Moment of Inertia for an Abnormally-Shaped Wheel

November 5, 2019

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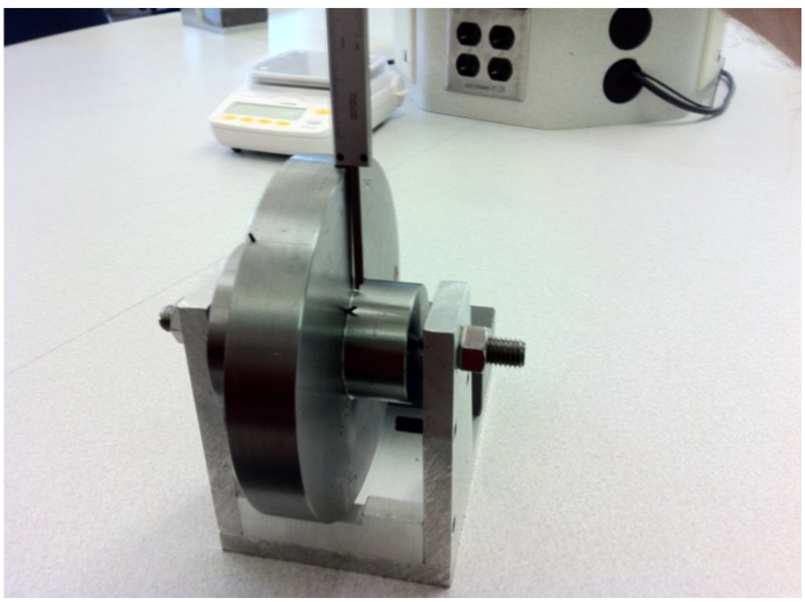
UNC Honor Pledge: I certify that no unauthorized assistance has been received or given in the completion of this work. Signature: ______________________________

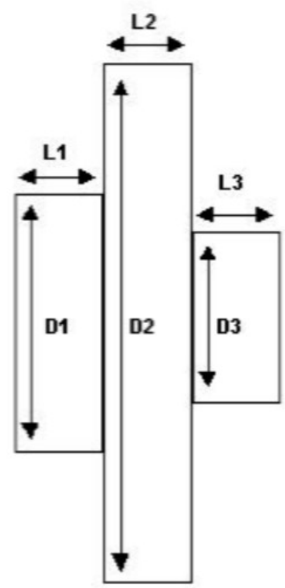
Abstract

In this experiment, we used torque, angular acceleration, and rotational inertia to compare the theoretical moment of inertia for a wheel with the experimentally calculated moment of inertia. The setup is pictured below in the Analysis section, but we did this by tying one end of a string to a fixed point on the wheel and wrapping the string around wheel, then tying the other end of the string to masses of different weights. The masses were then released and allowed to fall, and the time required to fall 0.4 meters was recorded. These measurements, coupled with measurements regarding the dimensions of the wheel, were used to calculate both the expected and actual moment of inertia for the wheel. We discovered that the formula for the torque of the wheel was $\tau = 103.81\alpha + 2.19$. Comparing this to the formula $\tau = I\alpha$, we found the moment of inertia for the wheel to be approximately 104.

Analysis

First, we needed to solve for the expected *I* value: the wheel that the mass hangs from is solid, but the wheel itself is an unusual shape. It can be simplified down to three separate disks put together, as shown in these pictures:





Moment of inertia is dependent on the mass of an object, and the way in which that mass is arranged. In order to calculate the theoretical moment of inertia, we used the formula below, and we found the moment of inertia to be 0.0115 ± 0.0002 kgm^2.

