

【不可使用計算機】*作答前請先核對試題、答案卷(試卷)與准考證之所組別與考科是否相符!!

每一題組全對才計分(5 分)，答錯倒扣 1 分。每一子題均為單選題，總共有 20 個題組，含 50 小題。

1. Please order the following functions in an increasing order asymptotically.

請使用答案卡作答

$N, \text{SQRT}(N), N^{1.5}, N^2, N \log N, N \log(\log N), N \log^2 N, N \log(N^2), 2^N, 29, N^2 \log N, N^3$

(1). What is the function in the 5th place?

(A) $\text{SQRT}(N)$ (B) $N \log(\log N)$ (C) $N \log(N^2)$ (D) $N^{1.5}$ (E) $N \log N$

(2). What is the function in the 9th place?

(A) $N \log^2 N$ (B) $N \log N$ (C) $N \log(N^2)$ (D) N^2 (E) $N^2 \log N$

(3). What is the function in the 12th place?

(A) $N \log(N^2)$ (B) N^2 (C) N^3 (D) 2^N (E) $N^2 \log N$

2. Given the following keys 89, 18, 49, 58 and 69, please write down the result using the hash function $h(X) = (X \bmod 10)$ with the following

(4). Open addressing hash table using linear probing ($F(i) = i$). What is the number in bucket 2?

(A) 69 (B) 18 (C) 49 (D) 58 (E) None

(5). Open addressing hash table using quadratic probing ($F(i) = 2 \cdot i^2$). What is the number in bucket 1?

(A) 69 (B) 18 (C) 49 (D) 58 (E) None

3. The following codes are for sorting.

```
template <class T>
```

```
void XSort(T *a, const int n)
```

```
{
    for (int j=2; j<=n; j++) {
        T temp=a[j];
        NCTUCS(temp, a, j-1);
    }
}
```

```
template <class T>
```

```
void NCTUCS(const T& e, T *a, int i)
```

```
{
    a[0]=e;
    while (e < a[i])
    {
        a[i+1]=a[i];
        i--;
    }
    a[i+1]=e;
}
```

(6). Assume that $n=5$, and the input sequence is 5, 4, 3, 2, 1. If $j=3$, what's the value in $a[3]$?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

(7). For the same setting above, if $j=4$, what's the value in $a[3]$?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

(8). According to the codes provided, what's the main concept of XSort?

(A) Insertion sort (B) Selection sort (C) Bubble sort (D) Quick sort (E) Radix sort

4. Assume that a max heap is implemented in an array and we input the set of numbers as "31, 41, 59, 26, 53, 58, 97".

(9). What's the final sequence of numbers in the array?

(A) (31, 58, 41, 26, 59, 53, 97) (B) (97, 59, 58, 53, 41, 31, 26) (C) (26, 31, 41, 53, 58, 59, 97) (D) (97, 53, 59, 26, 41, 58, 31) (E) (41, 59, 58, 31, 97, 53, 26)

(10). Suppose that we perform "DeleteMax" that removes the maximal number from the max heap and put this number in the last element of the array. What's the sequence of numbers in the array after we execute the first DeleteMax operation.

(A) (58, 41, 26, 59, 53, 31, 97) (B) (59, 53, 58, 26, 41, 31, 97) (C) (59, 58, 53, 41, 31, 26, 97) (D) (53, 58, 59, 41, 26, 31, 97) (E) (26, 31, 41, 53, 58, 59, 97)

(11). By iteratively using DeleteMax operation in the max heap, we could sort a set of numbers in an increasing order, which is referred to as "heap sort". What's the time complexity of heap sort?

(A) $O(N)$ (B) $O(\log N)$ (C) $O(N \log N)$ (D) $O(N \log(\log N))$ (E) $O(N^2)$

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5. AVL tree is a balanced tree structure. In the following, given a sequence of numbers, please build up one AVL tree and then traverse AVL tree built in a pre-order manner.

