國立臺灣大學97學年度碩士班招生考試試題

1—10 題為填充題，請依題號將答案填寫於答案卷上。

1. $\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}\begin{bmatrix}-1 & 1\\ 0 & 1\end{bmatrix}=$ ________ (5%)

2. $\begin{bmatrix}3 & 1 & 4\\ 2 & 2 & 3\\ 2 & 1 & 3\end{bmatrix}^{-1}=$ ________ (5%)

3. $\det(2I_n)=$ ________ (5%)

4. If $A\in\mathbf{R}^{8\times 7}$ and $rank(A)=3$, then $nullity(A^T)=$ ________ (5%)

5. If $\mathbf{u}=\begin{bmatrix}1\\ -1\\ -1\\ 1\\ 1\end{bmatrix}$, then $(I_5+2\mathbf{u}\mathbf{u}^T)(I_5+\mathbf{u}\mathbf{u}^T)^{-1}\mathbf{u}=$ ________ (5%)

6. If $\mathbf{u},\mathbf{v}\in\mathbf{R}^n$ and $rank(I_n+\mathbf{u}\mathbf{v}^T)=\begin{cases}n & if\ \mathbf{v}^T\mathbf{u}\neq[c]\\ d & if\ \mathbf{v}^T\mathbf{u}=[c]\end{cases}$

   then $(c,d)=$ ______ (5%)

7. The set $\{rank(the\ adjoint\ of\ A)\mid A\in\mathbf{R}^{7\times 7}\}$ contains ______ integers (5%)

8. If $\lambda_1,\lambda_2,\lambda_3,\lambda_4,\lambda_5$ are all the eigenvalues of the matrix

$$\begin{bmatrix}4 & -1 & 0 & 0 & 0\\ -1 & 3 & -1 & 0 & 0\\ 0 & -1 & 3 & -1 & 0\\ 0 & 0 & -1 & 3 & -1\\ 0 & 0 & 0 & -1 & 3\end{bmatrix}.$$

   Then $\lambda_1^2+\lambda_2^2+\lambda_3^2+\lambda_4^2+\lambda_5^2=$ ______ (5%)

9. If $S=\{\frac{|x+y-z|+|x-y+z|+|x+y+3z|}{|3x+y+4z|+|2x+2y+3z|+|2x+y+3z|}\mid x,y,z\in\mathbf{R}\ \&\ x^2+y^2+z^2\neq 0\}$,

   then the largest number in S is ________ (5%)

10. If $S=\{\frac{4x^2+2y^2+2z^2+4xy+2yz}{x^2+2y^2+2z^2+2xy+2yz}\mid x,y,z\in\mathbf{R}\ \&\ x^2+y^2+z^2\neq 0\}$, then the

    smallest number in S is ______ (5%)

見背面

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11. Suppose that $(K, \cdot, +)$ is a Boolean algebra. Prove that $\overline{(\overline{a})} = a$ for every $a \in K$, where $\overline{a}$ is the complement of a. (10%)

12. Suppose that $(R, +, \cdot)$ is a ring and $S \subset R$ is not empty. Prove that $(S, +, \cdot)$ is a subring of R
    (a) if for $a, b \in S$, $a+(-b) \in S$ and $a \cdot b \in S$, where $-b$ is the additive inverse of b; (10%)
    (b) if S is finite and for $a, b \in S$, $a+b \in S$ and $a \cdot b \in S$. (10%)

13. For each positive integer $n \geq 2$, define $\phi(n)$ to be the number of positive integers m with $\gcd(n, m)=1$, where $1 \leq m < n$. For example, $\phi(3)=2$, $\phi(4)=2$, and $\phi(p)=p-1$ if p is prime. Prove that if $n = p_1^{e_1} \times p_2^{e_2} \times p_3^{e_3}$, where p_1, p_2, p_3 are three distinct primes and e_1, e_2, $e_3 \geq 1$ are integers, then $\phi(n) = n \times (1-\frac{1}{p_1})(1-\frac{1}{p_2})(1-\frac{1}{p_3})$. (10%)

14. Suppose that $G=(V, E)$ is an undirected graph, where $V=\{v_1, v_2, \ldots, v_n\}$ and $n \geq 2$. For $1 \leq i \leq n$, let d_i be the degree of v_i. Prove that if $d_i + d_j \geq n-1$ for all v_i, $v_j \in V$ and $v_i \neq v_j$, then G is connected. (10%)

試題隨卷繳回