Inequality

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Chapter 1

Wikipedia's

1.1 Wikipedia/Inequality (Mathematics)

1.2 Wikipedia/Isoperimetric Inequality

"In mathematics, the *isoperimetric inequality* is a geometry inequality involving the perimeter of a set & its volume. In n-dimensional space $\mathbb{R}^n$ the inequality lower bounds the surface area or perimeter $\mathrm{per}(S)$ of a set $S \subset \mathbb{R}^n$ by its volume $\mathrm{vol}(S)$,

$$\mathrm{per}(S) \geq n\,\mathrm{vol}(S)^{1-\frac{1}{n}}\,\mathrm{vol}(B_1)^{\frac{1}{n}},$$

where $B_1 \subset \mathbb{R}^n$ is a unit sphere. The equality holds only when S is a sphere in $\mathbb{R}^n$.

On a plane, i.e., when $n = 2$, the isoperimetric inequality relates the square of the circumference of a closed curve & the area of a plane region it encloses. *Isoperimetric* literally means "having the same perimeter". Specifically in $\mathbb{R}^2$, the isoperimetric inequality states, for the length L of a closed curve & the area A of the planar region that it encloses, that $L^2 \geq 4\pi A$, & that equality holds iff the curve is a circle.

The *isoperimetric problem* is to determine a plane figure of the largest possible area whose boundary has a specified length. The closely related *Dido's problem* asks for a region of the maximal area bounded by a straight line & a curvilinear arc whose endpoints belong to that line. It is named after Dido, the legendary founder & 1st queen of Carthage. The solution to the isoperimetric problem is given by a circle & was known already in Ancient Greece. However, the 1st mathematically rigorous proof of this fact was obtained only in the 19th century. Since then, many other proofs have been found.

The isoperimetric problem has been extended in multiple ways, e.g., to curves on surfaces & to regions in higher-dimensional spaces. Perhaps the most familiar physical manifestation of the 3D isoperimetric inequality is the shape of a drop of water. Namely, a drop will typically assume a symmetric round shape. Since the amount of water in a drop is fixed, surface tension forces the drop into a shape which minimizes the surface area of the drop, namely a round sphere.

1.2.1 The isoperimetric problem in the plane

1.2.2 On a plane

1.2.3 On a sphere

1.2.4 In $\mathbb{R}^n$

1.2.5 In Hadamard manifolds

1.2.6 In a metric measure space

1.2.7 For graphs

1.2.7.1 Example: Isoperimetric inequalities for hypercubes

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1.2.7.1.2 Vertex isoperimetric inequality.

1.2.8 Isoperimetric inequality for triangles

Chapter 2

Hardy, Littlewood, and Pólya, 1952. Inequality

"Oh! the little more, & how much it is! & the little less, & what worlds away!" – Robert Browning

Preface to 1st Edition

"This book was planned & begun in 1929. Our original intention was that it should be 1 of the *Cambridge Tracts*, but it soon became plain that a tract[1] would be much too short for our purpose.

Our subjects in writing the book are explained sufficiently in the introductory chapter, but we add a note here about history & bibliography[2]. Historical & bibliographical questions are particularly troublesome[3] in a subject like this, which has applications in every part of mathematics but has never been developed systematically.

It is often really difficult to trace the origin of a familiar inequality. It is quite likely to occur 1st as an auxiliary proposition, often without explicit statement, in a memoir on geometry or astronomy; it may have been rediscovered, many years later, by half a dozen different authors; & no accessible statement of it may be quite complete. We have almost always found, even with the most famous inequalities, that we have a little new to add.

We have done our best to be accurate & have given all references we can, but we have never undertaken[4] systematic bibliographical research. We follow the common practice, when a particular inequality is habitually[5] associated with a particular mathematician's name; we speak of the inequalities of Schwarz, Hölder, & Jensen, though all these inequalities can be traced further back; & we do not enumerate[6] explicitly all the minor additions which are necessary for absolute completeness.

[1]**tract** [n] **1.** (*biology*) a system of connected organs or tissues along which materials or messages pass; **2. tract (of something)** an area of land, especially a large one; **3.** a short piece of writing, especially on a religious, moral or political subject, that is intended to influence people's ideas.

[2]**bibliography** [n] (plural **bibliographies**) the list of books, etc. that have been used by somebody writing an article, essay, etc.; a list of books or articles about a particular subject or by a particular author.

[3]**troublesome** [a] causing trouble, pain or difficulties.

[4]**undertake** [v] **1. undertake something** to make yourself responsible for something & start doing it; **2.** to agree or promise that you will do something.

[5]**habitual** [a] [only before noun] usual or typical of somebody/something.

[6]**enumerate** [v] (*formal*) **enumerate something** to name things on a list 1 by 1.

Bibliography

Hardy, G. H., J. E. Littlewood, and G. Pólya (1952). *Inequalities*. 2d ed. Cambridge, at the University Press, pp. xii+324.