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## EXPLORATION OF THE EFFECTS OF RESPONSE SPECTRUM MATCHING ON POWER SPECTRAL DENSITY FUNCTIONS

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### ABSTRACT

*Section 3.7.1, Revision 4, “Seismic Design Parameters,” issued December 2014, of the U.S. Nuclear Regulatory Commission document NUREG-0800, “Standard Review Plan for the Review of Safety Analysis Reports for Nuclear Power Plants: LWR Edition” (SRP), provides acceptance criteria for seismic design acceleration time histories, which establish two sets of criteria: (1) the calculated response spectra (RS) from the time histories envelop the design response spectra (DRS), and (2) as a secondary check, the power spectral density (PSD) functions envelop the minimum target PSD functions that are compatible with the DRS. SRP section 3.7.1 also indicates that the PSD functions of closely RS-enveloped (or RS-matched) time histories fluctuate significantly and randomly as a function of frequency, and the fluctuations may lead to unconservative results for the response of structures, systems, and components. This explains the previous difficulties in meeting both RS and PSD enveloping criteria during the development of seismic design acceleration time histories. Such difficulties can potentially be relieved by a recently published wavelet-based time-domain RS-matching algorithm in Nuclear Engineering and Design, 410(112384), because that algorithm uses a minimal number of wavelets to achieve RS convergence and so allows a well-separated treatment of power assurance and RS matching. This paper explores, through examples, how that algorithm affects the PSD functions of the time histories and,*

*more importantly, how it can help achieve both power sufficiency and the RS-matching criteria.*

**Keywords:** acceptance criteria; greedy wavelet method; power spectral density functions; response spectrum matching; seismic design; seismic time history analysis

### INTRODUCTION

Seismic acceleration time histories have been used widely in seismic soil-structure-interaction analysis, nonlinear analysis of complex structural systems, shake table tests, and advanced fragility analysis, among other engineering tasks. Except for situations for which actual recorded seismic ground motions are deemed appropriate, these time histories are often developed based on input response spectra (RS) or design RS (DRS) that cover many potential earthquake scenarios. One of the popular approaches is to perform RS matching, in which a recorded seed time history or a random seed time history is modified so that the resultant acceleration time history will have a computed RS close to the DRS by meeting certain acceptance/convergence criteria.

Most RS-matching algorithms do not explicitly consider the power spectral density (PSD) functions of the generated acceleration time histories, and PSD checks are usually conducted separately after reaching RS convergence. Section 3.7.1, Revision 4, “Seismic Design Parameters,” issued

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December 2014, of the U.S. Nuclear Regulatory Commission document NUREG-0800, “Standard Review Plan for the Review of Safety Analysis Reports for Nuclear Power Plants: LWR Edition” (SRP) [1], indicates that the RS-matched acceleration time histories produce PSD functions that are quite different in appearance from one individual function to another and that fluctuate significantly and randomly as a function of frequency. The SRP also recognizes that, as one tries to more closely envelop the DRS, the PSD functions tend to fluctuate more significantly and randomly. The fluctuations may lead to unconservative results for the response of structures, systems, and components. This paper provides examples on how RS-matched acceleration time histories behave differently regarding their PSD functions, based on a frequency domain method and a newly developed time domain wavelet-based method [2]. The latter method, named Greedy Wavelet Method (GWM), was developed to meet a research need for processing thousands of seed records to develop hundreds of RS-matched seismic acceleration time histories, which were used to estimate the uncertainties in in-structure RS due to uncertainties in input time series [3].

GWM shares some features with the widely used wavelet-based time domain code RspMatch09 [4], including the shared intention to add wavelets at such times that the effect of the wavelets on reducing the difference between the RS and DRS is maximized in each iteration. However, GWM, as a greedy algorithm and mostly implemented in the Python programming language, is fundamentally different from RspMatch09 and its predecessors [5–8]. GWM adds just one wavelet in each iteration to the seed record and does not need to solve an optimization problem in each iteration. The other methods all require solving optimization problems following the same basic procedure proposed by Kaul in 1978 [8]. GWM has been shown to use many fewer wavelets than RspMatch09 to achieve the same level of RS convergence. Another difference is that GWM does not enforce zero drift in the displacement during the process of adding wavelets, but instead, GWM provides several convenient and powerful tools through its graphical user interface for baseline correction of the converged acceleration time history. Nie et al. [2] provide the details on the GWM algorithm and its implementation.

In this paper, the focus is on the acceleration time histories while leaving out an explicit discussion of the velocity and displacement time histories. The time histories described in this paper may not have been baseline-corrected for simplicity. In many practical situations for which the deformation of the structures is the main concern in seismic time history analysis, the drifts in input seismic time histories usually do not cause any issue in the estimates of deformation-related seismic responses.

The examples described in this paper serve as visual cues to reinforce a recent conclusion that RS-matching criteria alone are not sufficient and that checking the PSD functions of the resultant acceleration time histories is a necessary part of ground motion development [9]. SRP section 3.7.1 [1] provides

detailed guidance and the associated procedures for PSD checks. It also provides acceptance criteria for seismic design acceleration time histories, which establish two sets of criteria: (1) the RS calculated from the time histories envelop the DRS, and (2) as a secondary check, the PSD functions envelop the minimum target PSD functions that are compatible with the DRS.

For well-selected seed records whose spectral shapes are close to the DRS in practice, GWM shows a potential to relieve the difficulties in meeting both RS-matching criteria and ensuring power sufficiency, because GWM uses a minimal number of wavelets to achieve RS convergence and consequently allows a well-separated treatment of power assurance and RS matching. Therefore, one of the important objectives of this paper is to explore, through examples, how GWM affects the PSD functions of the time histories while achieving the RS-matching criteria and to demonstrate how power assurance and RS matching can be separately treated and achieved.

This paper provides five examples, in progression, to demonstrate the advanced convergence characteristics of GWM and show how to use those characteristics to achieve the fast development of seismic design acceleration time histories that meet the RS-matching criteria and assure sufficient power. Summarized at the end of this paper are the insights obtained from these examples and a general approach based on the experience of the last example for meeting both RS convergence criteria and power assurance requirements.

## **EXAMPLE 1—TIME HISTORIES WITH 15 MAJOR FREQUENCIES**

Research Information Letter (RIL) 2019 01, “Assessment of Artificial Acceleration Time History Guidance in Standard Review Plan Section 3.7.1, ‘Seismic Design Parameters,’” issued August 2020 [9], includes a couple of synthetic seismic time histories to show that a time history closely matched to a DRS does not necessarily ensure that its PSD function envelops the DRS-compatible target PSD function. These time histories were generated using a frequency domain method, which iteratively multiplies the Fourier amplitude spectrum (FAS) of the time history by the frequency-dependent ratio of the DRS over RS, until some prescribed convergence criteria are met. The frequency domain algorithm is simple to implement and has been described by many in the literature. It is well recognized that frequency domain RS-matching methods are generally less capable than time domain methods in terms of convergence properties, while time domain methods are usually more complicated to implement.

