Diophantine Equation

Nguyễn Quản Bá Hồng*

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Abstract

A set of problems of Diophantine equations.

Contents

1 Wikipedia/Diophantine Equation

Finding all right triangles with integer side-lengths is equivalent to solving the Diophantine equation $a^2 + b^2 = c^2$.

"In mathematics, a *Diophantine equation* is a polynomial equation, usually involving 2 or more unknowns, s.t. the only solutions of interest are the integer ones. A *linear Diophantine equation* equates to a constant the sum of 2 or more monomials, each of degree 1. An *exponential Diophantine equation* is one in which unknowns can appear in exponents.

Diophantine problems have fewer equations than unknowns & involve finding integers that solve simultaneously all equations. As such systems of equations define algebraic curves, algebraic surfaces, or, more generally, algebraic sets, their study is a part of algebraic geometry that is called *Diophantine geometry*.

The word *Diophantine* refers to the Hellenistic mathematician of the 3rd century, Diophantus of Alexandria, who made a study of such equations & was 1 of the 1st mathematicians to introduce symbolism into algebra. The mathematical study of Diophantine problems that Diophantus initiated is now called *Diophantine analysis*.

While individual equations present a kind of puzzle & have been considered throughout history, the formulation of general theories of Diophantine equations (beyond the case of linear & quadratic equations) was an achievement of the 20th century." – Wikipedia/Diophantine equation

1.1 Examples of Diophantine Equation

1.2 Linear Diophantine Equations

1.3 Homogeneous Equations

1.4 Diophantine Analysis

1.5 Exponential Diophantine Equations

2 Phương Pháp Xét Tính Chia Hết

Bài toán 2.1 (Bình, 2021, Thí dụ 1, p. 6)**.** *Giải phương trình nghiệm nguyên* $3x + 17y = 159$.

*Independent Researcher, Ben Tre City, Vietnam
e-mail: nguyenquanbahong@gmail.com; website: https://nqbh.github.io.

Bài toán 2.2 (Bình, 2021, Thí dụ 2, p. 6)**.** *Tìm nghiệm nguyên của phương trình* $xy - x - y = 2$.

Bài toán 2.3 (Bình, 2021, Thí dụ 3, p. 7)**.** *Tìm nghiệm nguyên của phương trình* $2xy - x + y = 3$.

Tài liệu

Bình, Vũ Hữu (2021). *Phương Trình Nghiệm Nguyên & Kinh Nghiệm Giải*. Nhà Xuất Bản Giáo Dục Việt Nam, p. 224.