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Velocity and Acceleration Estimation for Optical Incremental Encoders

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Abstract: Optical incremental encoders are extensively used for position measurements in motion systems. The position measurements suffer from quantization errors. Velocity and acceleration estimations obtained by numerical differentiation largely amplify the quantization errors. In this paper, the time stamping concept is used to obtain more accurate position, velocity and acceleration estimations. Time stamping makes use of stored events, consisting of the encoder counts and their time instants, captured at a high resolution clock. Encoder imperfections and the limited resolution of the capturing rate of the encoder events result in errors in the estimations. In this paper, we propose a method to extend the observation interval of the stored encoder events using a skip operation. Experiments on a motion system show that the velocity estimation is improved by 54% and the acceleration estimation by 92%.

Keywords: Encoders; data acquisition; sensors; angular acceleration; angular velocity; angular position; quantization.

1. INTRODUCTION

Optical incremental encoders are widely used to apply feedback control on motion systems where the position is measured at a fixed sampling frequency. They are available in both rotational and linear form. The position accuracy is limited by the quantized position measurement of the encoder, i.e. it is limited by the number of slits on the encoder disk.

The quantization errors can be reduced using more expensive encoders with more increments at the expense of increased cost price. Velocity and acceleration information is often obtained by numerical differentiation of the quantized position signal. Direct differentiation mostly leads to signals that are not useful [Kadhim et al., 1992]. The quantization errors limit the performance in high accuracy control applications. Smart signal processing techniques can be used in combination with cheap low resolution encoders to obtain position estimations with the same accuracy as expensive high resolution encoders.

In literature, several methods have been proposed to improve the accuracy of velocity and acceleration estimations for optical incremental encoders. The methods can be divided into two kinds; fixed-time (clock-driven) methods and fixed-position (encoder-driven) methods.

For real-time control purposes, a fixed-time method is desired since the controller is generally evaluated at fixed-time intervals. Fixed-time velocity and acceleration estimations can be obtained using three different approaches; predictive postfiltering techniques, linear state observers and indirect measurement techniques.

Predictive postfiltering techniques perform a filtering on differentiated position signals. Euler based methods [Liu, 2002] and polynomial delayless predictive differentiators [Väliviita and Vainio, 1999] both disregard the variable rate of occurrence of the encoder events to estimate the velocity or acceleration. The transition based logic algorithm of [Lemkin et al., 1995] estimates only the velocity under the assumption that the sampling frequency is much larger than the rate of the encoder events.

Linear state observer techniques use the encoder position measurements, without the need for differentiation. Dual-sampling rate observers [Kovudhikulrungsri and Koseki, 2006] and Kalman filters [Bélanger, 1992] require accurate system models to be available. The non-model based observer of Tilli and Montanari [2001] switches between two estimation filters based on an estimation error, which is generally not available. Data based observers using neural networks [Chan, 1998] or fuzzy logic [Yusivar et al., 1999] estimate the velocity using only the position information, thus disregarding the non-constant time occurrence of the encoder events.

Indirect measurement techniques are based on analog or digital postprocessing of available position and/or velocity signals. Saito et al. [1988] estimate the velocity by a polynomial fitting through a number of encoder counts. No time information of the encoder events is taken into account and no acceleration information is obtained. Brown and Schneider [1987] use both encoder counts and their time instants to estimate the velocity. This is called the time stamping concept. However, simulations only are performed and at a practically unrealistic sampling frequency of 1 MHz. Furthermore, all encoder events are taken into account, which is not practically applicable.

Since for control purposes fixed-time methods are desired and since the encoder events have a fixed-position nature, occurring at a varying rate in time, a combination of the two approaches would be favorable. Therefore, in this paper, we propose a method to accurately estimate the velocity as well as the acceleration using the time stamping method of [Brown and Schneider, 1987]. The time stamping concept was used for position estimation in combination with the calibration and compensation of encoder errors in Merry et al. [2007]. The time stamping concept consists of capturing encoder events, consisting of the encoder transitions and their time instants, in hardware. Polynomial interpolation through the encoder events and extrapolation make it possible to estimate the encoder position, velocity and acceleration.