Using one of the examples in RIL 2019-01, example 1 of this paper consists of 15 major frequencies in the range of 0.3 hertz (Hz) to 24 Hz, linearly spaced on the log scale. The seed record used to develop this time history was constructed using random phase angles for the 15 corresponding sinusoids. Their initial amplitudes were determined so that a linear

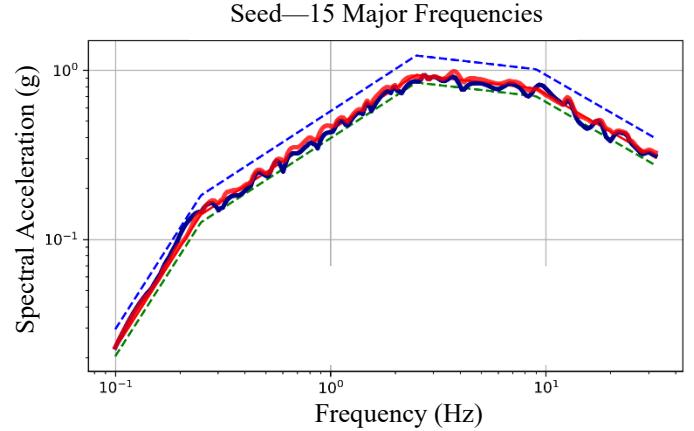
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combination of their RS fits the best to the horizontal DRS described in Regulatory Guide (RG) 1.60, Revision 2, “Design Response Spectra for Seismic Design of Nuclear Power Plants,” issued July 2014 [10]. This seed record was then matched using the frequency domain method to meet the RS-enveloping criteria in SRP section 3.7.1 [1], Option 1, Approach 2. The RS of the converged time history is shown as the blue curve in figure 1. The dashed piecewise-linear curves and the solid piecewise-linear curve, which is barely distinguishable in the figure because it is covered by the thick red and blue curves, designate the DRS and the 90% to 130% band following the criteria in Option 1, Approach 2.

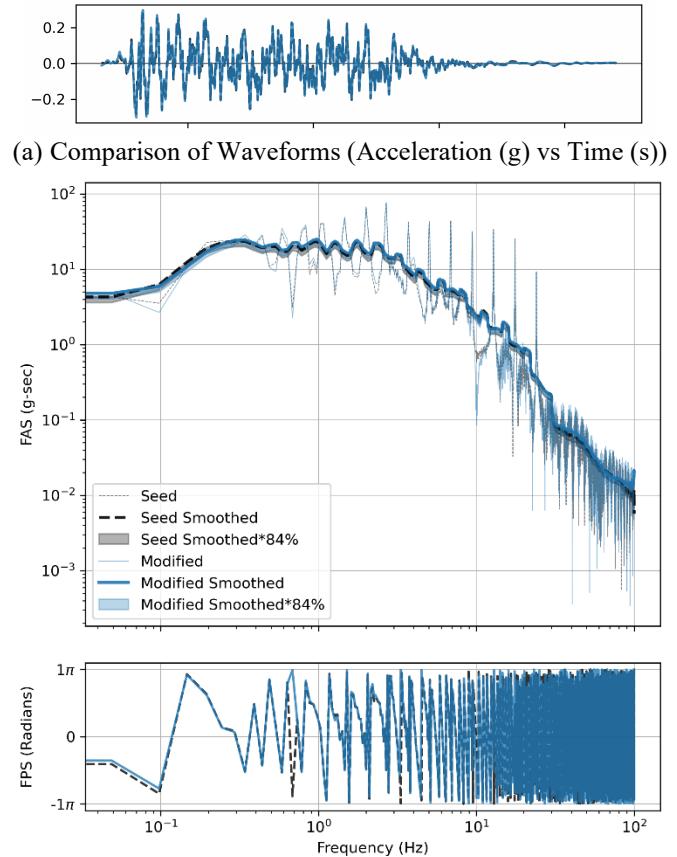
During the development of this paper, that seed record could not be found, so a direct application of GWM on that seed record was not possible. Reconstruction of the same seed record was also not feasible, because of the use of random phases for the 15 sinusoids. Therefore, the converged time history developed using the frequency domain method was used as the seed in GWM for further RS matching. The convergence criteria used by GWM were a range from -5% to 15% of the RG 1.60 horizontal DRS, which is slightly tighter than those used in the frequency domain method. Given that the seed record for the application of GWM is already closely matched to the DRS, GWM used only 19 wavelets to achieve the [-5%, 15%] convergence criteria. The RS of the converged time history using GWM is shown as the red curve in figure 1. It can be seen that the matched RS (in red) from GWM is slightly closer to the DRS than the one (in blue) that resulted from the frequency domain method.

Figure 2 shows a comparison of waveforms and of the Fourier amplitude spectra (FAS) and Fourier phase spectra (FPS). The waveforms show barely any visual differences, indicating that GWM has maintained the time domain nonstationary features of the time history very well. Minimal changes to the Fourier spectra can be observed. The peaks and valleys in FAS are well maintained. The large difference in the phase spectra around 0.7 Hz does not actually indicate a large change because a phase angle close to  $\pi$  is mathematically similar to a value close to  $-\pi$  on the complex plane. A direct comparison of the PSD functions is not performed for this example because the frequency valleys have been maintained by GWM. As RIL 2019-01 concluded, the time history generated by the frequency domain method is severely power deficient in the valleys. Therefore, GWM maintains the power deficiency in example 1 during its RS-matching process.

This example shows that both the frequency domain method and GWM as a time domain method would not remove the power deficiencies if they were present in the seed records (and the RS of the seed records are similar to the DRS). This reinforces a conclusion in RIL 2019-01 that RS-matching criteria alone are not sufficient and that checking the PSD functions of the resultant acceleration time histories is a necessary part of the ground motion development.



**FIGURE 1:** EXAMPLE 1—15 MAJOR FREQUENCIES: COMPARISON OF RS BEFORE AND AFTER APPLYING GWM (DASHED LINES FOR 130% OR 90% OF DRS)



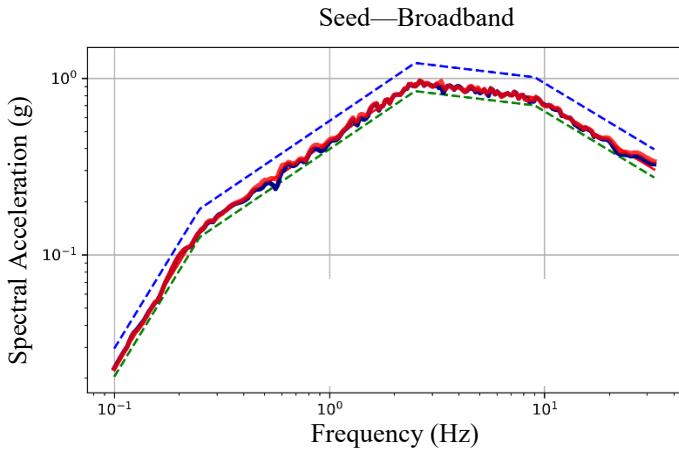
**(b) Comparison of Fourier Amplitude Spectra (FAS) and Fourier Phase Spectra (FPS) (Smoothed curves used a  $\pm 20\%$  smoothing frequency window)**

**FIGURE 2:** EXAMPLE 1—15 MAJOR FREQUENCIES: COMPARISON OF WAVEFORMS AND FOURIER SPECTRA BEFORE AND AFTER APPLYING GWM

## EXAMPLE 2—BROADBAND TIME HISTORIES

RIL 2019-01 [9] also describes a broadband time history that was generated to be power sufficient in the frequency range of interest. The same procedure as for example 1 was repeated, and GWM achieved RS convergence with 18 wavelets.

Figure 3 shows the RS comparisons, and figure 4 compares the waveforms and the Fourier spectra. The same observations for example 1 can be obtained for example 2: slightly improved RS convergence, essentially identical waveforms, and minimal changes to the Fourier spectra. In this case, GWM has maintained, during its RS-matching process, the sufficiency of power of the time history that was generated using the frequency domain method.

