Sobol Sensitivity Indices for the BTD Model

Setup packages.

In [1]:
```
require(data.table)
require(magrittr)
require(rpart)
require(sensitivity)
require(SobolSequence)

require(ggplot2)
```

```
Loading required package: data.table
Loading required package: magrittr
Loading required package: rpart
Loading required package: sensitivity
Registered S3 method overwritten by 'sensitivity':
  method    from
  print.src dplyr
Loading required package: SobolSequence
Loading required package: ggplot2
```

Read files.

Read design.

In [2]:
```
z.design <- fread("design-si.tsv")
z.design %>% dim
```

2500 64

In [3]:
```
n <- dim(z.design)[1]
k <- dim(z.design)[2] / 2
```

Read inputs.

In [4]:
```
z.inputs <- fread("inputs-si.tsv")
z.inputs %>% dim
```

85000 33

In [5]:
```
z.inputs %>% colnames
```

'Run' 'aversion to NPV deviation' 'base external investor ask rate'
'bioproduct long term price' 'bioproduct performance advantage'
'commercial capital cost input' 'commercial plant capacity'
'commercial plant capacity input' 'commercial process yield input'
'commercial variable operating cost input' 'expected continuity of government policy'
'government capital cost share' 'government production incentive' 'initial market size'
'management response time' 'market growth rate' 'minimum runway'
'number of missed stagegates allowed' 'payback period multiplier'
'pilot and demo response time' 'pilot capacity' 'piloting acceptable rate'
'piloting failure default recovery time' 'piloting failure distribution max' 'random stream'
'required internal return' 'required return multiplier' 'researching impact on piloting'
'startup piloting period' 'startup piloting rate' 'strategic value to external investors'
'target demo hours' 'target pilot hours'

Read outputs.

In [6]:
```
z.outputs <- rbind(
    fread("outputs-si-direct-niche.tsv"     )[, `:=`(Replacement="Direc
t"    , Scale="Niche"    )],
    fread("outputs-si-direct-commodity.tsv" )[, `:=`(Replacement="Direc
t"    , Scale="Commodity")],
    fread("outputs-si-perfadv-niche.tsv"    )[, `:=`(Replacement="Advant
aged", Scale="Niche"    )],
    fread("outputs-si-perfadv-commodity.tsv")[, `:=`(Replacement="Advant
aged", Scale="Commodity")]
)[order(Replacement, Scale, Run, Time)]
z.outputs <- z.outputs[, c(52:53, 1:51)]
z.outputs %>% dim
```

2720000 53

Take a simpler subset.

- Since the four base cases give very similar results, just use one of them.
- For now, just look at 2050.
- Only treat the `Cumulative Production` output.

In [7]:
```
y0 <- z.outputs[
    Replacement == "Advantaged" & Scale == "Niche" & Time == 2050,
    `Cumulative Production`
]
y0 %>% length
```

85000

Split the input into x_A and X_B.

In [8]:
```
xa <- z.inputs[1:2500]
xa %>% dim
```

2500 33

In [9]:
```
xb <- z.inputs[1:2500 + 2500]
xb %>% dim
```

2500 33

Split the output into the responses to the y_A, y_B, and $y_{B_{A_i}}$ results.

In [10]:
```
ya <- y0[1:n]
ya %>% length
```

2500

In [11]:
```
yb <- y0[1:n + n]
yb %>% length
```

2500

In [12]:
```
ybia <- matrix(y0[-(1:(2*n))], nrow = n, byrow = FALSE)
ybia %>% dim
```

2500 32

Package everything into a data table.

In [13]:
```
xyab <- NULL
for (i in 1:32) {
    temp <- data.table(
            Input = colnames(z.inputs)[1+i],
            xA = as.matrix(xa[, i + 1, with=FALSE]),
            xB = as.matrix(xb[, i + 1, with=FALSE]),
            yA = ya,
            yB = yb,
            yBiA = ybia[, i]
        )
    colnames(temp) <- c("Input", "xA", "xB", "yA", "yB", "yBiA")
    scale <- sqrt(((temp$yA - temp$yB) %*% (temp$yA - temp$yB))[1])
    xyab <- rbind(
        xyab, cbind(
            Sample = 0,
            temp[, .(Input, xA, xB, yA = yA / scale, yB = yB / scale, yB
iA = yBiA / scale)]
        )
    )
    for (j in 1:25) {
        temp2 <- temp[sample(1:nrow(temp), nrow(temp), replace = TRUE)]
        scale <- sqrt(((temp2$yA - temp2$yB) %*% (temp2$yA - temp2$yB))
[1])
        xyab <- rbind(
            xyab,
            cbind(
                Sample = j,
                temp2[, .(Input, xA, xB, yA = yA / scale, yB = yB / scal
e, yBiA = yBiA / scale)]
            )
        )
    }
}
xyab %>% head
```

