Discrete Mathematics for Computer Science
Toán Rời Rạc Cho Khoa Học Máy Tính

Nguyễn Quản Bá Hồng

UMT, HCMC – University of Management & Technology, Ho Chi Minh City

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Def: Discrete Mathematics

Definition (Discrete mathematics)

Discrete mathematics: study of countable, distinct, or separate mathematical structures.

Cf. Finite Mathematics vs. Discrete Mathematics vs. "Continuous Mathematics", including e.g., Calculus, Mathematical Analysis.

Note: Beyond the scope of this course: "Discontinuous Mathematics".

Example (Pixel)

Phones, computer monitors, televisions, modern screens, & Disney cartoons, animated films for kids & for adults, e.g., *Rick & Morty* (2013–).

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Some Critical Thinking Questions

Targets/Audiences. Typical super-lazy mentally lost & thus unmotivated undergraduate/graduate students majored in Natural Science, especially in Mathematics, Information Technology, Computer Science, & Engineering.

Some purpose-driven questions

- *Why do undergraduate or graduate students need to learn mathematics in general?*
- *Which type of mathematics do undergraduate or graduate students need to learn according to their majors?*
- *Why do CS-major students need to study Discrete Mathematics in particular?*

Topics in Discrete Mathematics

- **Theoretical Computer Science**: areas relevant to computing, e.g., study of algorithms & data structures, computability, complexity theory, automata theory, formal language theory, computational geometry, computer image analysis, etc.
- **Information Theory**: quantification of information, coding theory, analog signals/coding/encryption, etc.
- **Mathematical Logic**: truth table, mathematical proof, automated theorem proving, formal verification of software, etc.
- **Set Theory**: (naive set theory, not axiomatic one) main focus: countable (finite, infinitely countable) sets.
- **Combinatorics**: enumerative combinatorics (counting problems), generating functions, partition theory, etc.
  Watch *The Man Who Knew Infinity* (2015).

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Topics in Discrete Mathematics

- **Graph Theory**: study of graphs & networks, e.g., networks of communication, data organization, computational devices, flow of computation, etc.
- **Number Theory**: study of properties of (integer) numbers, e.g., cryptography, cryptanalysis.
- **Algebraic Structures**: discrete algebras, e.g., Boolean algebra used in logic gates & programming; relational algebra used in databases, etc.
- **Discrete Analogues of Continuous Mathematics**: discrete versions of continuous mathematics, e.g., discrete calculus, discrete probability theory, discrete optimization, discrete dynamical systems, etc.
- Others topics.

Motivations

- Learn Discrete Mathematics just for fun, to entertain yourself.

Example (*Good Will Hunting* (1997))

WILL HUNTING learned History, Sociology, Psychology Ψ, Advanced Mathematics, Combinatorial Discrete Mathematics to flirt hot girls in bars, & even Advanced Organic Chemistry for fun & to help her girlfriend.

- Learn "just enough" Discrete Mathematics to understand different branches of Computer Science.
  Main Goal: Focus strongly on writing programs, developing software, & building useful applications.
- If looking for research-oriented jobs, especially Theoretical Computer Science, then learn Discrete Mathematics much harder & deeper.
  Main Goal: Build some new useful theories, then find their theoretical- or practical real-world applications.

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References on Mathematics & Computer Science

On choosing Refs in general

How to choose "right/suitable" references, e.g., online courses, books, lecture notes, expository notes, other learning materials, etc.?

[NQBH]'s Lecture Note on Discrete Mathematics & beyond.

[Knu]** Donald Erwin Knuth. *The Art of Computer Programming*.

[GKP89]* Ronald L. Graham, Donald Erwin Knuth, Oren Patashmik. *Concrete Mathematics: A Foundation for Computer Science*.

[Ros19] Kenneth H. Rosen. *Discrete Mathematics & Its Applications*.

[WR21] Ryan T. White, Archana Tikayat Ray. *Practical Discrete Mathematics: Discover Math Principles that Fuel Algorithms for Computer Science & Machine Learning with Python*.

[Lib23] David Liben-Nowell. *Connecting Discrete Mathematics & Computer Science*.

References on Pedagogy & Psychology

On learning & teaching

How should we learn & teach Discrete Mathematics in particular & other types of Mathematics for Computer Science undergraduate students?