Encoder imperfections distort the velocity and acceleration estimations obtained using time stamping. A high resolution reference signal for the calibration of encoder errors is not always available. Furthermore, the number of events to be taken into account for the interpolation is limited during real-time experiments. For the estimation of both accurate velocity and acceleration signals, we propose a skip option for the event selection as an extension of the time stamping concept of Merry et al. [2007]. The skip option makes it possible to adjust the time span covered by the limited number of events without the need for additional events. The proposed method is an indirect measurement approach which is fully data-based, so no model of the system is required. Note that the required model for estimation of the position, velocity and acceleration is different from the model required for feedback controller design, e.g. the model required for estimation purposes should include friction, which is difficult to model accurately.

Experiments on a motion system show the applicability of the proposed method to obtain more accurate velocity and acceleration signals. Since the skip option and the capturing of the encoder events are implemented in hardware, the estimation of the signals is of the fixed-time kind. Therefore, the proposed method is applicable in real-time closed-loop experiments.

This paper is organized as follows. The time stamping concept is explained in more detail in Section 2. The position reconstruction method is briefly addressed in Section 3. In Section 4, the principle of skip in time stamping is introduced and its effects on the position reconstruction are described. The influence of skip is shown in Section 5 by means of experiments on a motion system. Finally, conclusions are drawn in Section 6.

2. TIME STAMPING CONCEPT

In most motion control applications the position is measured by reading out the quadrature encoder counter value at the fixed sampling rate T_c of the controller. This introduces even for ideal encoders a quantization error in the position measurement of maximally half an encoder count. The quantized signal contains the encoder counter value at the sample times of the controller t_c, as can be seen in Fig. 1.

A possibility for increasing the accuracy of the position information with the same resolution encoders is using the concept of time stamping [Brown and Schneider, 1987].

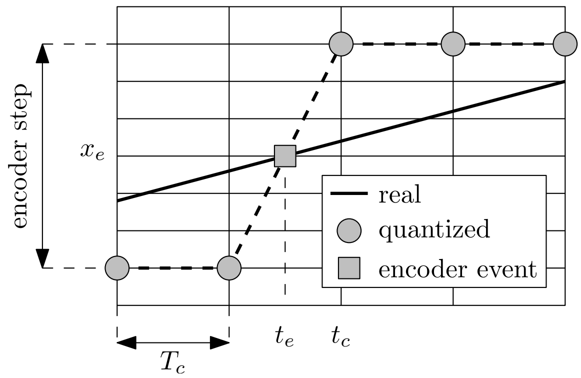

Fig. 1. Time stamping concept; the encoder transition x_e and time t_e are captured and stored as an event

The time stamping concept stores the time instants t_e of a number of encoder transitions together with their position x_e. The pair (t_e, x_e) is called an encoder event. The encoder events are captured by a high resolution clock at a sampling rate $T_e << T_c$.

3. POSITION RECONSTRUCTION

The use of encoder events for feedback control is not trivial since the encoder events are obtained at a variable rate proportional to the instantaneous velocity of the system during the measurement. To obtain a position estimation at the equidistant sampling times of the controller, a polynomial is fitted through a number of past encoder events. This polynomial is extrapolated to the desired time instant of the controller. Velocity and acceleration estimations are obtained by differentiation of the fitted polynomial with respect to time and extrapolation to the fixed sampling times of the controller.

For the position, velocity and acceleration estimations, a low order polynomial is fitted through a number of encoder events by the least squares method. Let n be the number of encoder events used in the fit, m the order of the fit, and k the index number of the events. Furthermore, let $p_{0,\ldots,m}$ be the polynomial coefficients to be estimated, $t_{1,\ldots,n}$ the time information of the encoder events, and $x_{1,\ldots,n}$ the position information of the encoder events. Now define the matrices $A \in \mathbb{R}^{n\times m+1}$, $P \in \mathbb{R}^{m+1}$, and $B \in \mathbb{R}^{n}$ as

$$A = \begin{bmatrix} t_{k-n+1}^{m} & t_{k-n+1}^{m-1} & \cdots & 1 \\ \vdots & \vdots & & \vdots \\ t_{k-1}^{m} & t_{k-1}^{m-1} & \cdots & 1 \\ t_{k}^{m} & t_{k}^{m-1} & \cdots & 1 \end{bmatrix}, \tag{1}$$

$$P = \begin{bmatrix} p_m & p_{m-1} & \cdots & p_0 \end{bmatrix}^T, \tag{2}$$

$$B = \begin{bmatrix} x_{k-n+1} & \cdots & x_{k-1} & x_k \end{bmatrix}^T. \tag{3}$$

To prevent numerical problems with the higher order terms in (1), the time variable t can be redefined to be zero every controller sample time t_c, i.e. $t := t - t_e$.