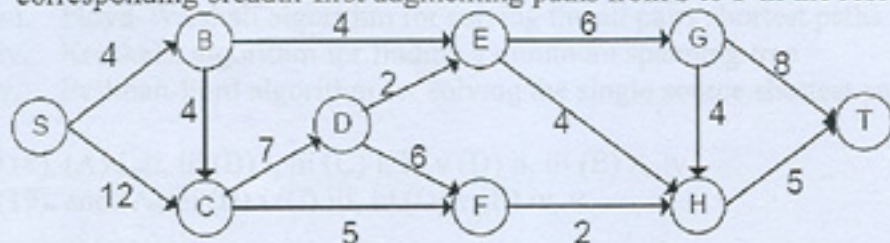
(12). What's the pre-order sequence after inserting "2, 1, 4, 5" into an initially empty AVL tree?

(A) (1, 2, 4, 5) (B) (2, 4, 1, 5) (C) (4, 2, 1, 5) (D) (5, 2, 1, 4) (E) (2, 1, 4, 5)

(13). What's the pre-order sequence after inserting "9, 3, 6, 7" into the above AVL tree?

(A) (4, 2, 1, 3, 6, 5, 9, 7) (B) (3, 2, 1, 4, 6, 9, 5, 7) (C) (1, 2, 3, 4, 5, 6, 7, 9)
(D) (9, 7, 5, 6, 3, 2, 1, 4) (E) (6, 5, 1, 9, 2, 3, 4, 7)

6. Use Edmonds-Karp algorithm to improve Ford-Fulkerson algorithm in the following network to find corresponding breadth-first augmenting paths from S to T in the residual network.



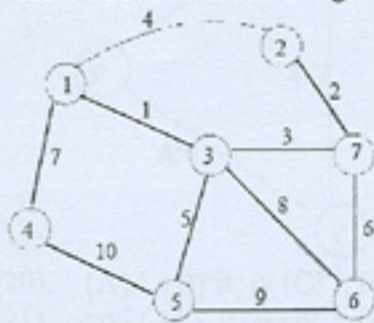
(14). For the first and second iterations

(A) S-B-E-G-T, S-B-C-F-H-T
(B) S-B-E-H-T, S-C-D-F-H-T
(C) S-B-E-G-T, S-C-F-H-T
(D) S-B-E-H-T, S-C-F-H-T
(E) S-B-C-D-E-G-T, S-C-F-H-T

(15). and for the third iteration

(A) S-C-D-F-H-T
(B) S-C-D-E-G-T
(C) S-C-D-F-H-T
(D) S-B-C-D-E-H-T
(E) S-B-C-F-H-T

7. Consider the following graph. Which edges, and in which order, are selected by Kruskal's algorithm? How about use Prim's algorithm if it starts at vertex 3? Select from the following 2 questions.



(16).

(A) (1,2), (2,7), (7,3), (3,6), (1,4), (3,5)
(B) (1,3), (2,7), (3,7), (3,5), (7,6), (1,4)
(C) (1,3), (3,7), (2,7), (3,6), (1,4), (3,5)
(D) (1,2), (2,7), (7,3), (1,4), (3,6), (3,5)
(E) (1,2), (3,7), (2,7), (1,3), (1,4), (3,5)

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(17).

- (A) (1,3), (7,2), (3,7), (3,5), (7,6), (1,4)
 (B) (1,3), (2,7), (3,5), (3,6), (1,4), (7,6)
 (C) (1,2), (3,7), (2,7), (3,6), (1,4), (3,5)
 (D) (1,3), (3,7), (7,2), (3,5), (7,6), (1,4)
 (E) (1,2), (3,7), (2,7), (1,3), (1,4), (3,5)

