An adaptive unscented Kalman filter for quaternion-based orientation estimation in low-cost AHRS

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Abstract

Purpose – This paper aims to develop an adaptive unscented Kalman filter (AUKF) formulation for orientation estimation of aircraft and UAV utilizing low-cost attitude and heading reference systems (AHRS).

Design/methodology/approach – A recursive least-square algorithm with exponential age weighting in time is utilized for estimation of the unknown inputs. The proposed AUKF tunes its measurement covariance to yield optimal performance. Owing to nonlinear nature of the dynamic model as well as the measurement equations, an unscented Kalman filter (UKF) is chosen against the extended Kalman filter, due to its better performance characteristics. The unscented transformation of the UKF is shown to equivalently capture the effect of nonlinearities up to second order without the need for explicit calculations of the Jacobians.

Findings – In most conventional AHRS filters, severe problems can occur once the system suddenly experiences additional acceleration, resulting in erroneous orientation angles. On the contrary in the high dynamic accelerative mode of the new proposed filter the errors would not suddenly increase, since the additional to cruise accelerations are being continuously estimated resulting in substantially more accurate orientation estimation. This feature causes the associated filter errors to gradually increase, in the event of continuous vehicle acceleration, up to a point of zero additional acceleration that subsequently causes a subsidence of the error back to zero.

Practical implications – The proposed filtering methodology can be implemented for orientation estimation of aircraft and UAV that are equipped with low-cost AHRSs.

Originality/value – Traditional AHRS algorithms utilize the accelerometers output for the computation of roll and pitch angles and magnetometer output for the heading angle. Moreover, these angles are also calculated from the gyroscopes output as well, but with errors that increase with time. Of course for some applications of AHRS system, orientation errors can be damped out with a proportional-integral controller. In such a case, the filter cut-off frequency is usually selected experimentally. But, for high accelerating vehicles utilizing AHRS, the system errors can become very large. A possible remedy to this problem could be to use more advanced nonlinear filter algorithms such as the one proposed.

Keywords Non-linear control systems, Filtration

Paper type Research paper

1. Introduction

Attitude and heading reference system (AHRS) is often required in aircraft, marine vessel, spacecraft and missiles. It measures the orientation of body frames relative to the navigation frame defined with three simple rotations angles, namely: heading, pitch and roll. For high altitude UAV, it is required that these measurements have accuracy order of 1.0 degree and a resolution of 0.5 degree (Whitmore *et al.*, 1997). These angels were traditionally measured with directional gyros (for the heading angle) and vertical gyros (for pitch and roll angles) (Kayton and Fried, 1969). The gyros are spinning wheels in gimbaled frames that use torquers at each gimbal to adjust and correct the attitude. Pendulum devices in vertical gyro and magnetic compass in directional gyros are also used in initialization process to erect the gyros and during operation, to remove accumulated errors (Kayton and Fried, 1969; Wie, 1998). Such mechanical-based systems are complex, expensive, bulky and heavy in addition to being prone to wear and failure.

To reduce cost and increase reliability; many researchers propose to replace traditional systems with solid-state strapdown AHRS that by nature contains more electronic than mechanics (Garg *et al.*, 1978). In the strapdown system, the outputs of a triad body-mounted rate gyro are integrated using Euler equations or quaternions to provide aircraft attitude angles. Since, strapdown inertial measurement systems have unbounded drift without external aiding, expensive inertial-grade rate gyros are required to achieve satisfactory precision over long periods lasting several hours. Therefore, many researchers Hong (2003); Wang *et al.* (2004) have been questing to come up with ways to replace the costly inertial-grade rate gyros of the strapdown attitude reference systems with low-cost solid-state miniature inertial sensors to minimize drift or bias with an estimating filter.

To observe and compensate for sensor drift, a process called augmentation is used. Assuming an aircraft to fly level with no acceleration, one can calculate the pith and roll angle from the output of the accelerometer and the heading angle from the magnetometer and presumably propagate these measures for the compensation of gyro drifts (Humphrey, 1997; Madani, 1998). When the system has acceleration, however,

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the calculated roll and pitch angles from the output of the accelerometers will be spurious. At this time, the output of the magnetometer that will be projected to level will have errors and thus the calculated heading angle will have large errors. Recently, adaptive filters are developed to solve this problem. An example is given in (Hong, 2003) utilizing cut-off frequency adapted with fuzzy logics. In (Wang *et al.*, 2004) an adaptive Kalman filter tunes its gain automatically based on the system dynamics sensed by the accelerometers to yield optimal performance. Besides these systems, other approaches are developed for attitude estimation using other sensors. In this regard, there is a good review in Lefferts *et al.* (1982) for spacecraft attitude determination with a general algorithm for orientation and gyro bias. There exists also good discussions on integration of magnetometer, air speed, and IMU in Guerrero Castellanos *et al.* (2000); multi antenna GPS, magnetometer IMU in (Gebre-Egziabher *et al.*, 2000); multi antenna GPS attitude determination in Lu (1995); two-antenna GPS and GPS/magnetometer in Yang (2001) and integration of GPS and air-data in Kornfeld *et al.* (1998).