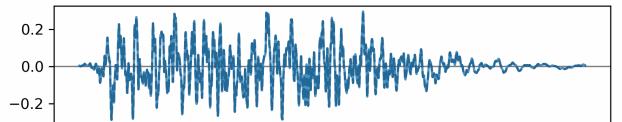


**FIGURE 3:** EXAMPLE 2—BROADBAND TIME HISTORIES: COMPARISON OF RS BEFORE AND AFTER APPLYING GWM (DASHED LINES FOR 130% OR 90% OF DRS)

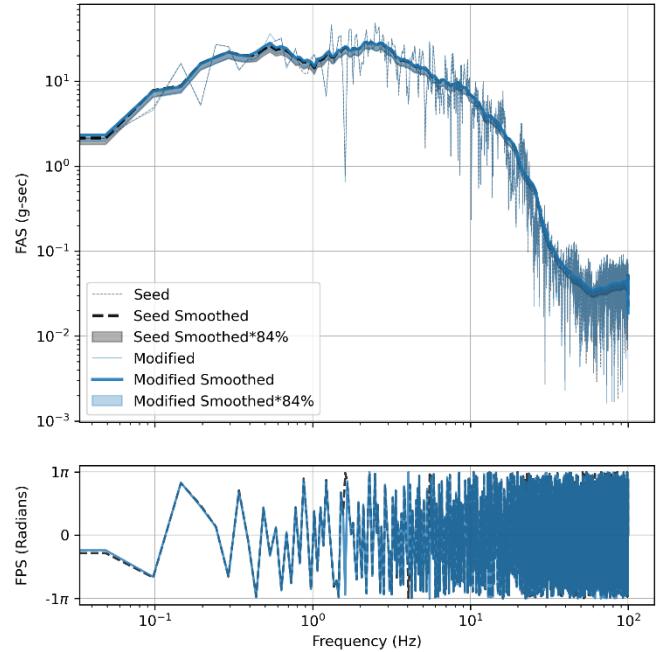
## EXAMPLE 3—BROADBAND WITH A LARGE FREQUENCY CUT

In example 3, the broadband time history converged using the frequency domain method is modified by reducing the FAS to 1% of its original values in a large frequency range of 5 Hz to 15 Hz, and then it is used as the seed record for RS matching using GWM. The goal of this example is to explore how GWM performs with a seed record that is deficient in power for a frequency range significantly wider than those frequency valleys in example 1. Only GWM is used in this and later examples in this paper, in line with the overall objective to explore how GWM can be used to meet both RS-matching criteria and power assurance.

Figure 5 shows the reduction of FAS in the frequency range of 5 Hz to 15 Hz. Figure 6 shows comparisons of the waveforms for the entire duration and a zoomed-in view of about 2 seconds (s). In these figures, the blue curves represent the original broadband time history, and the orange curves represent the time history after the frequency cut. These figures show the time history with a zero period acceleration (ZPA) of 1 g before a linear scaling to 0.3 g for the rest of the example.



(a) Comparison of Waveforms (Acceleration (g) vs Time (s))



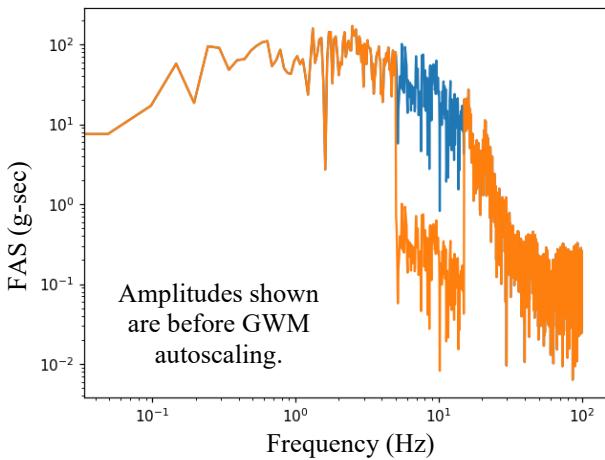
(b) Comparison of FAS and FPS (Smoothed curves used a  $\pm 20\%$  smoothing frequency window)

**FIGURE 4:** EXAMPLE 2—BROADBAND TIME HISTORIES: COMPARISON OF WAVEFORMS AND FOURIER SPECTRA BEFORE AND AFTER APPLYING GWM

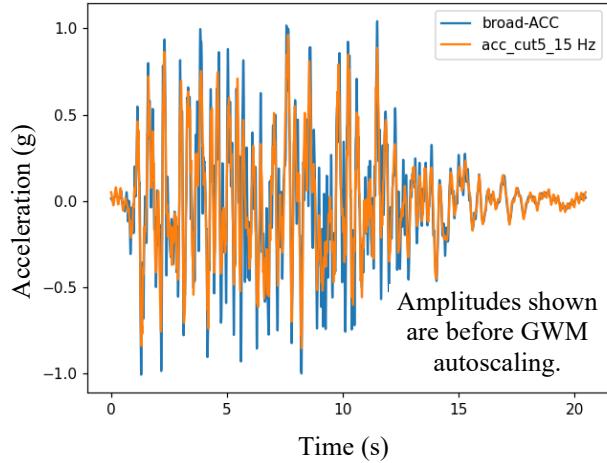
Figure 7 compares the RS before and after applying GWM, using the modified broadband time history as the seed record. It is clear that the RS of the seed has a large dip in the frequency range of 5 Hz–15 Hz, where the FAS was reduced to 1% of its original values. GWM achieved RS convergence using merely 30 wavelets, filling up the large RS dip of the seed. In this example, the seed record was not linearly scaled in GWM by a constant factor, because the RS of the seed record is already converged to the DRS outside of that frequency cut range.

Figure 8 compares the waveforms and Fourier spectra. Even though it has filled up the large dip in RS, GWM does not seem to cause a significant change to the waveform, showing the superb capability of GWM in maintaining the time domain nonstationary features of the seed record. In addition, GWM has significantly compensated for the frequency content that was cut from the broadband seed, without making large changes at other frequencies. However, the added frequency content in the frequency range of 5 Hz–15 Hz obviously is not enough to bring the FAS back to the same level as the original broadband seed time history, as shown in figure 4 and figure 5. The amplitudes shown in figure 8 (a) is smaller than figure 6 due to GWM autoscaling in example 2.

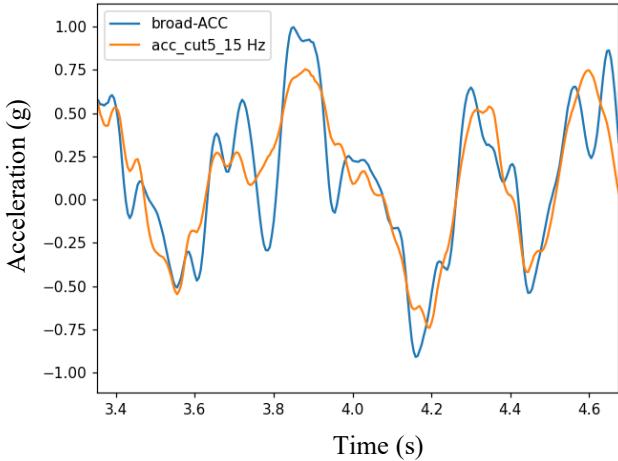
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**FIGURE 5:** EXAMPLE 3—BROADBAND WITH A LARGE FREQUENCY CUT: COMPARISON OF FAS BEFORE AND AFTER FREQUENCY CUT (TIME HISTORY OF A 1 G ZPA)

