Substitution & Change of Variables in Elementary Mathematics

Nguyễn Quản Bá Hồng*

August 24, 2022

Abstract

Various substitutions in elementary mathematics. This text is also a chapter in the book *Some Topics in Inequality* written by the author.

Contents

1 Basic Concepts

Start from some basic concepts:

Definition 1.1 (Change of variables)**.** *"In mathematics, a* change of variables *is a basic technique used to simplify problems in which the original variables are replaced with functions of other variables." – Wikipedia/change of variables*

"The intent is that when expressed in new variables, *the problem may become simpler, or equivalent to a better understood problem.* Change of variables is an operation that is related to *substitution*. However, these are different operations, as can be seen when considering differentiation (chain rule) or integration (integration by substitution)." – Wikipedia/change of variables

Definition 1.2 (Substitution (algebra))**.** *"In algebra, the operation of* substitution *can be applied in various contexts involving formal objects containing symbols (often called variables or indeterminates); the operation consists of systematically replacing occurrences of some symbol by a given value." – Wikipedia/substitution (algebra)*

"Substitution is a basic operation of computer algebra. It is generally called "subs" or "subst" in computer algebra systems." – Wikipedia/substitution (algebra)

Example 1.1 (Substitution in some CASs)**.**

Example 1.2 (Substitution in some programming languages)**.**

"A common case of substitution involves polynomials, where substitution of a numerical value for the indeterminate of a (univariate) polynomial amounts of evaluating the polynomial at that value. Indeed, this operation occurs so frequently that the notation for polynomials is often adapted to it." – Wikipedia/substitution (algebra)

Example 1.3 (Substitution in polynomial)**.** *Let* $P(x) \coloneqq \sum_{i=0}^{n} a_i x^i$ *be a polynomial of degree* n, $a_i \in \mathbb{R}$, $i = 0, \dots, n$, $a_n \neq 0$, $\deg P = n$. *Then the value of* $P(x)$ *when/at the point* $x = a$ *for some* $a \in \mathbb{R}$ *can be calculated by plugging the substitution* $x = a$ *into* $P(x)$, *i.e.,* $P(a) = \sum_{i=0}^{n} a_i a^i$, $\forall a \in \mathbb{R}$. *Similarly, with* $f : \mathbb{R} \to \mathbb{R}$ *be an arbitrary function, the composition function* $(P \circ f)(x) = P(f(x))$ *can be obtained by substituting "*$x = f(x)$*" into* $P(x)$, *i.e.,* $P(f(x)) = \sum_{i=0}^{n} a_i (f(x))^i$, $\forall x \in \mathbb{R}$.

*Independent Researcher, Ben Tre City, Vietnam
e-mail: nguyenquanbahong@gmail.com; website: https://nqbh.github.io.

2 Various Forms of Substitutions & Changes of Variables

2.1 Substitution in Solving Equations & Inequations

2.2 Substitution in Proving Inequality

2.3 Integration by Substitution

2.4 Trigonometric Substitution