If $n = m$, an exact fit is made through the events. For the least squares method $n > m$. The over-determined system of linear equations to be solved for the polynomial fit equals

$$AP = B.$$

The least squares method for calculating the polynomial coefficients can be formalized as

$$A^TAP = A^TB.$$

The polynomial coefficients are now obtained by

$$P = (A^TA)^{-1}A^TB. \tag{4}$$

The polynomial coefficients P of (4) have to be calculated in real-time. For this purpose, LU-factorization without pivoting is used. Since the position, the velocity and the acceleration estimations are required at the sampling times of the controller, the fitted polynomial with coefficients P is extrapolated to the desired time instant t_c. The extrapolation of the polynomial to the time instant t_c results in an estimated position $\hat{x}$, estimated velocity $\hat{\dot{x}}$, and estimated acceleration $\hat{\ddot{x}}$ as

$$\begin{aligned}\hat{x}(t)|_{t=t_c} &= p_m t_c^m + p_{m-1}t_c^{m-1} + \ldots + p_0,\\ \hat{\dot{x}}(t) &= \dot{\hat{x}}(t),\\ \hat{\ddot{x}}(t) &= \ddot{\hat{x}}(t).\end{aligned}$$

4. SKIP OPTION

The encoder events (t_e, x_e) suffer from errors due to encoder imperfections, such as a non-uniform slit distribution, misplacement of the sensor photodiodes, eccentricity of the encoder disc, etc. The encoder imperfections introduce an error between the real and observed encoder event. The time instant t_e of the encoder event can have an error of maximum T_e due to the limited resolution T_e of the high resolution clock.

For real-time experiments, n events are used in the polynomial fit. The errors in the encoder events act as disturbances on the position information. These disturbances are amplified in the velocity estimation and even more in the acceleration estimation. A possible solution would be to increase the number of events. However, in most hardware, the number of available events is limited. In this section, a skip option is proposed to extend the time span covered by the n events in the fit without the need for more events.

4.1 Skip

The skip option makes it possible to skip a fixed number of events in between two stored events. In Fig. 2, the skip option is shown graphically for a skip number of $\sigma = 2$ counts. The real signal and the quantized signal are shown by the solid and dashed line respectively. The arrows show the events to be discarded since the last stored event. The stored events are shown by the dark grey circles. The light grey circles are the discarded events. In between two dark circles always $\sigma = 2$ events are skipped.

The skip option performs a low-pass filtering on the encoder events with a spatial cut-off frequency that is dependent on the momentary velocity.

For a skip factor of σ (counts), the index numbers of the events to be stored and used for the polynomial fit can be calculated using

$$k_\sigma = 1 + i(\sigma + 1), i \in \mathbb{Z}^+, \tag{5}$$

where $\mathbb{Z}^+$ is the set of nonnegative natural numbers including zero.

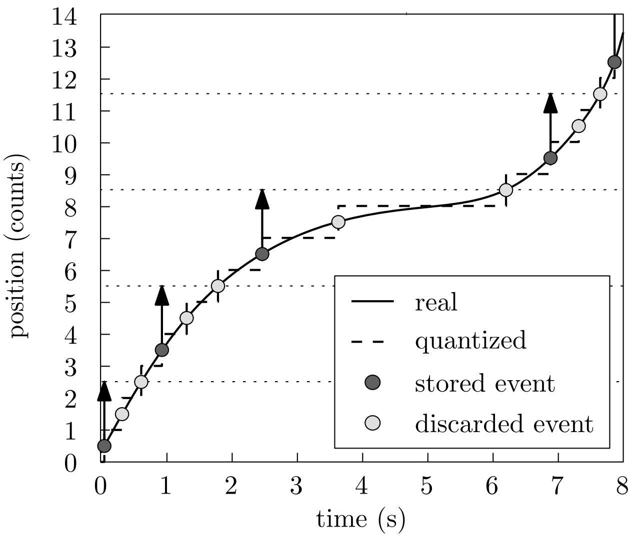

Fig. 2. Visualization of the skip option in time stamping for $\sigma = 2$ counts.

