Further Properties of Poisson Processes

You should solve these problems with as few calculations as possible, relying on properties of Poisson processes as much as possible.

1. Shocks occur to a system according to a Poisson process with rate $\lambda = 3$ per year. Suppose that the system survives each shock with probability $p = 0.9$, independent of other shocks. Let N_t denote the number of shocks have occurred by time t and let T denote the time at which the system fails.

a. Find an expression for and interpret $\mathrm{P}(T > t | N_t = k)$, where k is a nonnegative integer.
b. Compute and interpret $\mathrm{P}(T > t)$.
c. Compute and interpret $\mathrm{E}(T)$.
d. Interpret in words what the random variable N_T represents (the subscript is big T not little t), and find the distribution of this random variable. You should do this without any calculations.

2. Vehicles arrive at a particular intersection according to a Poisson process, with rate 2 per minute for cars, 1 per minute for trucks, and 0.5 per minute for motorcycles. Assume arrivals of the different vehicle types are independent. Let T be the time elapsed between now and the arrival of the next vehicle, and let I represent the type (1=car, 2=truck, 3=motorcycle) of the next vehicle to arrive.

a. Identify the distribution of T, find $\mathrm{E}(T)$, and compute the probability that a vehicle arrives within the next 30 seconds.
b. Find the distribution of I; that is, for each of the three vehicle types, find the probability that the next vehicle to arrive is of that type.
c. Given that the next vehicle to arrive is a motorcycle, find the conditional expected time until the next vehicle arrives, and the conditional probability that a vehicle arrives within the next 30 seconds.
d. Given that the next vehicle arrives within 30 seconds, compute the conditional probability that it is a motorcycle.
e. Explain in full detail how you could in principle simulate the arrival times of many vehicles and their types using only (1) a spinner and (2) an Exponential(1) random number generator.

3. In the summer, tornadoes hit a certain region according to a Poisson process with $\lambda = 2$ per month. The number of insurance claims filed after any tornado has a Poisson distribution with mean 30. The number of tornadoes is independent of the number of insurance claims. Let X be the total number of claims filed in three summer months.

a. Compute $\mathrm{E}(X)$.
b. Compute $\mathrm{SD}(X)$.
c. Use simulation to approximate $\mathrm{P}(X > 300)$.