Lab Report B-III: Radioactive Decay

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In this lab, we examined the distribution of γ particle count rates for a Cesium-137 sample, and approximated the mass and activity of the sample. The mass was found to be $(7.29 \pm 1.17) \times 10^{-21}$kg, and the activity was found to be 145.26 ± 22.96 counts/sec. After running a χ^2 Goodness of Fit test, we failed to reject the null hypothesis that the count rate data was Poisson distributed, noting a p-value of 0.5. Thus, the data is not significant at the 10% significance level, and is therefore consistent with the Poisson distribution.

I. INTRODUCTION

Although Cesium itself is stable, the synthetic isotope Cesium-137 is radioactive. It β-decays to ^{137}Ba, then decays to its ground state by emitting γ-rays. By measuring and analysing the resulting γ-rays, we can determine a few properties of the source. First, we can estimate the activity of the ^{137}Cs source by quantifying and calculating the radioactive emission of γ particles. Second, we can use the information regarding radioactivity to approximate the mass of the source. Finally, we can confirm the statistical nature of the radioactive source by computing the expected emissions using a Poisson distribution, comparing the expected values to the measured emissions, and performing χ^2 analysis.

II. THEORETICAL BACKGROUND

A. Radioactive Decay Law

For the sample of Cesium-137, there are N isotopes that decay. The number of radioactive decays ΔN over time Δt is proportional to number of isotopes, meaning the relationship can be written as

$$\Delta N = -\lambda N \Delta t, \tag{1}$$

where λ is a proportionality constant. The sign is negative because the number of decays comes directly from the isotopes. Integrating both sides and solving for N yields

$$N = N_0 e^{-\lambda t}, \tag{2}$$

with N_0 being the initial number of isotopes at $t = 0$.

We can then relate this to the activity of the sample. The quantity R refers to the decay rate, or activity, of the sample, and can be expressed as

$$R = \left| \frac{dN}{dt} \right|. \tag{3}$$

Taking the derivative of Eq. 2 with respect to t, plugging it into Eq. 3, and substituting

$$\lambda = \frac{\ln(2)}{t_{\frac{1}{2}}} \tag{4}$$

for λ in the resultant equation produces

$$R = \frac{\ln(2) N_0}{t_{\frac{1}{2}}} 2^{\frac{-t}{t_{\frac{1}{2}}}}, \tag{5}$$

where $t_{\frac{1}{2}}$ is the half-life length of Cesium-137. At $t = 0$, the term that is the number 2 raised to a value becomes 1, leaving a simple relationship between R and the initial number of isotopes in the sample N_0. Knowing R, one can then determine N_0. However, measuring R requires information about the attenuation constant, found next.

B. Count Dependence on Distance and Attenuation

The measured count rate ν for the particles emitted from the sample depends on distance from the sample. The sample emits particles in all directions, so one can visualize the emitted particles as a projection onto a three-dimensional sphere around the source. We used a circular detector of area A, so the detector counts a fraction of the total emitted particles. This fraction describes the ratio of A to the surface area S of the sphere, described by $4\pi r^2$. Increasing the radius will change the detector count at a rate inversely proportional to r^2. Knowing ν_0 the measured count rate over A, we can multiply this value by S/A (known as the proportionality constant) to find the count rate over the entire sphere, R.

The count rate also depends on matter in between the collector and the sample. The change in count rate $\Delta\nu$ is proportional to -$\nu\Delta x$, where x is the thickness of the matter (in this experiment it was plastic filters. Thus, ν can be found in a very similar fashion as N above. Solving for ν leaves

$$\nu = \nu_0 e^{-\alpha x}, \tag{6}$$

where α is the attenuation constant of the specific matter. Rearranging, one can find

$$-\ln(\frac{\nu}{\nu_0}) = \alpha x, \tag{7}$$

which forms a linear plot with slope of α. Rearranging Eq. 6, we see

$$\nu_0 = \nu e^{\alpha x}. \tag{8}$$

Relating ν_0 with R using the relationship and proportionality constant mentioned above, we see that

$$R = \frac{\nu_0}{t}\frac{S}{A}, \tag{9}$$

where t is the time interval of 10 seconds, and S/A is the proportionality constant. Plugging in Eq. 8, we see

$$R = R_0 \frac{4\pi r^2 e^{\alpha x}}{A}.?? \tag{10}$$

This allows us to solve for R, and in turn solve for N_0 using Eq. 5. Knowing the number of isotopes, we can then use dimensional analysis to find the mass of the Cesium sample.

III. METHODS

A. Procedure

There were three experiments performed. First, radioactive Cesium-137 contained in a small plastic case and housed in a lead brick shed was studied to determine the attenuation constant. The Vernier radiation monitor was held at a constant distance from the source, and plastic filters of various thickness were placed in front of the source. The count rate, in addition to the background count rate, were measured in order to determine the normalized rate coming from the source. Using Eq. 6, we found the attenuation constant.

Second, the Vernier radiation monitor was held at varying distances from the source to confirm the distance-dependence of the count rate. The setup was similar to the first experiment, except the plastic filters were not used. This was done to find the constant of proportionality mentioned in Sec. II.

Finally, the detector stayed in a constant location, and measured the count rate of the source over time. This was done to determine if the emission rate followed a Poisson distribution.

B. χ^2 Goodness of Fit Test

To determine if the number of decays per 10 seconds was statistically different from a Poisson distribution, we performed a χ^2 Goodness of Fit test. Our null hypothesis H_0 was "The number of decays per 10 seconds is Poisson distributed," and the alternative hypothesis H_A was "The number of decays per 10 seconds is not Poisson distributed." We binned the data from the last part of the procedure such that there were five bins with widths of five counts. To ensure we had at least five expected counts in each bin, the first and last bins were expanded to span 15 counts. The bins are reported in Table I.

