281L Lab A-2 Report

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(Physics 281)
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In this experiment, I found both the average wavelength of light emitted by a sodium doublet, and the difference in the two wavelengths emitted. The doublet emitted two wavelength sof light which were relatively quite close to each other. I found constants for the apparatus used, then used those to extrapolate data regarding light from the sodium doublet. The average wavelength was calculated to be 562 ± 37 nm, which is close to the expected value of 589 ± 1 nm, but does not overlap. The difference in wavelengths was calculated to be 0.39 ± 0.03 nm, which was on the same order of magnitude as the expected 0.59 ± 0.01 nm, but far from overlapping.

I. INTRODUCTION

Using a Michelson interferometer, such as the one in this experiment, one can use mirrors to split a beam of light evenly into two perpendicular beams. The interferometer has an adjustable arm that moves a mirror, resulting in projected patterns. Examining these patterns for one of the beams using the naked eye gives us information necessary in calculating the wavelengths of light produced. This information is very useful, because it is connected to many other properties of light, thus turning those properties into tools for gathering information.

II. THEORETICAL BACKGROUND

In order to eventually determine λ and $\Delta\lambda$, we need to know some properties of our interferometer. The mirror is adjusted using a knob, or micrometer, so we must understand how much the mirror moves when the knob is turned. The device projected the fringe pattern from a laser with known attributes onto a whiteboard, and the fringes moved when the knob was turned. The relationship between small changes in the knob's position and the number of fringes moving past a certain point is linear. Using a linear fit gives us a proportionality constant C. Knowing this relationship, we can then calculate the wavelength of sodium light. Once this too is known, the difference in wavelengths follows.

A. Calibrating the Micrometer

Small changes Δm to the micrometer result in small changes Δd to the mirror's position, proportional to some constant C. This can be written as

$$\Delta d = C\Delta m. \tag{1}$$

I calculated Δd for the HeNe laser used in this step by the equation

$$2\Delta d = \lambda N, \tag{2}$$

which relates a rotation Δm of the micrometer and the number of fringes N passing through an arbitrary point on the whiteboard. By recording the values for N as we varied Δm, and using them to plot Δd against Δm as seen in Equation 1, we were able to determine that $C = 1781 \pm 107$ using a linear fit. Now that this value is known, the wavelength λ of the Sodium doublet's light can be found.

B. Finding λ

We then repeated the previous step with the sodium lamp instead of the laser. By adjusting the micrometer and measuring the resulting N values, we were able to plot $2\Delta d$ against N (as it appears in Equation 2). The slope of the plotted line is λ_{Na}, which can be calculated using another linear fit. Doing so produced $\lambda_{Na} = 562.24 \pm 37$nm.

C. Finding $\Delta\lambda$

Finding the difference in the two wavelengths of light requires a derivation in order to easily relate λ and $\Delta\lambda$. Combining the waves of light causes an interference pattern to emerge, and the resulting light can be described using superposition:

$$f(x) = \cos\Big(\frac{2\pi x}{\lambda_{Na} + \Delta\lambda/2}\Big) + \cos\Big(\frac{2\pi x}{\lambda_{Na} - \Delta\lambda/2}\Big). \tag{3}$$

Knowing the trigonometric identity

$$\cos(u) + \cos(v) = 2\cos\Big(\frac{u+v}{2}\Big)\cos\Big(\frac{u-v}{2}\Big) \tag{4}$$

allows us to rewrite Equation 3 as

$$f(x) = 2\cos\Big(\frac{2\pi}{\lambda_{Na}}x\Big)\cos\Big(\frac{\pi\Delta\lambda}{\lambda_{Na}^2}x\Big). \tag{5}$$

The angular frequencies — the coefficients in front of x in the parameters for each cosinusoidal function — describe two waves within the graph of Equation 5. Figure 1,

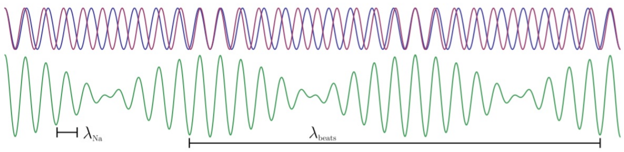

FIG. 1: The wave produced by the constructive and destructive interference between the two individual beams of light from the sodium lamp.

depicting the superposition of these two waves, appears in the Lab Manual [1] for Lab A-II.

The angular frequencies,

$$w_1 = \frac{2\pi}{\lambda_{Na}} \text{and } w_2 = \frac{\pi\Delta\lambda}{\lambda_{Na}^2}, \tag{6}$$

correspond to λ_{Na} and λ_{beats}, respectively. A "beat" is the longer envelope, stretching from one peak, past a second, and ending at a third peak. In our experiment, beats could be observed by rapidly spinning the micrometer knob. The fringe pattern produced shifted between high contrast and low contrast, both occurring at various peaks of the wave associated with λ_{beats}. A beat spans the journey from high contrast, to low contrast, and back to high contrast. This completes half of the wave. Observing this journey, one can measure B, the adjustment of Δd necessary to complete a full beat. Relating B back to w_2 will allow us to solve for λ_{beats}. We know that $\lambda = 2\pi/w$, so

$$\lambda_{\mathrm{beats}} = \frac{2\lambda^2}{\Delta\lambda}. \tag{7}$$

Because the light is reflected off a mirror, it traverses Δd twice. This means we must multiply B by 2 to set it equal to have of a wavelength. Doubling $2B$ gets us our full wavelength. Plugging $4B$ in for λ_{beats} and solving for $\Delta\lambda$ gives

$$\Delta\lambda = \frac{\lambda^2}{2B}. \tag{8}$$

Plugging in our measured values for B and the experimentally found value of λ gives $\Delta\lambda = 0.39 \pm 0.03$ nm.

