

Computer Science

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Chapter 1

Wikipedia's

1.1 Wikipedia/Computer Science

Fundamental areas of computer science. Programming language theory, Computational complexity theory, Artificial intelligence, Computer architecture. History, Outline, Glossary, Category.

Computer science is the study of computation, automation, & information. Computer science spans theoretical disciplines (such as algorithms, theory of computation, & information theory) to practical disciplines (including the design & implementation of hardware & software). Computer science is generally considered an area of academic research & distinct from computer programming.

Algorithms & data structures are central to computer science. The theory of computation concerns abstract models of computation & general classes of problems that can be solved using them. The fields of cryptography & computer security involve studying the means for secure communication & for preventing security vulnerabilities. Computer graphics & computational geometry address the generation of images. Programming language theory considers approaches to the description of computational processes, & database theory concerns the management of repositories of data. Human-computer interaction investigates the interfaces through which humans & computers interact, & software engineering focuses on the design & principles behind developing software. Areas such as operating systems, networks & embedded systems investigate the principles & design behind complex systems. Computer architecture describes the construction of computer components & computer-operated equipment. Artificial intelligence & machine learning aim to synthesize goal-orientated processes such as problem-solving, decision-making, environmental adaptation, planning & learning found in humans & animals. Within artificial intelligence, computer vision aims to understand & process image & video data, while natural-language processing aims to understand & process textual & linguistic data.

The fundamental concern of computer science is determining what can & cannot be automated. The **Turning Award** is generally recognized as the highest distinction in computer science.” – Wikipedia/computer science

1.1.1 History

1.1.2 Etymology

1.1.3 Philosophy

Epistemology of computer science

Paradigms of computer science

1.1.4 Fields

Theoretical computer science

Theory of computation.

Information & coding theory.

Data structures & algorithms.

Programming language theory & formal methods.

Computer systems & computational processes

Artificial intelligence.

Computer architecture & organization.

Concurrent, parallel & distributed computing.

Computer networks.

Computer security & cryptography.

Databases & data mining.

Computer graphics & visualization.

Image & sound processing.

Applied computer science

Computational science, finance & engineering.

Social computing & human–computer interaction.

Software engineering.

1.1.5 Discoveries

1.1.6 Programming paradigms

1.1.7 Academia

1.1.8 Education

Part I

The Art of Computer Programming

The Art of Computer Programming (TAOCP)

“At the end of 1999, these books were named among the best 12 physical-science monographs of the century by **American Scientists**, along with: Dirac on quantum mechanics, Einstein on relativity, Mandelbrot on fractals, Pauling on the chemical bond, Russell & Whitehead on foundations of mathematics, von Neumann & Morgensstern on game theory, Wiener on cybernetics, Woodward & Hoffmann on orbital symmetry, Feynmann on quantum electrodynamics, Smith on search for structure, & Einstein’s collected papers. Wow” “**historic**” **publisher’s brochure from the 1st edition of Vol. 1 (1968)**. A complimentary *downloadable PDF containing the collected indexes* is **available from the publisher** to registered owners of the 4-volume boxed set. This PDF also includes the complete indexes of Vols. 1, 2, 3, & 4A, as well as to Vol. 1 Fascicle 1 & to Vol. 4 Fascicles 5 & 6.”

eBook versions

“These volumes are now available also in portable electronic form, using PDF format prepared by the experts at **Mathematical Sciences Publishers**. Special care has been taken to make the search feature work well. Thousands of useful “clickable” cross-references are also provided – from exercises to their answers & back, from the index to the text, from the text to important tables & figures, etc.

Warning. Unfortunately, however, non-PDF versions have also appeared, against my recommendations, & those versions are frankly quite awful. A great deal of expertise & care is necessary to do the job right. If you have been misled into purchasing 1 of these inferior versions (e.g., a Kindle version), the publishers have told me that they will replace your copy with the PDF edition that I have personally approved. **Do not purchase eTAOCP in Kindle format if you expect the mathematics to make sense.** (The ePub format may be just as bad; I really don’t want to know, & I am really sorry that it was released). Please do not tell me about errors that you find in a non-PDF eBook; such mistakes should be reported directly to the publisher. Some non-PDF versions also masquerade as PDF. You can tell an authorized version because its copyright page (with the exception of Vol. 4 Fascicle 5) will say ‘Electronic version by Mathematical Sciences Publishers (MSP)’.

The authorized PDF versions can be purchased at www.informit.com/taocp. If you have purchased a different version of the eBook, & can provide proof of purchase of that eBook, you can obtain a gratis PDF version by sending email & proof of purchase to taocp@pearson.com.”

Volume 1

- *Fundamental Algorithms*, 3rd Edition (Reading, Massachusetts: Addison-Wesley, 1997), xx+650pp.
- *Volume 1 Fascicle 1*, MMIX: A RISC Computer for the New Millennium (2005), v+134pp.

Brochure

“I am overwhelmed by the wealth of exciting & fresh material you have managed to pack into the book, especially in view of the fact that it is only the 1st of 7 volumes! “Monumental” is the only word for it ... Moreover, it is written with a grace & humor that is, as you know, exceedingly rare in books on mathematics. I greatly enjoyed your dedication, your flow-chart for reading the series, your notes on the exercises; above all, your choice of illustrative material throughout & the clarity & brevity with which you explain everything.” – Martin Gardner, *Mathematical Games*, *Scientific American*

“This combined reference & text, *Fundamental Algorithms*, is the 1st volume of a planned 7-volume series. The series will provide a unified, readable, & theoretically sound summary of the present knowledge of computer programming techniques, plus a study of their historical development.