A data.table: 6 x 7

Sample	Input	xA	xB	yA	yB	yBiA
<dbl>	<chr>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>
0	aversion to NPV deviation	0.060	0.060	2.898561e-05	2.898561e-05	2.898561e-05
0	aversion to NPV deviation	1.360	1.360	0.000000e+00	0.000000e+00	0.000000e+00
0	aversion to NPV deviation	0.710	2.010	0.000000e+00	0.000000e+00	3.446320e-04
0	aversion to NPV deviation	2.010	0.710	0.000000e+00	0.000000e+00	0.000000e+00
0	aversion to NPV deviation	1.685	1.685	0.000000e+00	0.000000e+00	0.000000e+00
0	aversion to NPV deviation	0.385	0.385	0.000000e+00	0.000000e+00	0.000000e+00

Compute sensitivity indices.

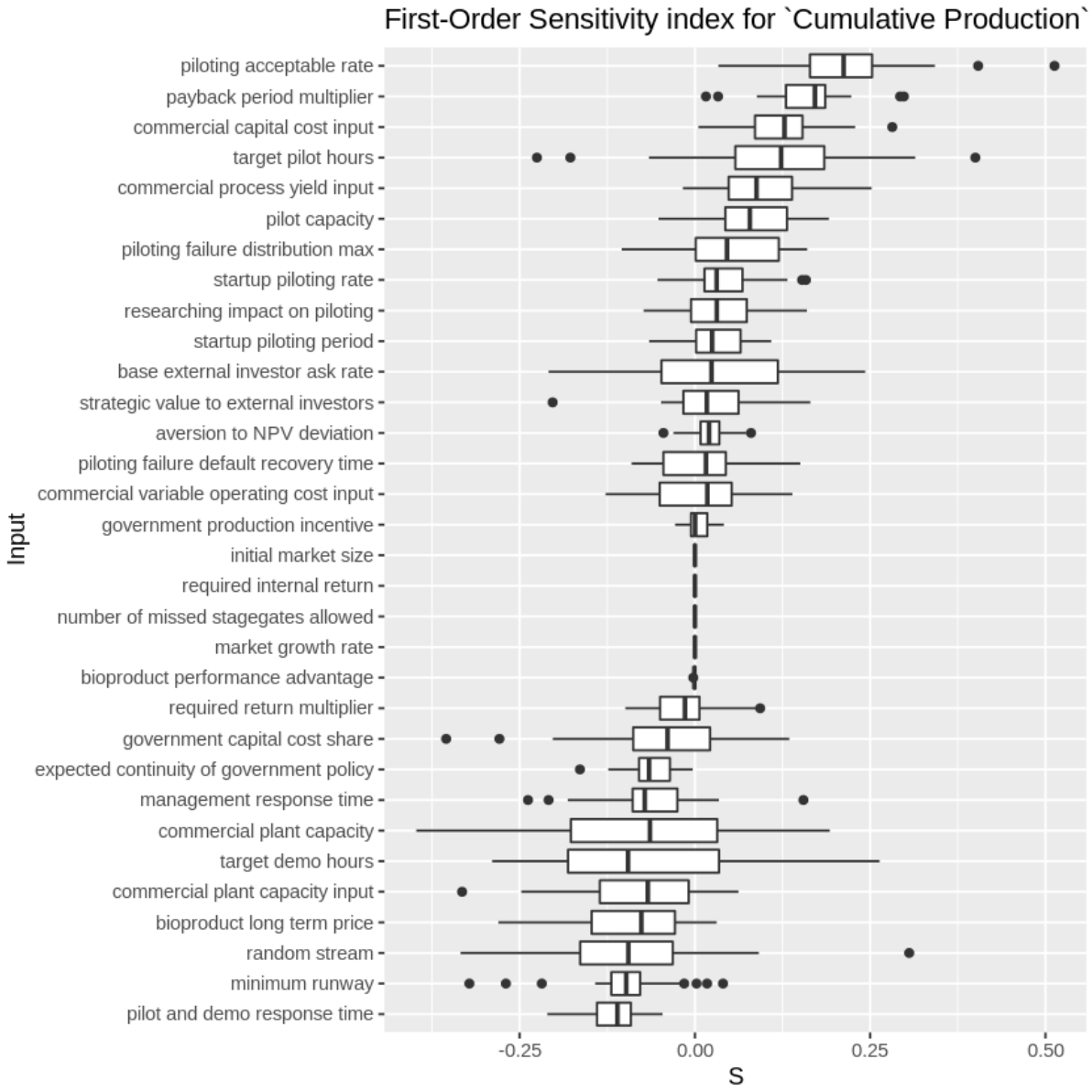
In [14]:
```
xyab.sensitivity <- xyab[, .(
    S = 1 - sum((yA - yBiA)^2),
    T =     sum((yB - yBiA)^2)
), by = .(Sample, Input)]
xyab.sensitivity[Sample == 0][order(-T)] %>% head
```