In addition, some studies have considered nonlinear filters to solve the drift/bias problem. The ability of the unscented Kalman filter (UKF) to accurately estimate nonlinearities makes it attractive for implementation in AHRS. The state and observation models in this problem are inherently nonlinear. The underlying intuition of the UKF is that with a fixed number of parameters it should be easier to approximate a Gaussian distribution than it is to approximate an arbitrary nonlinear function transformation (Julier and Uhlmann, 1996; van der Merwe, 2004). The state distribution is again represented by a Gaussian random variable, but is now specified using a minimal set of carefully chosen sample points, called the σ points which capture the true mean and covariance of the probability density function (PDF). Also when propagated through the true nonlinear system, captures the posterior mean and covariance accurately to the second order for any nonlinearity (Wan and van der Merwe, 2001; Julier and Uhlmann, 2004). With the UKF approach, error-prone Jacobian computations are avoided and all kinds of error models can be unified. Attitude determination based on UKF was discussed in Kraft (2003), Crassidis and Markley, 2003 and Ma and Jiang (2005). The main contribution of this paper is the development of an adaptive UKF for the attitude and gyro bias estimation problem. The adaptive UKF is tested through numerical simulation of a fully actuated rigid body with attitude sensors that provide two noisy vector measurements (magnetic field and acceleration) as well as a noisy angular velocity vector measurement. Acceleration is estimated through a recursive least-square (RLS) algorithm (Hykin *et al.*, 1977; Hykin, 2002). This estimation is applied to the measurements of the UKF with covariance of acceleration estimation that can be considered as covariance of measurements noise.

2. Unscented Kalman filter

The UKF was developed with the underlying assumption that approximating a Gaussian distribution is easier than approximating a nonlinear transformation (Julier and Uhlmann, 1996; van der Merwe, 2004). The UKF uses deterministic sampling to approximate the state distribution as a GRV. The σ points are chosen to capture the true mean and covariance of the state distribution. The σ points are propagated through the nonlinear system. The posterior mean and covariance are then calculated from the propagated σ points. The UKF determines the mean and covariance accurately to the second order, while the extended Kalman filter (EKF) is only able to obtain first-order accuracy (Wan and van der Merwe, 2001; van der Merwe, 2004). Therefore, the UKF provides better state estimates for nonlinear systems.

We consider the minimum-mean squared error estimate of the state vector of the nonlinear discrete time system:

$$x_k = f(x_{k-1}, u'_{k-1}, w_{k-1}) \quad y_k = h(x_k, v_k) + u_k \tag{1}$$

where $x_k \in \Re^{n_x}$ is the state of the system at time step k, $u_k \in \Re^{n_u}$ is the input vector, $u'_{k-1} \in \Re^{n_{u'}}$ is secondary input, $w_k \in \Re^{n_w}$ is noise process caused by disturbance and modeling errors, $y_k \in \Re^{n_y}$ is the observation vector and $v_k \in \Re^{n_y}$ is the additive measurement noise. It is assumed that the noise vector w_k, v_k are zero of mean and:

$$\begin{aligned} &E\left(v_i v_i^T\right) = \delta_{ij} R_i \quad E\left(w_i w_i^T\right) = \delta_{ij} Q_i, \quad \forall i,j \\ &E\left(v_i w_i^T\right) = 0 \end{aligned} \tag{2}$$

The Kalman filter propagates the first two moments of the distribution of x_k recursively and has a distinctive "predictive-corrector" structure. Let $\hat{x}_k$ be the estimate of x_k using the observation information up to and including time k. The covariance of this estimate is P_{x_k}. The recursive estimation for x_k can be expressed in the following from Kraft (2003); Crassidis and Markley (2003) and Ma and Jiang (2005):

$$\hat{x}_k = \hat{x}_k^- + K_k\left(y_k - y_k^-\right) \tag{3}$$

$$P_{x_k} = P_{x_k}^- - K_k P_{\tilde{y}_k} K_k^T \tag{4}$$

where $\hat{x}_k^-$ is the optimal prediction of the state at time k conditioned on all of the observed information up to and including time k-1, and $\hat{y}_k^-$ is the optimal prediction of the observation at time k. $P_{x_k}^-$ is the covariance of $\hat{x}_k^-$ and $P_{\tilde{y}_k}$ is the covariance of $r_k = y_k - \hat{y}_k^-$, termed the innovation or residual process. The optimal terms in this recursion are given by:

$$\hat{x}_k^- = E[f(x_{k-1}, u'_{k-1}, w_{k-1})] \tag{5}$$

$$\hat{y}_k^- = E[h(x_k, v_k) + u_k] \tag{6}$$

$$\begin{aligned} K_k &= P_{x_k y_k} P_{\tilde{y}_k}^{-1} \\ &= E\left[\left(x_k - \hat{x}_k^-\right)\left(y_k - \hat{y}_k^-\right)^T\right] E\left[\left(y_k - \hat{y}_k^-\right)\left(y_k - \hat{y}_k^-\right)^T\right]^{-1} \end{aligned} \tag{7}$$

EKF calculates these quantities (5)-(7) in linear function, but UKF calculates these quantities from a set of weighted samples (σ-points) that are deterministically calculated using the mean and square-root decomposition of the covariance matrix of x_{k-1}, w_{k-1} and v_k. When propagated through the nonlinear transformation, it captures the posterior covariance accurately (3rd order accuracy is achieved if the prior random variable has a symmetric distribution, such as the exponential family of PDF) (Kraft, 2003).

The pseudo-code for UKF follows. In UKF, state random variable (RV) is redefined as the concatenation of the original state plus the noise variables in an augmented state vector form $x_k^a = [x_k^T \quad w_k^T \quad v_k^T]^T$. The σ points selection scheme is applied to this new augmented state to calculate the corresponding σ -point set, $\{\chi_{k,i}^a; \quad i = 0, \ldots, 2L\}$ where $L = n_x + n_w + n_v$ and $\chi_{k,i}^a \in \Re^{2L+1}$ n_x, n_w and n_v are dimensions of state, noise process and noise of measurements, respectively. $\chi^a = [(\chi^a)^T \quad (\chi^w)^T \quad (\chi^v)^T]^T$ is the augmented σ points that are of dimension $L \times (2 \times L + 1)$. γ is a scaling parameter that determines the spread of the σ-points matrix around the prior mean. Q_k, R_k are covariances of the process and measurement noise processes, respectively.

