Poisson Processes

You should solve these problems with as few calculations as possible, relying on properties of Poisson processes as much as possible.

1. Calls arrive at a customer service center according to a Poisson process with with $\lambda = 3$ calls per minute. Compute and interpret the following quantities.

a. $\mathrm{E}(N_1)$.
b. $\mathrm{SD}(N_1)$.
c. $\mathrm{P}(N_1 = 2)$.
d. $\mathrm{P}(N_1 = 2, N_3 = 6)$.
e. $\mathrm{P}(N_1 = 2 | N_3 = 6)$.
f. $\mathrm{P}(N_3 = 6 | N_1 = 2)$.
g. $\mathrm{E}[N_{15} | N_{10} = 8]$
h. $\mathrm{E}[N_{10} | N_{15} = 8]$.
i. $\mathrm{Cov}(N_{10}, N_{15})$.
j. $\mathrm{Corr}(N_{10}, N_{15})$.

2. Arrivals of spam emails to your email spam filter follow a Poisson process with mean rate 1.5 spam emails per minute. For the parts below in addition to computing, denote the corresponding probability in terms of proper symbols and notation.

a. Compute the probability that exactly 4 spam emails arrive to the filter in the next 2 minutes.
b. Compute the conditional probability that more than 3 minutes elapse, starting now, before the next spam email arrives, given that the most recent spam email arrived 2 minutes ago.
c. Compute the probability that the next spam email arrives some time after 3 minutes but before 5 minutes from now.
d. Compute the probability that exactly one spam email arrives in the time interval from 3 minutes to 5 minutes from now.
e. If only 1 spam email arrives in the next 5 minutes, compute the conditional probability that it arrives in the next minute.
f. Compute the conditional probability that 2 spam emails arrive in the first minute, given that 5 spam emails arrive in the first 3 minutes.