A data.table: 6 x 4

Sample	Input	S	T
<dbl>	<chr>	<dbl>	<dbl>
0	target pilot hours	0.11649125	0.8268736
0	random stream	-0.10268546	0.6641606
0	target demo hours	-0.09407777	0.5960527
0	base external investor ask rate	0.06918549	0.5463625
0	piloting acceptable rate	0.21598993	0.5281508
0	government capital cost share	-0.06742518	0.4382778

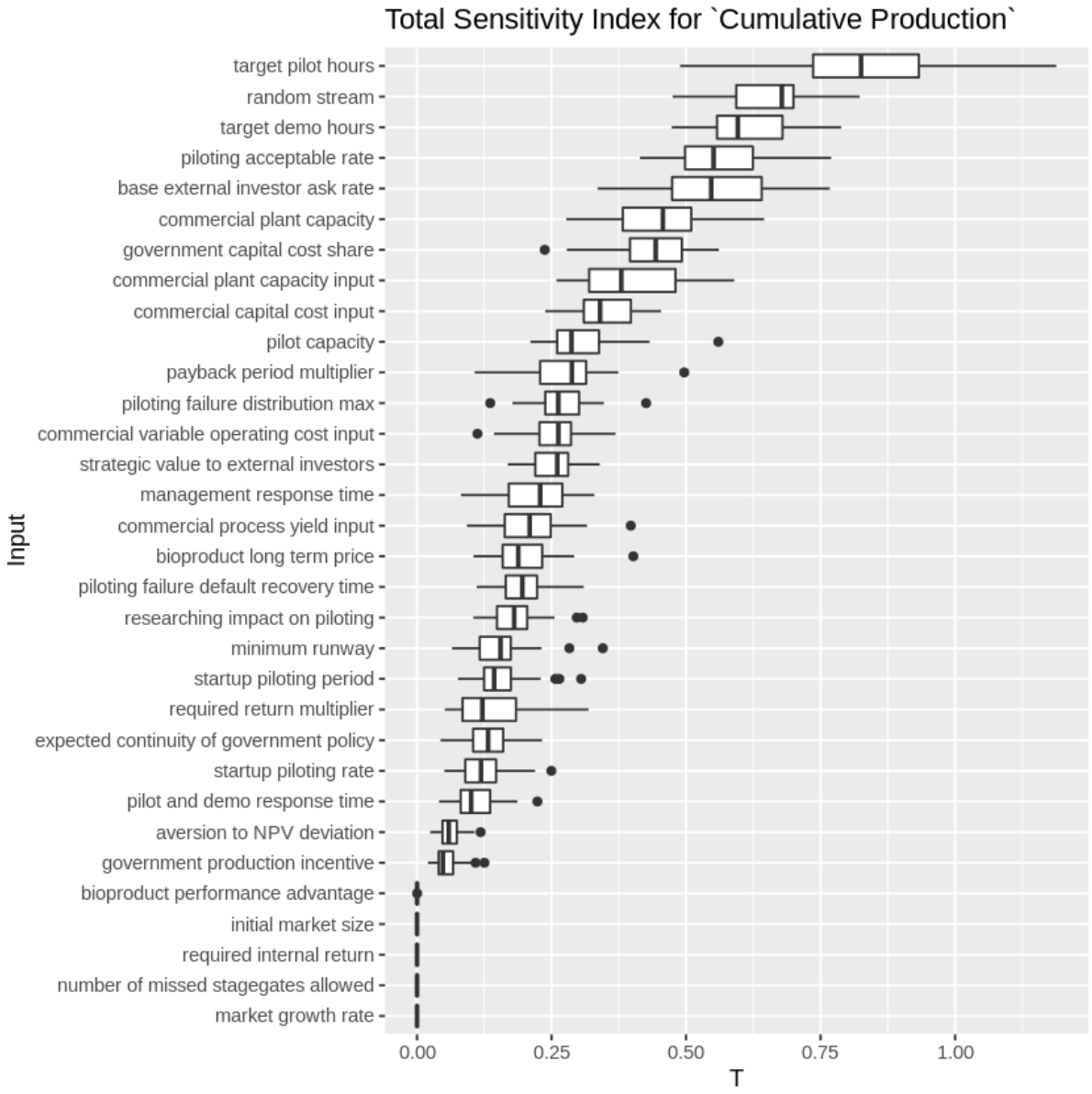
Plot first-order sensitivity index.