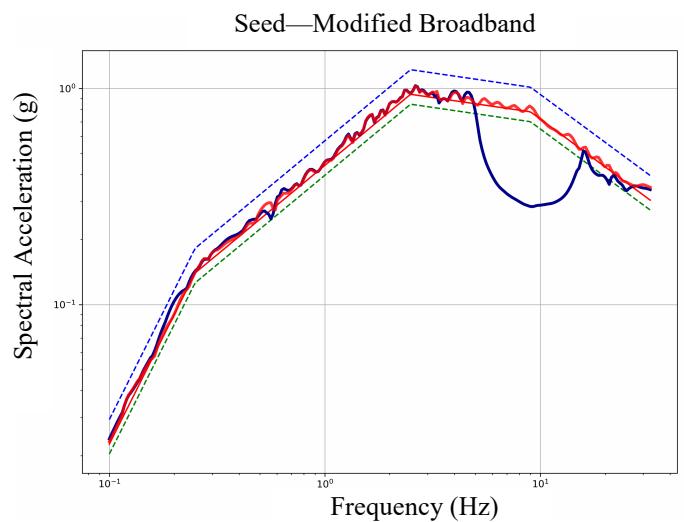


(a) Comparison of Waveforms

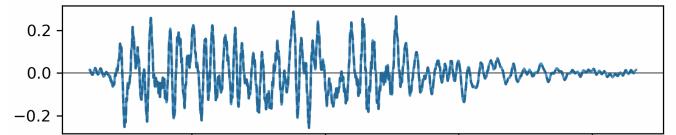


(b) Comparison of Waveforms (Zoomed-In)

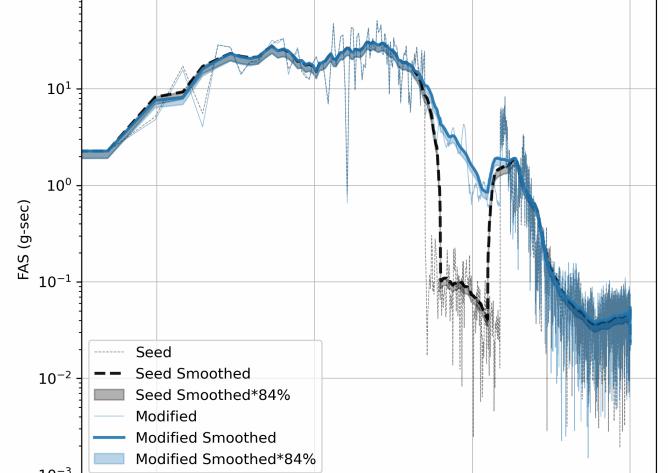
**FIGURE 6:** EXAMPLE 3—BROADBAND WITH A LARGE FREQUENCY CUT: COMPARISON OF WAVEFORMS (TIME HISTORY OF A 1 G ZPA)



**FIGURE 7:** EXAMPLE 3—BROADBAND WITH A LARGE FREQUENCY CUT: COMPARISON OF RS BEFORE AND AFTER APPLYING GWM (DASHED LINES FOR 130% OR 90% OF DRS)

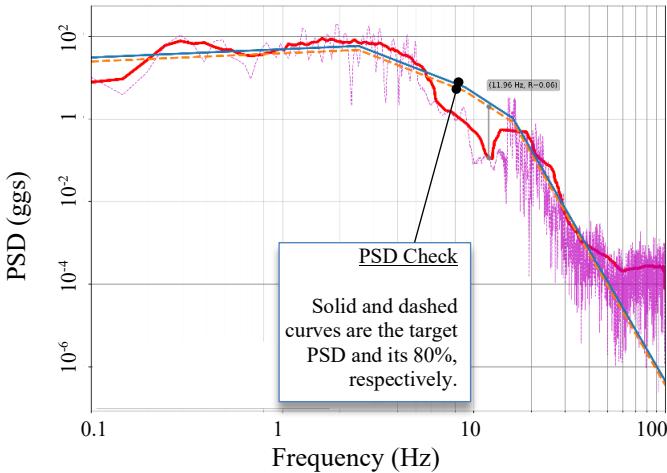


(a) Comparison of Waveforms (Acceleration (g) vs Time (s))



(b) Comparison of FAS and FPS (Smoothed curves used a ±20% smoothing frequency window)

**FIGURE 8:** EXAMPLE 3—BROADBAND WITH A LARGE FREQUENCY CUT: COMPARISON OF WAVEFORMS AND FOURIER SPECTRA BEFORE AND AFTER APPLYING GWM



**FIGURE 9:** EXAMPLE 3—BROADBAND WITH A LARGE FREQUENCY CUT: PSD CHECK (THE DASHED CURVE IS THE RAW PSD FUNCTION, WHILE THE THICK RED CURVE IS THE AVERAGED PSD FUNCTION USING A  $\pm 20\%$  SMOOTHING WINDOW.)

To further examine whether the resultant time history is power sufficient through a PSD check, the PSD function of the RS-matched time history was compared to the target PSD function for RG 1.60 horizontal DRS, which is provided in appendix A to SRP section 3.7.1 [1]. The PSD calculation was based on the strong motion duration, which was selected to be the time window from 1.4 s to 9.6 s (also the same window for the normalized Arias intensity to increase from 5% to 75% in this case). Following the SRP guidance, the strong motion duration is a time window that represents near maximum and nearly stationary power in the time history. Figure 9 shows the result of the PSD check, which indicates that RS-matched time history by GWM, using the modified broadband time history as the seed, is grossly power deficient in the frequency range of 5 Hz–15 Hz. Compared to the 80% of the target PSD function, the PSD level is roughly 20% overall, with a minimum of 6% at 12 Hz. In essence, GWM has maintained the power deficiency introduced by the frequency cut.

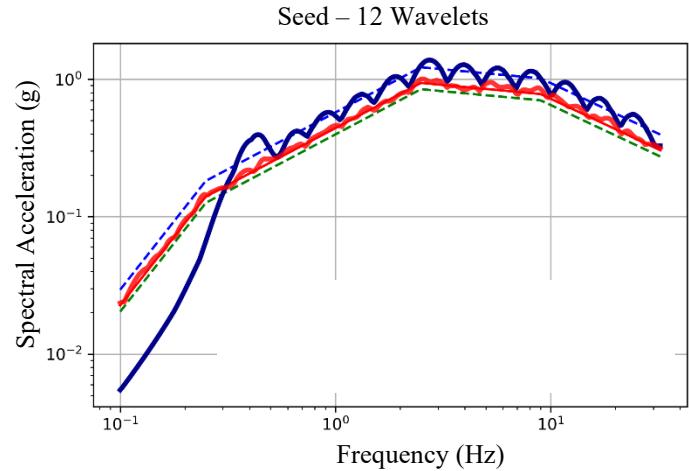
#### EXAMPLE 4—SEED OF 12 WAVELETS

Example 4 is an extreme example in the sense that the seed record does not resemble any reasonably expected recorded or random seismic acceleration time history. It is provided in this paper only for the purpose of demonstrating that GWM can achieve RS convergence even for seeds that are power deficient over the entire frequency range of interest, which is much larger than the 10 Hz frequency range in example 3. Therefore, this example provides a very strong justification for using GWM to separate the treatment of power assurance from RS matching, which is demonstrated later in example 5.

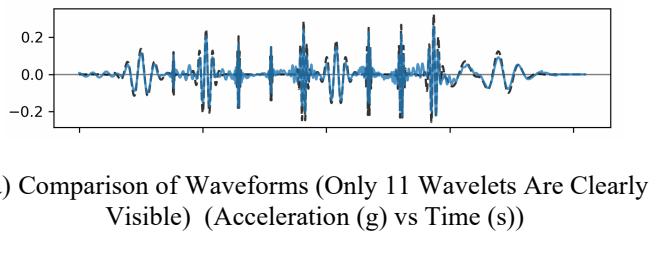
The seed time history for example 4 consists of 12 wavelets placed in a time history of a length about 41 s. This seed was constructed very differently from example 1, in which the 15 major frequencies actually represent 15 sinusoids that span the entire strong motion duration.

