

Data Structures & Algorithms – Cấu Trúc Dữ Liệu & Giải Thuật

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1 Vector in C++ STL

Resources – Tài nguyên.

1. [Geeks4Geeks/vector in C++ STL](#)

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Definition 1. C++ vector is a dynamic array that stores collection of elements of same type in contiguous memory. It has the ability to resize itself automatically when an element is inserted or deleted.

– C++ vector là một mảng động lưu trữ tập hợp các phần tử cùng loại trong bộ nhớ liên tiếp. Nó có khả năng tự động thay đổi kích thước khi một phần tử được chèn vào hoặc xóa.

1.1 Create vectors – Tạo vectors

Resources – Tài nguyên.

1. [Geeks4Geeks/8 ways to initialize vector in C++](#).

Before creating a vector, we must know that a vector is defined as the `std::vector` class template in the `<vector>` header file.

```
vector<T> v;
```

where T is the type of elements & v is the name assigned to the vector.

Now we are creating an instance of `std::vector` class. This requires us to provide the type of elements as template parameter.

```
1 #include <bits/stdc++.h>
2 using namespace std;
3
4 int main() {
5     // Creating an empty vector
6     vector<int> v1;
7     return 0;
8 }
```

We can also provide the values to be stored in the vector inside {} curly braces. This process is called *initialization*.

```
1 #include <bits/stdc++.h>
2 using namespace std;
3 void printVector(vector<int>& v) {
4     for (auto x: v) {
5         cout << x << " ";
6     }
7     cout << endl;
8 }
9
10 int main() {
11     // Creating a vector of 5 elements from initializer list
12     vector<int> v1 = {1, 4, 2, 3, 5};
13
14     // Creating a vector of 5 elements with default value
15     vector<int> v2(5, 9);
16
17     printVector(v1);
18     printVector(v2);
19     return 0;
20 }
```

Output:

```
1 4 2 3 5
9 9 9 9 9
```

Statement `vector<int> v1 = {1, 4, 2, 3, 5}` initializes a vector with given values. Statement `vector<int> v2(5, 9)` creates a vector of size 5 where each element initialized to 9.

Remark 1. Statement `vector<int> v = {v1, v2, ..., vn}` initializes a vector with given values. Statement `vector<int> v(n, a)` creates a vector of size $n \in \mathbb{N}^*$ where each element initialized to $a \in \mathbb{Z}$.

Initializing a vector means assigning some initial values to the `std::vector` elements. Here are 8 different ways to initialize a vector in C++.

1.1.1 Initialize vector in C++ by using initializer list – Sử dụng danh sách các giá trị khởi tạo

We can initialize a vector with the list of values enclosed in curly braces {} known as **initializer list**. The value of the list will be assigned sequentially i.e. 1st value will be assigned to the 1st element of vector, 2nd value to 2nd element, ..., n th value to n th element. Syntax:

```
vector<type> v = {val1, val2, val3, ...};
```

where val1, val2, val3, ... are the initial values, e.g.:

```
1 // C++ Program to initializ std::vector with initializer list
2 #include <bits/stdc++.h>
3 using namespace std;
4
5 int main() {
6     // Initializing std::vector with list of multiple values
7     vector<int> v = {11, 23, 45, 89};
8
9     for (auto i : v)
10         cout << i << " ";
11     return 0;
12 }
```

Output: 11 23 45 89.

Note 1. `for (auto i : v)` means for each element of the type that will be determined automatically in the vector `v`.

1.1.2 Initialize vector in C++ 1 by 1 initialization

Vector can be initialized by pushing value 1 by 1. In this method, an empty vector is created, & elements are added to it 1 by 1 using the **vector::push_back()** method. This method is mostly used to initialize vector after declaration. Syntax:

```
v.push_back(val);
```

where val is the value which we have to insert, e.g.:

```
1 // C++ Program to initialize std::vector by pushing values 1 by 1
2 #include <bits/stdc++.h>
3 using namespace std;
4
5 int main() {
6     vector<int> v;
7
8     // Pushing Value one by one
9     v.push_back(11);
10    v.push_back(23);
11    v.push_back(45);
12    v.push_back(89);
13
14    for (auto i : v)
15        cout << i << " ";
16    return 0;
17 }
```

Output: 11 23 45 89.