In [15]:
```
ggplot(xyab.sensitivity, aes(y = S, x = reorder(Input, S, FUN = mean)))
+
    geom_boxplot() +
    xlab("Input") +
    coord_flip() +
    ggtitle("First-Order Sensitivity index for `Cumulative Production`")
```



Plot total sensitivity index.

In [16]:
```
ggplot(xyab.sensitivity, aes(y = T, x = reorder(Input, T, FUN = mean)))
+
    geom_boxplot() +
    xlab("Input") +
    coord_flip() +
    ggtitle("Total Sensitivity Index for `Cumulative Production`")
```



Remember most sensitive variables.

In [17]:
```
xyab.best <- xyab.sensitivity[order(-T)][, unique(Input)][1:6]
xyab.best
```

'target pilot hours' 'random stream' 'target demo hours' 'piloting acceptable rate' 'base external investor ask rate' 'commercial plant capacity'

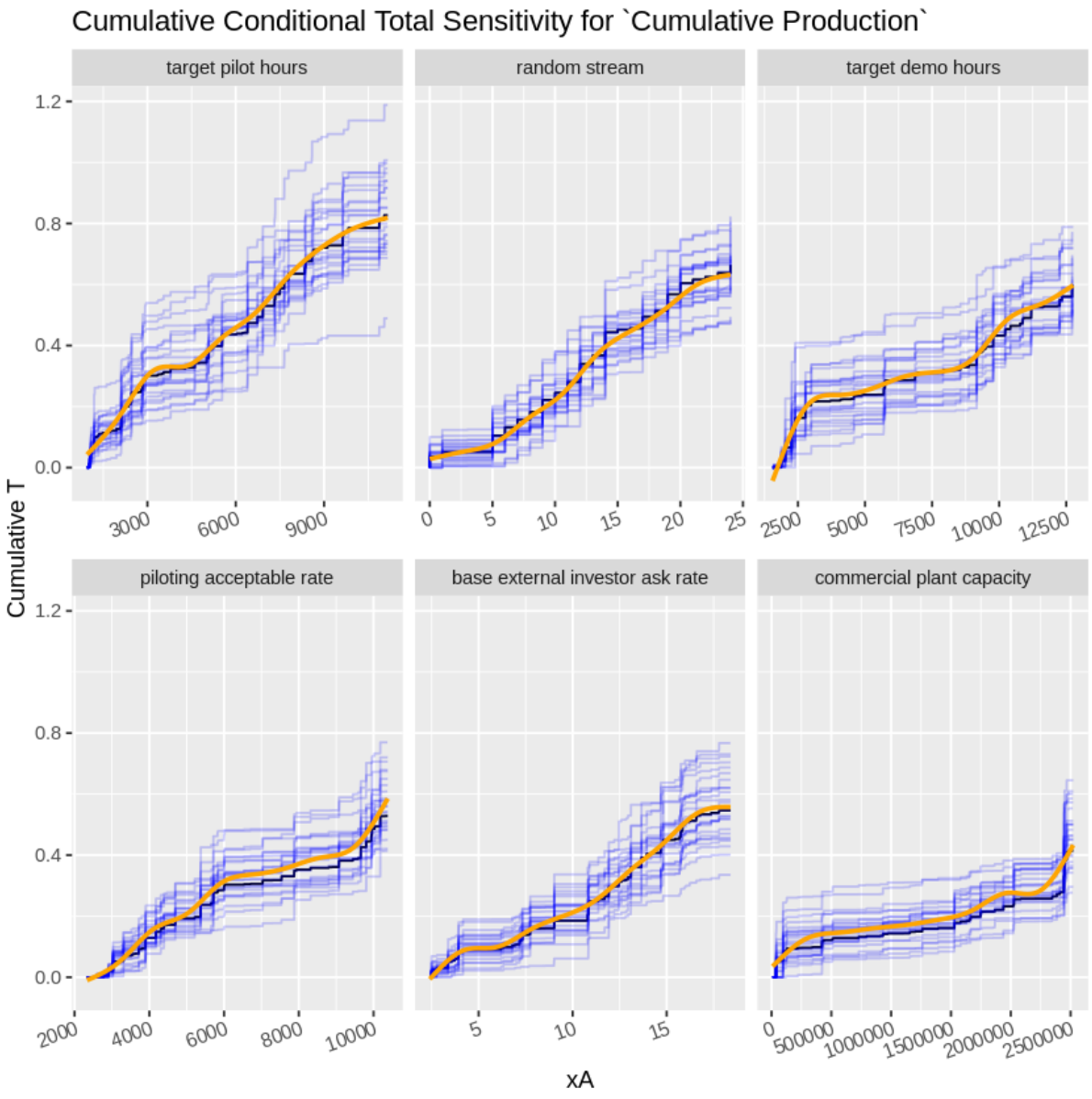
Sensitivity variances conditioned by input.