The point of view adopted by the author differs from many contemporary books about compute programming. The author does not try to teach the reader how to use somebody else’s subroutines, but is concerned rather with teaching the reader how to write better subroutines himself.

A reader who is interested primarily in programming rather than in the associated mathematics may stop reading each section as soon as the mathematics become recognizably difficult. On the other hand, a mathematically oriented reader will find a wealth of interesting material.

As a reference the series provides valuable information for system programmers, analyst programmers, & others in the computer & related software industries. All 7 volumes of this series may also be used in senior or graduate courses such as: Information Structures, Computer Science, Combinatorial Mathematics, Computer-Oriented Finite Mathematics, or Fundamentals of Symbolic Machine Language Programming.

Among the areas covered in Vol. 1 are the representation of information inside a computer; the structural interrelations between data elements & how to deal with them efficiently; plus applications to simulation, numerical methods, software

design, & other factors. Also included is an introduction to fundamental topics in discrete mathematics, of special importance in the study of computer programming techniques.

There are over 850 exercises, graded according to the level of difficulty from extremely simple questions to unsolved research problems. Answers are supplied for over 90% of the exercises. This enhances the value of the book for self-study, classroom use, & for reference. & it helps make it possible to organize the book so that it can be read by both mathematicians & non-mathematicians.” 634 pages, 71 figures, (1968), \$19.50

Fig. 28. Representation of polynomials using 4-directional links. Shaded areas of nodes indicate information irrelevant in the context considered.

Chapter 2

The Art of Computer Programming. Vol. 1: Fundamental Algorithms

This series of books is affectionately¹ dedicated to the Type 650 computer once installed at Case Institute of Technology, in remembrance² of many pleasant³ evenings.

Preface

“Here is your book, the one your thousands of letters have asked us to publish. It has taken us years to do, checking & rechecking countless⁴ recipes⁵ to bring you only the best, only the interesting, only the perfect. Now we can say, without a shadow of a doubt, that every single 1 of them, if you follow the directions to the letter, will work for you exactly as well as it did for us, even if you have never cooked before.” – McCall’s Cookbook (1963)

“THE PROCESS of preparing programs for a digital⁶ computer is especially attractive⁷, not only because it can be economically⁸ & scientifically⁹ rewarding¹⁰, but also because it can be an aesthetic¹¹ experience much like composing¹² poetry¹³ or music. This book is the 1st volume of a multi-volume set of books that has been designed to train the reader in various skills that go into a programmer’s craft¹⁴.

The following chapters are *not* meant to serve as an introduction to computer programming; the reader is supposed to have had some previous experience. The prerequisites¹⁵ are actually very simple, but a beginner requires time & practice in

¹**affectionately** [adv] in a way that shows caring feelings & love for somebody.

²**remembrance** [n] [uncountable] the act or process of remembering an event in the past or a person who is dead.

³**pleasant** [a] (**pleasanter, pleasantest**) (**more pleasant & most pleasant** are more common) enjoyable, pleasing or attractive, OPPOSITE: **unpleasant**.

⁴**countless** [a] [usually before noun] very many; too many to be counted or mentioned.

⁵**recipe** [n] **1. recipe (for something)** a set of instructions that tells you how to cook something & the ingredients you need for it; **2. recipe for something** a method or an idea that seems likely to have a particular result, SYNONYM: **formula**.

⁶**digital** [a] **1.** using a system of receiving & sending information as a series of the numbers 1 & 0, showing that an electronic signal is there or is not there; connected with computer technology; **2.** (of clocks, watches, etc.) displaying only the appropriate numbers, rather than pointing to numbers from a larger set of numbers; other information displayed in this way; **3.** connected with a finger or the fingers of the hand.

⁷**attractive** [a] **1.** (of a person or an animal) pleasant to look at, especially in a sexual way; making an animal interested in a sexual way, OPPOSITE: **unattractive**; **2.** (of a thing or a place) pleasant to look at or be in, OPPOSITE: **unattractive**; **3.** having features or qualities that make something seem interesting & worth having, SYNONYM: **appealing**, OPPOSITE: **unattractive**; **4.** (*physics*) involving the force that pulls things towards each other, OPPOSITE: **repulsive**.

⁸**economically** [adv] **1.** in a way that is connected with the trade, industry & development of wealth of a country, an area or a society; **2.** in a way that provides good service or value in relation to the amount of time or money spent.

⁹**scientifically** [adv] **1.** in a way that is connected with science; **2.** in a careful & organized way.

¹⁰**rewarding** [a] **1.** (of an activity) worth doing; that makes you happy because you think it is useful or important; **2.** producing a lot of money, SYNONYM: **profitable**.

¹¹**aesthetic** [a] (*North American English also esthetic*) **1.** concerned with beauty & art & the understanding of beautiful things; **2.** beautiful to look at; [n] (*North American English also esthetic*) **1.** [countable] **aesthetic (of something)** a set of principles that express the aesthetic qualities & ideas of a particular artist or a particular group of artists, writers, etc.; **2.** (**aesthetics**) [uncountable] the branch of philosophy that studies the principles of beauty, especially in art.

