

# Discrete Mathematics – Toán Rời Rạc

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Ngày 27 tháng 12 năm 2024

## Tóm tắt nội dung

This text is a part of the series *Some Topics in Advanced STEM & Beyond*:

URL: [https://nqbh.github.io/advanced\\_STEM/](https://nqbh.github.io/advanced_STEM/).

Latest version:

- *Discrete Mathematics – Toán Rời Rạc*.

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TEX: URL: [https://github.com/NQBH/advanced\\_STEM\\_beyond/blob/main/discrete\\_mathematics/NQBH\\_discrete\\_mathematics.tex](https://github.com/NQBH/advanced_STEM_beyond/blob/main/discrete_mathematics/NQBH_discrete_mathematics.tex).

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## 1

## 2 Miscellaneous

## 3 Wikipedia's

### 3.1 Wikipedia/discrete mathematics

“*Discrete mathematics* is study of **mathematical structures** that can be considered “discrete” (in a way analogous to **discrete variables**, having a **bijection** with  $\mathbb{N}$ ) rather than “continuous” (analogously to **continuous functions**). Objects studied in discrete mathematics include integers, **graphs**, & **statements** in **logic**. By contrast, discrete mathematics excludes topics in “continuous mathematics” e.g. real numbers, calculus or **Euclidean geometry**. Discrete objects can often be **enumerated** by integers; more formally, discrete mathematics has been characterized as branch of mathematics dealing with **countable sets** (finite sets or sets with same **cardinality** as  $\mathbb{N}$ ). However, there is no exact definition of term “discrete mathematics”.

Set of objects studied in discrete mathematics can be finite or infinite. Term *finite mathematics* is sometimes applied to parts of field of discrete mathematics that deals with finite sets, particularly those areas relevant to business.

Graphs e.g. these are among objects studied by discrete mathematics, for their interesting **mathematical properties**, their usefulness as models of real-world problems, & their importance in developing computer algorithms.

Research in discrete mathematics increased in latter half of 20th century partly due to development of **digital computers** which operate in “discrete” steps & store data in “discrete” bits. Concepts & notations from discrete mathematics are useful in studying & describing objects & problems in branches of computer science, e.g. **computer algorithms**, **programming languages**, **cryptography**, **automated theorem proving**, & **software development**. Conversely, computer implementations are significant in applying ideas from discrete mathematics to real-world problems.

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Although main objects of study in discrete mathematics are discrete objects, **analytic** methods from “continuous” mathematics are often employed as well.

In university curricula, discrete mathematics are discrete objects, **analytic** methods from “continuous” mathematics are often employed as well.

In university curricula, discrete mathematics appeared in 1980s, initially as a computer science support course; its contents were somewhat haphazard at time. Curriculum has thereafter developed in conjunction with efforts by **ACM** & **MAA** into a course that is basically intended to develop **mathematical maturity** in 1st-year students; therefore, it is nowadays a prerequisite for mathematics majors in some universities as well. Some high-school-level discrete mathematics textbooks have appeared as well. At this level, discrete mathematics is sometimes seen as a preparatory course, like **precalculus** in this respect.

**Fulkerson Prize** is awarded for outstanding papers in discrete mathematics.

### 3.1.1 Topics

1. Theoretical computer science. **Complexity** studies time taken by algorithms, e.g. this **quick sort**. **Theoretical computer science** includes areas of discrete mathematics relevant to computing. It draws heavily on **graph theory** & **mathematical logic**. Included within theoretical computer science is study of algorithms & data structures. **Computability** studies what can be computed in principle, & has close ties to logic, while complexity studies time, space, & other resources taken by computations. **Automata theory** & **formal language** theory are closely related to computability. **Petri nets** & **process algebras** are used to model computer systems, & methods from discrete mathematics are used in analyzing **VLSI** electronic circuits.

**Computational geometry** applies computer algorithms to representations of geometrical objects. **Computational geometry** applies algorithms to geometrical problems & representations of geometrical objects, while **computer image analysis** applies them to representations of images. Theoretical computer science also includes study of various continuous computational topics.

2. Information theory. **ASCII** codes for word “Wikipedia”, given here in **binary**, provide a way of representing word in **information theory**, as well as for information-processing algorithms. **Information theory** involves quantification of **information**. Closely related is **coding theory** which is used to design efficient & reliable data transmission & storage methods. Information theory also includes continuous topics e.g.: **analog signals**, **analog coding**, **analog encryption**.

3. Logic. **Mathematical logic** is study of principles of valid reasoning & **inference**, as well as of **consistency**, **soundness**, & **completeness**. E.g., in most systems of logic (but not in **intuitionistic logic**) **Peirce’s law**  $((P \rightarrow Q) \rightarrow P) \rightarrow P$  is a theorem. For classical logic, it can be easily verified with a **truth table**. Study of **mathematical proof** is particularly important in logic, & has accumulated to **automated theorem proving** & **formal verification** of software.