4.2 Position reconstruction

The events to be used for the position reconstruction with skip are determined by (5). In most control applications, the controller is sampled at a fixed sample interval. With skip it can occur that the last event before a controller interrupt is discarded. However, the last encoder event before a controller interrupt is the most recent measurement. Therefore, the last encoder event is always stored in the set of adjusted events. The set of elements $K \in \mathbb{R}^n$ to be stored equals

$$K = \begin{cases} [k \quad k_\sigma(N) \quad k_\sigma(N-1) \quad \cdots \quad k_\sigma(N-n+2)]^T, \\ \qquad\qquad\qquad \text{if } k \neq k_\sigma(N) \\ [k_\sigma(N) \quad k_\sigma(N-1) \quad \cdots \quad k_\sigma(N-n+1)]^T, \\ \qquad\qquad\qquad \text{if } k = k_\sigma(N), \end{cases}$$

where N is the last element of the vector with stored time stamps with skip. For the polynomial fit of (4), the matrices A and B equal with skip

$$A_\sigma = \left[t_K^m \; t_K^{m-1} \; \cdots \; 1\right], \quad B_\sigma = [x_K].$$

5. EXPERIMENTAL RESULTS

In this section, the results of the application of the time stamping concept to a motion system are discussed. Experiments are performed for different skip values σ and for sinusoidal reference profiles.

The experimental setup consists of the mechanical setup, an amplifier, a TUeDACs Microgiant data acquisition device [Griendt and Nijmweegen, 2006] and a computer, as shown in the block diagram of Fig. 3.

The mechanical setup, shown in Fig. 4, consists of a DC motor, which is connected to a rotating mass. On the DC motor, a HEDS-5540 encoder [HED, 2004] with a resolution of 100 slits/revolution ($1.5707 \cdot 10^{-2}$ rad/slit) is mounted. On the opposite site, a Heidenhain ROD-426 encoder with 5000 slits/revolution ($3.1416 \cdot 10^{-4}$ rad/slit) is connected to the rotating mass. The output of the Heidenhain encoder will be used as a reference to determine the improvement of the time stamping concept.

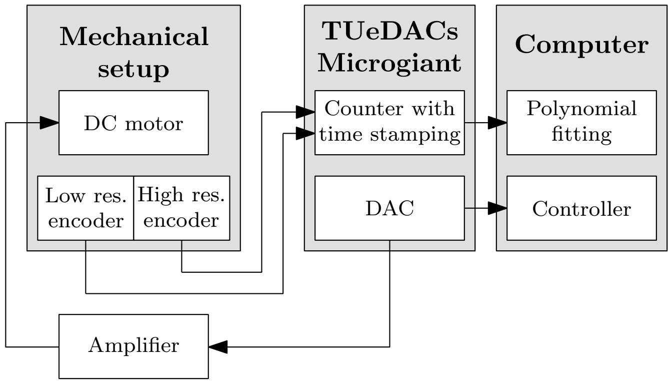

Fig. 3. Block diagram of the experimental setup.

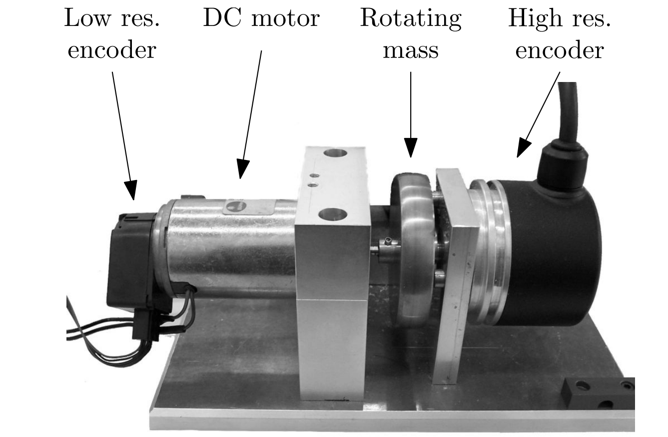

Fig. 4. The mechanical setup.