Figure 10 shows a comparison of the RS before and after applying GWM, using the seed record consisting of 12 wavelets. The blue curve in figure 10 shows the seed RS with 12 peaks, and the red curve shows the converged RS after GWM has added 61 wavelets to the seed record. Figure 11 compares the waveforms and the Fourier spectra. As shown in figure 11, only 11 wavelets are clearly visible in the time history because one wavelet is blended with two wavelets on the waveform. GWM has achieved RS convergence for such an extreme seed time history and has maintained relatively well the overall time domain nonstationary characteristics of the seed. The FAS comparison indicates that GWM has slightly lowered the FAS of the seed between a frequency range of 0.3 Hz and 33 Hz, while having greatly increased the frequency content outside of that range.

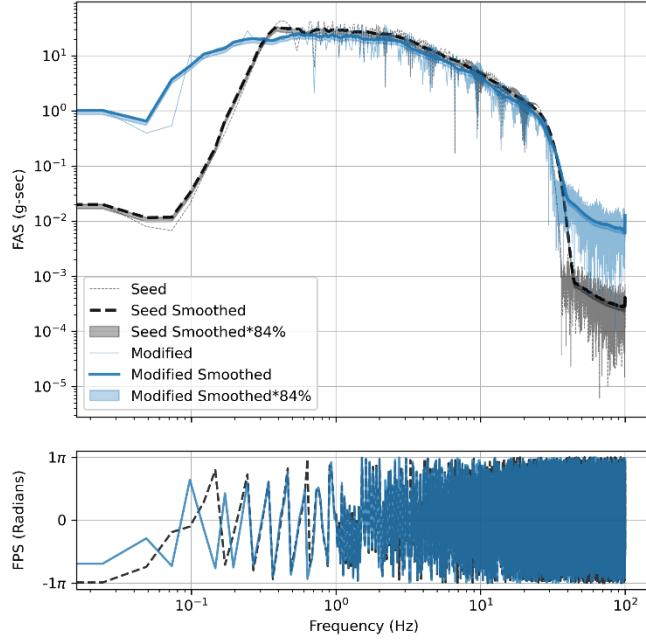
Importantly, because the FAS of both time histories show similar convex and smooth curves between 0.3 Hz and 33 Hz, a visual assessment of FAS cannot tell at all whether the seed record and the RS-matched time history are power deficient. Therefore, a PSD check comparing to the target PSD compatible with the DRS is necessary. As shown in figure 12, except for a small frequency region around 30 Hz, the RS-matched time history developed using the seed of 12 wavelets is severely power deficient at all other frequencies. This conclusion is consistent with an obvious impression that one can obtain from the waveforms shown in figure 11. However, this conclusion of power deficiency cannot be reached by the tightly matched RS, as shown by the red curve in figure 10. Additionally, power deficiency usually cannot be identified from waveforms, such as those shown in examples 1 and 3.



**FIGURE 10:** EXAMPLE 4—SEED OF 12 WAVELETS: COMPARISON OF RS BEFORE AND AFTER APPLYING GWM (DASHED LINES FOR 130% OR 90% OF DRS)

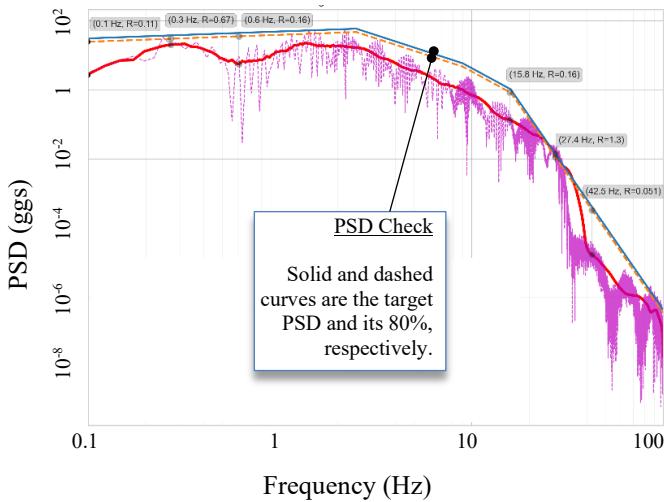


(a) Comparison of Waveforms (Only 11 Wavelets Are Clearly Visible) (Acceleration (g) vs Time (s))



(b) Comparison of FAS and FPS (Smoothed curves used a  $\pm 20\%$  smoothing frequency window)

**FIGURE 11:** EXAMPLE 4—SEED OF 12 WAVELETS: COMPARISON OF WAVEFORMS AND FOURIER SPECTRA BEFORE AND AFTER APPLYING GWM



**FIGURE 12:** EXAMPLE 4—SEED OF 12 WAVELETS: PSD CHECK (THE DASHED CURVE IS THE RAW PSD FUNCTION, WHILE THE THICK RED CURVE IS THE AVERAGED PSD FUNCTION USING A  $\pm 20\%$  SMOOTHING WINDOW.)

## EXAMPLE 5—MEETING BOTH RS MATCHING CRITERIA AND POWER ASSURANCE

From the previous four examples, it can be concluded that GWM can help meet tight RS convergence criteria while it does not significantly affect the PSD functions, either power deficient or power sufficient. This is a feature of GWM that can be leveraged to separate power assurance from RS matching. As reckoned by Nie et al. [2], because GWM requires only a very small number of wavelets to achieve RS convergence and consequently introduces a minimal amount of change to the seed time histories, one can first introduce sufficient power to the seed time histories based on the target PSD functions, and then apply GWM to achieve RS convergence criteria without being too concerned about making the final time histories deficient of power during the RS-matching process.

It is noted that these observations are based on the assumption that the initial seeds have spectral shapes close to the DRS. In general, when the spectral shape of a seed record is different from the DRS in some frequency ranges, GWM, as well as other methods such as the frequency domain method described in this paper, would need to introduce more changes to the seed, thereby increasing the effect on the power distribution of the resultant time history and possibly causing a beneficial effect on achieving power sufficiency in those frequency ranges. This observation is counterintuitive because a less conforming spectral shape of the seed record appears to help with power assurance in some cases during the RS-matching process.

In example 5, we first introduce sufficient power to a seed time history by replacing its FAS with a spectrum back calculated from the target PSD function that is compatible with the DRS, while keeping the same FPS. Using the target PSD function directly in this way ensures a power level across the entire frequency range of interest greater than the minimum target PSD function that is either 80% (as in appendix A to SRP section 3.7.1 [1]) or 70% (as in appendix B to SRP section 3.7.1) of the target PSD function. We then apply GWM on the FAS-replaced time history for RS matching until convergence has been achieved.

Example 5 uses the same DRS from PVP2023-105307, “Uncertainties in In-Structure Response Spectra Due to Uncertainties in Input Motion Amplitude and Phase Spectra” [3]. This DRS is the Western U.S. spectral shape for bin M7+D50-100 as described in appendix B to SRP section 3.7.1 [1], using the mid-bin properties  $M = 7.5$  and  $D = 75$  kilometers.  $M$  and  $D$  stand for earthquake magnitude and source-to-site distance, respectively. The corresponding target PSD function can also be found in appendix B to SRP section 3.7.1.

The seed time history was downloaded from the PEER database [11], based on the similarity of its spectral shape with the DRS. The filename for this seed in the PEER database is RSN577\_SMART1.45\_45O01EW.AT2.

