

## Spatial Poisson Processes, and Relatives of Poisson Processes

*You should solve these problems with as few calculations as possible, relying on properties of Poisson processes as much as possible.*

1. In a 2-d spatial Poisson process with intensity  $\lambda$ , let  $X$  represent the *nearest neighbor distance*, that is, the distance between an arbitrary point and the point of the process closest to it.
  - a. Find an expression for  $P(X > x)$ , for  $x > 0$ .
  - b. Find an expression for the probability density function (pdf) of  $X$ .
  - c. Find an expression for  $E(X)$ .
2. Starting at 9 a.m., customers arrive at a store according to a nonhomogeneous Poisson process with intensity function  $\lambda(t) = t^2$ , for  $t > 0$ , where the time is measured in hours. Find the probability mass function of the number of customers who enter the store by noon.
3. Suppose points are distributed in a 2-d region centered at the origin according to a nonhomogeneous, spatial Poisson process  $\{N_A\}$  with intensity function

$$\lambda(x, y) = e^{-(x^2+y^2)}$$

Let  $R$  be the distance from the origin to the nearest point. Compute  $P(R > 1)$ .