2.1 Pseudo-code for unscented Kalman filter

- Initialization

$$\hat{x}_0 = E[x_0], \quad P_{x_0} = E\left[(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T\right]$$

$$\hat{x}_0^a = E\left[x_0^a\right] = \left[\hat{x}_0^T \quad \bar{w}_0^T \quad \bar{v}_0^T\right]^T$$

$$P_0^a = \begin{bmatrix} P_{x_k} & 0 & 0 \\ 0 & Q_k & 0 \\ 0 & 0 & R_k \end{bmatrix}$$

- For $k = 1, \ldots, \infty$

1 Set $t = k - 1$
2 Calculate σ -points:

$$\chi_t^a = \left[\hat{x}_t^a \quad \hat{x}_t^a + \gamma S_{x_t}^a \quad \hat{x}_t^a - \gamma S_{x_t}^a\right]$$

3 Time-update equations:

$$\chi_{k|t}^x = f\left(\chi_t^x, u_t', \chi_t^w\right)$$

$$\hat{x}_k^- = \sum_{i=0}^{2L} w_i^m \chi_{i,k|t}^x$$

$$P_{x_k}^- = \sum_{i=0}^{2L} w_i^c \left(\chi_{i,k|t}^x - \hat{x}_k^-\right)\left(\chi_{i,k|t}^x - \hat{x}_k^-\right)^T$$

4 Measurement-update equations:

$$H_{k|t} = h\left(\chi_t^x, \chi_t^v\right)$$

$$h_k^- = \sum_{i=0}^{2L} w_i^m \mathrm{H}_{i,k|t}$$

$$y_k^- = h_k^- + u_k$$

$$P_{\tilde{y}_k} = \sum_{i=0}^{2L} w_i^c \left(\mathrm{H}_{i,k|t} - y_k^-\right)\left(\mathrm{H}_{i,k|t} - y_k^-\right)^T$$

$$P_{x_k y_k} = \sum_{i=0}^{2L} w_i^c \left(\chi_{i,k|t}^x - \hat{x}_k^-\right)\left(Y_{i,k|t} - y_k^-\right)^T$$

$$K_k = P_{x_k y_k P_{\tilde{y}_k}^{-1}}$$

$$\hat{x}_k = \hat{x}_k^- + K_k\left(y_k - y_k^-\right)$$

$$P_{x_k} = P_{x_k}^- - K_k P_{\tilde{y}_k}^{-1} K_k^T$$

where $\{w^t\}$ is a set of scalar weights:

$$w_0^m = \frac{\lambda}{L + \lambda}, \quad w_0^c = \frac{\lambda}{L + \lambda} + (1 - \alpha^2 + \beta)$$

and:

$$w_i^m = w_i^c = \frac{\lambda}{2(L + \lambda)},$$

$i = 1, \ldots, 2L, \lambda = \alpha^2(L + \kappa) - L$ and $\gamma = \sqrt{L + \lambda}$. The constant α determines the spread of the σ points around the prior mean. Typical range for α is $1e - 3 < \alpha \leq 1$. κ is a tertiary scaling factor and is usually set equal to 0. β is the secondary scaling factor used to emphasize the weighting on the zero's σ-point for the posterior covariance calculation. β can be used to minimize certain higher-order error terms based on known moments of the prior RV. For Gaussian priors, $\beta = 2$ is optimal.

This algorithm requires at each iteration to factorize square-root form of $P_k^a = S_k^a (S_k^a)^T$, however, this filter propagates the covariance of states and is usually very sensitive to round off errors causing numerically instability.

3. Residual-based input estimation

Assume that the Kalman filter measurement equation from equation (1) has the following form:

$$y_k = h(x_k, v_k) + u_k \tag{8}$$

where u_k is the deterministic input vector we need to estimate. Further, the residual process of the Kalman filter for all k can be defined by Maybeck (1994):

$$r_k = y_k - y_k^- = y_k - h_k^- - u_k \tag{9}$$

that is the difference between the current measurement value and the best prediction of it before the measurement is actually taken as the residual. The following sequence has been shown to be a white noise Gaussian sequence with zero mean and covariance (Hykin, 2002; Ma and Jiang, 2005):

$$P_{\tilde{y}_k} = R_{r_k} = \sum_{i=0}^{2L} w_i^c \left(\mathrm{H}_{i,k|t} - y_k^-\right)\left(\mathrm{H}_{i,k|t} - y_k^-\right)^T \tag{10}$$

Now we can define from equation (9):

$$z_k = y_k - h_k^- = u_k + r_k \tag{11}$$

where z_k is the measurement vector, u_k is an unknown signal to be estimated and r_k represents the measurement noise with known covariance R_{r_k}. Various methods for estimation of u_k are discussed in some references (Hykin *et al.*, 1977; Hykin, 2002). A nonlinear method was developed in (Wan and van der Merwe, 1999) based on recursive unscented transformation. But we will use a linear vector model, as follows:

$$\hat{u}_k = Z_k W_k \tag{12}$$

where $Z_k = [y_k y_{k-1} \ldots y_{k-N+1}]$ is the collection of observation vector from k-$N + 1$ until k. Here, we describe a method for the design of an RLS-type algorithm to cope with corresponding forms of nonstationary environmental conditions. According to Simon Haykin and Ali H. Sayed

