281L Lab A-1 Report

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Using a e/m apparatus, we determined both the charge to mass ratio of an electron and the magnitude of Earth's magnetic field at Chapel Hill. We recorded the diameter of the circular electron beam path for various acceleration voltages and currents, which we then used to derive an relation for linear fit. After applying this linear fit to the data, we found that the charge to mass ratio was $(-2.42 \pm 0.05) * 10^{11} \frac{\mathrm{C}}{\mathrm{kg}}$, which was on the same order of magnitude as the accepted value of $-1.76 * 10^{11} \frac{\mathrm{C}}{\mathrm{kg}}$, but their bounds of error do not overlap [3]. We found the calculated magnitude of the Earth's magnetic field B_e was $(5 \pm 3) * 10^{-5}$T, which does agree with the accepted value of B_e in Chapel Hill, NC which is $(4.49 \pm 0.01) * 10^{-5}$T [4]. Though more parameters of uncertainty should be accounted for in the calculation of the charge to mass ratio of the electron, we find our values in general agreement with prior literature.

I. INTRODUCTION

The Earth's magnetic field protects us from harmful solar activity even though its weak enough that we don't experience any major effects in our everyday lives. While we don't feel the magnetic field, free electrons are influenced by the field as they have small masses, m. The charge to mass ratio, q/m, of an electron can be found using magnetic fields similar to the Earth's, though specifically within a vacuum. This ratio is critical to understanding the properties of electrons. In this experiment, we used an e/m apparatus that used an electric potential to accelerate free electrons into a magnetic field, B_c, created by Helmholtz coils within the apparatus. Since this experiment was performed within the Earth's magnetic field, B_e, we also had to account for the effect it would have on the electrons. By incorporating the B_e in our calculations we could determine the magnitude and direction of B_e, and it also eliminates a large source of error in our calculations for q/m that would have been ignored; if B_e influences electrons on a daily basis then it would certainly interfere with some aspects of the experiment.

II. THEORETICAL BACKGROUND

The magnetic field $\vec{B}$ acting upon the free electrons is a sum of smaller individual fields, generated by electrons in the Helmholtz coils, and the Earth. It can be written as

$$\vec{B} = \vec{B}_c + \vec{B}_e. \tag{1}$$

Data collected in the experiment allows us to calculate $\vec{B}$ and $\vec{B}_c$. Once those values are known, one can then determine $\vec{B}_e$.

A. Solving for $\vec{B}$

We used the e/m apparatus to apply an electric potential drop V between an anode and a cathode, causing free electrons to travel at constant velocity v from anode to cathode. The energy from the potential drop was converted into kinetic energy, so we can relate the two by setting $-qV$ equal to $\frac{1}{2}mv^2$. Solving for velocity yields

$$v = \sqrt{\frac{-2qV}{m}}. \tag{2}$$

Remember that $\vec{B}$ acts upon these moving electrons; assuming $\vec{B}$ points down and the electrons move on a horizontal plane, $\vec{F}_B$ will cause the electrons to start moving in a uniform circular path by the right hand rule. Note

$$\vec{F}_B = q\vec{v} \times \vec{B}, \tag{3}$$

which relates $\vec{B}$ to the force $\vec{F}_B$ acting on the moving electrons q of velocity $\vec{v}$. Two important conclusions can be drawn. First, because of the aforementioned right hand rule, $\vec{v}$ and $\vec{B}$ are always perpendicular; thus, the magnitude of their cross product is equal to the product of v and B. Equation 2 can then be simplified to

$$F_B = qvB. \tag{4}$$

Second, the force $\vec{F}_B$ happens to be the centripetal force. It is equal to ma_c, or $m\frac{v^2}{r}$. Plugging this expression into Equation 2 for F_B yields $m\frac{v^2}{r} = qvB$, and solving for B gives

$$B = \frac{1}{r}\sqrt{\frac{-2mV}{q}}. \tag{5}$$

Note that $\frac{1}{r}$ is the same as $\frac{2}{d}$, as the diameter d of the circular electron path was measured in the experiment.

B. Solving for $\vec{B}_c$

Current was passed through the two Helmholtz coils, or loops. In order to find the magnetic field created by each loop, we can start with the equation describing an infinitesimal point on the loop's contribution to the B-field:

$$\mathrm{d}\vec{B} = \frac{\mu_0 I}{4\pi}\frac{\mathrm{d}\vec{s}\times\hat{r}}{r^2}, \tag{6}$$

where the figure below depicts the setup for integration. It has been parameterized to allow us to convert $\mathrm{d}\vec{s}$ to $\mathrm{d}\phi$. This figure appears in the Lab Manual [1] for Lab A-I.