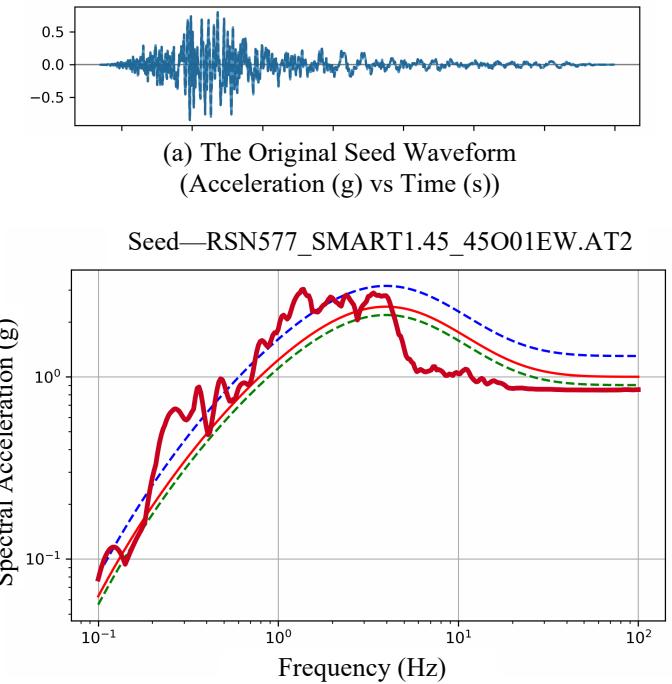
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The original seed record and a comparison of its RS with the DRS are shown in figure 13. It should be noted that the seed has been multiplied by a factor to make the overall shape of its RS closest to the DRS. This is the default option in GWM. The RS comparison seems to indicate that the seed does not have much power above about 5 Hz.

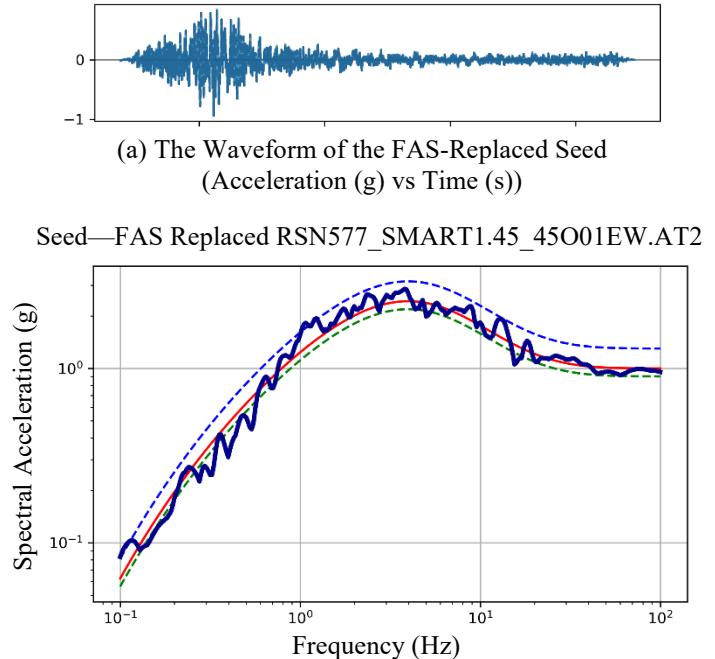
The FPS of the original seed is then used in combination with the target PSD function for bin M7+D50-100 to create a new seed time history. Figure 14 shows the waveform of the FAS-replaced seed and a comparison of its RS to the DRS. This new seed was also baseline corrected before further RS matching. The new seed generally maintains well the overall time domain nonstationary features of the original seed. Additionally, the RS of the new seed is much closer to the DRS than the original seed, because the PSD function of the new seed is the target PSD function that is compatible with the DRS. This can be confirmed by the very smooth FAS of the seed in the black dashed curve in figure 16.

GWM was then applied to the new seed, and it converged with merely 74 wavelets, as the new seed is already very close to the DRS. Figure 15 shows a tight comparison to the DRS. Figure 16 shows the comparisons of the waveforms and Fourier spectra of the new seed and RS-matched time history, indicating no major changes in both time domain features and frequency domain features. This confirms the capability of GWM for a well-separated treatment of PSD assurance and RS matching, as suggested by Nie et al. [2].

Figure 17 shows a PSD check of the RS-matched time history, which is generated based on the recorded seed RSN577\_SMART1.45\_45O01EW.AT2 with its FAS replaced using the target PSD function. It is clear that, over the frequency range of 0.4 Hz to around 30 Hz, the PSD function is greater than 70% of the target PSD function, indicating sufficient power over this frequency range of interest in practice. There is a power dip at 0.3 Hz, which is the lower frequency bound for a PSD check, as described in SRP section 3.7.1 [1], and this problem could be relatively easy to solve by adding a low-frequency wavelet close to the middle of the time history. To simplify this demonstration, this step was not performed. Additionally, 0.3 Hz is usually not a concern in practice because it is significantly below the dominant structural frequencies of nuclear power plants. Similarly, 30 Hz is close to the region where the zero-period acceleration occurs for this DRS, and an additional effort to increase the power level beyond 30 Hz was not warranted for the purpose of the demonstration. Importantly, the power level between 30 Hz and 45 Hz is only slightly lower than 70% of the target PSD function. In conclusion, the FAS-replaced, RS-matched time history in this example can be judged to have passed the secondary but necessary PSD check.

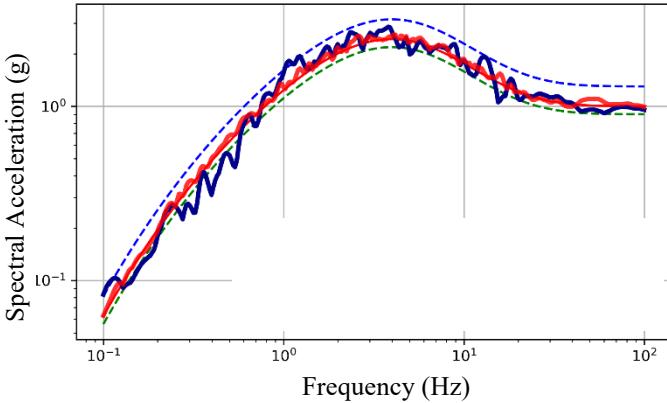


**FIGURE 13:** EXAMPLE 5—FAS-REPLACED RECORDED SEED: THE TIME HISTORY OF THE ORIGINAL SEED AND A COMPARISON OF ITS RS (DASHED CURVES FOR 130% OR 90% OF DRS)

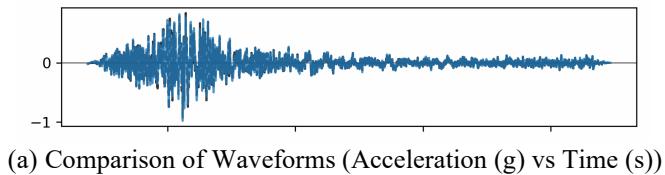


**FIGURE 14:** EXAMPLE 5—FAS-REPLACED RECORDED SEED: THE TIME HISTORY OF THE FAS-REPLACED SEED AND A COMPARISON OF ITS RS (DASHED CURVES FOR 130% OR 90% OF DRS)

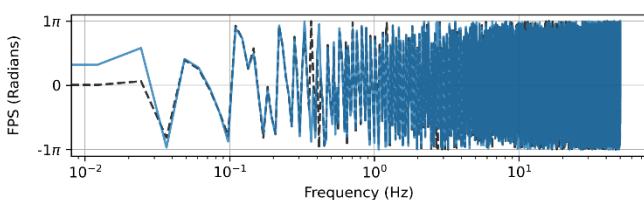
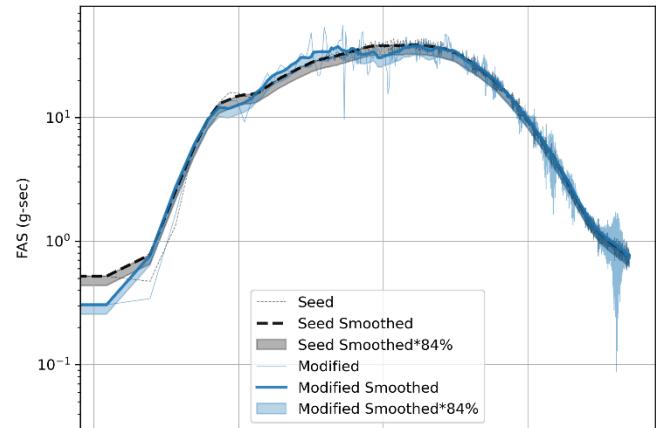
Seed—FAS Replaced RSN577\_SMART1.45\_45O01EW.AT2



