

Discrete Mathematics for Computer Science

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What is Discrete Mathematics?

Definition (Discrete mathematics)

Discrete mathematics: study of countable, distinct, or separate mathematical structures.

Cf. Finite Mathematics vs. Discrete Mathematics vs. “Continuous Mathematics”, including e.g., Calculus, Mathematical Analysis.

Example (Pixel)

Phones, computer monitors, televisions, modern screens, & Disney cartoons, animated films for kids & for adults, e.g., *Rick & Morty* (2013–).

Some Critical Thinking Questions

Targets. Typical super-lazy unmotivated undergraduate/graduate students majored in Natural Science.

Some purpose-driven questions

- *Why do undergraduate or graduate students need to learn mathematics?*
- *Which type of mathematics do undergraduate or graduate students need to learn?*
- *Why do CS-major students need to study Discrete Mathematics?*

Motivations

- Learn Discrete Mathematics just for fun, to entertain yourself.

Example (*Good Will Hunting* (1997))

WILL HUNTING learned History, Sociology, Psychology Ψ , Advanced Mathematics, Combinatorial Discrete Mathematics to flirt hot girls in bars, & even Advanced Organic Chemistry for fun & to help her girlfriend.

- Learn “just enough” Discrete Mathematics to understand different branches of Computer Science.
Main Goal: Focus strongly on writing programs, developing software, & building useful applications.
- If looking for research-oriented jobs, especially Theoretical Computer Science, then need to learn Discrete Mathematics much harder.
Main Goal: Build some new useful theories, then find their theoretical- or practical real-world applications.

References on Mathematics & Computer Science

On choosing Refs

How to choose “right/suitable” references, e.g., online courses, books, lecture notes, expository notes, other learning materials, etc.?

[NQBH]’s Lecture Note on Discrete Mathematics & beyond.

[GKP89]* RONALD L. GRAHAM, DONALD ERWIN KNUTH, OREN PATASHMIK. *Concrete Mathematics: A Foundation for Computer Science*.

[Lib23] DAVID LIBEN-NOWELL. *Connecting Discrete Mathematics & Computer Science*.

[Ros19] KENNETH H. ROSEN. *Discrete Mathematics & Its Applications*.

[WR21] RYAN T. WHITE, ARCHANA TIKAYAT RAY. *Practical Discrete Mathematics: Discover math principles that fuel algorithms for computer science & machine learning with Python*.

[GA08] ADAM M. GRANT, SUSAN J. ASHFORD. *The dynamics of proactivity at work*. Research in Organizational Behaviors 28 (2008) 3–34.

Targets. precision, robustness, creativity, usefulness, applicability, proactivity, valuable insight, deep comprehension, passion, novelty.

Some goal-driven rules in learning, teaching, & research

(will be adjusted according to UMT IT Depart.'s objectives & visions)

- Bonus points for proposing creative problems &/or solutions.
- Special points for projects combining Math + CS (+ Physics, Chemistry, &/or Biology) much harder or more useful than lectures.

Combinatorics using SciPy

Obviously, SciPy is not spicy at all like chicken wings in *Hot Ones* show.

Problem (Permutation, arrangement, combination)

Given $n, k \in \mathbb{N}^*$, $k \leq n$. Write Pascal/Python/C/C++ programs to compute the numbers of permutations P_n , of arrangements A_n^k , of combinations C_n^k .

Solution.

$P_n = n!$, $A_n^k = \frac{n!}{(n-k)!}$, $C_n^k = \frac{n!}{k!(n-k)!}$. Run `combinatorics.py`. □

Problem (Pascal triangle)

Given $n \in \mathbb{N}^*$. Write Pascal/Python/C/C++ programs to print the 1st $n + 1$ lines of the Pascal triangle.

Elementary Mathematics/Grade 6/Basic Plane Geometry

Problem (Count number of lines formed by some points)

Write Pascal/Python/C/C++ programs to count the number of lines formed by $n \in \mathbb{N}^$ distinguished points in (2D) plane.*

Hint. $C_n^2 - \sum_{i=1}^m C_{a_i}^2 + m = \frac{n(n-1)}{2} - \sum_{i=1}^m \frac{a_i(a_i-1)}{2} + m$ lines, where n given points is partitioned into exactly $m \in \mathbb{N}$ disjoint subsets A_i of collinear points, where $a_i := |A_i| = \text{card } A_i, \forall i = 1, \dots, m$.

Problem (Count number of intersections formed by some lines)

Write Pascal/Python/C/C++ programs to count the number of intersections of $n \in \mathbb{N}^$ distinguished lines in (2D) plane.*

Hint. Nếu trong n đường thẳng đã cho có đúng $m \in \mathbb{N}$ bộ lần lượt gồm a_1, \dots, a_m đường thẳng song song đôi một & $k \in \mathbb{N}$ bộ lần lượt gồm b_1, \dots, b_k đường thẳng đồng quy thì số giao điểm: $C_n^2 - \sum_{i=1}^m C_{a_i}^2 - \sum_{i=1}^k C_{b_i}^2 + k$
 $= \frac{n(n-1)}{2} - \sum_{i=1}^m \frac{a_i(a_i-1)}{2} - \sum_{i=1}^k \frac{b_i(b_i-1)}{2} + k$.

Discrete Mathematics vs. $DL \subset ML \subset AI$

How can Discrete Mathematics be useful in Artificial Intelligence (AI), Machine Learning (ML), & Deep Learning (DL)?