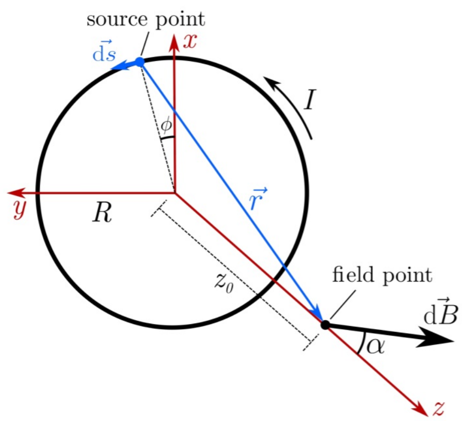

FIG. 1: An infinitely small component of current results in an infinitely small contribution to the magnetic field.

Some substitutions can be made. Arc length s is the product of loop radius R and the angle ϕ, so $\mathrm{d}\vec{s} = R\mathrm{d}\phi$. The vector $\hat{r}$ can be separated into its components, equal to $-Rcos(\phi)\hat{x} - Rsin(\phi)\hat{y} + R\hat{z}$. The radius r equals $\sqrt{R^2 + z_0^2}$. Making these substitutions leaves

$$\mathrm{d}\vec{B} = \frac{\mu_0 IR(z_0 cos(\phi)\hat{x} + z_0 sin(\phi)\hat{y} + R\hat{z})}{4\pi(R^2 + z_0^2)^{3/2}}\mathrm{d}\phi. \tag{7}$$

Setting up the integral in order to solve for $\vec{B}$ gives

$$\vec{B} = \frac{\mu_0 IR}{4\pi}\int_0^{2\pi}\frac{z_0 cos(\phi)\hat{x} + z_0 sin(\phi)\hat{y} + R\hat{z}}{(R^2 + z_0^2)^{3/2}}\mathrm{d}\phi. \tag{8}$$

The loops in the Helmholtz coil sit parallel to the ground, so $\vec{B}_c$ only has a z-component. This means $\vec{B}_x = \vec{B}_y = 0$, and

$$\vec{B}_z = \frac{\mu_0 IR^2}{4\pi(R^2 + z_0^2)^{3/2}}\int_0^{2\pi}\mathrm{d}\phi. \tag{9}$$

Substituting $\vec{B}_c$ for $\vec{B}_z$, multiplying by N because there are N turns in the coil, and solving the integral evaluates to

$$\vec{B}_c = \frac{\mu_0 NIR^2}{2(R^2 + z_0^2)^{3/2}}. \tag{10}$$

This can also be written in the form $\vec{B}_c = \beta I\hat{z}$, where

$$\beta = \frac{\mu_0 NR^2}{(R^2 + z_0^2)^{3/2}}. \tag{11}$$

C. Solving for $\vec{B}_e$

Plugging the solved-for values of $\vec{B}$ and $\vec{B}_c$ into Equation 1 results in

$$\frac{2}{d}\sqrt{\frac{-2mV}{q}} = \beta I\hat{z} + \vec{B}_e, \tag{12}$$

or

$$I = \frac{2}{d\beta}\sqrt{\frac{-2mV}{q}} + b, \tag{13}$$

where $b = -\frac{\vec{B}_e}{\beta}$.

III. METHODS

For this experiment, the e/m apparatus was first aligned with B_e. A dip needle was used to find the direction of B_e- in the x, y, and z directions- and then the apparatus was aligned such that the Helmholtz coils were perpendicular to said direction. This way B_c is parallel to B_e. The vertical component of B_e was not physically taken into account for this alignment- in order to protect the apparatus from falling over- but it was analytically included in the analysis of our results.

The e/m apparatus requires three separate power supplies, one for the AC Heater, one for the anode/Accelerating Voltage, and one for the Helmholtz coils, so an Eisco discharge tube power supply was used. Using banana cables, each component was then wired in. For the AC Heater Supply, the terminal with 6.3 V/1 A was used from the power supply. The anode supply was wired into at the 500 V/10 mA DC power supply output terminals and a voltmeter was wired in parallel to this component to more precisely measure the voltage. The Helmholtz coil supply was wired into the 20 V/5 A terminal and an ammeter was connected in series with this component in order to make sure that a current less than 2 A was always supplied. In Figure 2, a diagram from the e/m apparatus user manual [2] illustrates the wiring of the power supply to the apparatus described above.

Once all three components are wired into the power supply, an initial voltage to the anode was selected and an initial current below 2 A but above 1 A was chosen for the Helmholtz coils. Using these initial conditions, the diameter of the electron beam path was then measured using the built in ruler inside the vacuum. All three values were recorded and uncertainty for the measurements were considered and noted for later evaluation. Then, holding the voltage value constant, the current value was

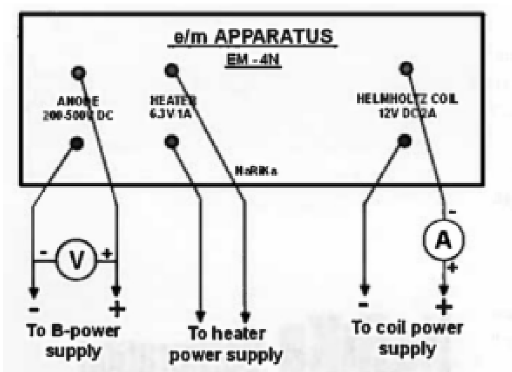

FIG. 2: The diagram above shows how each of the components of the e/m apparatus should be connected to the power supply. The anode connection had the voltmeter connected in parallel so that we could take measurements of the accelerating potential. The Helmholtz coils had the ammeter connected in series so that we could record the current measurements that influenced the electron beam path diameter and make sure that the current never exceeded 2 A.

increased in increments, still below 2 A, twelve times. Each time the current was increased, the diameter of the electron beam path was recorded. Once a wide spread of data was collected for one value of voltage, the voltage was increased and the process of recording varying currents and the resultant diameters was repeated four more times.

