

Introduction to Markov Chains



In each of the following, briefly explain why $\{X_n\}$ is a Markov chain, specify its transition matrix, and draw a state diagram.

1. Rihanna has three umbrellas, some at her office and some at home. If she is leaving home in the morning (or leaving work at night) and it is raining, she will take an umbrella if there is one there (otherwise, she gets wet). Assume that, independent of the past, it rains on each trip with probability 0.2.

Let X_n be the number of umbrellas at her current location.

2. Consider a retail store that sells a particular item (say a TV). Each day they experience customer demand of 0 units of the item with probability 0.6, 1 unit with probability 0.3, and 2 units with probability 0.1 (provided enough units are on the shelf), independently from day to day. The shelf space in the store has room for at most 5 units of the item. If at the end of the day there are fewer than 2 units left then the store restocks the shelf so that it has a total of 5 units on it at the start of the next day.

Let X_n be the number of units on the shelf at the *end* of day n , before restocking.

3. Consider the following simplified model for the spread of a disease in a population of 5 people. Each person either has the disease or not. In each time period, 2 of the 5 people are selected uniformly at random, and interact. If one of the two people selected has the disease and the other does not, then the disease is transmitted to the healthy person with probability 0.1. Otherwise, no transmission of the disease occurs.

Let X_n denote the number of people who have the disease at the end of the n th time period (after any disease transmission occurs).