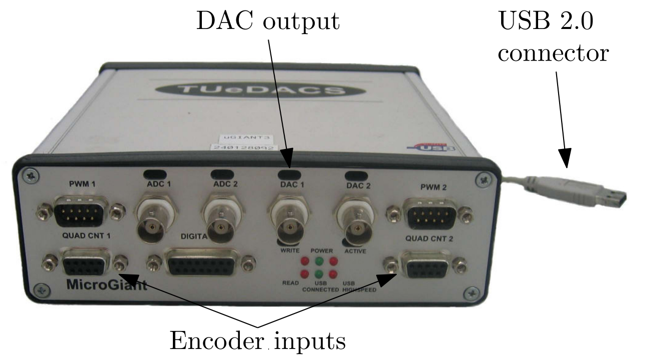

Fig. 5. The TUeDACs Microgiant data acquisition device.

The data acquisition device, shown in Fig. 5, is equipped with 32 bit quadrature counters with a maximum count frequency of 10 MHz and can capture encoder events with a time resolution of 50 ns [Griendt and Nijmweegen, 2006]. Up to five encoder events can be stored in a register. Furthermore, the Microgiant is equipped with a DAC output, which is used to drive the mechanical setup. The selection and storage of the encoder events when using skip is also performed in the Microgiant.

A fully preemptive Linux kernel hosts the real-time application at a fixed sampling rate of 1 kHz. The computer reads the time stamping registers of the Microgiant for the polynomial fitting and generates the control signal to the system in order to track a reference profile.

To compare the results for different skip values σ, the system must follow a known reference profile. Therefore, the system is feedback controlled. The time stamping concept is applied with a second order polynomial fit ($m = 2$) though five encoder events ($n = 5$). Experiments are performed without skip and with skip values $\sigma \in \{1, 2, 3, 4, 5, 10, 20\}$.

In order to evaluate the estimation accuracy, reference signals are made by off-line anti-causal filtering of the high resolution position measurement $x_{enc,hr}$ by a fifth order low-pass filter $L(s)$ with cut-off frequency at 50 Hz

$$X_{ref}(s) = L(s)X_{enc,hr}(s),$$
$$V_{ref}(s) = sX_{ref}(s),$$
$$A_{ref}(s) = sV_{ref}(s),$$

and

$$x_{ref} = \mathcal{L}^{-1}(X_{ref}(s)),$$
$$\dot{x}_{ref} = \mathcal{L}^{-1}(V_{ref}(s)),$$
$$\ddot{x}_{ref} = \mathcal{L}^{-1}(A_{ref}(s)),$$

where $\mathcal{L}$ denotes the Laplace operator. The estimation errors are defined as

$$e_x = x_{ref} - \hat{x}, \tag{6}$$
$$e_v = \dot{x}_{ref} - \hat{\dot{x}}, \tag{7}$$
$$e_a = \ddot{x}_{ref} - \hat{\ddot{x}}. \tag{8}$$

The skip option performs a time-independent spatial filtering on the encoder events. Therefore, for constant velocity setpoints the smallest errors are obtained for maximum skip values. The maximum skip value results in an observation window with the largest position history and thus maximally reduces the effect of the generally high frequent event errors.

Experiments are performed with sinusoidal reference signals $r(t) = A\sin(2\pi ft)$. The influence of varying amplitudes on the estimations is investigated. A changing frequency does not affect the optimal skip option since a changing frequency only affects the time properties of the signal and not the amplitude. The position estimations of time stamping with $\sigma = 0$ and $\sigma = 3$ are shown in Fig. 6 for $f = 1$ Hz and $A = \pi/2$ rad. The largest errors occur at the maxima of the reference signal, where the velocity equals zero and the time in between events is large.

The measured and the estimated velocities without skip and with $\sigma = 3$ are depicted in Fig. 7. The velocity obtained by differentiation of the quantized position (grey dotted) is clearly not useful for control purposes. With $\sigma = 3$ counts, the estimated velocity (black) results in a lower error than with $\sigma = 0$ counts (dark grey). The momentary oscillations caused by the event errors are filtered out by the skip option. The estimation error with $\sigma = 3$ is 54% more accurate than without skip ($\sigma = 0$).

The measured and estimated acceleration signals are shown in Fig. 8. The quantized acceleration obtained by two times differentiation of the quantized position is completely useless. The acceleration obtained with time stamping and $\sigma = 0$ counts shows large bursts and is not useful for control purposes. With a skip of $\sigma = 3$ counts, an acceleration signal that is useful for control purposes is estimated. The acceleration estimation with $\sigma = 3$ is 92% more accurate than with $\sigma = 0$ counts.