8. Which of following algorithms are greedy?

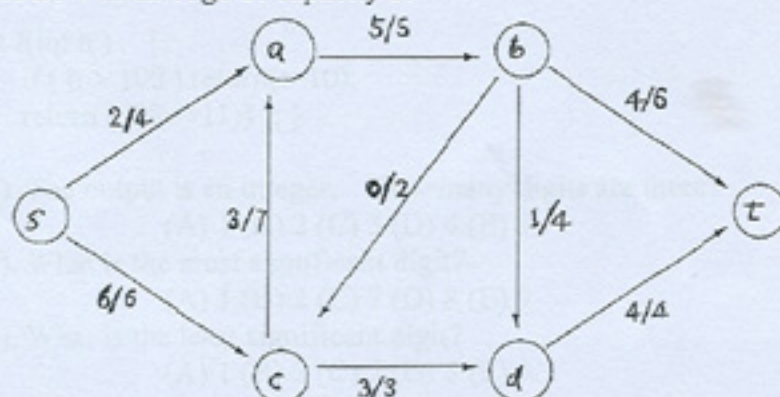
- i. Prim's algorithm for finding a minimum spanning tree
 ii. Dijkstra's algorithm for solving the single source shortest path
 iii. Floyd-Warshall algorithm for solving the all pairs shortest paths
 iv. Kruskal's algorithm for finding a minimum spanning tree
 v. Bellman-Ford algorithm for solving the single source shortest path

(18). (A) i, ii, iii (B) i, iii (C) i, ii, v (D) ii, iii (E) ii, iv

(19). and (A) iii (B) i (C) iii, iv (D) v (E) iv, v

9. Which of the followings are true?

- i. The Bellman-Ford algorithm is not suitable if the input graph has negative-weight edges.
 ii. Given a weighted, directed graph $G = (V, E)$ with no negative-weight cycles, the shortest path between every pair of vertices can be determined in $O(V^3)$ worst-case time.
 iii. There exists a polynomial time algorithm that finds the value of an s - t minimum cut in a directed graph.
 iv. Depth-first search of a graph is asymptotically faster than breadth-first search.
 v. The figure below describes a flow assignment in a flow network. The a/b notation describes a units of flow in an edge of capacity b .



(20). (A) i, (B) ii, iv (C) i, iii (D) ii (E) i, ii, iii

(21). and (A) i, v (B) iv, v (C) iv (D) iii, v (E) iii, iv

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10. Which of the followings are false?

- Let P be a shortest path from some vertex s to some other vertex t in a graph. If the weight of each edge in the graph is increased by one, P remains a shortest path from s to t .
- Let $G = (V, E)$ be a weighted graph and let M be a minimum spanning tree of G . The path in M between any pair of vertices u and v must be a shortest path in G .
- A graph $G = (V, E)$ is *bipartite* if the vertices V can be partitioned into two subsets L and R , such that every edge has one vertex in L and the other in R . Every tree is a bipartite graph.
- A maximum matching in a bipartite graph can be found using a maximum-flow algorithm.
- For any network and any maximal flow on this network there always exists an edge such that increasing the capacity on that edge will increase the network's maximal flow.

(22). (A) i, ii (B) ii, iv (C) i, iii (D) iii (E) i, ii, iii

(23). and (A) iv (B) iv, v (C) v (D) i, iv (E) iii, v

11. Consider the following statements, which are true? Select the answers from questions (24) and (25).

- Dynamic programming always provides polynomial time algorithms.
- Huffman coding for compression is a typical Dynamic programming algorithm.
- Dynamic programming uses tables to design algorithms.
- Optimal substructure is an important element of Dynamic programming algorithm.
- The single source shortest path problem has the property of optimal substructure.

(24). (A) i, ii (B) i, iii (C) iii, iv (D) i, iii, iv (E) i, v

(25). and (A) ii (B) ii, iii (C) i, vi (D) ii, v (E) v

12. Consider the following program.

```
void main() {
    printf("%4d", f(95)); }
```

```
int f(int n) {
    if ( n > 100 ) return(n-10);
    return ( f(f(n+11)) ); }
```

(26). The output is an integer. How many digits are there?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

(27). What is the most significant digit?

