Elementary Geometry

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Tóm tắt nội dung

[EN] This text is a collection of problems, from easy to advanced, about <*topic*>. This text is also a supplementary material for my lecture note on Elementary <Subject> grade <grade>, which is stored & downloadable at the following link: GitHub/NQBH/hobby/elementary <subject>/grade <grade>/lecture[1]. The latest version of this text has been stored & downloadable at the following link: GitHub/NQBH/hobby/elementary <subject>/grade <grade>/<topic>[2].

[VI] Tài liệu này là 1 bộ sưu tập các bài tập chọn lọc từ cơ bản đến nâng cao về <*topic vi*>. Tài liệu này là phần bài tập bổ sung cho tài liệu chính – bài giảng GitHub/NQBH/hobby/elementary <subject>/grade <grade>/lecture của tác giả viết cho <Subject> Sơ Cấp lớp <grade>. Phiên bản mới nhất của tài liệu này được lưu trữ & có thể tải xuống ở link sau: GitHub/NQBH/hobby/elementary <subject>/grade <grade>/<topic>.

Nội dung. Hình học sơ cấp.

Mục lục

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[1]URL: https://github.com/NQBH/hobby/blob/master/elementary_<subject>/grade_<grade>/NQBH_elementary_<subject>_grade_<grade>.pdf.

[2]URL: https://github.com/NQBH/hobby/blob/master/elementary_<subject>/grade_<grade>/<topic_name>/NQBH_<topic_name>.pdf.

1 Wikipedia's

1.1 Wikipedia/Congruence (Geometry)

"In geometry, 2 figures or objects are *congruent* if they have the same shape & size, or if one has the same shape & size as the mirror image of the other.

More formally, 2 sets of points are called *congruent* if, & only if, one can be transformed into the other by an isometry, i.e., a combination of rigid motions, namely a translation, a rotation, & a reflection. I.e., either object can be repositioned & reflected (but not resized) so as to coincide precisely with the other object. Therefore 2 distinct plane figures on a piece of paper are congruent if they can be cut out & then matched up completely. Turning the paper over is permitted.

In elementary geometry the word *congruent* is often used as follows. The word *equal* is often used in place of *congruent* for these objects. 2 line segments are congruent if they have the same length. 2 angles are congruent if they have the same measure. 2 circles are congruent if they have the same diameter. In this sense, *2 plane figures are congruent* implies that their corresponding characteristics are "congruent" or "equal" including not just their corresponding sides & angles, but also their corresponding diagonals, perimeters, & areas.

The related concept of similarity applies if the objects have the same shape but do not necessarily have the same size. (Most definitions consider congruence to be a form of similarity, although a minority require that the objects have different sizes in order to qualify as similar.)" – Wikipedia/congruence (geometry)

1.1.1 Determining congruence of polygons

"For 2 polygons to be congruent, they must have an equal number of sides (& hence an equal number – the same number - of vertices). 2 polygons with n sides are congruent iff they each have numerically identical sequences (even if clockwise for 1 polygon & counterclockwise for the other) side-angle-side-angle-... for n sides & n angles.

Congruence of polygons can be established graphically as follows:

1. Match & label the corresponding vertices of the 2 figures.
2. Draw a vector from 1 of the vertices of the 1 of the figures to the corresponding vertex of the other figure. *Translate* the 1st figure by this vector so that these 2 vertices match.
3. *Rotate* the translated figure about the matched vertex until 1 pair of corresponding sides matches.
4. *Reflect* the rotated figure about this matched side until the figures match.

If at any time the step cannot be completed, the polygons are not congruent." – Wikipedia/congruence (geometry)/determining congruence of polygons

1.1.2 Congruence of triangles

"See also: Wikipedia/solution of triangles. 2 triangles are congruent if their corresponding sides are equal in length, & their corresponding angles are equal in measure.

Symbolically, we write the congruency & incongruency of 2 triangles ΔABC & $\Delta A'B'C'$ as follows: $\Delta ABC \cong \Delta A'B'C'$, $\Delta ABC \ncong \Delta A'B'C'$. In many cases it is sufficient to establish the equality of 3 corresponding parts & use 1 of the following results to deduce the congruence of the 2 triangles.

Determining congruence. Sufficient evidence for congruence between 2 triangles in Euclidean space can be shown through the following comparisons:

- **SAS** (side-angle-side): If 2 pairs of sides of 2 triangles are equal in length, & the included angles are equal in measurement, then the triangles are congruent.
- **SSS** (side-side-side): If 3 pairs of sides of 2 triangles are equal in length, then the triangles are congruent.
- **ASA** (angle-side-angle): If 2 pairs of angle of 2 triangles are equal in measurement, & the included sides are equal in length, then the triangles are congruent.

`inserting ...`

" – Wikipedia/congruence (geometry)/congruence of triangles

1.1.3 Definition of congruence in analytic geometry

1.1.4 Congruent conic sections

1.1.5 Congruent polyhedra

1.1.6 Congruent triangles on a sphere

1.2 Wikipedia/Similarity (Geometry)

"In Euclidean geometry, 2 objects are *similar* if they have the same shape, or if one has the same shape as the mirror image of the other. More precisely, one can be obtained from the other by uniformly scaling (enlarging or reducing), possibly with additional translation, rotation, & reflection. I.e., either object can be rescaled, repositioned, & reflected, so as to coincide precisely with the other object. If 2 objects are similar, each is congruent to the result of a particular uniform scaling of the other.

E.g., all circles are similar to each other, all squares are similar to each other, & all equilateral triangles are similar to each other. On the other hand, ellipses are not all similar to each other, rectangles are not all similar to each other, & isosceles triangles are not all similar to each other. This is because 2 ellipses can have different width to height ratio, 2 rectangle can also have a different length to breadth ratio, & 2 isosceles triangle can have different base angles.

