Gaussian Distributions and Processes

1. Devi and Paxton are meeting. Arrival times are measured in minutes after noon, with negative times representing arrivals before noon. Devi's arrival time follows a Normal distribution with mean 20 and SD 15 minutes, and Paxton's arrival time follows a Normal distribution with mean 25 and SD 10 minutes.

   a. Assume the pairs of arrival times follow a Bivariate Normal distribution with correlation 0.8
      1. Compute the probability that Devi arrives first given that Paxton arrives at 12:10.
      2. Compute the probability that the first person to arrive has to wait more than 15 minutes for the second person to arrive.

   b. Assume the pairs of arrival times follow a Bivariate Normal distribution with correlation -0.7
      1. Compute the probability that Devi arrives first given that Paxton arrives at 12:10.
      2. Compute the probability that the first person to arrive has to wait more than 15 minutes for the second person to arrive.

2. The noise in a voltage signal is modeled by a Gaussian process with constant mean 0.9V. If we sample the noise at two nearby times, they will be correlated. Specifically, suppose the covariance of the noise at any two times that are t seconds apart is given by $0.04e^{-0.1|t|}$.

   a. Find $\mathrm{Cov}(X(3), X(8))$.
   b. Find $\mathrm{Var}(X(3))$ and $\mathrm{Var}(X(8))$. (Hint: how can you write a variance as a covariance?)
   c. Find $\mathrm{Corr}(X(3), X(8))$.
   d. Find $\mathrm{P}(X(3) > X(8) + 0.5)$.