Diophantine Equation

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Abstract

A set of problems of Diophantine equations.

Contents

1 Wikipedia/Diophantine Equation

Finding all right triangles with integer side-lengths is equivalent to solving the Diophantine equation $a^2 + b^2 = c^2$.

"In mathematics, a *Diophantine equation* is a polynomial equation, usually involving 2 or more unknowns, s.t. the only solutions of interest are the integer ones. A *linear Diophantine equation* equates to a constant the sum of 2 or more monomials, each of degree 1. An *exponential Diophantine equation* is one in which unknowns can appear in exponents.

Diophantine problems have fewer equations than unknowns & involve finding integers that solve simultaneously all equations. As such systems of equations define algebraic curves, algebraic surfaces, or, more generally, algebraic sets, their study is a part of algebraic geometry that is called *Diophantine geometry*.

The word *Diophantine* refers to the Hellenistic mathematician of the 3rd century, Diophantus of Alexandria, who made a study of such equations & was 1 of the 1st mathematicians to introduce symbolism into algebra. The mathematical study of Diophantine problems that Diophantus initiated is now called *Diophantine analysis*.

While individual equations present a kind of puzzle & have been considered throughout history, the formulation of general theories of Diophantine equations (beyond the case of linear & quadratic equations) was an achievement of the 20th century." – Wikipedia/Diophantine equation

1.1 Examples of Diophantine Equation

"In the following Diophantine equations, w, x, y, z are the unknowns & the other letters are given constants: • $ax+by = c$: a linear Diophantine equation. • $w^3+x^3 = y^3+z^3$: The smallest nontrivial solution in positive integers is $12^3+1^3 = 9^3+10^3 = 1729$. It was famously given as an evident property of 1729, a taxicab number (also named Hardy–Ramanujan number) by Ramanujan to Hardy while meeting in 1917. There are infinitely many nontrivial solutions. • For $n = 2$, there are infinitely many solutions (x, y, z): the Pythagorean triples. For larger integer values of n, Fermat's Last Theorem (initially claimed in 1637 by Fermat & proved by Andrew Wiles in 1995) states there are no positive integer solutions (x, y, z). • $x^2 - ny^2 = \pm 1$: This is Pell's equation, which is named after the English mathematician John Pell. It was studied by Brahmagupta in the 7th century, as well as by Fermat in the 17th century. • $\frac{4}{n} = \frac{1}{x} + \frac{1}{y} + \frac{1}{z}$: The Erdős–Strauss conjecture states that, for every positive integer $n \geq 2$, there exists a solution in x, y, z, all as positive integers. Although not usually stated in polynomial form, this example is equivalent to the polynomial equation $4xyz = yzn + xzn + xyn = n(xy + yz + zx)$. • $x^4 + y^4 + z^4 = w^4$:

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Conjectured incorrectly by Euler to have no nontrivial solutions. Proved by Elkies to have infinitely many nontrivial solutions, with a computer search by Frye determining the smallest nontrivial solution, $95800^4 + 217519^4 + 414560^4 = 422481^4$." – Wikipedia/Diophantine equation/example

1.2 Linear Diophantine Equations

1.3 Homogeneous Equations

1.4 Diophantine Analysis

1.5 Exponential Diophantine Equations

2 Phương Pháp Xét Tính Chia Hết

Bài toán 2.1 (Bình, 2021, Thí dụ 1, p. 6)**.** *Giải phương trình nghiệm nguyên* $3x + 17y = 159$.

Bài toán 2.2 (Bình, 2021, Thí dụ 2, p. 6)**.** *Tìm nghiệm nguyên của phương trình* $xy - x - y = 2$.

Bài toán 2.3 (Bình, 2021, Thí dụ 3, p. 7)**.** *Tìm nghiệm nguyên của phương trình* $2xy - x + y = 3$.

Tài liệu

Bình, Vũ Hữu (2021). *Phương Trình Nghiệm Nguyên & Kinh Nghiệm Giải*. Nhà Xuất Bản Giáo Dục Việt Nam, p. 224.