Plot the total sensitivity condition by the input values. Since the results are symmetric in `xA` vs `xB`, plot them both.

As a function of `xA`.

In [18]:

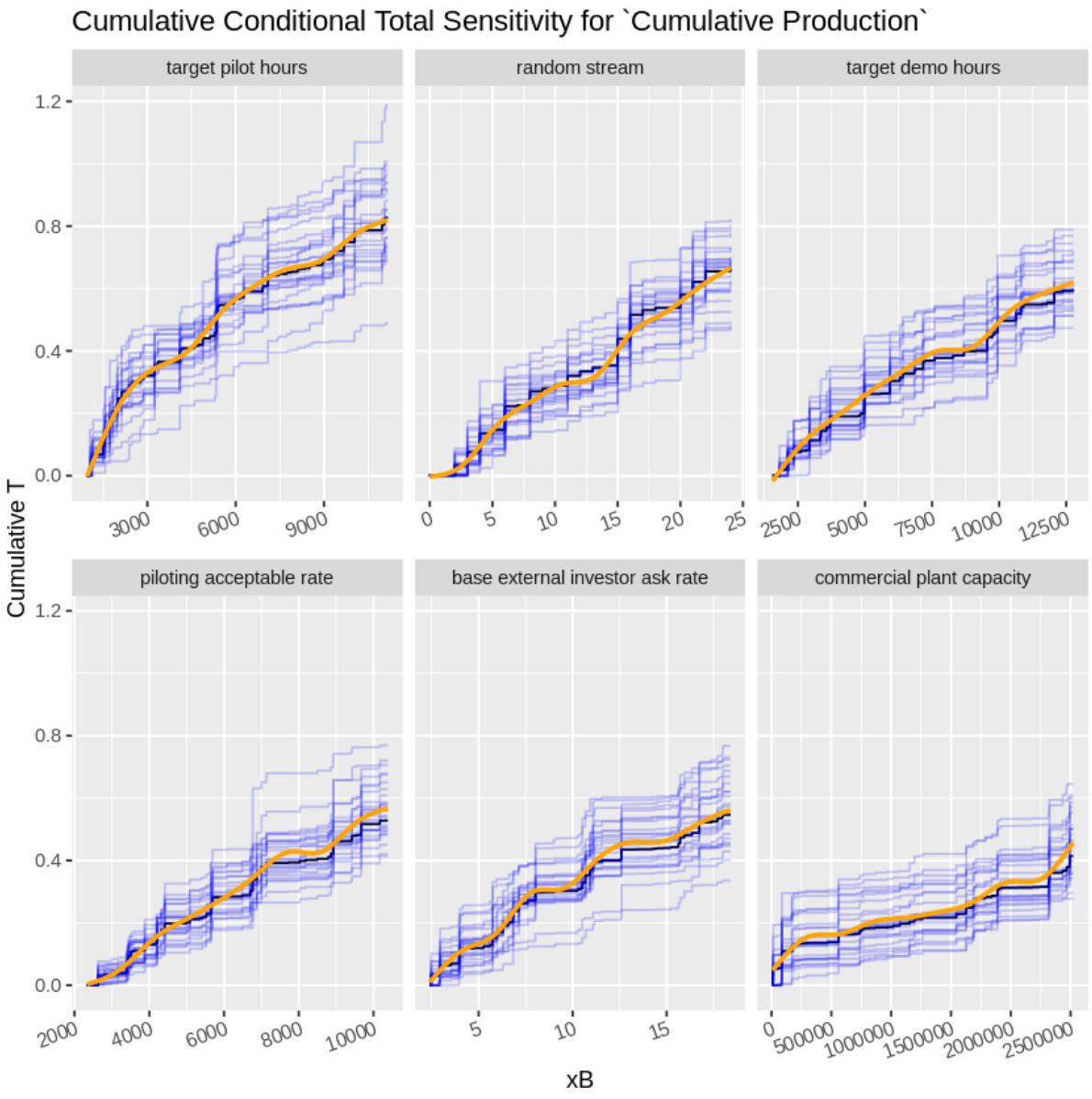
```
xyab.temp <- xyab[
        Input %in% xyab.best,
        .(Sample, Input=factor(Input, levels = xyab.best), xA, yB, yBiA)
    ][
        order(Input, Sample, xA)
    ][,
      .(xA, T = cumsum((yB - yBiA)^2)), by = .(Input, Sample)
     ]
ggplot(xyab.temp, aes(x = xA, y = T)) +
    geom_line(data = xyab.temp[Sample == 0], color = "black", aes(group
= Sample)) +
    geom_line(alpha = 0.2, color = "blue", aes(group = Sample)) +
    geom_smooth(color = "orange", method = "gam", formula = y ~ s(x, bs
= "cs")) +
    facet_wrap(Input ~ . , scales = "free_x") +
    theme(axis.text.x = element_text(angle = 20, hjust=1)) +
    ylab("Cumulative T") +
    ggtitle("Cumulative Conditional Total Sensitivity for `Cumulative Pr
oduction`")
```