We used the MatLab function Poisscdf.m, which provides a cumulative distribution for Poisson-distributed data, to determine the probability of a random measurement falling into the above bins. This probability was multiplied by $N = 240$, the total number of measurements taken, to find the number of expected counts.

Bin #	Width (Counts)
1	0-14
2	15-19
3	20-24
4	25-29
5	30-44

TABLE I: Bins

The number of degrees of freedom for this test is given by $dof = k - l - 1$, where k is the number of bins and l is the number of parameters estimated from the data. Because we only used the mean of the data, μ, to run the test, we had $5 - 1 - 1 = 3$ degrees of freedom.

The actual χ^2 test was performed using the MatLab function Chi2cdf.m.

C. Uncertainty Analysis

The number of counts ν is Poisson-distributed, as shown Sec. V, so the variance is the same as the mean: $u_\nu^2 = \mu$. Therefore, uncertainty is $\sigma_\nu = \sqrt{\mu}$. When plotting $-\ln \nu/\nu_0$, we propagated uncertainty through quadrature.

Uncertainty in the measurements of r, the distance of the sample from the detector, was due largely to parallax. While we were fairly confident in the recorded value, we felt the true measurement could have fallen with 0.005 m of our measurement. Because of this, we assumed a triangle distribution with points $a = r - 0.005$m, $b = r + 0.005$m, and $c = r$. This yields an uncertainty of $\sigma_r = 0.2$ m. We found uncertainty in values of $1/r^2$ via quadrature.

The thickness x of the plastic between the sample and the detector was measured with calipers and had uncertainty of $\sigma_x = 0.001$m.

Our data was plotted using the LinFit.m function in MatLab, which only accounts from uncertainty along one axis. As such, σ_x does not factor into the final uncertainty analyses.

IV. RESULTS

Fig. 1 demonstrates the linear dependence of ν on r^{-2}, with a correlation coefficient of $R^2 = 0.9448$. From Fig. 2, plotted using the measurement model in Eq. 7, $\alpha = 6 \pm 61$ Np/m.

Our χ^2 Goodness of Fit test yielded a p-value of 0.51211.

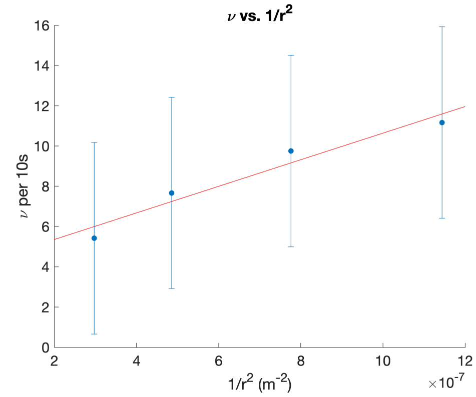

FIG. 1: This plot has a slope of $7 \pm 8 \times 10^6$ and an R^2 value of 0.9448, indicating a strong linear relationship.

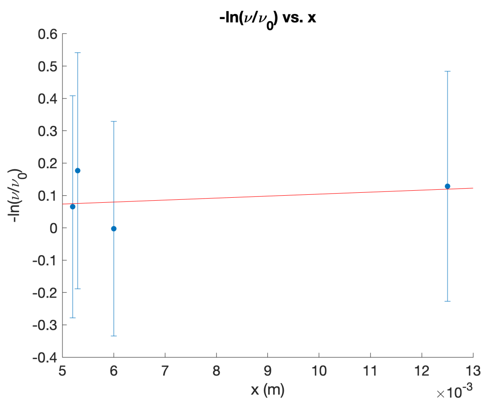

FIG. 2: This plot, modeled after Eq. 7, has the equation $-\ln(\nu/\nu_0) = (6 \pm 61)x + 0.04 \pm 0.47$, where the slope is $\alpha = 6 \pm 61$ Np/m.

V. DISCUSSION AND CONCLUSIONS

A. χ^2 Goodness of Fit Test

Our p-value from the test was 0.51211. Since this is greater than all of the common thresholds, we chose to compare it to the largest one, $p = 0.1$. As such, we fail to reject the null hypothesis. The observed distribution of counts is not significant at the 10% significance level, and is therefore consistent with the Poisson distribution.

Because the counts are shown to be Poisson distributed, the variance in these measurements is given by the mean of the data, μ.

B. Finding the Activity and Mass

Fig. 1 shows the linear dependence of ν on r^{-2}, which corroborates the relationship seen in Sec. II B. The activity R was calculated using Eq. **??**, and found to be 145.26 ± 22.96 counts/sec. The uncertainty was propagated through quadrature. Using the methods described in Sec. II, we found the mass of the Cesium sample to be $(7.29 \pm 1.17) \times 10^{-21}$kg. The uncertainty was propagated through quadrature.

VI. ACKNOWLEDGMENTS

The experiment was performed by C.S. and N.M., and N.M. recorded the data. The Abstract, Methods, and Results sections were written in tandem. C.S. wrote the Discussions and Conclusions section, and N.M. wrote the Introduction and Theoretical Background sections. Data was fit using the LinFit.m MatLab function provided by Ben Levy.

[1] Lab B-III Manual `https://sakai.unc.edu/access/content/group/41081125-b301-4ad9-8173-970601749ad6/Lab%20Manuals/Lab%20B-III%3A%20Radioactive%20Decay%20of%20Caesium-137/Lab%20B-III%20Radioactive%20Decay%20of%20Caesium-137.pdf`