Inequality

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Wikipedia's

1.1 Wikipedia/Inequality (Mathematics)

Chapter 2

Hardy, Littlewood, and Pólya, 1952. Inequality

"Oh! the little more, & how much it is! & the little less, & what worlds away!" – Robert Browning

Preface to 1st Edition

"This book was planned & begun in 1929. Our original intention was that it should be 1 of the *Cambridge Tracts*, but it soon became plain that a tract[1] would be much too short for our purpose.

Our subjects in writing the book are explained sufficiently in the introductory chapter, but we add a note here about history & bibliography[2]. Historical & bibliographical questions are particularly troublesome[3] in a subject like this, which has applications in every part of mathematics but has never been developed systematically.

It is often really difficult to trace the origin of a familiar inequality. It is quite likely to occur 1st as an auxiliary proposition, often without explicit statement, in a memoir on geometry or astronomy; it may have been rediscovered, many years later, by half a dozen different authors; & no accessible statement of it may be quite complete. We have almost always found, even with the most famous inequalities, that we have a little new to add.

We have done our best to be accurate & have given all references we can, but we have never undertaken[4] systematic bibliographical research. We follow the common practice, when a particular inequality is habitually[5] associated with a particular mathematician's name; we speak of the inequalities of Schwarz, Hölder, & Jensen, though all these inequalities can be traced further back; & we do not enumerate[6] explicitly all the minor additions which are necessary for absolute completeness.

[1]**tract** [n] **1.** (*biology*) a system of connected organs or tissues along which materials or messages pass; **2. tract (of something)** an area of land, especially a large one; **3.** a short piece of writing, especially on a religious, moral or political subject, that is intended to influence people's ideas.

[2]**bibliography** [n] (plural **bibliographies**) the list of books, etc. that have been used by somebody writing an article, essay, etc.; a list of books or articles about a particular subject or by a particular author.

[3]**troublesome** [a] causing trouble, pain or difficulties.

[4]**undertake** [v] **1. undertake something** to make yourself responsible for something & start doing it; **2.** to agree or promise that you will do something.

[5]**habitual** [a] [only before noun] usual or typical of somebody/something.

[6]**enumerate** [v] (*formal*) **enumerate something** to name things on a list 1 by 1.

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Bibliography

Hardy, G. H., J. E. Littlewood, and G. Pólya (1952). *Inequalities*. 2d ed. Cambridge, at the University Press, pp. xii+324.