black tree, a red node cannot have a red child.
   C. In a red-black tree, every path from the root to any external (or leaf node) contains equal number of black nodes.
   D. The height of a red-black tree with n element is at most $\Theta(\log n)$.
   E. After inserting a new node to a red-black tree, at most two rotations are needed to re-balance the tree.

16. For a connected undirected graph with v vertices and e edges, which of the following statement(s) is (are) true?
   A. $v\text{-}1 \le e \le (v^2\text{-}v)/2$.
   B. The time complexity of breadth-first-search is $\Theta(v^2)$ given its adjacency matrix.
   C. Its breadth-first spanning tree is unique.
   D. Its depth-first spanning tree always has a smaller branch factor than its breadth-first spanning tree.
   E. The branch factor of its spanning tree can be as large as its degree.

17. Suppose sparse matrices are on average of size n by n with n non-zero entries, and we store a sparse matrix by several lists: one list per row. Which of the following statement(s) is (are) true?
   A. One such sparse matrix takes $\Theta(n)$ space.
   B. Accessing an entry takes $\Theta(n)$ time on average.
   C. Accessing an entry takes $\Theta(n^2)$ time in the worst case.
   D. In the worst case, the multiplication of two such sparse matrices takes $\Theta(n^2)$ time.
   E. In the best case, the multiplication of two such sparse matrices takes $\Theta(n^2)$ time.

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18. In C/C++, a 2D array is implemented by using the array-of-arrays representation. Assume that each pointer and integer takes two bytes. Now we have an array **int** $x[2][3]$, where the index starts from 0 (so the last valid element is x[1][2]), and the address of x is 108. Based on the following memory table, which of the following statement(s) is (are) true?

Address	100	102	104	106	108	110	112	114	116	118
Content	102	116	114	104	116	108	110	106	110	100

A. The content of $x[0]$ is 116.
B. The content of $x[0][0]$ is 108.
C. The content of $x[1][2]$ is 116.
D. The address of $x[1][2]$ is 112.
E. Memory cell at address 106 is not used by the array.

19. Suppose that we have a hash table with 19 buckets, where each bucket holds only one key-element pair. The hash function is $h(key) = (key \text{ modulo } 19)$. Consider the following quadratic probing scheme: first we check bucket no. $h(key)$. When collision, next we check bucket no. $(h(key)+1)$ modulo 19, $(h(key)-1)$ modulo 19, $(h(key)+4)$ modulo 19, $(h(key)-4)$ modulo 19, …, $(h(key)+t^2)$ modulo 19, $(h(key)-t^2)$ modulo 19,…, in order. Which of the following statement(s) is (are) true?

A. This scheme will check every bucket eventually with $t \le 9$.
B. If the hash table contains 14 elements, an unsuccessful look-up checks less than 3 buckets on average.
C. A successful look-up takes $\Theta(1)$ time on average when the hash table is half-full.
D. A successful look-up takes $\Theta(\log n)$ time in the worst case when the hash table is half-full with n pairs.
E. Deleting an element given its key can be done in $\Theta(1)$ time in the worst case.

20. Define $s(n)$ as the number of nodes checked for looking up node n. As an example, for a tree with only one node 13, $s(13) = 1$. Define the efficiency of a BST with n nodes as:
$E(n) = \sum_i s(i)$, where i is a node in the BST. Which of the following statement(s) is (are) true?

A. $E(n)$ can be as large as $\Theta(n^2)$.
B. $E(n)$ can be as small as $\Theta(n)$.
C. $\min[E(5)] = 11$.
D. For a BST with n nodes and the smallest $E(n)$, s of the smallest node is at least $\Theta(\log n)$.
E. For a BST with n nodes and the smallest $E(n)$, s of the smallest node is at most $\Theta(n)$.