Discrete Time Markov Chains: Long Run Behavior ⟩

1. Rihanna has three umbrellas, some at her office and some at home. If she is leaving home in the morning (or leaving work at night) and it is raining, she will take an umbrella if there is one there; otherwise, she gets wet. Assume that, independent of the past, it rains on each trip with probability 0.2. In the long run, on what proportion of trips does Rihanna get wet? Hint: we have previously shown that X_n, the number of umbrellas at her current location, is a Markov chain with the following transition matrix.

```
state_names = 0:3

P = rbind(
  c(0, 0, 0, 1),
  c(0, 0, 0.8, 0.2),
  c(0, 0.8, 0.2, 0),
  c(0.8, 0.2, 0, 0)
)
```

	0	1	2	3
0	0.0	0.0	0.0	1.0
1	0.0	0.0	0.8	0.2
2	0.0	0.8	0.2	0.0
3	0.8	0.2	0.0	0.0

2. Every day for lunch you have either a sandwich (state 1), a burrito (state 2), or pizza (state 3). Pizza costs $5, burrito $7, and sandwich $9. Suppose your lunch choices from one day to the next follow a MC with transition matrix

```
state_names = c("sandwich", "burrito", "pizza")

P = rbind(
  c(0, 0.5, 0.5),
  c(0.1, 0.4, 0.5),
  c(0.2, 0.3, 0.5)
)
```

	sandwich	burrito	pizza
sandwich	0.0	0.5	0.5
burrito	0.1	0.4	0.5

	sandwich	burrito	pizza
pizza	0.2	0.3	0.5

a. If your lunches follow this pattern for a year, on what proportion of days do you have a burrito for lunch? Pizza? Sandwich? What is your average daily lunch cost?

b. What is the difference between the questions in part (a) and the kinds of questions you have previously considered for this MC?

3. For each of the following transition matrices, draw a well labeled state diagram. Then identify the transient and recurrent states and the irreducible closed sets of the corresponding Markov chains.

a.
$$\mathbf{P} = \begin{bmatrix} & 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0.2 & 0 & 0.8 & 0 \\ 2 & 0.1 & 0.2 & 0.3 & 0.4 & 0 \\ 3 & 0 & 0.6 & 0 & 0.4 & 0 \\ 4 & 0.3 & 0 & 0 & 0 & 0.7 \end{bmatrix}$$

b.
$$\mathbf{P} = \begin{bmatrix} & 0 & 1 & 2 & 3 & 4 & 5 \\ 0 & 2/3 & 0 & 0 & 1/3 & 0 & 0 \\ 1 & 0 & 1/2 & 0 & 0 & 1/2 & 0 \\ 2 & 0 & 0 & 1/3 & 1/3 & 1/3 & 0 \\ 3 & 1/2 & 0 & 0 & 1/2 & 0 & 0 \\ 4 & 0 & 1/2 & 0 & 0 & 1/2 & 0 \\ 5 & 1/2 & 0 & 0 & 1/2 & 0 & 0 \end{bmatrix}$$