**FIGURE 15:** EXAMPLE 5—FAS-REPLACED RECORDED SEED: COMPARISON OF RS BEFORE AND AFTER APPLYING GWM (DASHED CURVES FOR 130% OR 90% OF DRS)

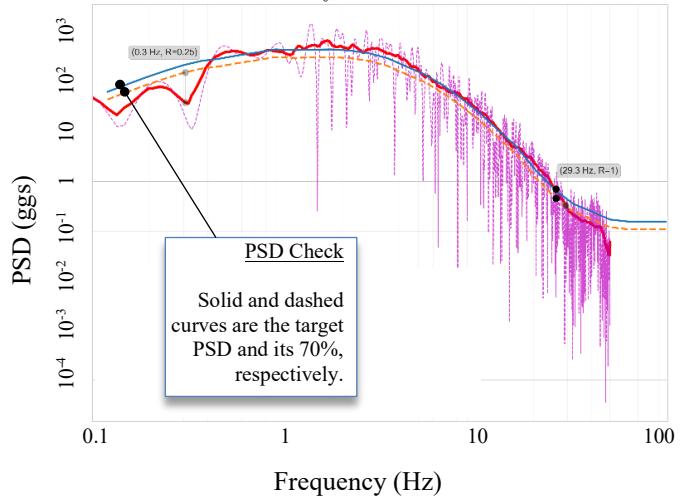


(a) Comparison of Waveforms (Acceleration (g) vs Time (s))



(b) Comparison of FAS and FPS (Smoothed curves used a  $\pm 20\%$  smoothing frequency window)

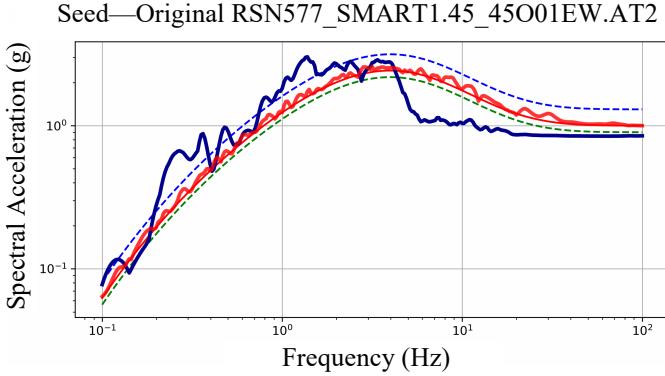
**FIGURE 16:** EXAMPLE 5—FAS-REPLACED RECORDED SEED: COMPARISON OF WAVEFORMS AND FOURIER SPECTRA BEFORE AND AFTER APPLYING GWM



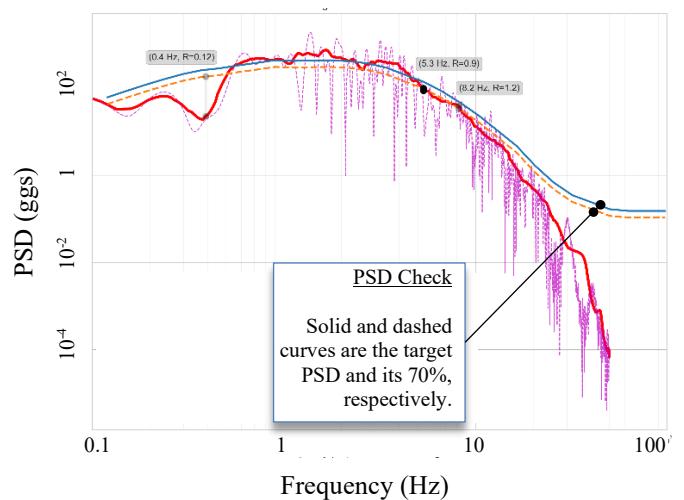
**FIGURE 17:** EXAMPLE 5—FAS-REPLACED RECORDED SEED: PSD CHECK (THE DASHED CURVE IS THE RAW PSD FUNCTION, WHILE THE THICK RED CURVE IS THE AVERAGED PSD FUNCTION USING A  $\pm 20\%$  SMOOTHING WINDOW.)

A reasonable question to ask here is what would have happened if the original seed had been used. For a case using the original seed RSN577\_SMART1.45\_45O01EW.AT2, figure 18 shows the comparisons of the RS of the converged time history with the DRS, and figure 19 shows the result of the PSD check. Although the PSD function has a dip at 0.4 Hz and is lower than the minimum target PSD function at all frequencies higher than 8.2 Hz, the overall shape of the PSD function is not much worse than the case described above. This reflects a common observation that many recorded seeds do lead to reasonably good results with PSD checks, and with some efforts, such as using a small factor to uniformly increase the magnitudes of the entire time history, the PSD function can be made acceptable without introducing too much conservatism. It is noted that the power deficiencies observed in examples 1, 3, and 4 in this paper are merely to demonstrate how GWM behaves when there are power deficiencies in the seed records, but they are not intended to indicate a common situation in practice.

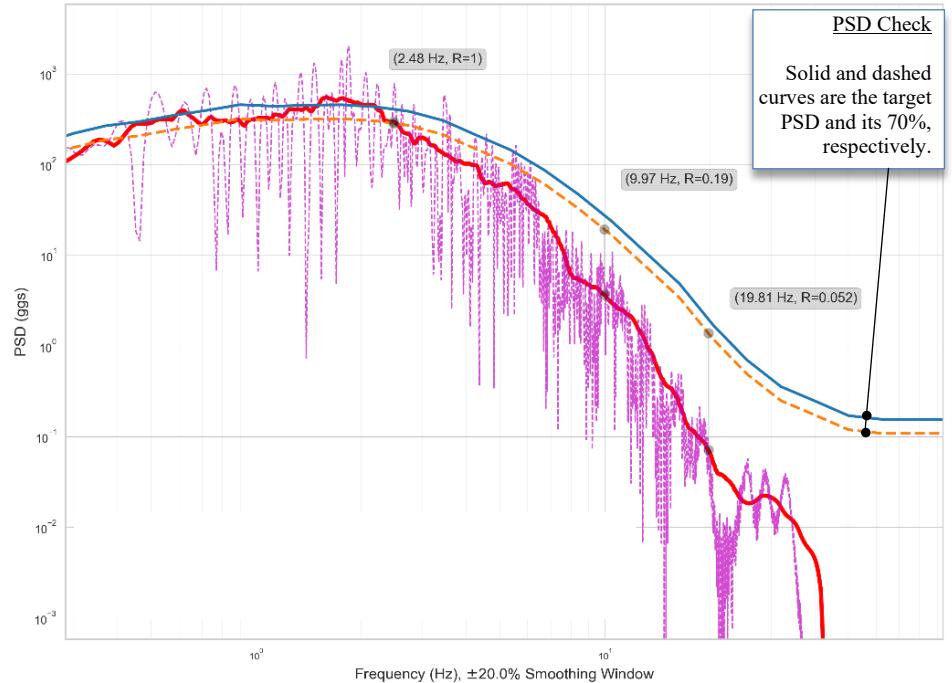
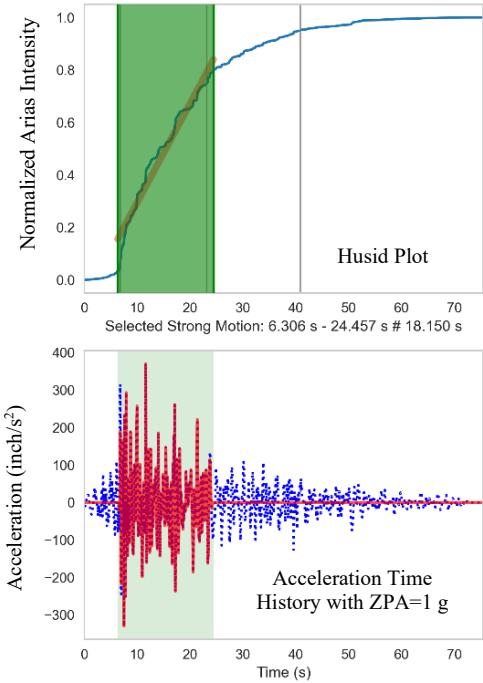
However, some seeds do lead to larger power deficiencies for which simple measures such as applying a constant factor would not be able to resolve the issue. Figure 20 shows such a case in which a constant factor to increase the power at higher frequencies would introduce too much conservatism at frequencies lower than about 2.5 Hz, where power is already adequate.



