Turbulence

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Contents

Chapter 1

Fluid Mechanics 101

1.1 [CFD] The k-ϵ Turbulence Model

Aidan Wimshurst, *CFD Fundamentals*, Jun 2019.

1.1.1 Overview

What is the standard k-ϵ model? How has the model evolved over time & what variant am I using? What are the *damping functions* & why are they needed? What are the *high-Re* & *low-Re* formulations of the k-ϵ model?

1.1.2 The Eddy Viscosity Hypothesis

The Reynolds Stress in the RANS equation needs to be modeled to close the equations:

$$\partial_t(\rho\mathbf{u}) + \nabla\cdot(\rho\mathbf{u}\mathbf{u}) = -\nabla p + \nabla\cdot\left[\mu(\nabla\mathbf{u} + (\nabla\mathbf{u})^\top)\right] + \rho\mathbf{g} - \nabla\left(\frac{2}{3}\mu(\nabla\cdot\mathbf{u})\right) - \underbrace{\nabla\cdot(\overline{\rho\mathbf{u}'\mathbf{u}'})}_{\text{Reynolds-stress}}.$$

The most common approach is the Boussinesq hypothesis

$$\underbrace{-\overline{\rho\mathbf{u}'\mathbf{u}'}}_{\text{Reynolds stress}} = \mu_t \underbrace{(\nabla\mathbf{u} + (\nabla\mathbf{u})^\top)}_{\text{Mean Velocity Gradients}} - \frac{2}{3}\rho k\mathbf{I} - \frac{2}{3}(\nabla\cdot\mathbf{u})\mathbf{I}.$$

So μ_t needs to be calculated to close the equations.

1.1.3 Mixing Length Models

Older models used a *mixing length* l_m approach to calculate the *eddy viscosity* μ_t: $\mu_t = \rho k^{1/2} l_m$ or $\mu_t = \rho l_m^2 |\partial_y U|$. The *Prandtl mixing length hypothesis* supposes that $l_m = \kappa y$, $\kappa = 0.41$. The wall blocks the maximum size of the eddies.

1.1.4 Van Driest Mixing Model

The *viscosity in the viscous sub-layer* dampens the eddies & reduces their size. The *Van Driest model* accounts for this damping: $l_m = \kappa y\left[1 - \exp(-y^+/A^+)\right]$, $A^+ = 26.0$.

1.1.5 The Scale of Turbulence

The mixing length is specified *algebraically*, $\mu_t = \rho k^{1/2} l_m$. We would like to solve a *transport equation* instead. Instead of l_m, solve for ϵ: $\mu_t = C_\mu \frac{\rho k^2}{\epsilon}$. **Note.** We can calculate the mixing length from the turbulence dissipation rate. $l_m = \frac{C_\mu k^{3/2}}{\epsilon}$.

1.1.6 Transport Equation for k

The transport equation for k is the *same* in RNG, realizable & standard k-ϵ models

$$\underbrace{\partial_t(\rho k)}_{\text{Time}} + \underbrace{\nabla\cdot(\rho\mathbf{u}k)}_{\text{Convection}} = \underbrace{\nabla\cdot\left[\left(\mu + \frac{\mu_t}{\sigma_k}\right)\nabla k\right]}_{\text{Diffusion}} + \underbrace{P_k + P_b - \rho\epsilon + S_k}_{\text{Sources+Sinks}}.$$

P_k is the production due to mean velocity shear. P_b is the production due to buoyancy. S_k is a user-defined source.

1.1.7 Transport Equation for ϵ

$$\underbrace{\partial_t(\rho\epsilon)}_{\text{Time}} + \underbrace{\nabla\cdot(\rho\mathbf{u}\epsilon)}_{\text{Convection}} = \underbrace{\nabla\cdot\left[\left(\mu+\frac{\mu_t}{\sigma_\epsilon}\right)\nabla\epsilon\right]}_{\text{Diffusion}} + \underbrace{C_1\frac{\epsilon}{k}(P_k+C_3P_b) - C_2\rho\frac{\epsilon^2}{k} + S_\epsilon}_{\text{Source+Sinks}}.$$

The model coefficients C_1, C_2, & C_3 vary between models. Once the transport equations for k & ϵ have been solved, we can compute μ_t by $\mu_t = C_\mu\frac{\rho k^2}{\epsilon}$.

1.1.8 Model Coefficients

The model coefficients for the standard k-ϵ model have evolved through time.

Model	σ_k	σ_ϵ	C_1	C_2	C_μ
Jones & Launder (1972)	1.0	1.3	**1.55**	**2.0**	0.09
Launder & Spalding (1974)	1.0	1.3	**1.44**	**1.92**	0.09
Launder & Sharma (1974)	1.0	1.3	**1.44**	**1.92**	0.09

The Launder & Sharma (1974) coefficients are the most up-to-date. They are used Fluent, OpenFOAM, CFX & Star.