(Hykin *et al.*, 1977), a state-space model for exponentially weighted RLS algorithm may be described as follows:

$$W_k = FW_{k-1} + n_{k-1} \quad z_k = Z_k W_k + r_k \tag{13}$$

Here, n_{k-1}, r_k are system and measurements noise with zero mean and covariance's $Q_{n,k-1} = q_{k-1}I, R_{r_k}$, respectively. W_k are tap-weights and in this form are the state vectors to be estimated. In the special case that $F = aI$, and $a \approx 1$. In stationary environments, covariance $Q_{n,k-1}$ is zero for all k, in which case $P_{w_k} = P_{w_{k-1}}$, and the modified RLS algorithm reduces to its standard form (without exponential weighting). Under this condition, P_{w_k} equals the inverse of the deterministic correlation matrix Φ_k of the measurement vector:

$$\Phi_k = \sum_{j=k-N+1}^{k} y_j^T y_j \tag{14}$$

By applying Kalman filter algorithm for system (12) we will have (Hykin *et al.*, 1977; Hykin, 2002):

$$\begin{aligned} &W_k^- = F\hat{W}_{k-1} \quad P_k^- = FP_{k-1}F + Q_{n,k-1} \\ &S_k = Z_k P_k^- Z_k^T + R_{r_k} \quad K_{w_k} = P_k^- Z_k^T S_k^{-1} \\ &\hat{W}_k = W_{k-1}^- + K_{w_k}\left(z_k - Z_k W_{k-1}^-\right) \end{aligned} \tag{15}$$

This algorithm has a single variable parameter, namely, q_{k-1}. The difference between Haykin algorithm and above algorithm is in measurement covariance. In many applications we do not have R_{w_k} for this reason in Haykin algorithm Assume that $R_{w_k} = I$. But here we presume to know have R_{w_k} a priori, and use it. Also determinations of q_{k-1} (gain of noise process covariance) are discussed in many papers (Hu *et al.*, 2003; Moose *et al.*, 1987; Myers and Tapley, 1976; Moose, 1975; Kirlin and Moghaddamjoo, 1986). Here, we use a new information process, denoted as s_k and defined for all k by Maybeck (1994):

$$s_k = K_{w_k}\left(z_k - Z_k W_{k-1}^-\right) \tag{16}$$

The covariance of new information process at steady state condition is Hide *et al.* (2004):

$$Q_{n,k} \approx \frac{1}{N_1} \sum_{j=k-N+1}^{k} s_j s_j^T \tag{17}$$

Considering a diagonal matrix with identical elements on the diagonals, recursive estimation of $Q_{n,k}$ with a forgetting factor can be:

$$Q_{n,k+1} = \left(1 - \frac{1}{T}\right) Q_{n,k} + \frac{1}{T} s_j s_j^T, \quad T > 1 \tag{18}$$

If we take a trace from both side of the above equation (18) we will have

$$q_{k+1} = \left(1 - \frac{1}{T}\right) q_k + \frac{1}{NT} s_j^T s_j \tag{19}$$

This equation allows us to adapt the gain of the noise covariance matrix. For this reason this algorithm works well worked in non stationary environments.

4. Attitude propagation based on gyroscopes measurements

As is typical for all modern INS systems, to the eliminate problems with infinite angular rates caused by the nose-down initial attitude at launch, the estimation algorithm formulates the problem in terms of the quaternion parameters (Battin, 1987). In quaternion transformation the orientation is written as a four-space vector with the magnitude being constrained to always be unity.

Denoting gyro measurements as $\omega_{ib}^b = [\omega_x \quad \omega_y \quad \omega_z]^{\mathrm{T}}$ with ω_x, ω_y, ω_z being three-axis angular rate in body frame, and assuming that ω_{in}^b is very small relative to ω_{ib}^b. Then we have:

$$\omega_{nb}^b = \omega_{ib}^b - \omega_{in}^b = [p \quad q \quad r]^{\mathrm{T}} \tag{20}$$

The differential equation, relating to the quaternion with the body angular rates is propagation, is:

$$\dot{q} = \frac{1}{2}\Omega\left(\omega_{nb}^b\right)q, \quad \Omega\left(\omega_{nb}^b\right) = \begin{bmatrix} 0 & r & -q & p \\ -r & 0 & p & q \\ q & -p & 0 & r \\ -p & -q & -r & 0 \end{bmatrix} \tag{21}$$

If the direction of ω_{nb}^b is constant over the time interval of interest or if the rotation vector defined by:

$$\Delta\theta = \int_{t-\Delta t}^{t} \omega_{nb}^b(t')\mathrm{d}t' \tag{22}$$

is small, for then small time of sampling or constant angular velocity from t to $t + \Delta t$ one can write:

$$\Delta\theta = \omega_{nb}^b \Delta t \tag{23}$$

in this case the solution to equation (21) would be:

$$q_k = M(\Delta\theta)q_{k-1} \tag{24}$$

where:

$$M(\Delta\theta) = \cos\left(\left|\frac{\Delta\theta}{2}\right|\right) I_{4\times4}\Bigg| + \frac{\sin(|\Delta\theta/2|)}{|\Delta\theta|}\Omega(\Delta\theta) \tag{25}$$

Equation (24) is a recursive equation for the propagation of the quaternions.

5. Sensor models

We use a simple but realistic model for gyro operation developed bye Farrenkop (Farrnkopf, 1978) will applied to the HEAO mission by Hoffman and McElroy (1978). In this model the angular velocity is related to the gyroscope output vector u, according to the following equation (Lefferts *et al.*, 1982):

$$\omega_{\mathbf{ib}}^{\mathbf{b}} = u - \mathbf{b} - \eta_1 \tag{26}$$

The vector **b** is the drift-rate bias and η_1 is the drift-rate noise. η_1 is assumed to be a Gaussian white noise process with zero mean and covariance Q_1.