III. METHODS

For this experiment, the interferometer was setup as shown in Figure 2, an image from the Lab Manual [1] for Lab A-II.

First, we carefully adjusted the vertical and horizontal components of the mirror's position in order to align the pattern it projected onto the whiteboard when the laser was pointed at the mirror.

Next, we solved for C by spinning the micrometer knob and observing the resulting fringe pattern motion when the beam splitter projected the laser beam onto the whiteboard. To count the number of fringes, we drew a dot on the whiteboard, and counted the number of fringes that moved past that dot. This was done ten times.

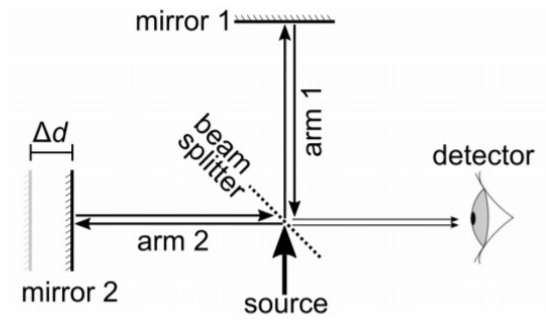

FIG. 2: The beam splitter of the interferometer, a half-silvered mirror, split the incoming beams of light into two perpendicular beams of light.

Then, we replaced the laser with the sodium lamp. The light wasn't bright enough to be observed when projected onto the whiteboard, so we examined the detector, as shown in Figure 2. Similarly to the previous step, we counted the number of fringes moving past a certain arbitrary point in the detector's view. The knob had to be spun carefully, as it was very sensitive. This step was used to calculate the average wavelength of light produced by the sodium doublet. These measurements were taken nine times.

Finally, we investigated the behavior of the fringes in the detector's view as they varied between high and low contrast. The micrometer knob had to be spun much more quickly in order to notice these variations. This was completed seven times, one of which was an outlier that was thrown out due to an apparent error. After collecting the data, plotting the relationships to extrapolate values, and using the equations described in the Theoretical Background, we were able to solve for λ and $\Delta\lambda$.

IV. RESULTS

The data from the first part of the experiment — finding the relation between Δm and Δd — was plotted linearly in accordance with Equation 1. The uncertainty in N was estimated to be ± 1. C was found by calculating the slope of the trendline for the plot, and its uncertainty was 107 units. This was found using the LINEST function in Excel.

Solving for the average wavelength of sodium light yielded $\lambda = 562 \pm 37$ nm. The uncertainty in our measurement of Δm was ± 0.3. The value of λ was found by looking at the slope of the trendline for the plot of $2C\Delta m$ versus N. The uncertainty was calculated using the LINEST function. Unfortunately, the experimentally-calculated wavelength does not overlap with the known wavelength of 589 ± 1 nm.

Solving for the difference in wavelengths produced a value of $\Delta\lambda = 0.39 \pm 0.03$ nm. When finding the beats, our uncertainty in Δm was ± 50. The uncertainty in $2B$ was found using the standard deviation, because we averaged our $2B$ values before plugging them into Equation 8. The uncertainty in $\Delta\lambda$ was propagated through quadrature. Our experimentally-calculated value does not overlap with the known difference of 0.59 ± 0.01 nm.

V. DISCUSSION AND CONCLUSIONS

During the experiment, it was difficult to obtain precise measurements. Counting the fringes was difficult because of the high sensitivity of the micrometer knob. Measuring the beats was difficult because it was hard to notice the difference in low and high contrast. There was one of the data points that we had to throw out, as it was an outlier; I struggled to keep track of the contrast during that iteration of the measurement.

Although our experimentally-found values do not overlap with the known values, they are still on the same orders of magnitude. Considering the uncertainty and difficulty with some measurements in the experiment, our obtained values were surprisingly close to the known values.

VI. ACKNOWLEDGMENTS

During the experiment, I worked with my lab partner Emma to perform the experiment, and we passed data on to our other partner, Megan, who transcribed the data in Microsoft Excel and calculated some of our values with uncertainty.

[1] PHYS 281L Lab A-II Manual `https://sakai.unc.edu/access/content/group/41081125-b301-4ad9-8173-970601749ad6/Lab\%20Manuals/Lab\%20A-II\%3A\%20Sodium\%20Doublet/Lab\%20A-II\%20Sodium\%20Doublet.pdf`