HW-1
Regular languages and FA

Igumnov Oleg

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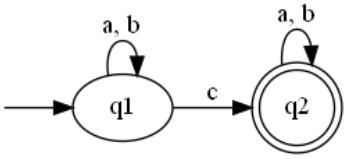
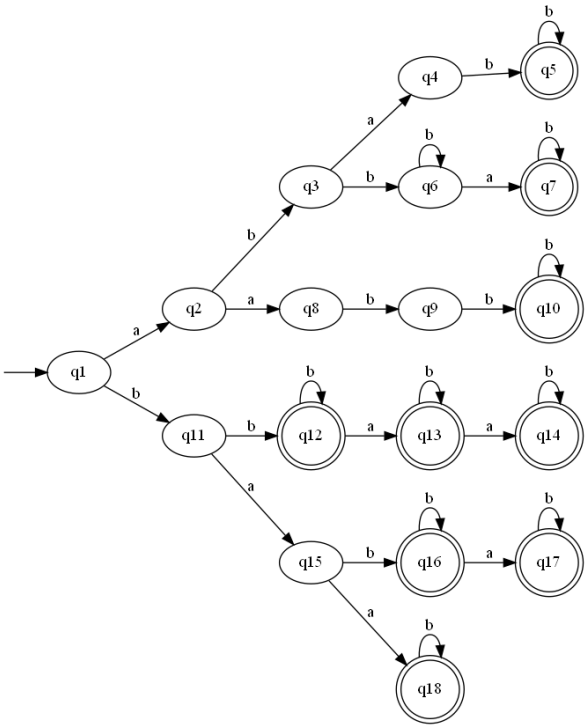
1 Task №1. Construct DFA recognizing language

