Brownian Motion

1. Let $\{W(t), t \geq 0\}$ be a standard Brownian motion, and let $X(t) = 0.5W(4t)$.

a. Find the autocovariance function of the process $\{X(t), t \geq 0\}$.

b. Is the process $\{X(t), t \geq 0\}$ a standard Brownian motion? Justify your answer.

2. Suppose the net *losses* of a gambler who plays a large number of games of roulette can be reasonably modeled as a Brownian motion with drift $\mu = 0.01$ per minute and scale parameter $\sigma = 0.2$ per minute. Let $B(t)$ denote the net loss of the gambler after t minutes of roulette play. For the parts below in addition to computing, denote the corresponding probability in terms of proper symbols and notation.

a. Compute the conditional probability that the gambler's net loss after 10000 minutes is greater than 105, given that the net loss after 1000 minutes is equal to 5.

b. Compute the probability that the gambler's net loss after 10000 minutes is more than 4 times the net loss after 5000 minutes.