1.1.9 Damping Functions

In the mixing length model, the mixing length was damped close to the wall (to account for viscosity) $l_m = \kappa y\left[1-\exp(-y^+/A^+)\right]$, $A^+ = 26.0$. In the k-ϵ model, the *model coefficients* C_1, C_2, & C_μ are damped. The damping functions are termed f_1, f_2, & f_μ. Hence, the equations can be applied in the viscous sub-layer ($y^+ < 5$). This is called a *low-Re formulation*.

$$f_1 = 1,\ f_2 = 1-0.3\exp(-\mathrm{Re}_T^2),\ f_\mu = \exp\left(-\frac{3.4}{\left(1+\frac{\mathrm{Re}_T}{50}\right)^2}\right).$$

Re_T is the *turbulent Reynolds number* $\mathrm{Re}_T = \frac{\rho k^2}{\mu\epsilon}$. Re_T is the Reynolds number that characterizes the strength of the near wall turbulence relative to viscosity

$$\mathrm{Re}_T = \frac{\text{Turbulent Forces}}{\text{Viscous Forces}} = \frac{\rho k^{1/2} l_m}{\mu} \text{ i.e. } \sim \frac{\rho U L}{\mu}.$$

Substitute for the mixing length $\mathrm{Re}_T = \frac{\rho k^{1/2}}{\mu}\frac{k^{3/2}}{\epsilon} = \frac{\rho k^2}{\mu\epsilon}$. When Re_T is small, viscous effects dominate. Re_T is small when we are in the viscous sub-layer (a *low-Re formulation*).

1.1.10 The Damping Function f_μ

In a *low-Re formulation* the turbulent/eddy viscosity is computed from k & ϵ: $\mu_t = f_\mu C_\mu\frac{\rho k^2}{\epsilon}$. Launder & Sharma (1974) propose: $f_\mu = \exp\left(-\frac{3.4}{\left(1+\frac{\mathrm{Re}_T}{50}\right)^2}\right)$. Physically, the damping function is reducing the turbulent viscosity μ_t in *every cell* in the mesh (not just wall adjacent cell). Hence, the laminar viscosity will dominate the diffusion term in the momentum equations: $\cdots + \nabla\cdot\left[(\mu+\mu_t)(\nabla\mathbf{u}+(\nabla\mathbf{u})^\top)\right]+\cdots$. The damping function is applied to every cell in the mesh. Far from the wall, $f_\mu = 1$ & we return to the *high-Re* formulation.

1.1.11 The Damping Function f_1

The damping function f_1 is applied to the *production* of ϵ:

$$\partial_t(\rho\epsilon) + \nabla\cdot(\rho\mathbf{u}\epsilon) = \nabla\cdot\left[\left(\mu+\frac{\mu_t}{\sigma_\epsilon}\right)\nabla\epsilon\right] + C_1\frac{\epsilon}{k}(f_1P_k+C_3P_b) - C_2\rho\frac{\epsilon^2}{k} + S_\epsilon.$$

Launder & Jones (1972) find no noticeable improvement in implementing a damping function for f_1. $f_1 = 1$.

1.1.12 The Damping Function f_2

The damping function f_2 is applied to the *dissipation* of ϵ

$$\partial_t(\rho\epsilon) + \nabla\cdot(\rho\mathbf{u}\epsilon) = \nabla\cdot\left[\left(\mu + \frac{\mu_t}{\sigma_\epsilon}\right)\nabla\epsilon\right] + C_1\frac{\epsilon}{k}(P_k + C_3P_b) - f_2C_2\rho\frac{\epsilon^2}{k} + S_\epsilon.$$

Physically, f_2 reduces the dissipation of ϵ near the wall. Hence we got *more dissipation* of k near the wall. This is the expected action of viscosity. $f_2 = 1 - 0.3\exp(-\mathrm{Re}_T^2)$.

1.1.13 Summary

In a low-Re formulation, damping functions are applied to the model coefficients C_μ, C_1, & C_2. Hence, we can resolve the mesh down to the viscous sub-layer ($y^+ < 5$). Nowadays, the k-ω SST model is preferred for these applications. Hence, the k-ϵ model is usually preferred for *high-Re* application ($y^+ > 30$), where separation, reattachment are not present.

Refs.

- W. P. Jones, B. E. Launder. The prediction of laminarization with a 2-equation model of turbulence. *Int. Journal of Heat & Mass Transfer*, 1972.
- B. E. Launder, D. B. Spalding. The numerical computation of turbulent flows. *Comp. Methods in App. Mech & Engineering*, 1974.
- B. E. Launder, B. I. Sharma. Application of the energy dissipation model of turbulence to the calculation of flow near a spinning disc. *Letters in Heat & Mass Transfer*, 1974.

Chapter 2

YouTube/József Nagy

2.1 YouTube/József Nagy/CFD basics

YouTube/József Nagy/Introduction to CFD.

2.1.1 YouTube/József Nagy/CFD basics/how to run your 1st simulation in OpenFOAM – Part 1 – tutorial

Goals. Case setup. Initial values. Mesh (e.g., boundaries). Simulate 75s of a flow in an elbow (2D) (tri-mesh, quad-mesh, refined quad-mesh). Postprocessing.

`icoFoam`. 'Transient solver for incompressible, laminar flow of Newtonian fluids': incompressible, transient, laminar, Newtonian fluids, single phase, isothermal, PISO-loop.

2.2 YouTube/József Nagy/CFD Intermediate