The drift rate bias is itself not a static quantity but is driven by a second Gaussian white-noise process, the gyro drift-rate ramp noise:

$$\dot{b} = \eta_2 \tag{27}$$

Here, η_2 also has zero mean and covariance Q_2. The two noise processes are assumed to be uncorrelated. In general, u, b, η_1, η_2 will be a linear combination of the outputs of the three or more gyros, which need not be aligned along the body axes. An alternative equation for drift-rate bias is:

$$\dot{b} = \frac{-1}{\tau} b + \frac{1}{\tau} \eta_2 \tag{28}$$

which gives rise to an exponentially correlated noise term in the model. Allowing seven to be infinit which is adequate for most applications. In this paper, we considered model destined in equations (25)–(26) and (28) for rate gyroscopes. The parameters of gyros are given experimentally in laboratory. The model considered for accelerometers and magnetometer can be of the form:

$$a_m = f^b + v_f, \quad m_m = m^b + v_m \tag{29}$$

Where a_m *and* m_m, are measurements of the accelerometers and magnetometer. These sensors can measure the specific forces and magnetic filed in own body frames. Here, we suppose that this sensor are calibrated in the laboratory and can only have a white noise error with covariance R_a, R_m, respectively.

6. Modeling of specific force in navigation frame

The equation for the ground velocities in the navigation frame according Titerton and Weston (1997) is:

$$\dot{v}^n = C_b^n f^b - \left(\Omega_{en}^n + 2\Omega_{ie}^n\right) v^n + g^n \tag{30}$$

Here, v^n is ground speed in navigation frame, C_b^n is transformation matrix from body to navigation frame, g^n and is the gravity vector in navigation frame. In this equation centrifugal acceleration are very small and can be neglected in many applications.

If aircraft is in a steady and level flight, the left hand of equation (30) will be zero. So we will have:

$$f^b \approx -C_n^b g^n + v_f \tag{31}$$

We can use this equation indeed for the measurement equation in Kalman filter. According to Wahba's *et al.* (1966), If we have two vectors in two coordinates Frames one can find the relative orientation of these frames. So in non acceleration mode we have two vectors in two coordinates and this problem is full observable. Also we have gyro outputs and this measurement allows us to have a good estimate of the orientation. But if there is acceleration in the system, the acceleration will cause a drift from the nominal and value.

Next, we define a new measurement equation:

$$y_k = C_b^n\left(f_k^b + v_f\right) + g_k^n + u_k \tag{32}$$

Where $u_k = -\dot{v}^n - (\Omega_{en}^n + 2\Omega_{ie}^n)v^n$ is an unknown input that will be estimated. We know that the measurement y_k is zero at all times. The measurement equation defined above has have an unknown input that can be estimated with the algorithm described in Section 3 of this paper.

7. Description

In order to estimate the quaternions and bias vector, we define an augmented state vector as follows:

$$x_k = \begin{bmatrix} q_k \\ b_k \end{bmatrix} \tag{33}$$

Substituting equation (26) in equation (23) and subsequently using equation (24) yields:

$$q_k = M\left(\Delta t\left(u - b - \eta - \omega_{in}^b\right)\right) q_{k-1} \tag{34}$$

Also by discretizing equation (28), we will get:

$$b_k = F_b b_{k-1} + G_b \eta_2 \tag{35}$$

Equation (33) together with equations (34) and (35) form our nonlinear dynamic system.

For measurement equation, we use the magnetometer output (29) without any compensation together with equation (32) for the second measurement equation.

8. Simulation and results

For the purpose of this analysis, a numerical simulation of a typical high performance general aviation, Navion type, aircraft is performed using an in-house developed software. Subsequently, the simulation results are distorted for the filtering applications. Table I shows the corresponding noise characteristics augmented to the simulation results in order to provide for the required sensors (measured data) outputs for subsequent estimation process.

Also a value of $\tau = 40$ s is taken for the correlation time in the drift-rate bias equation (28). Further, the aircraft is simulated in low as well as high dynamic accelerating modes by imposing various control actions over the craft while recording the simulation responses. Figures 2 and 6 show the actual generated accelerations during the simulated low and high dynamic maneuvers which are next polluted for filtering application in accordance to the values of Table I. Similarly the other pertinent simulation response parameters are augmented with sensor errors for the filtering and estimation process.

Figure 1 shows the errors of the orientation angles in low dynamic acceleration mode which is behaving in a nice fashion. Figure 2 shows the actual acceleration inputs being estimated, while their corresponding estimation errors are shown in Figure 3. As evidenced from the results, the filter is working very well in the low dynamic phase of flight.

Table I Bias and random error standard deviations augmented to simulated data

	Data type	Added biases	Random errors
1	Gyroscopes output	$\begin{bmatrix} 3.5 \\ -0.02.4^{*}t \\ 0.2 \end{bmatrix}$ deg /s	0.2 deg/sec for all gyros
2	η_2, white noise	0	2 deg/sec
3	Accelerometers	0	14 mg
4	Magnetometers	0	100 (nT)

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Figure 1 Orientations errors (low dynamic)

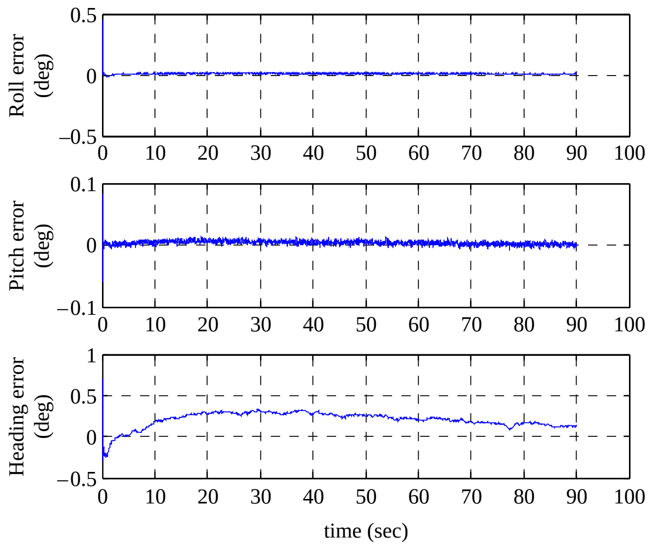


Figure 2 Actual acceleration inputs (low dynamic)