1.2 Initialize vector in C++ by with a single value

We can initialize all the elements of the vector to a single value. We create a vector of a specified size & initialize all elements to the same value using vector constructor. Syntax:

```
vector<type> v(n, val);
```

where $n \in \mathbb{N}$ is the size & val is the initial value, e.g.:

```
1 // C++ Program to initializ the std::vector with specific value
2 #include <bits/stdc++.h>
3 using namespace std;
4
```

```

5  int main() {
6      // Initializing all the elements of a vector using a single value
7      vector<int> v(5, 11);
8
9      for (auto i : v)
10         cout << i << " ";
11     return 0;
12 }

```

Output: 11 11 11 11 11.

1.2.1 Initialize vector in C++ from an array

We can also initialize a vector using plain old static arrays using vector constructor. This works by copying all the elements of the array to the newly created vector. Syntax:

```
vector<type> v(arr, arr + n);
```

where `arr` is the array name & $n \in \mathbb{N}$ is the size of the array, e.g.:

```

1  // C++ Program to initializ the std::vector from another array
2  #include <bits/stdc++.h>
3  using namespace std;
4
5  int main() {
6      int arr[] = {11, 23, 45, 89};
7      int n = sizeof(arr) / sizeof(arr[0]);
8
9      // initialize the std::vector v by arr
10     vector<int> v = {arr, arr + n};
11
12     for (auto i : v)
13         cout << i << " ";
14     return 0;
15 }

```

Output: 11 23 45 89.

1.2.2 Initialize vector in C++ from another vector

We can also initialize a newly created vector from an already created vector if they are of same type. Syntax:

```
vector<type> v2(v1.begin(), v1.end());
```

where `v1` is the already existing vector, e.g.:

```

1  // C++ Program to initializ the std::vector from another vector
2  #include <bits/stdc++.h>
3  using namespace std;
4
5  int main() {
6      vector<int> v1 = {11, 23, 45, 89};
7
8      // initialize the vector v2 from vector v1
9      vector<int> v2(v1.begin(), v1.end());
10
11     for (auto i : v2)
12         cout << i << " ";
13     return 0;
14 }

```

Output: 11 23 45 89.

1.2.3 Initialize vector in C++ by from any STL container

Vectors are flexible containers that can be initialized by any other already existing containers e.g. set, multiset, map, etc. if they are of same type. Syntax:

```
vector<type> v(first, last);
```

where `first`, `last` are the iterator to the 1st element & the element just after the last element in the range of STL container.

1.2.4 Initialize vector in C++ by using `std::fill()` function

We can also use the `std::fill` function to initialize the whole or a part of a vector to the same value. Syntax:

```
fill(first, last, val);
```

where `first`, `last` are the iterator to the 1st element & the element just after the last element in the range of STL container & `val` is the value to be initialized with, e.g.:

```
1 // C++ Program to initialize the std::vector using std::fill() method
2 #include <bits/stdc++.h>
3 using namespace std;
4
5 int main() {
6     vector<int> v(5);
7
8     // initialize vector v with 11
9     fill(v.begin(), v.end(), 11);
10
11     for (auto i : v)
12         cout << i << " ";
13     return 0;
14 }
```

Output: 11 11 11 11 11.