(A) 1 (B) 2 (C) 7 (D) 8 (E) 9

(28). What is the least significant digit?

(A) 1 (B) 2 (C) 7 (D) 8 (E) 9.

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13. Given n items of known weights $w[1], \dots, w[n]$ and values $v[1], \dots, v[n]$ and a knapsack of capacity W , find the most valuable subset of the items that fit into the knapsack. This is the so called Knapsack Problem. Let $V[i, j]$ be the value of an optimal solution by selecting from the first i items with capacity j .
- (29). If j is smaller than $w[i]$, then $V[i, j]$ is equal to which of the followings.
 (A) $V[i, j-1]$ (B) $w[i]$ (C) $V[i-1, j+1]$ (D) $w[i-1]$ (E) $V[i-1, j]$.
- (30). If j is not smaller than $w[i]$, then $V[i, j]$ is equal to which of the followings.
 (A) $\max\{V[i, j-1], v[i]\}$
 (B) $\max\{V[i-1, j], V[i-1, j-w[i]]+w[i]\}$
 (C) $\max\{V[i-1, j+1], v[i]+V[i-1, j-w[i]]\}$
 (D) $\max\{V[i-1, j-1], V[i-1, j-w[i]]+v[i]\}$
 (E) $\max\{V[i-1, j], V[i-1, j-w[i]]+v[i]\}$.
14. Given a weighted graph and two vertices s and t , we want to find a longest path from s to t . Which of the following statements are WRONG about this problem? Select the correct answers from the following 2 questions.
- This problem has a polynomial time algorithm.
 - This problem can be solved by transforming it into shortest path problem.
 - This problem can be solved by using Dynamic Programming.
 - For a connected weighted graph the longest path has at least 3 edges.
 - For a graph with n vertices, if the longest path has $n-1$ edges, then it passes through each vertex exactly once.
- (31). (A) i, ii (B) i, iii (C) ii, iii (D) iii, iv (E) ii, v.
 (32). (A) iii, v (B) v (C) iv (D) ii, iv, v (E) iii.
15. Given 5 matrices with dimensions, 12×5 , 5×10 , 10×2 , 2×5 and 5×4 , what is the minimum number of scalar multiplications to multiply these 5 matrices?
- (33). How many digits does the answer have?
 (A) 2 (B) 3 (C) 4 (D) 5 (E) 6.
- (34). The most significant two digits are:
 (A) 10 (B) 11 (C) 32 (D) 35 (E) 36.
- (35). The last digit of the answer is:
 (A) 5 (B) 6 (C) 7 (D) 8 (E) 9.

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16. Please finish the definition of Big-O

 $f(n)=O(g(n))$ if and only if there exist (36) c and n_0 such that (37) for all n (38).(36). (A) positive integers (B) positive constants (C) constants (D) integers
(E) real numbers.(37). (A) $f(n)<cg(n)$ (B) $f(n)\leq cg(n)$ (C) $f(n)>cg(n)$ (D) $f(n)\geq cg(n)$
(E) $g(n)<f(n)<cg(n)$.(38). (A) $n\geq c$ (B) $n\leq c$ (C) $n\geq n_0$ (D) $n\leq n_0$ (E) $n_0\leq n\leq c$.

17. Please solve the following recurrence relations.

(39). $T(n)=2T(n/2)+n$, $T(1)=1$ (A) $\Theta(\log n)$ (B) $\Theta(n)$ (C) $\Theta(n\log n)$ (D) $\Theta(n\log^2 n)$ (E) $\Theta(n^2)$ (40). $T(n)=4T(n/2)+n$, $T(1)=1$ (A) $\Theta(\log n)$ (B) $\Theta(n)$ (C) $\Theta(n\log n)$ (D) $\Theta(n\log^2 n)$ (E) $\Theta(n^2)$ (41). $T(n)=3T(n/4)+n\log n$, $T(1)=1$ (A) $\Theta(\log n)$ (B) $\Theta(n)$ (C) $\Theta(n\log n)$ (D) $\Theta(n\log^2 n)$ (E) $\Theta(n^2)$

