Discrete Mathematics for Computer Science
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What is Discrete Mathematics?

Definition (Discrete mathematics)

Discrete mathematics: study of countable, distinct, or separate mathematical structures.

Cf. Finite Mathematics vs. Discrete Mathematics vs. "Continuous Mathematics", including e.g., Calculus, Mathematical Analysis.

Example (Pixel)

Phones, computer monitors, televisions, modern screens, & Disney cartoons, animated films for kids & for adults, e.g., *Rick & Morty* (2013–).

Some Critical Thinking Questions

Targets. Typical super-lazy unmotivated undergraduate/graduate students majored in Natural Science.

Some purpose-driven questions

- *Why do undergraduate or graduate students need to learn mathematics?*
- *Which type of mathematics do undergraduate or graduate students need to learn?*
- *Why do CS-major students need to study Discrete Mathematics?*

Motivations

- Learn Discrete Mathematics just for fun, to entertain yourself.

Example (*Good Will Hunting* (1997))

WILL HUNTING learned History, Sociology, Psychology Ψ, Advanced Mathematics, Combinatorial Discrete Mathematics to flirt hot girls in bars, & even Advanced Organic Chemistry for fun & to help her girlfriend.

- Learn "just enough" Discrete Mathematics to understand different branches of Computer Science.
  Main Goal: Focus strongly on writing programs, developing software, & building useful applications.
- If looking for research-oriented jobs, especially Theoretical Computer Science, then need to learn Discrete Mathematics much harder.
  Main Goal: Build some new useful theories, then find their theoretical- or practical real-world applications.

References on Mathematics & Computer Science

On choosing Refs

How to choose "right/suitable" references, e.g., online courses, books, lecture notes, expository notes, other learning materials, etc.?

[NQBH]'s Lecture Note on Discrete Mathematics & beyond.

[GKP89]* Ronald L. Graham, Donald Erwin Knuth, Oren Patashmik. *Concrete Mathematics: A Foundation for Computer Science*.

[Lib23] David Liben-Nowell. *Connecting Discrete Mathematics & Computer Science*.

[Ros19] Kenneth H. Rosen. *Discrete Mathematics & Its Applications*.

[WR21] Ryan T. White, Archana Tikayat Ray. *Practical Discrete Mathematics: Discover math principles that fuel algorithms for computer science & machine learning with Python*.

References on Pedagogy & Psychology

[GA08] Adam M. Grant, Susan J. Ashford. *The dynamics of proactivity at work*. Research in Organizational Behaviors 28 (2008) 3–34.

Targets. precision, robustness, creativity, usefulness, applicability, proactivity, valuable insight, deep comprehension, passion, novelty.

Some goal-driven rules in learning, teaching, & research

(will be adjusted according to UMT IT Depart.'s objectives & visions)

- Bonus points for proposing creative problems &/or solutions.
- Special points for projects combining Math + CS (+ Physics, Chemistry, &/or Biology) much harder or more useful than lectures.

Combinatorics using SciPy

Obviously, SciPy is not spicy at all like chicken wings in *Hot Ones* show.

Problem (Permutation, arrangement, combination)

Given $n, k \in \mathbb{N}^\star$, $k \leq n$. *Write* Pascal/Python/C/C++ *programs to compute the numbers of permutations* P_n, *of arrangements* A_n^k, *of combinations* C_n^k.

Solution.

$P_n = n!, A_n^k = \frac{n!}{(n-k)!}, C_n^k = \frac{n!}{k!(n-k)!}$. Run `combinatorics.py`. □

Problem (Pascal triangle & Newton binomial expansion)

Given $m, n \in \mathbb{N}^\star$. *Write* Pascal/Python/C/C++ *programs to print the 1st* $n+1$ *lines of the Pascal triangle & Newton binomial expansion of* $(a+b)^n, (a+b+c)^n, \left(\sum_{i=1}^m a_i\right)^n$, $\forall a, b, c, a_i \in \mathbb{R}$, $\forall i = 1, \ldots, m$.

Problem (Count number of lines formed by some points)

Write Pascal/Python/C/C++ *programs to count the number of lines formed by* $n \in \mathbb{N}^\star$ *distinguished points in (2D) plane.*

Hint. $C_n^2 - \sum_{i=1}^m C_{a_i}^2 + m = \frac{n(n-1)}{2} - \sum_{i=1}^m \frac{a_i(a_i-1)}{2} + m$ lines, where n given points is partitioned into exactly $m \in \mathbb{N}$ disjoint subsets A_i of collinear points, where $a_i := |A_i| = \operatorname{card} A_i$, $\forall i = 1, \ldots, m$.

Problem (Count number of intersections formed by some lines)

Write Pascal/Python/C/C++ *programs to count the number of intersections of* $n \in \mathbb{N}^\star$ *distinguished lines in (2D) plane.*

Hint. Nếu trong n đường thẳng đã cho có đúng $m \in \mathbb{N}$ bộ lần lượt gồm $a_1, \ldots, a_m$ đường thẳng song song đôi một & $k \in \mathbb{N}$ bộ lần lượt gồm $b_1, \ldots, b_k$ đường thẳng đồng quy thì số giao điểm: $C_n^2 - \sum_{i=1}^m C_{a_i}^2 - \sum_{i=1}^m C_{b_i}^2 + k = \frac{n(n-1)}{2} - \sum_{i=1}^m \frac{a_i(a_i-1)}{2} - \sum_{i=1}^k \frac{b_i(b_i-1)}{2} + k$.

Further & Beyond

Discrete Mathematics vs. DL $\subset$ ML $\subset$ AI

How can Discrete Mathematics be useful in Artificial Intelligence (AI), Machine Learning (ML), & Deep Learning (DL)?