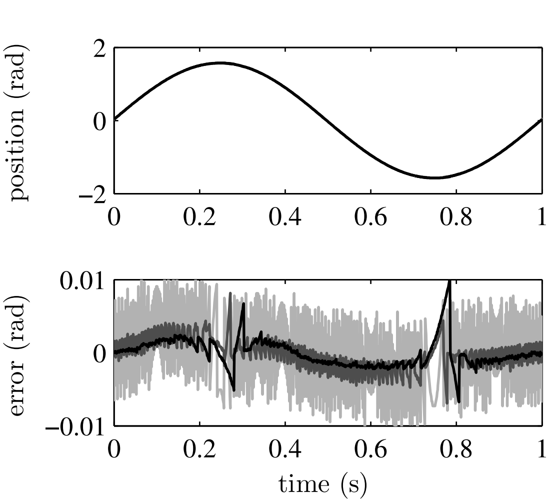

Fig. 6. Position signals, reference (dashed), quantized measurement (light grey), estimated with $\sigma = 0$ counts (dark grey) and with $\sigma = 3$ counts (black).

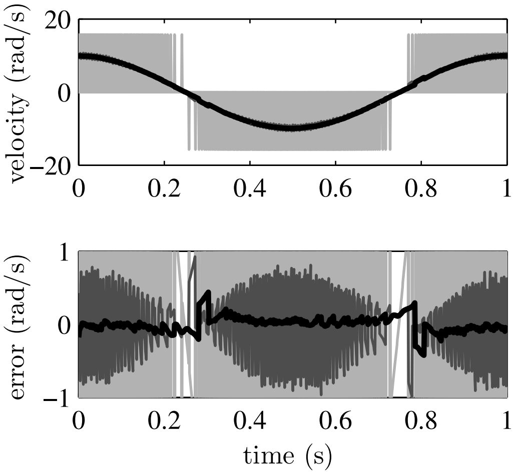

Fig. 7. Velocity signals, reference (dashed), quantized measurement (light grey), estimated with $\sigma = 0$ counts (dark grey) and with $\sigma = 3$ counts (black).

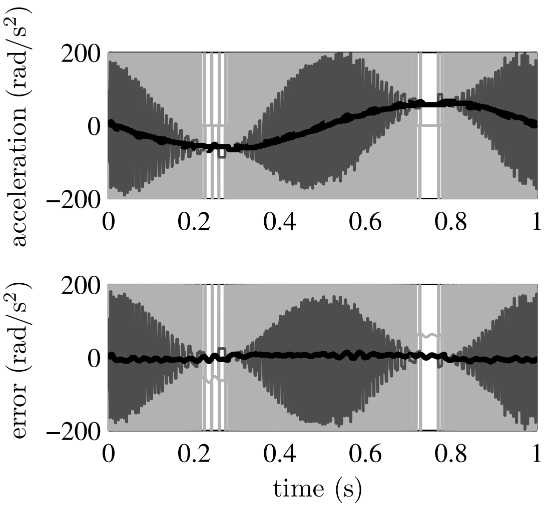

Fig. 8. Acceleration signals, reference (dashed), quantized measurement (light grey), estimated with $\sigma = 0$ counts (dark grey) and with $\sigma = 3$ counts (black).

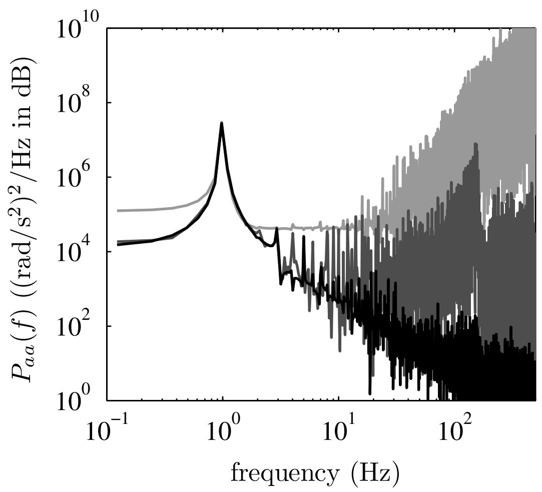

Fig. 9. PSDs of the measured and estimated accelerations, reference (light grey), estimated with $\sigma = 0$ counts (dark grey) and with $\sigma = 3$ counts (black).