18. Please finish the following implementation of circular queue.

class Queue

{ private:

int orz, a, b;

int data[3];

public:

Queue()

{ orz=0;

a=-1;

b=0; }

void Enqueue(int x)

{ if (IsFull())

{ cout<<"Queue is full"<<endl;

return; }

a=(a+1)%3;

data[a]=x;

if ((a+1)%3==b) orz=1; }

int Dequeue(void)

{ int x;

if (IsEmpty())

{ cout<<"Queue is empty"<<endl;

return -1; }

orz=0;

x=data[(42)];

b%=3;

return x; }

int IsEmpty(void)

{ return (orz==0 && ((a+1)%3==b) ? (43); }

int IsFull(void)

{ return (orz==1) ? (44); }

};

(42). (A) b (B) b++ (C) ++b (D) b-- (E) --b

(43). (A) 1:0 (B) 0:1 (C) 2:3 (D) 3:2 (E) 1:2

(44). (A) 1:0 (B) 0:1 (C) 2:3 (D) 3:2 (E) 1:2

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19. Consider the following failure function and a string "abcabcacab".

$$f(j) = \begin{cases} \text{largest } k < j \text{ such that } p_0 p_1 \cdots p_k = p_{j-k} p_{j-k+1} \cdots p_j & \text{if such a } k \geq 0 \text{ exists} \\ -1 & \text{otherwise} \end{cases}$$

Please answer the following questions.

(45). $f(5) = ?$

- (A) -1 (B) 0 (C) 1 (D) 2 (E) 3

(46). $f(6) = ?$

- (A) -1 (B) 0 (C) 1 (D) 2 (E) 3

(47). $f(7) = ?$

- (A) -1 (B) 0 (C) 1 (D) 2 (E) 3

20. Please answer the following questions.

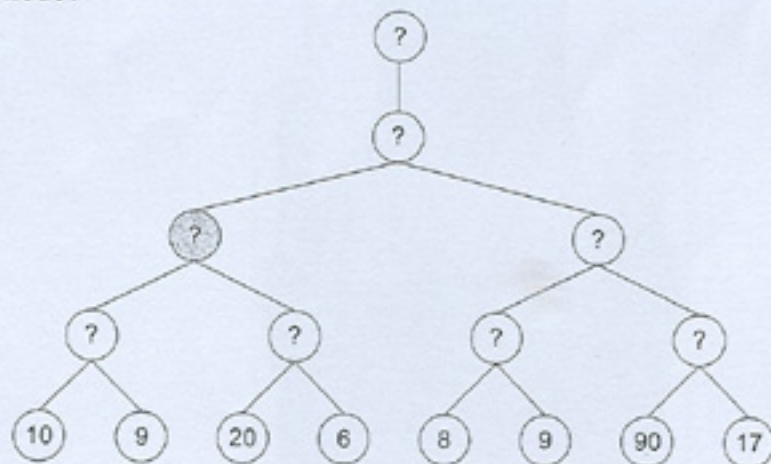
(48). Consider the case of inserting 10, 8, 12, 6, 4, 2, 11, 1 into an empty min heap. Please show the first visited node when using LVR to traverse the resultant tree.

- (A) 11 (B) 10 (C) 8 (D) 12 (E) 6

(49). Consider the case of inserting 10, 8, 12, 6, 4, 2, 11, 1 into an empty binary search tree. Please show the first visited node when using LVR to traverse the resultant tree.

- (A) 8 (B) 6 (C) 4 (D) 2 (E) 1

(50). Consider the following (partial) loser tree. Suppose that the node with smaller value wins. What is the value of the grey node?



- (A) 6 (B) 8 (C) 9 (D) 10 (E) 20