¹²**compose** [v] **1. (be composed of something)** to be made or formed from several substances, parts of people; **2.** (not used in the progressive tenses) **compose something** to combine together to form a whole, SYNONYM: **make something up**; **3.** to write a piece of music; **4.** to write something, especially a poem.

¹³**poetry** [n] [uncountable] a collection of poems; poems in general, SYNONYM: **verse**.

¹⁴**craft** [n] **1.** [countable, uncountable] an activity involving a special skill at making things with your hands; **2.** [singular] all the skills needed for a particular activity; **3.** (plural **craft**) [countable] a boat or ship; [v] [usually passive] **craft something** to make something using a special skill, SYNONYM: **fashion**.

¹⁵**prerequisite** [n] [usually singular] something that must exist or happen before something else can happen or be done, SYNONYM: **precondition**.

order to understand the concept of a digital computer. The reader should possess:

- a) Some idea of how a stored-program digital computer works; not necessarily the electronics¹⁶, rather the manner¹⁷ in which instructions¹⁸ can be kept in the machine¹⁹'s memory²⁰ & successively²¹ executed²².
- b) An ability to put the solutions to problems into such explicit²³ terms that a computer can “understand” them. (These machines have no common sense²⁴; they do exactly²⁵ as they are told, no more & no less. This fact is the hardest concept to grasp²⁶ when one 1st tries to use a computer.)
- c) Some knowledge of the most elementary²⁷ computer techniques, such as looping²⁸ (performing²⁹ a set of instructions³⁰ repeatedly³¹), the use of subroutines³², & the use of indexed³³ variables³⁴.

¹⁶**electronics** [n] **1.** [uncountable] the branch of science & technology that studies electric currents in electronic equipment; **2.** [uncountable] the use of electronic technology; the making of electronic products; **3.** [plural] the electronic circuits & components used in electronic equipment.

¹⁷**manner** [n] **1.** [singular] the way that something is done or happens; **2.** [singular] the way that somebody behaves towards other people; **3.** (**manners**) [plural] behavior that is considered to be polite in a particular society or culture; **4.** (**manners (of somebody/something)**) [plural] the habits & customs of a particular group of people; **all manner of somebody/something** [idiom] many different types of people or things; **in the manner of somebody/something** [idiom] in a style that is typical of somebody/something.

¹⁸**instruction** [n] **1.** (**instructions**) [plural] detailed information on how to do or use something, SYNONYM: **direction**; **2.** [countable, usually plural] something that somebody tells you to do, SYNONYM: **order**; **3.** [countable] (*computing*) a code in a program that tells a computer to perform a particular operation; **4.** [uncountable] the act of teaching something to somebody.

¹⁹**machine** [n] **1.** (often in compounds) a piece of equipment with moving parts that is designed to do a particular job. The power used to work a machine may be electricity, steam, gas, etc. or human power; **2.** a group of people who operate in an efficient way within an organization.

²⁰**memory** [n] (plural **memories**) **1.** [countable, uncountable] your ability to remember things; the part of your mind in which you store things that you remember; **2.** [uncountable] **in/within ... memory** the period of time that somebody is able to remember events; **3.** [countable] a thought of something that you remember from the past; **4.** [uncountable, countable] the part of a computer where data are stored; the amount of space in a computer for storing data; **5.** [uncountable] **memory (of somebody)** what is remembered about somebody after they have died; **from memory** [idiom] without reading or looking at notes; **in memory of somebody, to the memory of somebody** [idiom] intended to show respect & remind people of somebody who has died; **within/in living memory** [idiom] at a time, or during the time, that is remembered by people still alive.

²¹**successive** [a] [only before noun] following immediately one after the other, SYNONYM: **consecutive**.

²²**execute** [v] **1.** [usually passive] to kill somebody, especially as a legal punishment; **2.** **execute something** to do a piece of work, perform a duty, put a plan into action, etc.; **3.** **execute something** (*computing*) carry out an instruction or program; **4.** **execute something** (*law*) to follow the instructions in a legal document; to make a document legally valid.

²³**explicit** [a] **1.** saying something clearly & exactly; **2.** showing or referring to sex in a very obvious or detailed way.

²⁴**common sense** [n] [uncountable] the ability to think about things in a practical way & make sensible decisions.

²⁵**exactly** [adv] used to emphasize that something is correct in every way or in every detail, SYNONYM: **precisely**.

²⁶**grasp** [v] **1.** to understand something completely; **2.** **grasp an opportunity** to take an opportunity without hesitating & use it; **3.** **grasp somebody/something** to take a firm hold of somebody/something, SYNONYM: **grip**; [n] [usually singular] **1.** a person's understanding of a subject; **2.** a firm hold of somebody/something or control over somebody/something; **3.** the ability to get or achieve something.

²⁷**elementary** [a] **1.** connected with the 1st stages of a course of study, or the 1st years at school; **2.** of the most basic kind; **3.** very simple & easy.