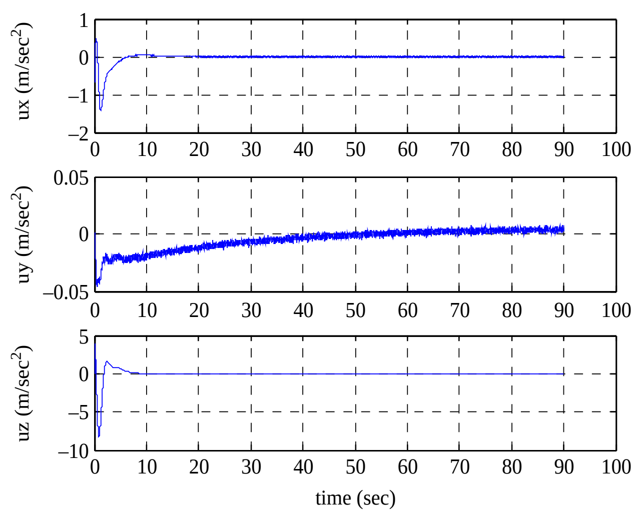
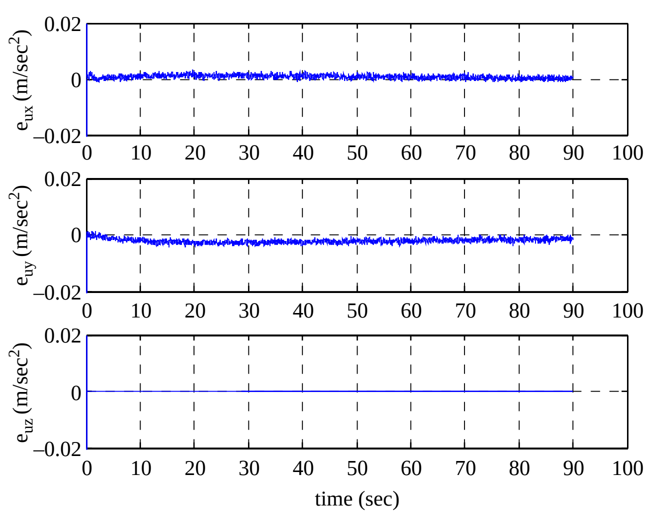


Figure 3 Input estimation errors (low dynamic)



The Final result of the low dynamic mode is shown in Figure 4 that corresponds to the actual gyroscope biases and their filter estimations. Similar graphs for the high dynamic mode of flight are shown in Figures 5-8, respectively. It is interesting to

Figure 4 Actual and estimated biases (low dynamic)

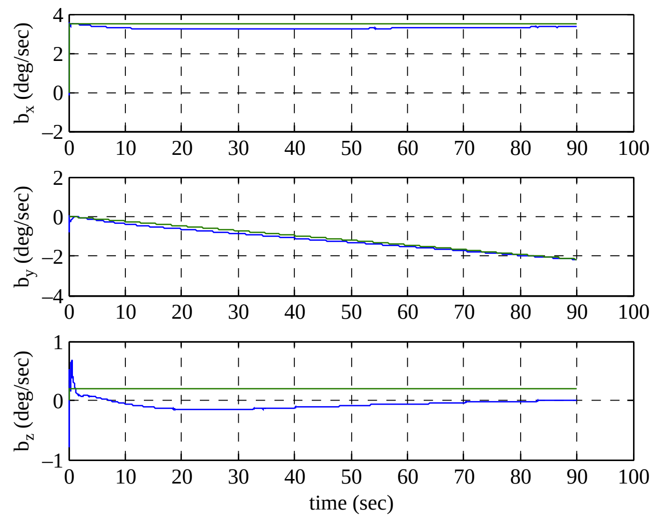


Figure 5 Orientations errors (high dynamic)

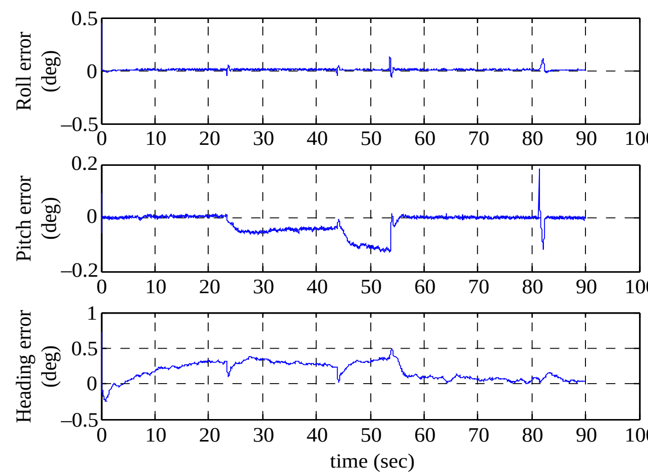


Figure 6 Actual acceleration inputs (high dynamic)

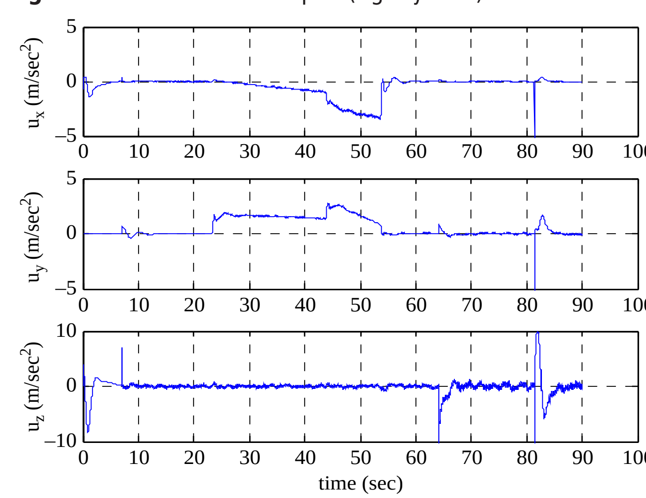


Figure 7 Input estimation errors (high dynamic)