**Logical formulas** are discrete structures, as are **proofs**, which form finite **trees** or, more generally, **directed acyclic graph** structures (with each **inference step** combining 1 or more **premise** branches to give a single conclusion). **Truth values** of logical formulas usually form a finite set, generally restricted to 2 values: true & false, but logic can also be continuous-valued, e.g., **fuzzy logic**. Concepts e.g. infinite proof trees or infinite derivation trees have also been studied, e.g., **infinitary logic**.

4. Set theory. **Set theory** is branch of mathematics that studies **sets**, which are collections of objects, e.g. {blue, white, red} or (infinite) set of all **prime numbers**. **Partially ordered sets** & sets with other **relations** have applications in several areas.

In discrete mathematics, **countable sets** (including **finite sets**) are main focus. Beginning of set theory as a branch of mathematics is usually marked by **GEORGE CANTOR**’s work distinguishing between different kinds of **infinite set**, motivated by study of trigonometric series, & further development of theory of infinite sets is outside scope of discrete mathematics. Indeed, contemporary work in **descriptive set theory** makes extensive use of traditional continuous mathematics.

5. Combinatorics. **Combinatorics** studies ways in which discrete structures can be combined or arranged. **Enumerative combinatorics** concentrates on counting number of certain combinatorial objects – e.g., **12fold way** provides a unified framework for counting **permutations**, **combinations**, & **partitions**. **Analytic combinatorics** concerns enumeration (i.e., determining number) of combinatorial structures using tools from **complex analysis** & probability theory. In contrast with enumerative combinatorics which uses explicit combinatorial formulae & **generating functions** to describe results, analytic combinatorics aims at obtaining **asymptotic formulae**. **Topological combinatorics** concerns use of techniques from **topology** & **algebraic topology/combinatorial topology** in **combinatorics**. Design theory is a study of **combinatorial designs**, which are collections of subsets with certain intersection properties. **Partition theory** studies various enumeration & asymptotic problems related to **integer partitions**, & is closely related to **q-series**, **special functions**, & **orthogonal polynomials**. Originally a part of number theory & analysis, partition theory is now considered a part of combinatorics or an independent field. **Order theory** is study of **partially ordered sets**, both finite & infinite.

6. Graph theory. **Graph theory** has close links to **group theory**. This **truncated tetrahedron** graph is related to **alternating group**  $A_4$ . **Graph theory**, study of **graphs** & **networks**, is often considered part of combinatorics, but has grown large enough & distinct enough, with its own kind of problems, to be regarded as a subject in its own right. Graphs are 1 of prime objects of study in discrete mathematics. They are among most ubiquitous models of both natural & human-made structures. They can model many types of relations & process dynamics in physical, biological & social systems. In computer science, they can represent networks of communication, data organization, computational devices, flow of computation, etc. In mathematics, they are useful in geometry & certain parts of topology, e.g. **knot theory**. **Algebraic graph theory** has close links with group theory & **topological graph theory** has close links to topology. There are also **continuous graphs**; however, for most part, research in graph theory falls within domain of discrete mathematics.

7. Number theory. **Ulam spiral** of numbers, with black pixels showing prime numbers. This diagram hints at patterns in **distribution** of prime numbers. **Number theory** is concerned with properties of numbers in general, particularly integers. It has applications to **cryptography** & **cryptanalysis**, particularly with regard to **modular arithmetic**, **diophantine equations**, linear & quadratic congruences, prime numbers & **primality testing**. Other discrete aspects of number theory include **geometry of numbers**. In **analytic number theory**, techniques from continuous mathematics are also used. Topics that go beyond discrete objects include **transcendental numbers**, **diophantine approximation**, **p-adic analysis** & **function fields**.
8. Algebraic structures. Main article: [Wikipedia/abstract algebra](#). **Algebraic structures** occur as both discrete examples & continuous examples. Discrete algebras include: **Boolean algebra** used in **logic gates** & programming; **relational algebra** used in **databases**; discrete & finite versions of groups, rings, & fields are important in **algebraic coding theory**; discrete **semigroups** & **monoids** appear in theory of **formal languages**.
9. Discrete analogues of continuous mathematics. There are many concepts & theories in continuous mathematics which have discrete versions, e.g. **discrete calculus**, **discrete Fourier transforms**, **discrete geometry**, **discrete logarithms**, **discrete differential geometry**, **discrete exterior calculus**, **discrete Morse theory**, **discrete optimization**, **discrete probability theory**, **discrete probability distribution**, **difference equations**, **discrete dynamical systems**, & **discrete vector measures**.
  - Calculus of finite differences, discrete analysis. In **discrete calculus** & **calculus of finite differences**, a function defined on an interval of integers is usually called a **sequence**. A sequence could be a finite sequence from a data source or an infinite sequence from a **discrete dynamical system**. Such a discrete function could be defined explicitly by a list (if its domain is finite), or by a formula for its general term, or it could be given implicitly by a **recurrence relation** or **difference equation**. Difference equations are similar to **differential equations**, but replace **differentiation** by taking difference between adjacent terms; they can be used to approximate differential equations or (more often) studied in their own right. Many questions & methods concerning differential equations have counterparts for difference equations. E.g., where there are **integral transforms** in **harmonic analysis** for studying continuous functions for analogue signals, there are **discrete transforms** for discrete functions or digital signals. As well as **discrete metric spaces**, there are more general **discrete topological spaces**, **finite metric spaces**, **finite topological spaces**.  
**Time scale calculus** is a unification of theory of **difference equations** with that of **differential equations**, which has applications to fields requiring simultaneous modeling of discrete & continuous data. Another way of modeling such a situation is notion of **hybrid dynamical systems**.
  - Discrete geometry. **Discrete geometry** & combinatorial geometry are about combinatorial properties of *discrete collections* of geometrical objects. A long-standing topic in discrete geometry is **tiling of plane**.  
 In **algebraic geometry**, concept of a curve can be extended to discrete geometries by taking **spectra** of **polynomial rings** over **finite fields** to be models of **affine spaces** over that field, & letting **subvarieties** or spectra of other rings provide curves that lie in that space. Although space in which curves appear has a finite number of points, curves are not so much sets of points as analogues of curves in continuous settings. E.g., every point of form  $V(x - c) \subset \text{Spec} K[x] = \mathbb{A}^1$  for  $K$  a field can be studied either as  $\text{Spec} K[x]/(x - c) \cong \text{Spec} K$ , a point, or as spectrum  $\text{Spec} K[x]_{(x-c)}$  of **local ring at  $(x - c)$** , a point together with a neighborhood around it. Algebraic varieties also have a well-defined notion of **tangent space** called **Zariski tangent space**, making many features of calculus applicable even in finite settings.
  - Discrete modeling. In **applied mathematics**, **discrete modeling** is discrete analogue of **continuous modeling**. In discrete modeling, discrete formulae are fit to **data**. A common method in this form of modeling is to use **recurrence relation**. **Discretization** concerns process of transferring continuous models & equations into discrete counterparts, often for purposes of making calculations easier by using approximations. **Numerical analysis** provides an important example.