1.2.5 Initialize vector in C++ by using `std::iota()` function

The `std::iota()` function from the `<numeric>` library allows us to initialize a vector with consecutive values starting from the given value. Syntax:

```
std::iota(first, last, val);
```

where `first`, `last` are the iterator to the 1st element & the element just after the last element in the range of the vector & `val` refers to the starting value, e.g.:

```
1 // C++ Program to initializ the std::vector using std::iota()
2 #include <bits/stdc++.h>
3 using namespace std;
4
5 int main() {
6     vector<int> v(5);
7
8     // using std::iota() to initialize vector v with 11
9     iota(v.begin(), v.end(), 11);
10
11     for (auto i : v)
12         cout << i << " ";
13     return 0;
14 }
```

Output: 11 12 13 14 15.

1.3 Insert elements – Chèn phần tử

An element can be inserted into a vector using `vector insert()` method which takes linear time. But for the insertion at the end, the `vector push_back()` method can be used, which is much faster, taking only constant time.

```
1 #include <bits/stdc++.h>
2 using namespace std;
3
4 int main() {
5     vector<char> v = {'a', 'f', 'd'};
6
7     // Inserting 'z' at the back
8     v.push_back('z');
9
10    // Inserting 'c' at index 1
```

```

11     v.insert(v.begin() + 1, 'c');
12
13     for (int i = 0; i < v.size(); i++)
14         cout << v[i] << " ";
15     return 0;
16 }

```

Output: a c f d z.

1.4 Access or update elements – Tiếp cận hoặc cập nhật các phần tử

Just like arrays, vector elements can be accessed using their index inside the `[]` subscript operator. While accessing elements, we can also update the value of that index using assignment operator `=`. The `[]` subscript operator doesn't check whether the given index exists in the vector or not. So, there is another member method `vector::at()` for safely accessing or update elements.

```

1  #include <bits/stdc++.h>
2  using namespace std;
3
4  int main() {
5      vector<char> v = {'a', 'c', 'f', 'd', 'z'};
6
7      // accessing & printing values
8      cout << v[3] << endl;
9      cout << v.at(2) << endl;
10
11     // updating values using indexes 3 & 2
12     v[3] = 'D';
13     v.at(2) = 'F';
14
15     cout << v[3] << endl;
16     cout << v.at(2);
17     return 0;
18 }

```

1.5 Find vector size – Tìm cỡ/kích thước của vector

1 of the common problems with arrays was to keep a separate variable to store the size information. Vector provides the solution to this problem by providing `size()` method.

```

1  #include <bits/stdc++.h>
2  using namespace std;
3
4  int main() {
5      vector<char> v = {'a', 'c', 'f', 'd', 'z'};
6
7      // finding size
8      cout << v.size();
9      return 0;
10 }

```

Output: 5.

1.6 Traverse vector – Duyệt vector

Vector in C++ can be traversed using indexes in a loop. The indexes start from 0 & go up to a vector size - 1. To iterate through this range, we can use a loop & determine the size of the vector using the `vector::size()` method.

```

1  #include <bits/stdc++.h>
2  using namespace std;
3
4  int main() {
5      vector<char> v = {'a', 'c', 'f', 'd', 'z'};
6
7      // traversing vector using range based for loop
8      for (int i = 0; i < v.size(); i++)
9          cout << v[i] << " ";

```

```

10     return 0;
11 }

```

Output: a c f d z.

We can also use a range-based loop for simple traversal.

1.7 Delete elements – Xóa phần tử

An element can be deleted from a vector using `vector::erase()` but this method needs iterator to the element to be deleted. If only the value of the element is known, then `find()` function is used to find the position of this element.

For the deletion at the end, the `vector::pop_back()` method can be used, & it is much faster, taking only constant time.

```

1  #include <bits/stdc++.h>
2  using namespace std;
3
4  int main() {
5      vector<char> v = {'a', 'c', 'f', 'd', 'z'};
6
7      // deleting last element 'z'
8      v.pop_back();
9
10     // deleting element 'f'
11     v.erase(find(v.begin(), v.end(), 'f'));
12
13     for (int i = 0; i < v.size(); i++) {
14         cout << v[i] << " ";
15     }
16     return 0;
17 }

```

Output:

```

a c f d
a c d

```

1.8 Other operations – Các thao tác khác

Vector is 1 of the most frequently used containers in C++. It is used in many situations for different purposes. The following examples aim to help you master vector operations beyond the basics.