²⁸**loop** [n] **1.** a shape like a curve or circle made by a line curving right around; **2.** a piece of rope, wire, etc. in the shape of a curve or circle; **3.** a long, narrow piece of film or tape on which the pictures & sound are repeated continuously; **4.** (*computing*) a set of instructions that is repeated again & again until a particular condition is satisfied; **5.** a complete circuit for electrical current; **6.** (*British English*) a railway line or road that leaves the main track or road & then joins it again; [v] **1.** [transitive] **loop something + adv./prep.** to form or bend something into a loop; **2.** [intransitive] + **adv./prep.** to move in a way that makes the shape of a loop; **loop the loop** [idiom] to fly or make a plane fly in a circle going up & down.

²⁹**perform** [v] **1.** [transitive] **perform something** to do something, such as a piece of work, task or duty, SYNONYM: **carry something out**; **2.** [intransitive] + **adv./prep.** to work or function well or badly. In this meaning, where there is no adverb or preposition, **perform** means ‘perform well’; **3.** [transitive, intransitive] **perform (something)** to entertain an audience by playing a piece of music, acting in a play, etc.

³⁰**instruction** [n] **1.** (**instructions**) [plural] detailed information on how to do or use something, SYNONYM: **direction**; **2.** [countable, usually plural] something that somebody tells you to do, SYNONYM: **order**; **3.** [countable] (*computing*) a code in a program that tells a computer to perform a particular operation; **4.** [uncountable] the act of teaching something to somebody.

³¹**repeatedly** [adv] many times; again & again.

³²**subroutine** [n] (also **subprogram**) (*computing*) a set of instructions which perform a task within a program.

³³**index** [n] **1.** (plural **indexes**) **index (to something)** (in a book or set of books) an alphabetical list of names, subjects, etc. with the numbers of the pages on which they are mentioned; **2.** (**indexes, indices**) a number in a system or scale that represents the average value of particular prices, shares, etc. compared with a previous or standard value; **3.** (**indices**) **index of something** a sign or measure that something else can be judged by; **4.** (in compounds) a number that gives the value of a physical quality in terms of a standard formula; **5.** (usually **indices** [plural] *mathematics*) a small number written above another number to show how many times the other number must be multiplied by itself. In the equation $4^2 = 16$, the number 2 is an index; **6.** (**indexes**) (*computing*) a list of items, each of which identifies a particular record in a computer file or database & contains information about its address; [v] **1.** **index something** to record names, subjects, etc. in an index; **2.** **index something** to provide an index to something; **3.** [usually passive] **index something (to something)** to link salaries, prices, etc. to the level of prices of food & other goods so that they both increase at the same rate.

³⁴**variable** [n] **1.** an element or a feature that is likely to vary or change, OPPOSITE: **constant**; **2.** a property that is measured or observed in an experiment or a study; a property that is adjusted in an experiment, OPPOSITE: **constant**; **3.** (*mathematics*) a quantity in a calculation that can take any of a set of different numerical values, represented by a symbol such as x , OPPOSITE: **constant**; [a] **1.** often changing; likely to change, SYNONYM: **fluctuating**, OPPOSITE: **constant**; **2.** not the same in all parts or cases; not having a fixed pattern, SYNONYM: **diverse**. When **variable** is used to describe the quality of something, the tone is slightly disapproving, meaning that some parts of it are good & some are bad, SYNONYM: **inconsistent, mixed**, OPPOSITE: **consistent, uniform**; **3.** that can be changed to meet different needs or suit different conditions, OPPOSITE: **fixed**; **4.** (*mathematics*) (of a quantity) that can take any of a set of different numerical values, represented by a symbol such as x , OPPOSITE: **constant**.

- d) A little knowledge of common computer jargon³⁵ – “memory,” “registers³⁶,” “bits³⁷,” “floating³⁸ point,” “overflow³⁹,” “software⁴⁰.” Most words not defined in the text are given brief definitions⁴¹ in the index at the close of each volume.

These 4 prerequisites can perhaps be summed up into the single requirement that the reader should have already written & tested at least, say, 4 programs for at least 1 computer.

I have tried to write this set of books in such a way that it will fill several needs. In the 1st place, these books are reference works that summarize⁴² the knowledge that has been acquired⁴³ in several important fields. In the 2nd place, they can be used as textbooks⁴⁴ for self-study⁴⁵ or for college⁴⁶ courses⁴⁷ in the computer & information⁴⁸ sciences⁴⁹. To meet both of these objectives, I have incorporated⁵⁰ a large number of exercises into the text & have furnished⁵¹ answers for most of them. I have also made an effort to fill the page with facts rather than with vague⁵², general commentary⁵³.

This set of books is intended^{54 55} for people who will be more than just casually⁵⁶ interested⁵⁷ in computers, yet it is by

³⁵**jargon** [n] [uncountable] (*often disapproving*) words or expressions that are used by a particular profession or group of people, & are difficult for others to understand.

³⁶**register** [v] **1.** [transitive, intransitive] to record the name of somebody/something on an official list; **2.** [transitive] **register something** to make your opinion known officially or publicly; **3.** [intransitive] + **noun** (of a measuring instrument) to show or record an amount; **4.** [transitive] **register something** to achieve a particular score or result; **5.** [transitive] **register something** to notice something & remember it; [n] **1.** [countable] **register (of something)** an official list or record of names or items; a book that contains such a list; **2.** [countable, uncountable] (*linguistics*) the level & style of a piece of writing or speech, that is usually appropriate to the situation that it is used in; **3.** [countable] (*computing*) (in electronic devices) a location in a store of data, used for a particular purpose & with quick access time.

