FACULTY OF IT-HCMUS

Mathematical Method in Visual Data Analysis

Lecturer: Assoc Prof. Lý Quốc Ngọc

HCMc, 10-2022

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Lecture 3:
Apply Metric space to VDA

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3.1. Drawing based on Fractal geometry

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3.1. Drawing based on Fractal geometry

3.1.1. Computing Fractals from Iterated Function Systems

Consider IFS $\{R^2, w_n : n = 1,2,\ldots,N\}$

$$w_i \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} a_i & b_i \\ c_i & d_i \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} e_i \\ f_i \end{bmatrix} = A_i x + t_i$$

p_i associated with w_i

$$p_i \approx \frac{\left|\det \mathrm{A_i}\right|}{\sum_{\mathrm{i=1}}^{\mathrm{N}} \left|\det \mathrm{A_i}\right|} = \frac{\left|a_i d_i - b_i c_i\right|}{\sum_{\mathrm{i=1}}^{\mathrm{N}} \left|a_i d_i - b_i c_i\right|}, i = 1,2,\ldots,N$$

3.1. Drawing based on Fractal geometry

Deterministic Algorithm

Let, $\{X; w_1, w_2, \ldots w_N\}$ be an IFS,

Compute directly a sequence of sets $A_n = W^{on}(A)$, starting from an initial set $A_0 \subset R^2$

$$A_{n+1} = \bigcup_{j=1}^{N} w_j(A_n), \quad n = 1,2,\ldots$$

Sequence $\{A_n\}$ converges to the attractor A of the IFS in the Hausdorff metric.

3.1. Drawing based on Fractal geometry

Deterministic Algorithm

```
1. Initialize s[M,M], t[M,M]
2. Setup the values of IFS
       a[1]=0.5;b[1]=0;c[1]=0;d[1]=0.5;e[1]=1;f[1]=1
       a[2]=0.5;b[2]=0;c[2]=0;d[2]=0.5;e[2]=50;f[2]=1
       a[3]=0.5;b[3]=0;c[3]=0;d[3]=0.5;e[3]=50;f[3]=50
3. Input the initial set A(0) into t[M.M]
4. Repeat
5. For i=1 to M  /Apply W to  A(n) to make A(n+1) in s[i,j]/
6.        For j=1 to M
7.              If t[i,j]=1 then
8.                 s[a[1]*i+b[1]*j+e[1], c[1]*i+d[1]*j+f[1]]=1
9.                 s[a[2]*i+b[2]*j+e[2], c[2]*i+d[2]*j+f[2]]=1
10.                s[a[3]*i+b[3]*j+e[3], c[3]*i+d[3]*j+f[3]]=1
11.             End if
12.        End /for j/
13. End  /for i/
```

3.1. Drawing based on Fractal geometry

Deterministic Algorithm

```
For i=1 to M
    For j=1 to M
        t[i,j]=s[i,j]  /Put A(n+1) into the array t[i,j]/
        s[i,j] =0      /Reset the array s[i,j] to 0/
        If t[i,j]=1 then
            setpixel(i,j)  /Plot A(n+1)/
        End if
    End /for j/
End  /for i/
Until A(n+1)=W(A(n+1))
```

3.1. Drawing based on Fractal geometry

Random Iteration Algorithm

Let $\{X; w_1, w_2, \ldots w_N\}$ be an IFS, where $p_i > 0$ has been assigned to $w_i > 0$, $\sum p_i = 1$

Choose $x_0 \in X$ and then choose recursively, independently,

$$x_n \in \{w_1(x_{n-1}), w_2(x_{n-1}), \ldots w_N(x_{n-1})\}, n = 1,2,3,\ldots$$

Where the probability of the event $x_n = w_i(x_{n-1})$ is p_i

Sequence $\{x_n\}$ be constructed converges to the attractor of the IFS.

3.1. Drawing based on Fractal geometry

Random Iteration Algorithm

1. Initialize s[M,M], t[M,M]
2. Setup the values of IFS
   a[1]=0.5;b[1]=0;c[1]=0;d[1]=0.5;e[1]=1;f[1]=1
   a[2]=0.5;b[2]=0;c[2]=0;d[2]=0.5;e[2]=50;f[2]=1
   a[3]=0.5;b[3]=0;c[3]=0;d[3]=0.5;e[3]=50;f[3]=50
3. x=0;y=0;numits=N /Initialize (x,y) and the number of iterations/
4. For n=1 to numits /Random Iteration/
5. k=int(3*rnd-0.0001)+1 /Choose one of the numbers 1,2,3/
6. newx=a[k]*x+b[k]*y+e[k];
7. newy=c[k]*x+d[k]*y+f[k];
8. x= newx ;
9. y=newy;
10. setpixel(x,y);
11. End /for n/

3.2. Fractal image compression

Method

Based on theorem of contraction mapping sequence in metric space Hausdorff

Let the Original image be O

The compression image will be $\{w_n\}$

The ideal uncompressed will be A

The real uncompressed be $W(B)$