[Tru]: Giản Tư Trung's Hat-trick. (+ other books of IRED)

- *Đúng Việc: 1 Góc Nhìn Về Câu Chuyện Khai Minh*.
- *Sư Phạm Khai Phóng: Thế Giới, Việt Nam, & Tôi*.
- *Quản Trị Bằng Văn Hóa: Cách Thức Kiến Tạo & Tái Tạo Văn Hóa Tổ Chức*.

[Pol14] George Pólya. *How To Solve It: A New Aspect of Mathematical Method*.

[GA08] Adam M. Grant, Susan J. Ashford. *The dynamics of proactivity at work*. Research in Organizational Behaviors 28 (2008) 3–34.

Teaching & learning methodologies

Targets/Criteria. precision, robustness, creativity, usefulness, applicability, proactivity, valuable insight, deep comprehension, passion, novelty.

Some goal-driven rules in learning, teaching, & research

(will be adjusted according to UMT IT Depart.'s objectives & visions)

- Bonus points for proposing creative problems &/or solutions.
- Special points for projects combining Math + CS (+ Physics, Chemistry, &/or Biology) much harder or more useful than lectures.

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Combinatorics using SciPy: Problems

Important Note: Obviously, SciPy is not spicy at all like any chicken wings in *Hot Ones* show.

Recall from Elementary Mathematics Grade 10/combinatorics:

Problem (Permutation, arrangement, combination)

Given $n, k \in \mathbb{N}^\star$, $k \leq n$. Write Pascal/Python/C/C++ *programs to compute the numbers of permutations P_n, of arrangements A_n^k, of combinations C_n^k.*

Solution.

$P_n = n!, A_n^k = \dfrac{n!}{(n-k)!}, C_n^k = \dfrac{n!}{k!(n-k)!}$. Run `combinatorics.py`. □

Combinatorics using SciPy: Problems

Problem (Pascal triangle & Newton binomial expansion)

Given $m, n \in \mathbb{N}^\star$. *Write* Pascal/Python/C/C++ *programs to print the 1st* $n+1$ *lines of the Pascal triangle & Newton binomial expansion of* $(a+b)^n, (a+b+c)^n, (\sum_{i=1}^m a_i)^n$, $\forall a, b, c, a_i \in \mathbb{R}$, $\forall i = 1, \ldots, m$.

Recall from Elementary Mathematics Grade 6/plane geometry:

Problem (Count number of lines formed by some points)

Write Pascal/Python/C/C++ *programs to count the number of lines formed by* $n \in \mathbb{N}^\star$ *distinguished points in (2D) plane.*

Problem (Count number of intersections formed by some lines)

Write Pascal/Python/C/C++ *programs to count the number of intersections of* $n \in \mathbb{N}^\star$ *distinguished lines in (2D) plane.*

Solution.

$C_n^2 - \sum_{i=1}^m C_{a_i}^2 + m = \dfrac{n(n-1)}{2} - \sum_{i=1}^m \dfrac{a_i(a_i-1)}{2} + m$ lines, where n given points is partitioned into exactly $m \in \mathbb{N}$ disjoint subsets A_i of collinear points, where $a_i := |A_i| = \operatorname{card} A_i$, $\forall i = 1, \ldots, m$. □

Solution.

Nếu trong n đường thẳng đã cho có đúng $m \in \mathbb{N}$ bộ lần lượt gồm $a_1, \ldots, a_m$ đường thẳng song song đôi một & $k \in \mathbb{N}$ bộ lần lượt gồm $b_1, \ldots, b_k$ đường thẳng đồng quy thì số giao điểm:

$$C_n^2 - \sum_{i=1}^m C_{a_i}^2 - \sum_{i=1}^m C_{b_i}^2 + k$$
$$= \frac{n(n-1)}{2} - \sum_{i=1}^m \frac{a_i(a_i-1)}{2} - \sum_{i=1}^k \frac{b_i(b_i-1)}{2} + k.$$ □

Further & Beyond

More results, problems, & practical applications of Discrete Mathematics in Number Theory, Graph Theory, Generating Functions, Discrete Probability Theory, Asymptotics, etc.

Discrete Mathematics vs. DL $\subset$ ML $\subset$ AI

How can Discrete Mathematics be useful in Artificial Intelligence (AI), Machine Learning (ML), & Deep Learning (DL), especially Artificial Neural Networks (ANNs)?

Acknowledgment

All types of feedback & contributions are welcome.

Thanks

Thank You for your valuable time & attention.

I appreciate all.