³⁷**bit** [n] **1.** [countable] the smallest unit of information used by a computer; **2.** (**a bit**) [singular] (used as an adverb) (*especially British English, rather informal*) rather; **3.** (**a bit**) [singular] (used as an adverb) a small amount; a little; **4.** [countable] **bit of something** (*especially British English, rather informal*) a small piece or part of something; **5.** [countable] **bit of something** (*especially British English, rather informal*) a small amount of something; **bit by bit** [idiom] (*rather informal*) a piece or part at a time; gradually; **every bit as good, bad, etc. (as somebody/something)** [idiom] (*rather informal*) just as good, bad, etc.; equally good, bad, etc.

³⁸**float** [v] **1.** [intransitive] + **adv./prep.** to move slowly on or in water or in the air; **2.** [intransitive] to stay on or near the surface of a liquid & not sink; **3.** [transitive] **float something** (+ **adv./prep.**) to make something move on or near the surface of a liquid; **4.** [transitive] **float something** to suggest an idea or a plan for other people to consider; **5.** [transitive] **float something** to sell shares in a company or business to the public for the 1st time; **6.** [transitive, intransitive] (*economics*) if a government floats its country's money or allows it to float, it allows its value to change freely according to the value of the money of other countries.

³⁹**overflow** [v] **1.** [intransitive, transitive] to be so full that the contents go over the sides; **2.** [intransitive] **overflow (with something)** (of a place) to have too many people in it; **3.** [intransitive, transitive] **overflow (into something)** | **overflow (something)** to spread beyond the limits of a place or container that is too full; [n] **1.** [uncountable, singular] a number of people or things that do not fit into the space available; **2.** [uncountable, singular] the action of liquid flowing out of a container, etc. that is already full; the liquid that flows out; **3.** (also **overflow pipe**) [countable] a pipe that allows extra liquid to flow away safely when a container is full; **4.** [countable, usually singular] (*computing*) a fault that happens because a number or data item is too large for the computer to represent it exactly.

⁴⁰**software** [n] [uncountable] the programs used by a computer for doing particular jobs.

⁴¹**definition** [n] **1.** [countable] an exact statement or description of the nature, extent or meaning of something; **2.** [countable] a statement of the exact meaning of a word or phrase, especially in a dictionary; **3.** [uncountable] the action or process of stating the exact meaning of a word or phrase; **by definition** [idiom] as a result of what something is.

⁴²**summarize** [v] (*British English also summarise*) [transitive, intransitive] **summarize (something)** to give a summary of something.

⁴³**acquire** [v] **1.** **acquire something** to learn or develop a skill, habit or quality; **2.** **acquire something** to obtain something by buying or being given it; **3.** **acquire something** to come to have a particular reputation.

⁴⁴**textbook** [n] (*North American English also text*) a book that teaches a particular subject & that is used especially in schools & colleges.

⁴⁵**self-study** [n] [uncountable] the activity of learning about something without a teacher to help you; [a] designed to help students to learn about something without a teacher to help them.

⁴⁶**college** [n] **1.** [countable, uncountable] (often in names) (in the US) a university where students can study for a degree after they have left school; **2.** [countable, uncountable] (often in names) (in Britain) a place where students go to study or to receive training after they have left school; **3.** [countable, uncountable] (often in names) 1 of the separate institutions that come British universities, such as Oxford & Cambridge, are divided into; **4.** [countable, uncountable] (often in names) (in the US) 1 of the main divisions of some large universities; **5.** [countable + singular or plural verb] (usually in names) an organized group of professional people with special interests, duties or powers.

⁴⁷**course** [n] **1.** [countable] a series of classes or lectures on a particular subject; **2.** [countable] (*especially British English*) a period of study at a college or university that leads to an exam or a qualification; **3.** [singular] the way that something develops or should develop; **4.** (also **course of action**) [countable] a way of acting in or dealing with a particular situation; **5.** [countable, usually singular] the general direction in which somebody's ideas or actions are moving; **6.** [uncountable, countable, usually singular] a direction or route followed by a ship or an aircraft, or by another moving object; **7.** [countable] **course (of something)** a series of medical treatments.

⁴⁸**information** [n] [uncountable] **1.** facts or details about somebody/something that are provided or learned; **2.** data that are stored, analyzed or passed on by a computer; **3.** what is shown by a particular arrangement of things.

⁴⁹**information sciences** [n] (also **informatics**) [uncountable] (*computing*) the study of processes for storing & obtaining information.

⁵⁰**incorporate** [v] **1.** to include something so that it forms a part of something; **2.** [usually passive] (*business*) to create a legally recognized company.

⁵¹**furnish** [v] **1.** **furnish something (with something)** to put furniture in a house, room, etc.; **2.** (*formal*) to supply something; to supply or provide somebody/something with something.

⁵²**vague** [a] (**vaguer, vaguest**) not having or giving enough information or details about something.

⁵³**commentary** [n] (plural **commentaries**) **1.** [countable] **commentary (on something)** a written explanation or discussion of something such as a theory or book; **2.** [countable, uncountable] **commentary (on something)** a criticism or discussion of something.