### 3.1.2 Challenges

Much research in **graph theory** was motivated by attempts to prove: all maps can be **colored** using **only 4 colors** so that no areas of same color share an edge. **KENNETH APPEL** & **WOLFGANG HAKEN** proved this in 1976.

History of discrete mathematics has involved a number of challenging problems which have focused attention within areas of field. In graph theory, much research was motivated by attempts to prove **4 color theorem**, 1st stated in 1852, but not proved until 1976 (by **KENNETH APPEL** & **WOLFGANG HAKEN**, using substantial computer assistance).

In logic, **2nd problem** on **DAVID HILBERT**'s list of open **problems** presented in 1900 was to prove: axioms of arithmetic are consistent. **Gödel's 2nd incompleteness theorem**, proved in 1931, showed: this was not possible – at least not within arithmetic itself. **Hilbert's 10th problem** was to determine whether a given polynomial **Diophantine equation** with integer coefficients has an integer solution. In 1970, **YURI MATIYASEVICH** proved: this **could not be done**.

Need to **break** German codes in **World War II** led to advances in **cryptography** & **theoretical computer science**, with **1st programmable digital electronic computer** being developed at England's **Bletchley Park** with guidance of **ALAN TURING** & his seminal work, *On Computable Numbers*. **Cold War** meant: cryptography remained important, with fundamental advances e.g. **public-key cryptography** being developed in following decades. **Telecommunication industry** has also motivated advances in discrete mathematics, particularly in graph theory & **information theory**. **Formal verification** of statements in logic has been necessary for **software development** of **safety-critical systems**, & advances in **automated theorem proving** have been driven by this need.

**Computational geometry** has been an important part of **computer graphics** incorporated into modern **video games** & **computer-aided design** tools.

Several fields of discrete mathematics, particularly theoretical computer science, graph theory, & **combinatorics**, are important in addressing challenging **bioinformatics** problems associated with understanding **tree of life**.

Currently, 1 of most famous open problems in theoretical science is **P = NP problem**, which involves relationship between **complexity classes P & NP**. **Clay Mathematics Institute** has offered a \$1 million USD prize for 1st correct proof, along with prizes for **6 other mathematical problems**.” – [Wikipedia/discrete mathematics](#)

## 3.2 Wikipedia/outline of discrete mathematics

“**Discrete** is study of **mathematical structures** that are fundamentally **discrete** rather than **continuous**. In contrast to real numbers that have property of varying “smoothly”, objects studied in discrete mathematics – e.g. integers, graphs, & statements in logic – do not vary smoothly in this way, but have distinct, separated values. Discrete mathematics, therefore, excludes topics in “continuous mathematics” e.g. calculus & analysis.

Included below are many of standard term used routinely in university-level courses & in research papers. This is not, however, intended as a complete list of mathematical terms; just a selection of typical **terms of art** that may be encountered.

- **Logic**: Study of correct reasoning.
- Modal logic – Type of formal logic
- Set theory – Branch of mathematics that studies sets

### 3.2.1 Discrete mathematical disciplines

### 3.2.2 Concepts in discrete mathematics

### 3.2.3 Mathematicians associated with discrete mathematics

” – [Wikipedia/outline of discrete mathematics](#)