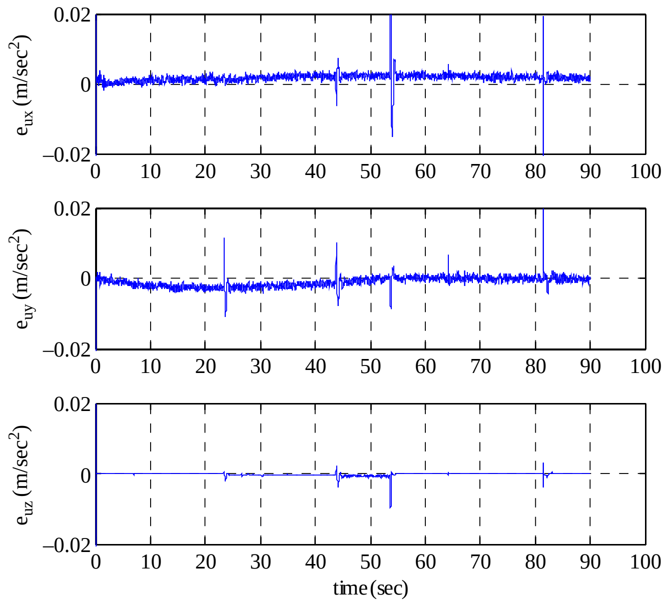
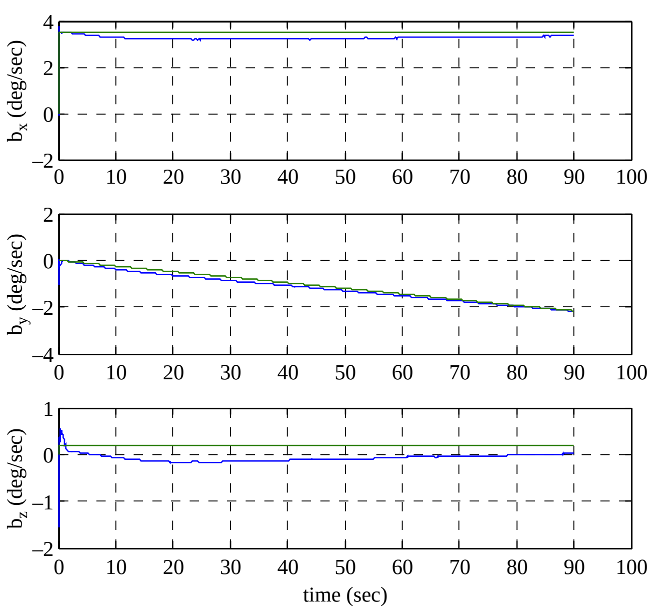


Figure 8 Actual and estimated biases (high dynamic)



see that for this high dynamic mode of simulation, where the vehicle accelerations are much larger than before (Figure 6) the filter estimation errors have remained bounded (Figure 7) and consequently the orientation angles have still been predicted accurately with bounded estimation errors shown in Figure 5. Overall, the results of the proposed filtering scheme are very promising for UAV type configurations, considering application of low-cost, large errors, AHRS sensors.

Table II shows the mean square values of the orientation angle errors calculated during high and low dynamics flights.

Table II Results of simulation

Mean square error (°)	Low dynamic	High dynamic
Roll	0.0145	0.0277
Pitch	0.0356	0.1099
Heading	0.2836	0.5174

9. Conclusions

An adaptive unscented Kalman filter is developed for orientation estimations of aircrafts and UAV's that utilize low-cost AHRS. The proposed filter estimates the additional accelerations generated over steady cruising flights and so will be able to predict the roll and pitch orientation angles with very good accuracy. The filter is shown to have an excellent performance in low dynamic operating conditions and a desirable performance in high accelerating mode.

For most conventional AHRS filters, severe problems can occur once the system suddenly experiences additional acceleration, resulting in erroneous orientation angles, that requires modification of the filter gains for corrections. On the contrary in the high dynamic mode of the new proposed filter the errors have not suddenly increased since, the additional to cruise, accelerations are being continuously estimated resulting in a substantial more accurate orientation estimation. This feature causes the associated filter errors to gradually increase, in the event of continuous vehicle acceleration, up to a point of zero additional acceleration that will subsequently causes a subsidence of the errors back to zero. Overall, the results indicate that, the new proposed filtering scheme can further facilitate utilization of low-cost MEMS sensors while maintaining estimation accuracy at acceptable levels for many aerospace applications.

References

Battin, R.H. (1987), *An Introduction to the Mathematics and Methods of Astrodynamics*, AIAA Education series, American Institute of aeronautics and Astronautics, New York, NY.

Crassidis, L. and Markley, E.L. (2003), "Unscented filtering for spacecraft attitude estimation", *Journal of Guidance. Conlrol, and Dynamics*, Vol. 26 No. 4, p. 536542.

Farrnkopf, R.L. (1978), "Analytic steady – state accuracy solutions for two common spacecraft attitude estimators", *Journal of Guidance and Control*, Vol. 1, pp. 282-4.

Garg, S.C., Morrow, L.D. and Mamen, R. (1978), "Strapdown navigation technology: a literature survey", *J. Guid. Contr. Dyn.*, Vol. 1 No. 3, pp. 287-91.

Gebre-Egziabher, D., Elkaim, G.H., Powell, J.D. and Parkinson, B.W. (2000), "A gyro-free quaternion-based attitude determination system suitable for implementation using low cost sensors", paper presented at IEEE Plans, March, 2000, San Diego, California, CA.

