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# Nonlinear Analysis

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### Erratum

Erratum to "Some properties on the surfaces of vector fields and its application to the Stokes and Navier–Stokes problems with mixed boundary conditions" [Nonlinear Anal. 113 (2015) 94–114]



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(7) in (3.3):

$$v_{\tau}|_{\Gamma_7} = 0, \quad \left(-pn + \nu \frac{\partial v}{\partial n}\right)\Big|_{\Gamma_7} = \phi_7$$

and

(7) in (3.4):

$$v_{\tau}|_{\Gamma_7} = 0, \quad \left(-pn - \frac{1}{2}|v|^2n + \nu \frac{\partial v}{\partial n}\right)\Big|_{\Gamma_7} = \phi_7$$

must be corrected, respectively, by

$$v_{\tau}|_{\Gamma_7} = 0, \quad \left(-p + \nu \frac{\partial v}{\partial n} \cdot n\right)\Big|_{\Gamma_7} = \phi_7 \cdot n$$

and

$$v_{\tau}|_{\Gamma_7} = 0, \quad \left(-p - \frac{1}{2}|v|^2 + \nu \frac{\partial v}{\partial n} \cdot n\right)\Big|_{\Gamma_7} = \phi_7 \cdot n.$$

## Explanation:

DOI of original article: http://dx.doi.org/10.1016/j.na.2014.09.017.  $E\text{-}mail\ address:\ math.inst@star-co.net.kp}$ . Using Theorem 2.2, in variational formulations we assumed that the flow on  $\Gamma_7$  is orthogonal. Let us consider (7) in (3.3). Using this condition, we come to a variational formulation. Inversely, when the solution v is smooth enough, converting from variational formulations to original problems we have

$$\left(-pn + \nu \frac{\partial v}{\partial n}, u\right)_{\Gamma_7} = \langle \phi_7, u \rangle_{\Gamma_7} \quad \forall u, u_\tau = 0$$

on  $\Gamma_7$ . Usually,  $v_{\tau}=0$  does not give  $\frac{\partial v}{\partial n} \cdot \tau=0$ . If there is not the condition  $u_{\tau}=0$ , then from the formula above we can get  $-pn+\nu\frac{\partial v}{\partial n}=\phi_7$  on  $\Gamma_7$ , thus (7) in (3.3). But owing to  $u_{\tau}=0$  we only get  $(-pn+\nu\frac{\partial v}{\partial n},n)_{\Gamma_7}=\langle\phi_7,n\rangle_{\Gamma_7}$ , which is the new condition. We can see the same for condition (7) in (3.4). This means our variational formulations are for the problems with new conditions instead (7) in (3.3) or (7) in (3.4). Thus, the conditions of original problems must be corrected.

Then, the new condition of (3.3) is rather different from "do nothing" condition  $-pn + \nu \frac{\partial v}{\partial n} = 0$ . "Do nothing" boundary condition results from variational principle and does not have a real physical meaning but is rather used in truncating large physical domains to smaller computational domains by assuming parallel flow. If the flow is parallel in a near the boundary, then the new condition is same with "do nothing" condition. In point of view of pure mathematics, to reflect correctly "do nothing" condition in variational formulation we can use other variational formulation, which will be mentioned in other papers.