2 2D Vector in C++

Resources – Tài nguyên.

1. [Geeks4Geeks/2D vector in C++](#).

A 2D vector is a vector of the vectors, i.e., each element is a vector in itself. It can be visualized as a matrix where each inner vector represents a row, & the number of rows represents a row, & the number of rows represents the maximum columns. A 2D vector is dynamically resizable in both dimensions. Syntax:

```
vector<vector<data_type>> v;
```

where `data_type` is the type of elements & `V` is the name assigned to the 2D vector.

2.1 Creating a 2D vector – Tạo 1 vector 2D

In C++, we can create/declare a 2D vector by using the vector container defined in the C++ Standard Template Library (STL). We can simply create a 2D vector by creating a vector with the vector data type.

Just like vectors, a 2D vector can be created & initialized in multiple ways:

1. **Default.** An empty 2D vector can be created using the declaration:

```
vector<vector<data_type>> v;
```

It can be filled in later on in the program.

2. **With user defined size & default value.** A vector of a specific size can also be declared & initialized to the given value as default value.

```
vector<vector<T>> v(n, vector<T>(m, value));
```

where $n \in \mathbb{N}^*$ is the number of rows, $m \in \mathbb{N}^*$ is the number of columns, `val` is the new default value for all of the elements of the vector.

3. **Using initializer list.** Vector can also be initialized using a list of values enclosed in `{}` braces separated by comma. The list must be nested according to the 2 dimensions as it helps in determining the row size & column size.

```
vector<vector<T>> v = {{x1, x2, ...}, {y1, y2, ...}, ...};
```

E.g.:

```
1  #include <bits/stdc++.h>
2  using namespace std;
3
4  void printV(vector<vector<int>>& v) {
5      for (auto i: v) {
6          for (auto j: i) {
7              cout << j << " ";
8          }
9          cout << endl;
10     }
11     cout << endl;
12 }
13
14 int main() {
15     // an empty 2D vector
16     vector<vector<int>> v1;
17
18     // 2D vector with initial size and value
19     vector<vector<int>> v2(2, vector<int>(3, 11));
20
21     // a 2D vector initialized with initializer list
22     vector<vector<int>> v3 = {
23         {1, 2, 3},
24         {4, 5, 6},
25     };
26
27     printV(v1);
28     printV(v2);
29     printV(v3);
30     return 0;
31 }
```

Output:

```
11 11 11
11 11 11
```

```
1 2 3
4 5 6
```

Basic operations of 2D vector:

1. Inserting elements in a 2D vector
2. Accessing & updating elements
3. Deleting elements
4. Traversing the vector

2.2 Inserting elements in a 2D vector

In 2D vectors, there are 2 types of insertion:

1. Insert a new row.
2. Insert a value in an existing row.

These can be inserted at any given position using `vector_insert()` & at the end using `vector push_back()`. As vector can dynamically grow, each row can have different size like **Java's jagged arrays**, e.g.:

```
1  #include <bits/stdc++.h>
2  using namespace std;
3
4  int main() {
5      vector<vector<int>> v = {{1, 2, 3}, {4, 5, 6}};
6
7      // insert a new row at the end
8      v.push_back({7, 8, 9});
9
10     // insert value in 2nd row at 2nd position
11     v[1].insert(v[1].begin() + 1, 10);
12
13     for (int i = 0; i < v.size(); i++) {
14         for (int j = 0; j < v[i].size(); j++) {
15             cout << v[i][j] << " ";
16         }
17         cout << endl;
18     }
19     return 0;
20 }
```

Output:

```
1 2 3
4 10 5 6
7 8 9
```

2.3 Accessing & uploading elements

As 2D vectors are organized as matrices with row & column, we need 2 indexes to access an element: 1 for the *row number i* & other for the *column number j*. We can then use any access method e.g. **[] operator** or **vector at()** method.