**FIGURE 18:** EXAMPLE 5—ORIGINAL RECORDED SEED: COMPARISON OF RS BEFORE AND AFTER APPLYING GWM (DASHED CURVES FOR 130% OR 90% OF DRS)



**FIGURE 19:** EXAMPLE 5—ORIGINAL RECORDED SEED: PSD CHECK (THE DASHED CURVE IS THE RAW PSD FUNCTION, WHILE THE THICK RED CURVE IS THE AVERAGED PSD FUNCTION USING A  $\pm 20\%$  SMOOTHING WINDOW.)



**FIGURE 20:** EXAMPLE 5—A RECORDED SEED LEADING TO POOR PSD CHECK RESULT. (THE DASHED CURVE IS THE RAW PSD FUNCTION, WHILE THE THICK RED CURVE IS THE AVERAGED PSD FUNCTION USING A  $\pm 20\%$  SMOOTHING WINDOW.)

## SUMMARY

This paper provides five examples, briefly summarized below, to demonstrate GWM's computational advantages that can be leveraged to facilitate the development of seismic design acceleration time histories. These examples have shown that GWM can achieve tight RS-matching criteria without significant changes to the time domain nonstationary

characteristics of the seed record and to its PSD function, either power deficient or power sufficient, as long as the original seed record has a spectral shape close to the DRS. This is achievable because GWM requires only a very small number of wavelets to achieve RS convergence and consequently introduces a minimal amount of change to the seed time histories.

**Example 1** shows a seismic acceleration time history developed using a frequency domain RS-matching method and

a seed record of 15 sinusoids. This time history includes power deficiencies, shown as frequency valleys on FAS. The application of GWM on this time history has further improved the RS convergence, while barely causing any change to the time domain nonstationary characteristics and essentially maintaining the frequency valleys.

**Example 2** shows a seismic acceleration time history developed to be a broadband time history using the frequency domain method. The application of GWM has further improved the RS convergence, while essentially maintaining the same characteristics of the time history both in the time domain and in the frequency domain.

**Example 3** uses the same broadband time history as in example 2 but with its frequency content between 5 Hz and 15 Hz reduced to 1% of the original values. The modified time history was used as the seed record, and GWM achieved RS convergence with 30 wavelets, caused minimal changes to the time domain nonstationary characteristics, and maintained the power deficiency in that frequency range (although more power was added).

**Example 4** shows an extreme seed consisting of only 12 wavelets. The application of GWM reached RS convergence with 61 wavelets and caused insignificant changes to the waveform and insignificant changes to the FAS in the dominant frequency range of 0.3 Hz to 33 Hz. A PSD check revealed that an RS-matched time history using GWM maintained power deficiency across the entire frequency range of interest, confirming the deficient waveform of essentially just 12 wavelets.

**Example 5** uses a recorded seismic acceleration time history as the initial seed, but with its FAS replaced using the target PSD function that is compatible with the DRS. The application of GWM on the FAS-replaced seed achieved RS convergence with 74 wavelets. A PSD check confirmed power sufficiency over the frequency range of interest.

Example 5 effectively demonstrates how the computational advantages of GWM can be leveraged to separate power assurance from RS matching during the development of synthetic seismic design acceleration time histories, and consequently to satisfy both RS and PSD criteria without the difficulties revealed in SRP section 3.7.1 [1]. Example 5 lays out a feasible and convenient approach: (1) Identify a seed record with a spectral shape close to the DRS. (2) Keep its FPS and replace its FAS using the target PSD function that can be developed to be compatible with the DRS. (3) As necessary, perform baseline correction and ensure zero values at both ends of the FAS-replaced time history. (4) Perform RS matching using GWM. (5) Perform a PSD check to confirm power sufficiency. The PSD function of the acceleration time history

developed using this approach should be adequate for most frequencies of interest. Minor issues in the very low frequencies and very high frequencies can be remedied in GWM by adding additional wavelets within the strong motion duration.

## REFERENCES

1. U.S. Nuclear Regulatory Commission (NRC) (2014), “Standard Review Plan for the Review of Safety Analysis Reports for Nuclear Power Plants: LWR Edition,” NUREG-0800, “Seismic Design Parameters,” Section 3.7.1, Revision 4, Washington, DC.
2. Nie, J.R., V. Graizer, and D. Seber (2023), “A greedy algorithm for wavelet-based time domain response spectrum matching,” *Nuclear Engineering and Design*, **410**(1123843). (Agencywide Documents Access and Management System Accession No. ML23139A173)
3. Nie, J.R., J. Xu, V. Graizer, and D. Seber (2023), “Uncertainties in In-Structure Response Spectra Due to Uncertainties in Input Motion Amplitude and Phase Spectra,” *American Society of Mechanical Engineers Pressure Vessels and Piping Conference* (PVP2023-105307), Atlanta, GA. (ML23095A243)
4. Atik, L., and N. Abrahamson (2010), “An improved method for nonstationary spectral matching,” *Earthquake Spectra*, **26**(3), 601–617.
5. Hancock, J., J. Watson-Lamprey, N.A. Abrahamson, J.J. Bommer, A. Markatis, E. McCoyh, and R. Mendis (2006), “An improved method of matching response spectra of recorded earthquake ground motion using wavelets,” *J. Earthq. Eng.*, **10**(s1), pp. 67–89.
6. Abrahamson, N.A. (1992), “Non-stationary spectral matching,” *Seismol. Res. Lett.*, **63** (1), p. 30
7. Lilhanand, K. and W.S. Tseng (1988), “Development and application of realistic earthquake time histories compatible with multiple-damping design spectra,” *Proc. of the Ninth World Conf. on Earthquake Engineering*, Tokyo, Japan.
8. Kaul, M.K. (1978). “Spectrum-consistent time-history generation,” *J. Eng. Mech. Div.*, **104**(4), pp. 781–788.
9. NRC, Research Information Letter RIL 2019-01, Revision 1 (2020), “Assessment of Artificial Acceleration Time History Guidance in Standard Review Plan Section 3.7.1, ‘Seismic Design Parameters,’” Washington, DC. (ML20239A900)
10. NRC, Regulatory Guide 1.60, Revision 2 (2014), “Design Response Spectra for Seismic Design of Nuclear Power Plants,” Washington, DC. (ML13210A432)
11. PEER NGA-West2 database:  
<https://ngawest2.berkeley.edu/>.