Exponential distributions

1. Xiomara and Rogelio each leave work at noon to meet the other for lunch. The amount of time, X, that it takes Xiomara to arrive is a random variable with an Exponential distribution with mean 10 minutes. The amount of time, Y, that it takes Rogelio to arrive is a random variable with an Exponential distribution with mean 20 minutes. Assume that X and Y are independent. Let $L = \max(X, Y) - \min(X, Y)$ be the amount of time, in minutes, that the first person to arrive has to wait for the second person to arrive.

   Solve the following without doing any calculus, using properties of Exponential distributions as much as possible.

   a. Compute the conditional probability that Xiomara has to wait more than 15 minutes for Rogelio to arrive, given that Xiomara arrives first.
   b. Compute the conditional probability that Rogelio has to wait more than 15 minutes for Xiomara to arrive, given that Rogelio arrives first.
   c. Compute and interpret $\mathrm{P}(L > 15)$.

2. The lifetime of a laptop battery has an Exponential distribution with mean 6 hours, the lifetime of a cell phone batter has an Exponential distribution with mean 7 hours, and the lifetime of a tablet battery has an Exponential distribution with mean 9 hours. Assume the lifetimes are independent.

   Solve the following without doing any calculus, using properties of Exponential distributions as much as possible.

   a. Find the probability that all three last at least 5 hours.
   b. Find the probability that each of the batteries is the first to run out.
   c. Find the probability that all three batteries last at least 5 hours and the laptop battery is the first to run out.
   d. Find the probability that all three batteries last at least 5 hours given that the laptop battery is the first to run out.