As a function of `xB`.

In [19]:

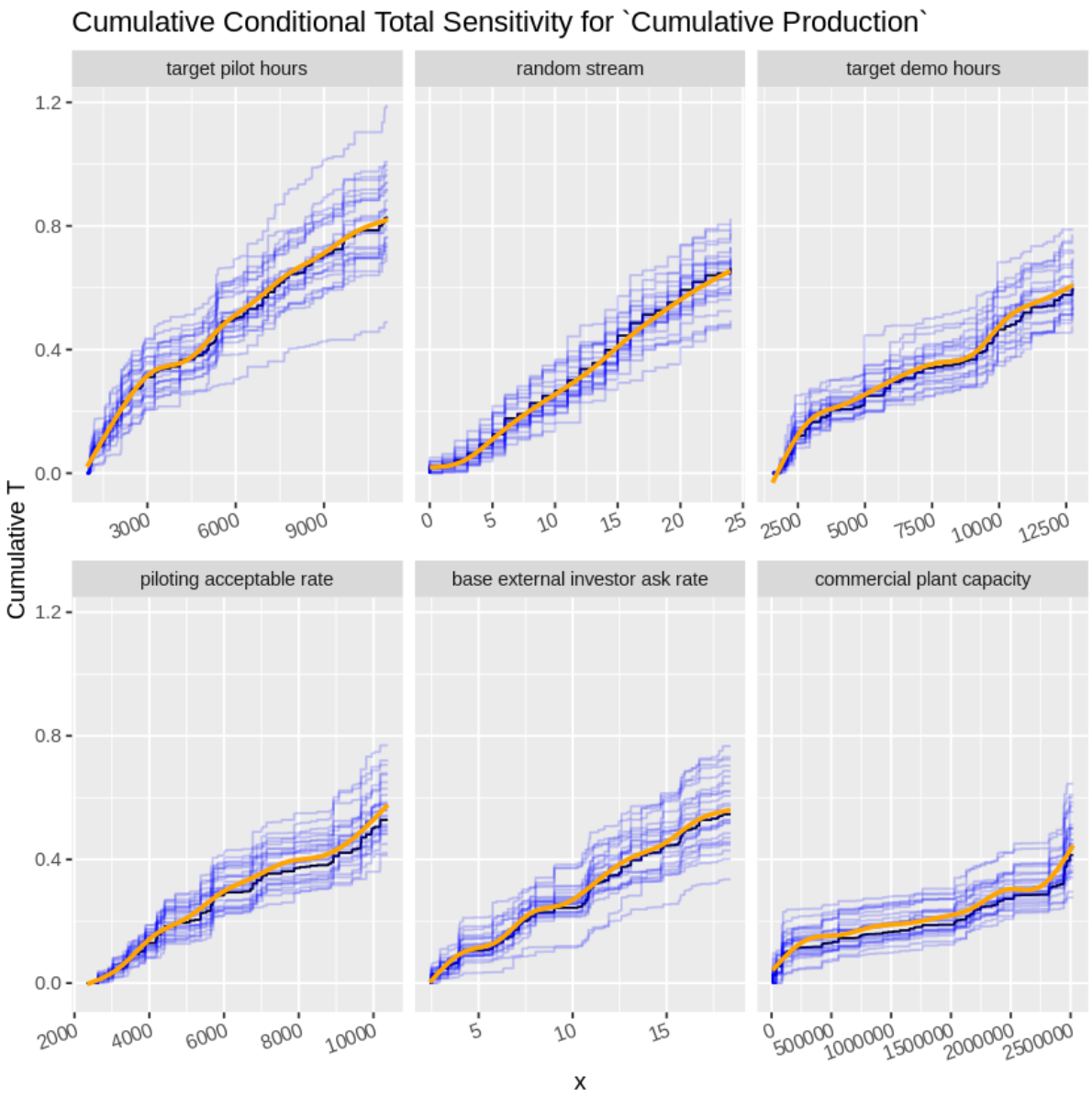
```
xyab.temp <- xyab[
        Input %in% xyab.best,
        .(Sample, Input=factor(Input, levels = xyab.best), xB, yB, yBiA)
    ][
        order(Input, Sample, xB)
    ][,
      .(xB, T = cumsum((yB - yBiA)^2)), by = .(Input, Sample)
     ]
ggplot(xyab.temp, aes(x = xB, y = T)) +
    geom_line(data = xyab.temp[Sample == 0], color = "black", aes(group
= Sample)) +
    geom_line(alpha = 0.2, color = "blue", aes(group = Sample)) +
    geom_smooth(color = "orange", method = "gam", formula = y ~ s(x, bs
= "cs")) +
    facet_wrap(Input ~ . , scales = "free_x") +
    theme(axis.text.x = element_text(angle = 20, hjust=1)) +
    ylab("Cumulative T") +
    ggtitle("Cumulative Conditional Total Sensitivity for `Cumulative Pr
oduction`")
```



