Introduction to Markov Chain Monte Carlo (MCMC) Methods

1. Consider the integral

$$\int_0^1 \int_1^2 \exp\left(-\frac{2}{3}\left(x^2 + y^2 - 0.5xy\right)\right) dx\, dy$$

   a. Describe in detail a Monte Carlo approach to approximating the integral using only Uniform(0, 1) random numbers.
   b. Write a few lines of code to carry out your approximation and report the value. Compare to an analytic solution (use an integral calculator like WolframAlpha).

2. Roll a fair six-sided die 100 times. What is the conditional expected value of the sum of the rolls given that no two consecutive rolls are the same? Describe in detail how you can use an MCMC procedure to approximate this expected value.