Guerrero Castellanos, J.F., Lesecq, S., Marchand, N. and Delamare, J. (2000), "A low-cost air data attitude heading reference system for the tourism airplane applications", 4th IEEE Sensors.

Hide, C., Moore, T. and Smith, M. (2004), "Adaptive Kalman filtering alighnments for integrating GPS and low cost INS", available at: 0-7803-8416-4/04/$20.00@2004 IEEE

Hoffman, D.P. and McElroy, T.T. (1978), "HEAO attitude reference design", AAS Paper 78-120, March.

Hong, S.K. (2003), "Fuzzy logic based closed-loop strapdown attitude system for unmanned aerial vehicle (UAV)", available at: www.elsevier.com (accessed June 4)

Hu, C., Chen, W., Chen, Y. and Liu, D. (2003), "Adaptive Kalman filtering for vehicle navigation", *Journal of Global Positioning Systems*, Vol. 2 No. 1, pp. 42-7.

Humphrey, I. (1997), "Inertial navigation system for a micro unmanned aircraft", *Proceedings of the AIAA-97-3567 Paper, AIAA Guidance, Navigation, and Control Conference, New Orleans*.

Hykin, S., Sayed, A.H., Zeidler, J.R., Yee, P. and Wei, P.C. (1977), "Adaptive tracking of linear time-variant systems by extended RLS algorithms", *IEEE Transactions on Signal Processing*, Vol. 45 No. 5.

Hykin, S. (2002), *Adaptive Filter Theory*, 4th ed., Prentice-Hall, Inc., Upper Saddle River, NJ.

Julier, S.J. and Uhlmann, J.K. (1996), *A General Method for Approximating Nonlinear Transformations of Probability Distributions*, OX1 3PI UK, Tech. Rep., November 1996, Department of Engineering Science, University of Oxford, Oxford.

Julier, S.J. and Uhlmann, J.K. (2004), "Unscented filtering and nonlinear estimation", *Proceedings of the IEEE*, Vol. 92 No. 3, pp. 401-22.

Kayton, M. and Fried, W.R. (1969), *Avionics Navigation Systems*, Wiley, New York, NY.

Kirlin, R.L. and Moghaddamjoo, A. (1986), "Robust adaptive Kalman filtering for systems with unknown step inputs and non-gaussian measurement errors", *IEE Trans, On Acoustics, Speech, and Signal Processing*, Vol. ASSP-34 No. 2, April.

Kornfeld, R.P., Hansman, R.J. and Deyst, J.J. (1998), "Single-antenna GPS-based aircraft attitude determination", *Journal of The Institute of Navigation*, Vol. 45 No. 1.

Kraft, E. (2003), "A quaternion-based unscented Kalman filter for orientation tracking", *Proceedings of Fusion, Cairns, Australia, July*.

Lefferts, E.J., Markley, F.L. and Shuster, M.D. (1982), "Kalman filtering for spacecraft attitude estimation", *J. Guidance*, Vol. 5 No. 5.

Lu, G. (1995), "Development of a GPS multi-antenna system for attitude determination", PhD thesis, available at: www.ensu.ucalgary.ca/GradTheses.html (accessed January)

Ma, G-F. and Jiang, X-Y. (2005), "Unscented Kalman filter for spacecraft attitude estimation and calibration using magnetometer measurements", *Proceedings of the Fourth International Conference on Machine Learning and Cybernetics, Guangzhou, 18-21 August*.

Madani, I. (1998), "An investigation into a reduced sensor fit for unmanned aircraft", MSc thesis, Cranfield University, Cranfield.

Maybeck, P.S. (1994), *Stochastic Models, Estimation, and Control, Volume I and II*, Academic Press, Mathematics in Science and Engineering, New York, NY.

Moose, R.L., Sistanizadeh, M.K. and Skagfjord, G. (1987), "Adaptive state estimation for a system with unknown input and measurement bias", *IEEE Journal of Oceanic Engineering*, Vol. OE-12, I.

Moose, R.L. (1975), "An adaptive state estimation solution to the maneuvering target problem", *IEEE Trans. Automatic Control*, Vol. AC-20, pp. 359-62.

Myers, K.A. and Tapley, B.D. (1976), "Adaptive sequential estimation with unknown noise statistics", *IEEE Trans. on Automatic Control*, August.

Titerton, D.H. and Weston, J.L. (1997), *Strapdown Inertial Navigation Technology*, Peter Peregrinus Ltd, London.

van der Merwe, R. (2004), "Sigma-point Kalman filters for probabilistic inference in dynamic state-space models", PhD thesis, OGI School of Science & Engineering, Oregon Health & Science University, Portland, OR, April.

Wahba, G. *et al.*, (1966), "Problem 65-1 (Solution)", *SIAM, Review*, Vol. 8, pp. 384-6.

Wan, E.A. and van der Merwe, R. (1999), "Noise-regularized adaptive filtering for speech enhancement", available at: www.ece.ogi.edu/ ~ ericwan

Wan, E.A. and van der Merwe, R. (2001) in Haykin, S. (Ed.), *Kalman Filtering and Neural Networks Haykin*, Wiley, New York, NY, eh. 7.

Wang, M., Yang, Y., Hatch, R.R. and Zhang, Y. (2004), "Adaptive filter for a miniature MEMS based attitude and heading reference system", *IEEE*, April 26-29, pp. 26-9.

Whitmore, S.A., Fife, M. and Brasher, L. (1997), "Development of strapdown attitude system for an ultrahigh altitude flight experiment", NASA TM-4775, January.

Wie, B. (1998), *Space Vehicle Dynamics and Control*, AIAA Education Series, AIAA Inc., Reston, VA.

Yang, Y. (2001), "Tightly integrated attitude determination methods for low-cost inertial narigation: two-antenna GPS and GPS/magnetometer", PhD dissertation, Department of Electrical Engínecling. University of California, Riverside, CA, June.

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