⁵⁴**intend** [v] **1.** to have a plan, result or purpose in your mind when you do something; **2.** **intend something as something** | **intend something to be something** to plan that something should have a particular meaning or use.

⁵⁵**intended** [a] **1.** meant or designed to be something or to be used by somebody; **2.** [only before noun] that you are trying to achieve or reach.

⁵⁶**casual** [a] **1.** [usually before noun] without paying attention to detail; **2.** [usually before noun] not showing much care or thought; **3.** [usually before noun] (of a relationship) lasting only a short time & without deep affection; **4.** [usually before noun] (*British English*) (of work) not permanent; not regular; **5.** not formal; **6.** [only before noun] happening by chance; doing something by chance.

⁵⁷**interested** [a] **1.** [not usually before noun] giving your attention to something because you enjoy finding out about it or doing it; showing

no means only for the computer specialist⁵⁸. Indeed, 1 of my main goals has been to make these programming⁵⁹ techniques more accessible⁶⁰ to the many people working in other fields who can make fruitful use of computers, yet who cannot afford⁶¹ the time to locate⁶² all of the necessary information that is buried⁶³ in technical⁶⁴ journals⁶⁵.

We might call the subject of these books “nonnumerical analysis.” Computers have traditionally⁶⁶ been associated^{67 68} with the solution⁶⁹ of numerical⁷⁰ problems such as the calculation⁷¹ of the roots⁷² of an equation⁷³, numerical interpolation⁷⁴ & integration⁷⁵, etc., but such topics are not treated here except in passing⁷⁶. Numerical computer programming

interest in something & finding it exciting; **2.** [usually before noun] in a position to gain from a situation or be affected by it.

⁵⁸**specialist** [n] **1.** a doctor who has specialized in a particular area of medicine; **2. specialist (in something)** a person who is an expert in a particular area of work or study; [a] [only before noun] **1.** connected with a doctor who has specialized in a particular area of medicine; **2.** having or involving detailed knowledge of a particular topic or area of study.

⁵⁹**programming** [n] [uncountable] **1.** the process of writing & testing programs for computers; **2. programming (of something)** the activity of planning which television or radio programmes to broadcast; the programmes that are broadcast; **3.** factors, ranging from genetic to social, that instruct a person or animal to behave in a certain way.

⁶⁰**accessible** [a] **1.** that can be reached, entered, used or obtained; **2.** easy to understand.

⁶¹**afford** [v] **1.** [no passive] (usually used with *can*, *could* or *be able to*, especially in negative sentences or questions) to have enough money or time to be able to buy or to do something; **2.** [no passive] **afford to do something** (usually used with *can* or *could*, especially in negative sentences & questions) if you say that you cannot afford to do something, you mean that you should not do it because it will cause problems for you if you do; **3. (formal)** to provide somebody with something.

⁶²**locate** [v] **1.** [transitive] **locate somebody/something** to find the exact position of somebody/something; **2.** [transitive] **locate something + adv./prep.** to put or build something in a particular place; **3.** [intransitive] + **adv./prep.** (*especially North American English*) to start a business in a particular place.

⁶³**bury** [v] **1. bury somebody/something** to place something in the ground, especially a dead body in a grave; **2.** [often passive] **bury somebody/something** to cover something with soil, rocks, leaves, etc.; **3. bury something** to ignore or hide a feeling, a mistake, etc.

⁶⁴**technical** [a] **1.** [usually before noun] connected with the use of science or technology; involving the use of machines; **2.** [usually before noun] connected with a particular type of activity, or the skills & processes needed for it; **3.** [usually before noun] (of language, writing or ideas) requiring knowledge & understanding of a particular subject; **4.** connected with the details of a law or set of rules.

⁶⁵**journal** [n] **1.** a newspaper or magazine that deals with a particular subject or profession; **2.** a written record of the things you do or see every day.

⁶⁶**traditionally** [adv] **1.** according to what has always or usually happened in the past; **2.** according to the beliefs, customs or way of life that have existed for a long time among a particular group of people; according to what is believed.

⁶⁷**associate** [v] **1.** [transitive] to make a connection between people or things in your mind, SYNONYM: **connect**, **relate**, **link**, OPPOSITE: **dissociate**; **2.** [intransitive] **associate with somebody** to spend time with somebody, especially somebody that others do not approve of; **3.** [transitive] **associate yourself with something** to show that your support or agree with something, OPPOSITE: **dissociate**; [n] **1.** a person that you work with, do business with or spend a lot of time with; **2. (Association) associate (of something)** an associate member of an organization; [a] [only before noun] **1.** (often in titles) of a lower rank; having fewer rights in a particular profession or organization; **2.** joined or connected with a profession or an organization.

⁶⁸**associated** [a] **1.** if 1 thing is associated with another, the 2 things are connected because they happen together or 1 thing causes the other, SYNONYM: **connected**; **2.** if a person is associated with a person, organization or idea, they support it; **3.** (of a company) connected or joined with another company or companies.

⁶⁹**solution** [n] **1.** [countable] a way of solving a problem or dealing with a difficult situation, SYNONYM: **answer**; **2.** [countable] **solution (to/for/of something)** an answer to a problem in mathematics; **3.** [countable, uncountable] a liquid in which a substance has been dissolved, so that the substance has become part of the liquid; the state of being dissolved in a liquid.