With `xA` and `xB` interleaved.

In [20]:

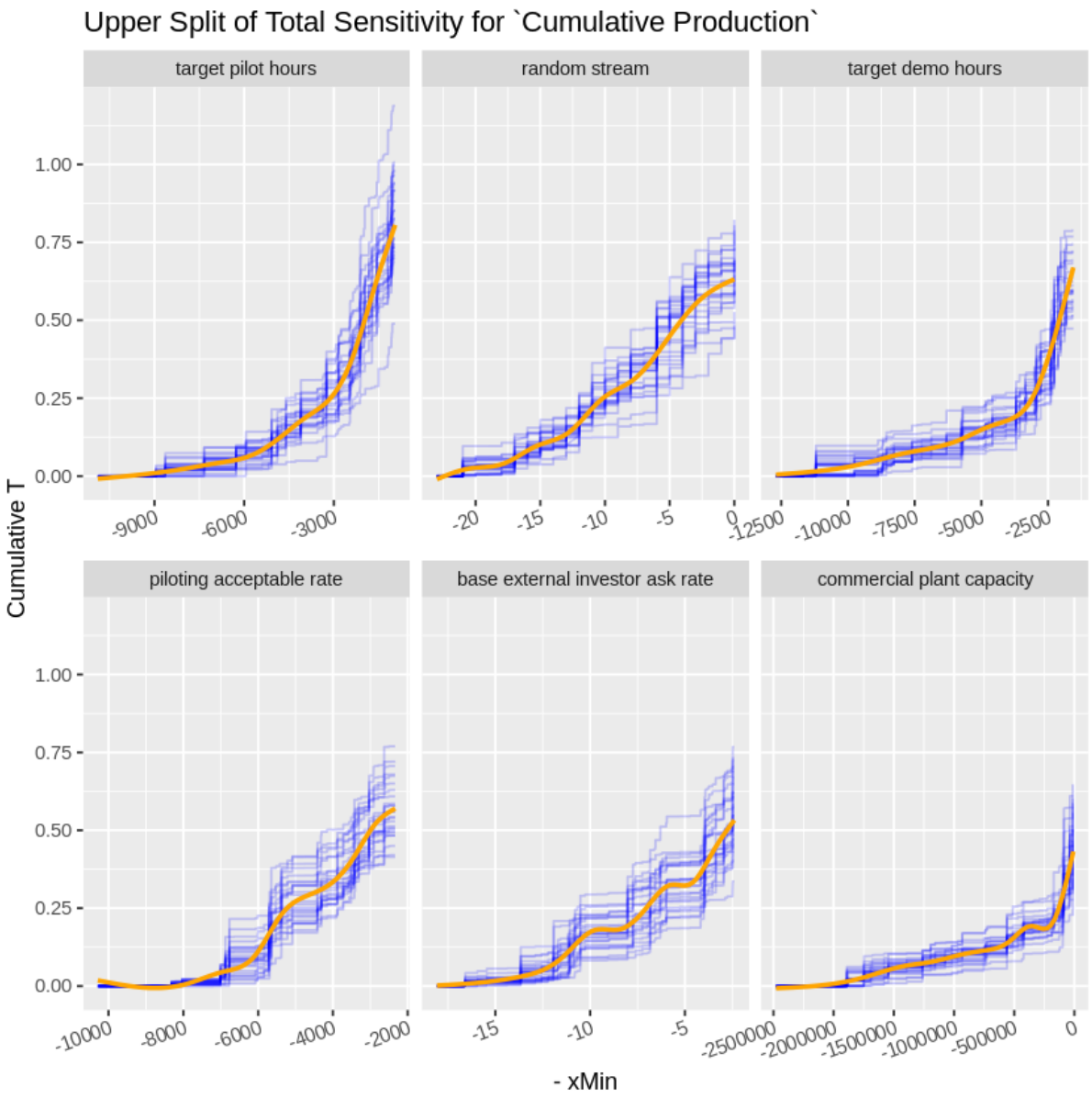
```
xyab.temp <- rbind(xyab[, .(Input, Sample, x = xA, yB, yBiA)], xyab[, .
(Input, Sample, x = xB, yB, yBiA)])[
        Input %in% xyab.best,
        .(Sample, Input=factor(Input, levels = xyab.best), x, yB = yB /
sqrt(2), yBiA = yBiA / sqrt(2))
    ][
        order(Input, Sample, x)
    ][,
      .(x, T = cumsum((yB - yBiA)^2)), by = .(Input, Sample)
     ]
ggplot(xyab.temp, aes(x = x, y = T)) +
    geom_line(data = xyab.temp[Sample == 0], color = "black", aes(group
= Sample)) +
    geom_line(alpha = 0.2, color = "blue", aes(group = Sample)) +
    geom_smooth(color = "orange", method = "gam", formula = y ~ s(x, bs
= "cs")) +
    facet_wrap(Input ~ . , scales = "free_x") +
    theme(axis.text.x = element_text(angle = 20, hjust=1)) +
    ylab("Cumulative T") +
    ggtitle("Cumulative Conditional Total Sensitivity for `Cumulative Pr
oduction`")
```



