Introduction to Continuous Time Markov Chains

1. Let $\{X(t), t \geq 0\}$ be a continuous time Markov chain with state space $S = \{1, 2, 3, 4\}$ and transition rate matrix Q:

$$\mathbf{Q} = \begin{bmatrix} & 1 & 0 & 1 \\ 2 & & 2 & 0 \\ 0 & 3 & & 3 \\ 4 & 0 & 4 & \end{bmatrix}$$

   a. Find the diagonal entries of $\mathbf{Q}$.
   b. Explain in full detail how you could simulate the process for a long time using only (1) a coin, and (2) an Exponential(1) spinner.

2. A system is composed of 5 machines. A machine operates for an Exponentially distributed amount of time with rate $\mu = 1$ and then fails. When a machine fails it undergoes repair; repair times are Exponential distributed with rate $\lambda = 2$. Let $X(t)$ represent the number of machines operating at time t; then $\{X(t)\}$ is a CTMC. Find the rate matrix of the CTMC.