⁷⁰**numerical** [a] (also *less frequent numeric*) [usually before noun] connected with numbers; expressed in numbers.

⁷¹**calculation** [n] [countable, uncountable] **1.** the act or process of using numbers to find out an amount; **2.** the process of using your judgment to decide what the results would be of doing something.

⁷²**root** [n] **1.** [countable] the part of a plant that grows under the ground & absorbs water & minerals that it sends to the rest of the plant; **2.** [countable, usually singular] **root of something** the main cause of something, such as a problem or difficult situation; **3.** [countable, usually plural] the basis of something; **4. (roots)** [plural] the feelings or connections that you have with a place because you have lived there or your family came from there; **5.** [countable] (*linguistics*) the part of a word that has the main meaning & that its other forms are based on; a word that other words are formed from; **6.** [countable] **root (of something)** (*mathematics*) a quantity which, when multiplied by itself a particular number of times, produces another quantity; **root & branch** [idiom] thorough & complete; **take root** [idiom] **1.** (of a plant) to develop roots; **2.** (of an idea) to become widely accepted; [v] [intransitive] (of a plant) to grow roots; **root something/somebody out** [phrasal verb] to find a person or thing that is causing a problem & remove or get rid of them.

⁷³**equation** [n] **1.** [countable] (*mathematics*) a statement showing that 2 amounts or values are equal; **2.** [countable] (*chemistry*) a statement using symbols to show the changes that happen in a chemical reaction; **3.** [uncountable, singular] the act of making something equal or considering something as equal; **4.** [countable, usually singular] a situation in which several factors must be considered & dealt with.

⁷⁴**interpolation** [n] [uncountable, countable] **1. (formal)** a remark that interrupts a conversation; the act of making a remark that interrupts a conversation; **2. (formal)** a thing that is added to a piece of writing; the act of adding something to a piece of writing, SYNONYM: **insertion**; **3. (mathematics)** the act of adding a value into a series by calculating it from surrounding known values.

⁷⁵**integration** [n] **1.** [uncountable, countable] the act or process of combining 2 or more things so that they work together; **2.** [uncountable] the act or process of mixing people who have previously been separated, usually because of color, race or religion; **3.** [uncountable, countable] **integration (of something)** (*mathematics*) the process of finding an integral or integrals.

⁷⁶**passing** [n] [uncountable] **1. the passing of time/the years** the process of time going by; **2. passing (of somebody/something)** the fact of something ending or of somebody dying; **3. the passing of something** the act of making something become a law; **in passing** [idiom] done or said while you are giving your attention to something else.

is an extremely⁷⁷ interesting & rapidly⁷⁸ expanding⁷⁹ field, & many books have been written about it. Since the early 1960s, however, computers have been used even more often for problems in which numbers occur only by coincidence⁸⁰; the computer's decision-making⁸¹ capabilities⁸² are being used, rather than its ability⁸³ to do arithmetic^{84 85}. We have some use for addition⁸⁶ & subtraction⁸⁷ in nonnumerical problems, but we rarely feel any need for multiplication⁸⁸ & division⁸⁹. Of course, even a person who is primarily⁹⁰ concerned⁹¹ with numerical computer programming will benefit⁹² from a study of the nonnumerical techniques, for they are present in the background of numerical programs as well.

The results of research in nonnumerical analysis are scattered^{93 94} throughout numerous⁹⁵ technical journals. My approach has been to try to distill⁹⁶ this vast literature by studying the techniques that are most basic, in the sense that they can be applied to many types of programming situations⁹⁷. I have attempted^{98 99} to coordinate¹⁰⁰ the ideas into more or less of a "theory," as well as to show how the theory applies to a wide variety¹⁰¹ of practical problems.

Of course, "nonnumerical analysis" is a terribly¹⁰² negative name for this field of study; it is much better to have a

⁷⁷**extremely** [adv] (usually with adjectives & adverbs) to a very high degree.

⁷⁸**rapidly** [adv] in a short period of time or at a fast rate.

⁷⁹**expand** [v] **1.** [intransitive, transitive] to become greater in size, number or importance; to make something greater in size, number or importance; **2.** [transitive] **expand something** to write something such as a scientific formula in a longer form; **expand on/upon something** [phrasal verb] to add more details & give more information about something.

⁸⁰**coincidence** [n] **1.** [countable, uncountable] the fact of 2 things happening at the same time by chance, often in a surprising way; **2.** [singular, uncountable] **coincidence of A with/& B** the fact of things being present in the same place at the same time; **3.** [singular, uncountable] **coincidence of something** the fact of 2 or more opinions, etc. being the same.

⁸¹**decision-making** [n] (also **decision making**) [uncountable] the process of deciding about something important, especially in a group of people or in an organization.

⁸²**capability** [n] (plural **capabilities**) **1.** [countable, uncountable] the ability or qualities necessary to do something; **2.** [countable] the power or weapons that a country has for war or for military action.

⁸³**ability** [n] (plural **abilities**) **1.** [singular] the fact that somebody/something is able to do something, OPPOSITE: **inability**; **2.** [uncountable, countable] a level of skill or intelligence.