$$I_{\text{wheel}} = \frac{M}{8}\frac{\sum_i L_i D_i^4}{\sum_i L_i D_i^2}$$

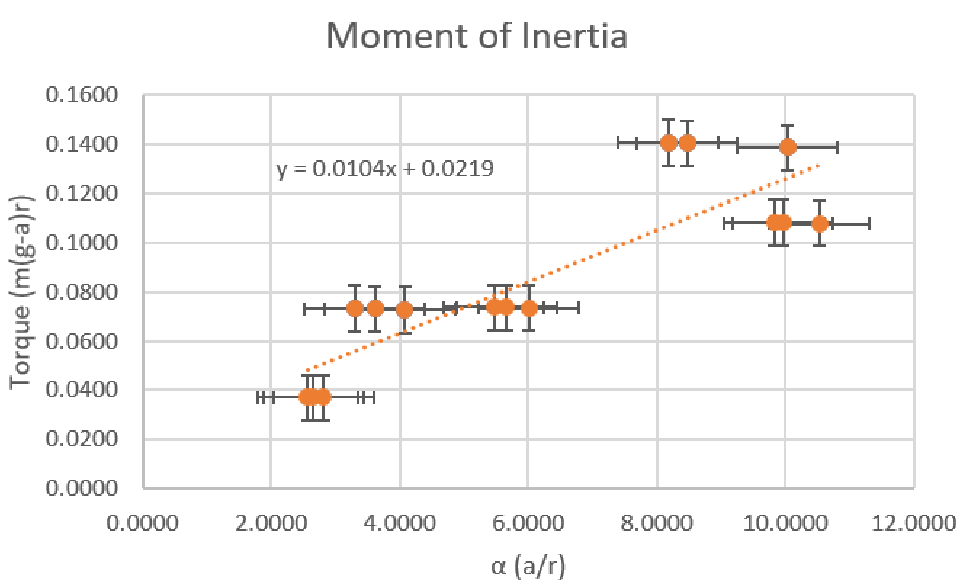
To find the individual contributions of each part of the wheel to the total moment of inertia, we used the equation below. The first disk was 5.7% of the total *I*, the second disk was 93.1%, and the third disk was 1.1%. The variable *d* is the diameter of each disk, as one can see in one of the photos above.

$$\frac{I_1}{I_{total}} = \frac{d_1^4}{d_1^4+d_2^4+d_3^4}$$

Next, we solved for the experimental *I* value. The part of the string in contact with the wheel does not slip, so the magnitude of the linear acceleration for a single point on the hanging part of the string is the same as the magnitude of the tangential acceleration for a point on the string in contact with the wheel. In order to compute the linear acceleration, we recorded the time it took for the mass to drop a certain distance after being released from rest and used one dimensional kinematics to determine the acceleration of the mass. The collected data appears in the table below.

Distance (m)	Time (s)	Acceleration (m/s^2)
0.40	1.11	0.65
0.40	1.02	0.77
0.40	1.13	0.63
0.40	1.70	0.28
0.40	1.60	0.31
0.40	1.78	0.25
0.40	2.73	0.11
0.40	2.70	0.10
0.40	2.75	0.10
0.40	1.45	0.38
0.40	1.41	0.40
0.40	1.46	0.38
0.40	2.40	0.14
0.40	2.36	0.14
0.40	2.29	0.15

To plot the data regarding torque and angular acceleration, we used a scatter plot and linearized the data:



The slope of this graph is the moment of inertia for the wheel. So, our experimental calculations yield a moment of inertia equal to 0.0104 ± 0.002 kgm^2. The y-intercept of this graph is the frictional torque, which is 0.02 ± 0.01 kg $\frac{m^2}{s^2}$. To ascertain these values, we used the LINEST function in Excel.

Discussion

The real and expected moment of inertia values were close to each other. The experimental value was 0.0104 ± 0.002 kgm^2 and the expected value was 0.0115 ± 0.0002 kgm^2. We found that the uncertainties overlap, so the experimental value is reasonable. The small difference could be due to our wheel being greased recently. Because the real value was lower than the expected, and a lower moment of inertia means a torque yields a higher acceleration, our wheel accelerated more quickly. As mentioned earlier, the y-intercept on the graph of our experimental data was the force known in this scenario as frictional torque. While this value does seem a little low, it makes sense given the condition the wheel system was in. A disk that is easier to accelerate and takes a while to slow to a stop (which describes our wheel) must have a small coefficient of kinetic friction.

Each disk's contribution to the total moment of inertia was surprising; despite only having a diameter twice the size of the first disk, the second disk made up over 16 times as much of the moment of inertia, at 93.1%. This is pretty surprising considering the smallest disk had a diameter of one quarter of the largest disk, yet only contributed to around a percent of the total *I*.

When one makes a timing error in this experiment, it can either increase or decrease the calculated acceleration, which is reflected in an even larger error after plugging in the acceleration to multiple formulas. Because some of these formulas deal with such small numbers, slight timing and distance measuring mistakes can cause the experimentally calculated moment of inertia to be quite off. Fortunately, our measurements were fairly accurate, as the actual and expected moment of inertia values were close. Real-life applications of moment of inertia and torque include the way dancers and skaters bring their arms and legs in or out to control rotation while spinning, or the way divers use the same method in conjunction with gravity-induced torque to control their rotation while falling down towards the water. Additionally, one can relate this to the expected motion of our galaxy; based on the location of mass, the spiral arms of our galaxy are moving much more quickly than they are expected to. This incongruence between real and expected lead to the discovery of dark energy and dark matter, which help explain this phenomenon.