Mathematical Optimization – Toán Tối Ưu

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Tóm tắt nội dung

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- *Mathematical Optimization – Toán Tối Ưu*.
PDF: URL: https://github.com/NQBH/advanced_STEM_beyond/blob/main/optimization/NQBH_mathematical_optimization.pdf.
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1. [AH01]. Grégoire Allaire, Antoine Henrot. *On some recent advances in shape optimization.*
2. [Aze20]. Hideyuki Azegami. *Shape Optimization Problems.*
3. [BW23]. Catherine Bandle, Alfred Wagner. *Shape Optimization: Variations of Domains & Applications.*
4. [DZ01; DZ11]. Michael C. Delfour, Jean-Paul Zolésio. *Shapes & Geometries.*
5. [HM03]. J. Haslinger, R. A. E. Mäkinen. *Introduction to Shape Optimization.*
6. [MP10]. Bijan Mohammadi, Olivier Pironneau. *Applied Shape Optimization for Fluids.*
7. [MZ06]. Marwan Moubachir, Jean-Paul Zolésio. *Moving Shape Analysis & Control.*
8. Stephan Schmidt. Master course: *Shape & Geometry.* Humboldt University of Berlin. [written in German, taught in English & German].
9. [SZ92]. Jan Sokołowski, Jean-Paul Zolésio. *Introduction to Shape Optimization.*

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10. [Wal15]. Shawn W. Walker. *The Shapes of Things.*

Differential equations on surfaces. Differential geometry is useful for understanding mathematical models containing geometric PDEs, e.g., surface/manifold version of the standard Laplace equation, which requires the development of the surface gradient & surface Laplacian operators – the usual gradient ∇ & Laplacian $\Delta = \nabla \cdot \nabla$ operators defined on a surface (manifold) instead of standard Euclidean space $\mathbb{R}^n$. *Advantage*: provide alternative formulas for geometric quantities, e.g., the summed (mean) curvature, that are much clearer than the usual presentation of texts on differential geometry.

Differentiating w.r.t. Shape. The approach to differential geometry is advantageous for developing the framework of *shape differential calculus* – the study of how quantities change w.r.t. changes of independent "shape variable".

Example 1 ([Wal15], Sect. 1.2.1, pp. 1–2). *Let $f = f(r,\theta)$ be a smooth function defined on the disk $B_{R,2}(0,0)$ of radius R in terms of polar coordinates. The integral of f over $B_{R,2}(0,0)$ $J := \int_{B_{2,R}(0,0)} f \,\mathrm{d}\mathbf{x} = \int_0^{2\pi}\int_0^R f(r,\theta)\,\mathrm{d}r\,\mathrm{d}\theta$ depends on R. Assume f also depends on R, i.e., $f = f(r,\theta,R)$ with a physical example: J is the* net flow rate *of liquid through a pipe with cross-section Ω, then f is the flow rate per unit area & could be the solution of a PDE defined on Ω, e.g., a Navier–Stokes fluid flowing in a circular pipe. Advantageous to know the* sensitivity *of J w.r.t. R, e.g., for optimization purposes. Differentiate J w.r.t. R:*

$$\frac{d}{dR}J = \int_0^{2\pi}\left(\frac{d}{dR}\int_0^R f(r,\theta;R)r\,\mathrm{d}r\right)\mathrm{d}\theta = \int_0^{2\pi}\int_0^R f'(r,\theta;R)r\,\mathrm{d}r\,\mathrm{d}\theta + \int_0^{2\pi} f(R,\theta;R)\,\mathrm{d}\theta.$$

The dependence of f on R can more generally be viewed as dependence on $B_{R,2}(0,0)$, i.e., $f(\cdot;R) \equiv f(\cdot;B_{R,2}(0,0))$. Rewriting d/dRJ using Cartesian coordinates $\mathbf{x}$:

$$\frac{d}{dR}J = \int_{B_{R,2}(0,0)} f'(\mathbf{x};\Omega)\,\mathrm{d}\mathbf{x} + \int_{S_{R,2}(0,0)} f(\mathbf{x};\Omega)\,\mathrm{d}S(\mathbf{x}), \tag{1}$$

where $\mathrm{d}\mathbf{x}$ is the volume measure, $\mathrm{d}S(\mathbf{x})$ is the surface area masure.

4 Topology Optimization – Tối Ưu Tôpô

5 Miscellaneous

Tài liệu

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