The value of the accessed element can be changed by assigning a new value using **= operator**, e.g.:

```
1  #include <bits/stdc++.h>
2  using namespace std;
3
4  int main() {
5      vector<vector<int>> v = {{1, 2, 3}, {4, 5, 6}};
6
7      // access 3rd element in 2nd row
8      cout << "3rd element in 2nd row: " << v[1][2] << endl;
9
10     // access 2nd element in 1st row
11     cout << "2nd element in 1st row: " << v[0][1] << endl;
12
13     // updating the 2nd element in 1st row
14     v[0][1] = 9;
15     cout << "2nd element in 1st row after updating: " << v[0][1] << endl;
16
17     return 0;
18 }
```

Output:

```
3rd element in 2nd row: 6
2nd element in 1st row: 2
2nd element in 1st row after updating: 9
```

2.4 Deleting elements from a 2D vector

Similar to insertion, there are 2 types of deletion in 2D vector:

1. Delete a row
2. Delete a value in an existing row.

Elements can be deleted using `vector::erase()` for a specific position or range & using `vector::pop_back()` to remove the last element, e.g.:

```
1  #include <bits/stdc++.h>
2  using namespace std;
3
4  int main() {
5      vector<vector<int>> v = {{1, 2, 3}, {4, 5, 6}};
6
7      // delete the 2nd row
8      v.erase(v.begin() + 1);
9
10     // delete 2nd element in 1st row
11     v[0].erase(v[0].begin() + 1);
12
13     for (int i = 0; i < v.size(); i++) {
14         for (int j = 0; j < v[i].size(); j++)
15             cout << v[i][j] << " ";
16         cout << endl;
17     }
18
19     return 0;
20 }
```

Output: 1 3.

2.5 Traversing 2D vectors

Traversing a 2D vector involves iterating through rows & columns using nested loops & access the elements by indexes, e.g.:

```
1  #include <bits/stdc++.h>
2  using namespace std;
3
4  int main() {
5      vector<vector<int>> v = {{1, 2, 3}, {4, 5, 6}};
6
7      // loop through rows
8      for (int i = 0; i < v.size(); i++) {
9          // loop through columns
10         for (int j = 0; j < v[i].size(); j++)
11             cout << v[i][j] << " ";
12         cout << endl;
13     }
14
15     return 0;
16 }
```

Output:

```
1 2 3
4 5 6
```

C++ provides more methods to traverse 2D vector.

2.6 Finding size of 2D vector

Finding the size of a 2D vector involves finding its row size & column size which can be done using the `vector::size()` method. The size vector used on the outer vector gives the number of rows in the 2D vector while using them on the inner vector gives the number of columns in that row (as all rows can have different number of columns). Syntax:

```
//finding the number of rows
int rows = vec.size();
```

```
//finding the number of columns
int rows = vec[0].size();
```

where `vec` is the name of the vector for which the size is to be determined. Since each element of a 2D vector is a vector itself we can use the `size()` method on the elements `vec[0]` to find the size of each row separately, e.g.:

```
1  #include <iostream>
2  #include <vector>
3  using namespace std;
4
5  int main() {
6      // creating a 2D vector
7      vector<vector<int>> vec = {
8          {1, 2, 3},
9          {4, 5, 6},
10         {7, 8, 9}
11     };
12
13     // finding the number of rows (size of the outer vector)
14     int rows = vec.size();
15     cout << "Number of rows: " << rows << endl;
16
17     // finding the number of columns (size of any inner vector first row)
18     int cols = vec[0].size();
19     cout << "Number of columns: " << cols << endl;
20
21     return 0;
22 }
```

Output:

```
Number of rows: 3
Number of columns: 3
```

2.7 Common operations & applications

Apart from the basic operations, there are many operations that can be performed on 2D vectors.

3 Miscellaneous