Candidate locations for splitting into regimes with different sensitvities.

In [21]:

```
ggplot(
    xyab[
        Input %in% xyab.best,
        .(Sample, Input=factor(Input, levels = xyab.best), xMin=-mapply(
min, xA, xB), yB, yBiA)
    ][
        order(Input, Sample, xMin)
    ][,
      .(xMin, T = cumsum((yB - yBiA)^2)), by = .(Input, Sample)
    ][,
     ],
        aes(x = xMin, y = T)
    ) +
    geom_line(alpha = 0.2, color = "blue", aes(group = Sample)) +
    geom_smooth(color = "orange", method = "gam", formula = y ~ s(x, bs
= "cs")) +
    facet_wrap(Input ~ . , scales = "free_x") +
    theme(axis.text.x = element_text(angle = 20, hjust=1)) +
#   scale_x_reverse() +
    xlab("- xMin") +
    ylab("Cumulative T") +
    ggtitle("Upper Split of Total Sensitivity for `Cumulative Production
`")
```


Upper Split of Total Sensitivity for `Cumulative Production`
target pilot hours
random stream
target demo hours
piloting acceptable rate
base external investor ask rate
commercial plant capacity
Cumulative T
1.00
0.75
0.50
0.25
0.00
-9000
-6000
-3000
-20
-15
-10
-5
0
-12500
-10000
-7500
-5000
-2500
-10000
-8000
-6000
-4000
-2000
-15
-10
-5
-2500000
-2000000
-1500000
-1000000
-500000
0
- xMin

In [22]:

```
ggplot(
    xyab[
        Input %in% xyab.best,
        .(Sample, Input=factor(Input, levels = xyab.best), xMax=mapply(max, xA, xB), yB, yBiA)
    ][
        order(Input, Sample, xMax)
    ][,
      .(xMax, T = cumsum((yB - yBiA)^2)), by = .(Input, Sample)
     ],
        aes(x = xMax, y = T)
    ) +
    geom_line(alpha = 0.2, color = "blue", aes(group = Sample)) +
    geom_smooth(color = "orange", method = "gam", formula = y ~ s(x, bs = "cs")) +
    facet_wrap(Input ~ . , scales = "free_x") +
    theme(axis.text.x = element_text(angle = 20, hjust=1)) +
    ylab("Cumulative T") +
    ggtitle("Lower Split of Total Sensitivity for `Cumulative Production`")
```