⁸⁴**arithmetic** [n] [uncountable] **1.** the type of mathematics that deals with the use of numbers in counting & calculation; **2.** the use of numbers in counting & calculation; [a] (*mathematics*) **1. arithmetic progression/series** a series in which the interval between each term & the next remains constant; **2. arithmetic mean = mean**; **3. = arithmetical**.

⁸⁵**arithmetical** [a] (also **arithmetic**) connected with arithmetic.

⁸⁶**addition** [n] **1.** [uncountable, countable] the act of adding something to something else, OPPOSITE: **removal**; **2.** [countable] **addition (to something)** a thing that is added to something else; **3.** [uncountable, countable] the process of adding 2 or more numbers together to find their total; **in addition (to somebody/something)** [idiom] used to introduce a new fact or argument.

⁸⁷**subtraction** [n] [uncountable, countable] the process of taking a number or amount away from another number or amount.

⁸⁸**multiplication** [n] [uncountable] **1.** the act or process of multiplying 1 number by another, OPPOSITE: **division**; **2.** the process of increasing very much in number or amount.

⁸⁹**division** [n] **1.** [uncountable, countable, usually singular] the process or result of dividing into separate parts; the process or result of dividing something or sharing it out; **2.** [uncountable] the process of providing 1 number by another; **3.** [countable, usually plural, uncountable] a disagreement or difference in people's opinions of ways of life, especially between members of a society or an organization; **4.** [countable] **division (of something)** a part of something into which it is divided; **5.** [countable + singular or plural verb] (abbr., **Div.**) a large & important unit or section of an organization.

⁹⁰**primarily** [adv] mainly, SYNONYM: **chiefly**.

⁹¹**concerned** [a] **1.** worried & feeling concern about something; **2.** interested in something; **as/so far as somebody/something is concerned, as/so far as somebody/something goes** [idiom] used to give facts or an opinion about a particular aspect of something.

⁹²**benefit** [n] **1.** [countable, uncountable] a helpful & useful effect that something has; an advantage that something provides; **2.** [uncountable, countable] (*British English*) money provided by the government to people who need financial help because they are unemployed, sick, etc.; **give somebody the benefit of the doubt** [idiom] to accept that somebody has told the truth or has not done something wrong because you cannot prove that they have not told the truth/have done something wrong; [v] **1.** [intransitive] to be in a better position because of something; **2.** [transitive] **benefit somebody/something** to be useful or provide an advantage to somebody/something.

⁹³**scatter** [v] **1.** [intransitive, transitive] (*physics*) to change direction or spread in many directions; to make something change direction or spread in many directions; **2.** [transitive, often passive] **scatter something (on/over/around something)** to throw or drop things in different directions so that they cover an area; **3.** [transitive, often passive] to be found spread over an area rather than all together; **4.** [intransitive, transitive] (of people or animals) to move very quickly in different directions; to make people or animals do this, SYNONYM: **disperse**; [n] (also **scattering**) [singular] **scatter (of something)** a small amount of something or a small number of people or things spread over an area.

⁹⁴**scattered** [a] **scattered (throughout/across/around something)** spread far apart over a wide area, SYNONYM: **dispersed**.

⁹⁵**numerous** [a] [usually before noun] existing in large numbers, SYNONYM: **many**.

⁹⁶**distil** [v] (*North American English also distill*) **1. distil something (from something)** to make a liquid pure by heating it until it becomes a gas, then cooling it & collecting the drops of liquid that form; **2. distil something** to make something such as a strong alcoholic drink in this way; **3. distil something (from/into something)** to get the essential meaning or ideas from thoughts, information or experiences.

⁹⁷**situation** [n] **1.** all the circumstances & things that are happening at a particular time & in a particular place; **2.** the area or place where something is located.

⁹⁸**attempt** [n] **1.** [countable, uncountable] an act of trying to do something difficult, often with no success; **2.** [countable] an act of trying to kill somebody; [v] to try to do or provide something, especially something difficult.

⁹⁹**attempted** [a] [only before noun] (of a crime, etc.) that somebody has tried to do but without success.

¹⁰⁰**coordinate** [v] (*British English also co-ordinate*) to organize the different parts of an activity & the people involved in it so that it works well; **coordinate with somebody** [phrasal verb] to reach an agreement with other people about how to work together effectively; [n] (*British English also co-ordinate*) 1 of the numbers or letters used to fix the position of a point on a map or graph.

¹⁰¹**variety** [n] (plural **varieties**) **1.** [singular] **variety (of something)** a number or range of different things of the same general type; **2.** [uncountable] the quality of not being the same in all parts or not doing the same thing all the time, SYNONYM: **diversity**; **3.** [countable] a type of a thing, e.g. a plant or language, that is different from others in the same general group. In biology, a **variety** is a category below a **species** & **subspecies**, used especially to describe plants.

¹⁰²**terribly** [adv] **1.** (*especially British English*) very; **2.** very much; very badly.

positive, descriptive¹⁰³ term that characterizes the subject. “Information processing” is too broad a designation for the material I am considering, & “programming techniques” is too narrow.

” – Knuth, 1997, Preface, pp. v–

¹⁰³**descriptive** [a] **1.** describing what something is like, rather than saying what it should be like or what category it belongs to; **2.** saying or showing clearly what something is like; giving a clear account of something.

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