After collecting the data, using Equation 13, the data was plotted linearly to find the q/m ratio and the magnitude and direction of B_e

IV. RESULTS

The data was linearly fitted in accordance with Equation 13, as shown in Figure 3 below. It should be noted that in the aforementioned plot, $x = \frac{2}{d\beta}\sqrt{2V}$ for simplification purposes. The uncertainties in V and d were propagated through quadrature to determine the uncertainty in x. The uncertainties of N and R were considered negligible enough to ignore since they already meet a manufacture's tolerance. The resulting fitted line was $I = (2.03 \pm 0.04) * 10^{-6}x + (0.05 \pm 0.03)$, with the uncertainties being determined by the Microsoft Excel LINEST function.

Solving for the value of $\frac{q}{m}$ from the slope of the fitted line, we found the charge to mass ratio of an electron was $(-2.42 \pm 0.05) * 10^{11} \frac{\text{C}}{\text{kg}}$. The uncertainty was determined by propagating the slope uncertainty found by the Microsoft Excel LINEST function through quadrature.

Solving for the value of B_e from the y-intercept of the fitted line, we found that the strength of the magnetic field was $(5 \pm 3) * 10^{-5}$T, with its direction running anti-parallel to that of the Helmholtz coils. The uncertainty was determined by propagating the uncertainty of the y-

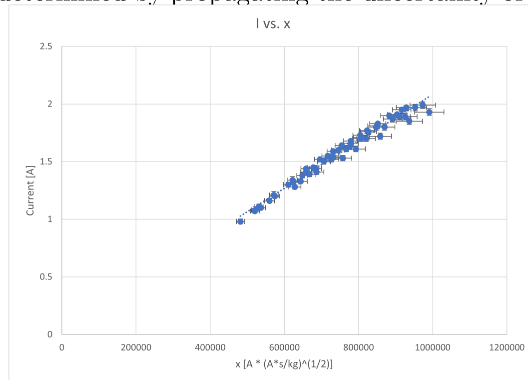

FIG. 3: Line of best fit relating I to x, where $x = \frac{2}{d\beta}\sqrt{2V}$

intercept and θ found by the Microsoft Excel LINEST function through quadrature.

V. DISCUSSION AND CONCLUSIONS

Overall, our results demonstrate a general agreement with values from literature.

The accepted value for the charge to mass ratio of an electron is $-1.76 * 10^{11} \frac{\text{C}}{\text{kg}}$ [3]. This is along the same order of magnitude as the answer that we found, but is not included in our bounds of error. The diameter of the beam seemed to hop around when I changed direction, which could indicate a larger uncertainty in the diameter measurements than the ± 0.2cm that was used in our analysis. The variation in radius of the Helmholtz coil could have also had a non-negligible contribution to the uncertainty.

The accepted value for the Earth's magnetic field at Chapel Hill is $(4.49 \pm 0.01) * 10^{-5}$T [4]. The bounds of error our answer does overlap with that of the accepted. However, it should be noted that the bounds of error of our answer were rather large, mostly because it was hard to gauge the dip needle reading from the video. There was also some "slop" noticed in the current dial.

VI. ACKNOWLEDGMENTS

Data analysis and writing of the Results and Discussion and and Conclusions sections were done by Megan. The Abstract, Introduction, and Methods section were written by Emma. The Theoretical Background section was written by Nathan. Clarifying questions were answered by Ben Levy.

[1] PHYS 281L Lab A-I Manual, https://sakai.unc.edu/access/content/group/41081125-b301-4ad9-8173-970601749ad6/Lab\%20Manuals/Lab\%20A-I\%3A\%20Charge\%20To\%20Mass\%20Ratio\%20of\%20Electrons/Lab\%20A-I\%20Charge-To-Mass\%20Ratio\%20of\%20Electrons.pdf

[2] NadaScientific: e/m Apparatus manual, http://nadascientific.com/pub/media/PDF/Catalog/N99-B10-7355_Manual.pdf

[3] THz/Infrared Double Resonance Two-Photon Spectroscopy of HD+ for Determination of Fundamental Constants F. L. Constantin atoms 5(4), 1-18 (2017)

[4] Center, N. (n.d.). NCEI geomagnetic calculators. Retrieved February 09, 2021, from https://www.ngdc.noaa.gov/geomag/calculators/magcalc.shtmligrfwm