The power spectral densities (PSDs) of the measured and estimated acceleration signals, depicted in Fig. 9, show the influence of time stamping and the skip option. The PSD of the quantized acceleration is for all frequencies located above the PSDs of the estimated accelerations. The PSD of the estimation with $\sigma = 3$ counts clearly shows the reduction of the high frequencies in the estimation.

The estimation errors for a frequency $f = 1$ Hz and a varying amplitude $A \in \{\pi/16,\ \pi/4,\ \pi/2,\ \pi\}$ are shown in Fig. 10. For varying amplitude, the optimal skip value varies. A change in the amplitude changes the amount of encoder counts over one period. Since the skip values perform a spatial low-pass filtering based on the encoder counts, a change in the amplitude changes the cut-off frequency for constant skip value. The optimal skip setting, i.e. the skip setting to obtain the smallest estimation error, is dependent on the amplitude. For increasing amplitude the optimal skip value also increases.

For a skip of $\sigma = 3$ counts, every fourth subsequent count is stored. For a movement in one direction this corresponds to storing the same event of the four quadrature signals of one encoder slit. A skip value of $\sigma = 3$ counts therefore eliminates all event errors that are caused by phase errors between the quadrature signals.

6. CONCLUSIONS

The position measurements of optical incremental encoders suffer from quantization errors. Velocity and accel-

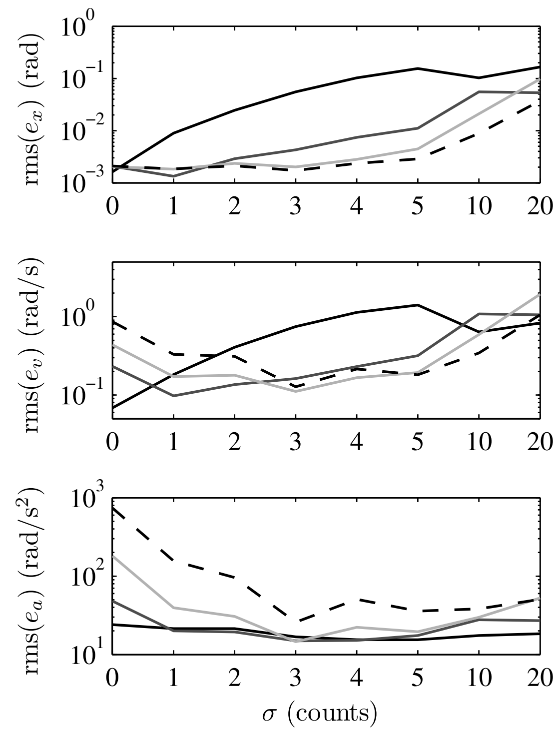

Fig. 10. Estimation errors with various skip values σ for sinusoidal reference with varying amplitude A, $\pi/16$ rad (black), $\pi/4$ rad (dark grey), $\pi/2$ rad (light grey) and π rad (dashed).

eration estimations obtained by numerical differentiation of the quantized position measurements show large spikes. The time stamping concept uses encoder events, consisting of the counter value and the corresponding time instant. Through the stored events a polynomial is fitted and extrapolated to the desired time instant. Differentiation of the fitted polynomial and extrapolation lead to velocity and acceleration estimations that are applicable for control purposes.

Encoder imperfections and the limited resolution with which the encoder events are captured lead to errors on the encoder events. These errors result in oscillations in the velocity and acceleration estimations. Increasing the time span covered by the stored encoder events reduces the oscillations in the estimations. However, the amount of events to be stored is limited. Therefore, a skip option is proposed to discard a fixed number of events in between stored events. The skip option increases the covered time span without the need for additional events. The skip option performs a spatial low-pass filtering on the encoder counts.

Experiments show the improvement of the velocity and acceleration estimations with a skip of three events in comparison to differentiation of the quantized position measurement and in comparison to the time stamping concept without skip. Compared to time stamping without skip, the velocity estimation is improved by 54% and the acceleration estimation by 92%.

The optimal skip value is independent on the velocity of rotation or the frequency of oscillation. However, it is dependent on the amplitude of oscillation since this changes the amount of events per period.

Future research includes the derivation of explicit conditions for the optimal number of events, the optimal fit order and the optimal skip value for various operating conditions.

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