If 2 angles of a triangle have measures equal to the measures of 2 angles of another triangle, then the triangles are similar. Corresponding sides of similar polygons are in proportion, & corresponding angles of similar polygons have the same measure.

2 congruent shapes are similar, with a scale factor of 1. However, some school textbooks specifically exclude congruent triangles from their definition of similar triangles by insisting that the sizes must be different if the triangles are to qualify as similar." – Wikipedia/similarity (geometry)

1.2.1 Similar triangles

"2 triangles, ΔABC & $\Delta A'B'C'$ are similar iff corresponding angles have the same measure: this implies that they are similar iff the lengths of corresponding sides are proportional. It can be shown that 2 triangles having congruent angles (*equiangular triangles*) are similar, i.e., the corresponding sides can be proved to be proportional. This is known as the *AAA similarity theorem*. Note that the "AAA" is a mnemonic: each 1 of the 3 A's refers to an "angle". Due to this theorem, several authors simplify the definition of similar triangles to only require that the corresponding 3 angles are congruent.

There are several criteria each of which is necessary & sufficient for 2 triangles to be similar:

- Any 2 pairs of congruent angles, which in Euclidean geometry implies that all 3 angles are congruent: If $\widehat{BAC}$ is equal in measure to $\widehat{B'A'C'}$, & $\widehat{ABC}$ is equal in measure to $\widehat{A'B'C'}$, then this implies that $\widehat{ACB}$ is equal in measure to $\widehat{A'C'B'}$ & the triangles are similar.
- All the corresponding sides are proportional: $\frac{AB}{A'B'} = \frac{BC}{B'C'} = \frac{CA}{C'A'}$. This is equivalent to saying that 1 triangle (or its mirror image) is an enlargement of the other.
- Any 2 pairs of sides are proportional, & the angles included between these sides are congruent: $\frac{AB}{A'B'} = \frac{BC}{B'C'}$ & $\widehat{ABC}$ is equal in measure to $\widehat{A'B'C'}$.

This is known as the *SAS similarity criterion*. The "SAS' is a mnemonic: each 1 of the 2 S's refers to a "side"; the A refers to an "angle" between the 2 sides. Symbolically, we write the similarity & dissimilarity of 2 triangles ΔABC & $\Delta A'B'C'$ as follows: $\Delta ABC \backsim \Delta A'B'C'$, $\Delta ABC \not\backsim \Delta A'B'C'$. There are several elementary results concerning similar triangles in Euclidean geometry:

- Any 2 equilateral triangles are similar.
- 2 triangles, both similar to a 3rd triangle, are similar to each other (transitivity of similarity of triangles).
- Corresponding altitudes of similar triangles have the same ratio as the corresponding sides.
- 2 right triangles are similar if the hypotenuse & one other side have lengths in the same ratio. There are several equivalent conditions in this case, such as the right triangles having an acute angle of the same measure, or having the lengths of the legs (sides) being in the same proportion.

Given a triangle ΔABC & a line segment $\overline{DE}$ one can, with a ruler & compass, find a point F s.t. $\Delta ABC \backsim \Delta DEF$. The statement that point F satisfying this condition exists is Wallis's postulate & is logically equivalent to Euclid's parallel postulate. In hyperbolic geometry (where Wallis's postulate is false) similar triangles are congruent.

In the axiomatic treatment of Euclidean geometry given by George David Birkhoff (see Birkhoff's axioms) the SAS similarity criterion given above was used to replace both Euclid's parallel postulate & the SAS axiom which enabled the dramatic shortening of Hilbert's axioms.

Similar triangles provide the basis for many synthetic (without the use of coordinates) proofs in Euclidean geometry. Among the elementary results that can be proved this way are: the angle bisector theorem, the geometric mean theorem, Ceva's theorem, Menelaus's theorem & the Pythagorean theorem. Similar triangles also provide the foundations for right triangle trigonometry." – Wikipedia/similarity (geometry)/similar triangles

1.2.2 Other similar polygons

"The concept of similarity extends to polygons with ≥ 3 sides. Given any 2 similar polygons, corresponding sides taken in the same sequence (even if clockwise for 1 polygon & counterclockwise for the other) are proportional & corresponding angles taken in the same sequence are equal in measure. However, proportionality of corresponding sides is not by itself sufficient to prove similarity for polygons beyond triangles (otherwise, e.g., all rhombi would be similar). A sufficient condition for similarity of polygons is that corresponding sides & diagonals are proportional. For given n, all regular n-gons are similar." – Wikipedia/similarity (geometry)/other similar polygons

1.2.3 Similar curves

"Several types of curves have the property that all examples of that type are similar to each other. These include: Lines (any 2 lines are even congruent). line segment. circles. parabolas. hyperbolas of a specific eccentricity. Ellipses of a specific eccentricity. catenaries. Graphs of the logarithm function for different bases. Graphs of the exponential function for different bases. Logarithmic spirals are self-similar." – Wikipedia/similarity (geometry)/similar curves

1.2.4 In Euclidean space

1.2.5 Area ratio & volume ratio

1.2.6 Similarity with a center

1.2.7 In general metric spaces

1.2.8 Topology

1.2.9 Self-similarity

1.2.10 Psychology

"The intuition for the notion of geometric similarity already appears in human children, as can be seen in their drawings." – Wikipedia/similarity (geometry)/psychology

2 Some Topics in Geometry & Beyond

- Congruent Triangles – Các Tam Giác Bằng Nhau.
- Similar Triangles – Các Tam Giác Đồng Dạng.