Markov Chains

Every day for lunch you have either a sandwich (state 1), a burrito (state 2), or pizza (state 3). Suppose your lunch choices from one day to the next follow a Markov chain with transition matrix

$$\mathbf{P} = \begin{bmatrix} 0 & 0.5 & 0.5 \\ 0.1 & 0.4 & 0.5 \\ 0.2 & 0.3 & 0.5 \end{bmatrix}$$

Suppose today is Monday and consider your upcoming lunches.

- Monday is day 0, Tuesday is day 1, etc.
- You start with pizza on day 0 (Monday).
- Let T be the first time (day) you have a sandwich. (Note: it is possible for T to be greater than 4.)
- Let V be the number of times (days) you have a burrito this five-day work week.
- Pizza costs $5, burrito $7, and sandwich $9.
- Let X_n be the *cost* of your lunch on day n.
- Let $W = X_0 + \cdots + X_4$ be your total lunch cost for this five-day work week.

Write code to setup and run a simulation to investigate the following.

1. Approximate the marginal distribution, along with the expected value and standard deviation, of each of the following
   1. X_4
   2. T
   3. V
   4. W
2. Approximate the joint distribution, along with the correlation, of each of the following
   1. X_4 and X_5.
   2. T and V
   3. T and W
   4. W and V
3. Approximate the conditional distribution of V given $T = 4$, along with its (conditional) mean and standard deviation.
4. Your choice. Choose at least one other joint, conditional, or marginal distribution to investigate. You can work with X_n, T, V, W, but you are also welcome to define other random variables in this context. You can also look at time frames other than a single week.

For each of the approximate distributions, display the results in an appropriate plot, and write a sentence or two describing in words in context some of the main features.