

# Discrete Time Markov Chains: Absorbing States and First Step Analysis



1. Consider the following simplified model for the spread of a disease in a population of 5 people. Each person either has the disease or not. In each time period, 2 of the 5 people are selected uniformly at random, and interact. If one of the two people selected has the disease and the other does not, then the disease is transmitted to the healthy person with probability 0.1. Otherwise, no transmission of the disease occurs.

If one person initially has the disease, what is the expected number of time periods until everyone has the disease?

2. At a local two year college, 60% of freshmen become sophomores, 25% remain freshmen (because they don't have enough credits), and 15% drop out (and don't re-enroll). 70% of sophomores graduate and transfer to a four year college, 20% remain sophomores (because they don't have enough credits), and 10% drop out (and don't re-enroll). Answer the following questions by defining an appropriate Markov chain.
  - a. What proportion of new students eventually graduate?
  - b. On average how many years does a student stay enrolled at the college?

