Discrete Time Markov Chains: Joint, Conditional, and Marginal Distributions

Every day for lunch you have either a sandwich (state 1), a burrito (state 2), or pizza (state 3). Suppose your lunch choices from one day to the next follow a MC with transition matrix

$$\mathbf{P} = \begin{bmatrix} 0 & 0.5 & 0.5 \\ 0.1 & 0.4 & 0.5 \\ 0.2 & 0.3 & 0.5 \end{bmatrix}$$

Suppose today is Monday and consider your upcoming lunches.

- Monday is day 0, Tuesday is day 1, etc.
- You start with pizza on day 0 (Monday).
- Let T be the first time (day) you have a sandwich.
- Let V be the number of times (days) you have a burrito this five-day work week.
- Pizza costs \$5, burrito \$7, and sandwich \$9.

1. Compute and interpret in context $\mathrm{P}(T > 4)$.
2. Find the marginal distribution of V, and interpret in context $\mathrm{P}(V = 2)$.
3. Compute the expected total cost of your lunch this work week (Monday through Friday). Interpret this value as a long run average in context.
4. Describe in detail how, in principle, you could use physical objects (coins, dice, spinners, cards, boxes, etc) to perform by hand a simulation to approximate $\mathrm{E}(V|T = 4)$. Note: this is NOT asking you to compute $\mathrm{E}(V|T = 4)$ or how you would compute it using matrices/equations. Rather, you need to describe in words how you would set up and perform the simulation, and how you would use the simulation results to approximate $\mathrm{E}(V|T = 5)$.