Stationary Distributions and Long Run Behavior of CTMCs

1. The weather in a certain city can be in one of 3 states: sunny (1), cloudy (2), or rainy (3). Suppose the weather evolves over time according to a continuous time Markov chain with the following transition rate matrix. Rates are all per day (24 hours). (Diagonals left blank on purpose.)

$$\mathbf{Q} = \begin{bmatrix} & 0.25 & 0 \\ 0.8 & & 0.4 \\ 2.0 & 1.5 & \end{bmatrix}$$

a. In the long run, in what state does the city's weather spend the highest fraction of time? Explain your reasoning without doing any calculations.
b. Set up by hand the system of equations you would solve to find the stationary distribution for the continuous time chain. (You don't need to solve the system by hand.)
c. Use software to compute the stationary distribution.
d. Roughly, how much time does it take for the CTMC to converge — close enough for practical purposes — to its stationary distribution? Answer this question by using software to compute $\mathbf{P}_t$ for different values of t and comparing to the stationary distribution.

2. A newborn baby spends its time in one of 3 states: eat (1), play (2), or sleep (3). Suppose the baby moves from state to state according to a discrete time Markov chain with unique stationary distribution $(0.4071, 0.2566, 0.3363)$. Assume that the time the baby spends in each state has an Exponential distribution, independent of time spent in previous states, with mean: 30 minutes for eating, 1 hour for playing, and 2 hours for sleeping. Compute the long run fraction of time the baby spends in each state.