1.1 $L = \{\omega \in \{a, b, c\}^* || \omega|_c = 1\}$



1.2 $L = \{\omega \in \{a, b\}^* || \omega|_a \leq 2, |\omega|_b \geq 2\}$

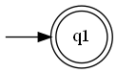


1.3 $L = \{\omega \in \{a, b\}^* || \omega|_a \neq |\omega|_b\}$

It is impossible to construct DFA, since here we need to remember amount

1.4 $L = \{\omega \in \{a, b\}^* | \omega\omega = \omega\omega\omega\}$

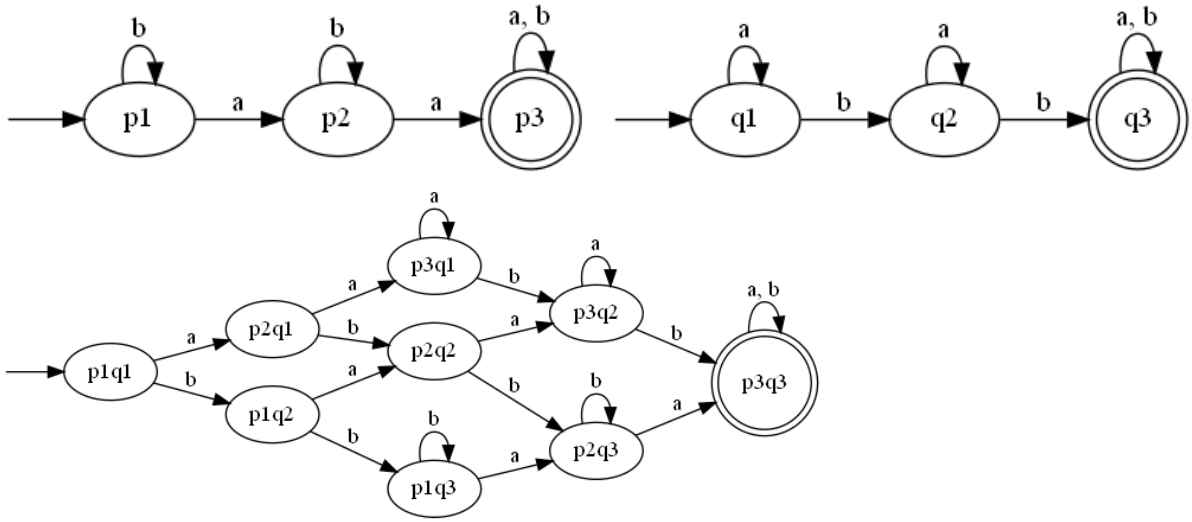
$\omega = \lambda$



2 Task №2. Construct FA using Cartesian product

2.1 $L_1 = \{\omega \in \{a, b\} | |\omega|_a \geq 2 \wedge |\omega|_b \geq 2\}$

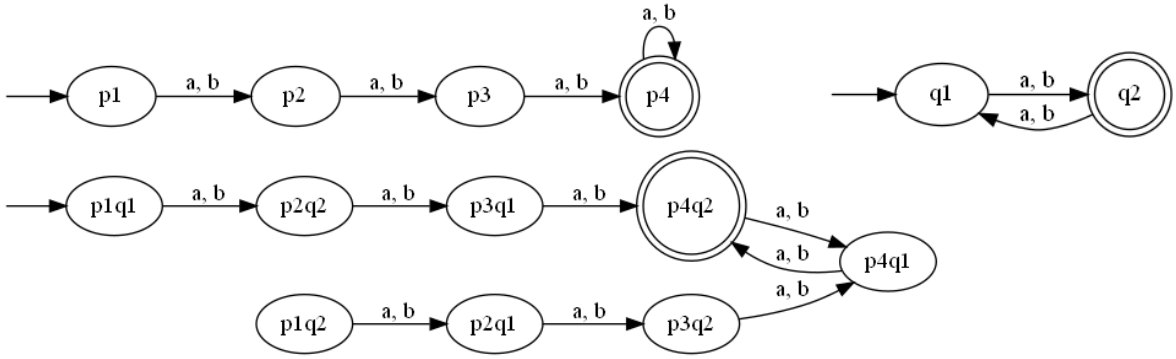
Let's construct 2 DFA:



$\Sigma = \Sigma_1 \cup \Sigma_2 = \{a, b\}$
$Q = Q_1 \times Q_2 = \{p_1q_1, p_1q_2, p_1q_3, p_2q_1, p_2q_2, p_2q_3, p_3q_1, p_3q_2, p_3q_3, \}$
$s = (s_1, s_2) = (p_1, q_1)$
$T = T_1 \times T_2 = \{p_3q_3\}$
$\delta(p_1q_1, a) = p_2q_1, \delta(p_1q_1, b) = p_1q_2, \delta(p_1q_2, a) = p_2q_2, \delta(p_1q_2, b) = p_1q_3, \delta(p_1q_3, a) = p_2q_3, \delta(p_1q_3, b) = p_1q_3, \delta(p_2q_1, a) = p_3q_1, \delta(p_2q_1, b) = p_2q_2, \delta(p_2q_2, a) = p_3q_2, \delta(p_2q_2, b) = p_2q_3, \delta(p_2q_3, a) = p_3q_3, \delta(p_2q_3, b) = p_2q_3, \delta(p_3q_1, a) = p_3q_1, \delta(p_3q_1, b) = p_3q_2, \delta(p_3q_2, a) = p_3q_2, \delta(p_3q_2, b) = p_3q_3, \delta(p_3q_3, a) = p_3q_3, \delta(p_3q_3, b) = p_3q_3$

2.2 $L_2 = \{\omega \in \{a, b\}^* | |\omega| \geq 3 \wedge |\omega|$ **odd**$\}$

Let's construct 2 DFA:



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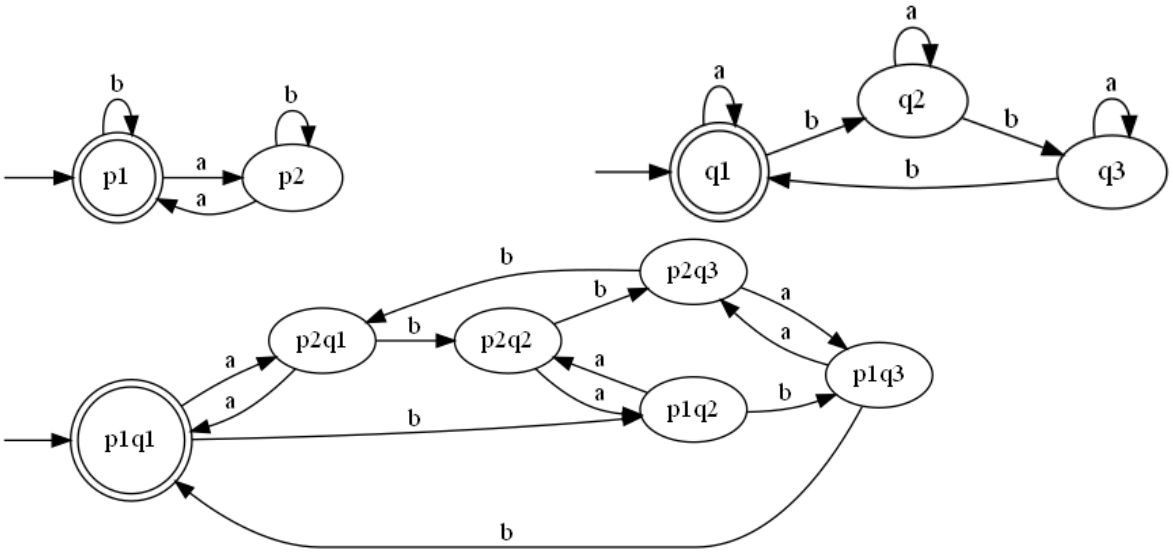
2.3 $L_3 = \{\omega \in \{a, b\}^* || \omega|_a$ **even** $\wedge\ |\omega|_b \vdots 3\}$

Let's construct 2 DFA:



$\Sigma = \Sigma_1 \cup \Sigma_2 = \{a, b\}$
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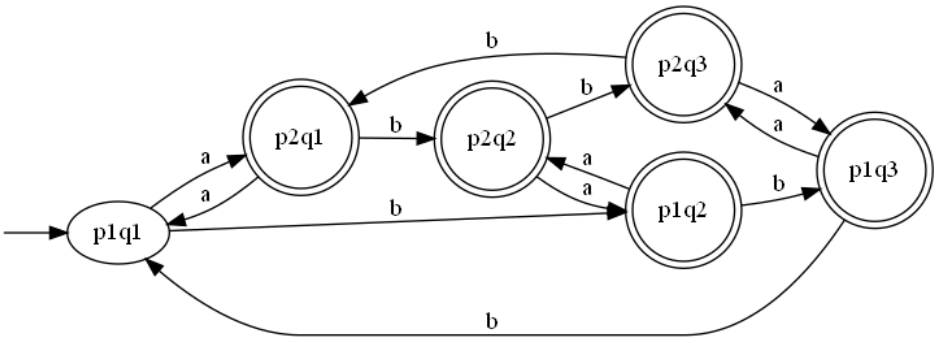
2.4 $L_4 = \overline{L_3}$



$T = \{p_1q_2, p_1q_3, p_2q_1, p_2q_2, p_2q_3\}$

2.5 $L_5 = L_2 \backslash L_3$

$L_5 = L_2 \backslash L_3 = L_2 \cap \overline{L_3} = L_2 \cap L_4$
$T = T_2 \times (Q_4 \backslash T_4) = \{p_4 q_2 r_1 t_1\}$
$\Sigma = \Sigma_2 \cup \Sigma_4 = \{a, b\}$
$s = (s_2, s_4) = (p_1, q_1, r_1, t_1)$
$Q = Q_2 \times Q_4 = \{...\}$
$|Q| = 48$

3 Task №3. Construct minimal DFA by regular expression

3.1 $(ab + aba)^* a$

3.2 $a(a(ab)^* b)^* (ab)^*$

3.3 $(a + (a + b)(a + b)b)^*$

3.4 $(b + c)((ab)^* c + (ba)^*)^*$

3.5 $(a + b)^+ (aa + bb + abab + baba)(a + b)^+$

4 Task №4. Determine whether the language is regular or not

if it's regular construct FA else proof that language is irregular using pumping lemma

4.1 $L = \{(aab)^n b (aba)^m | n \geq 0, m \geq 0\}$

4.2 $L = \{uaav | u \in \{a, b\}^*, v \in a, b^*, |u|_b \geq |v|_a\}$

4.3 $L = \{a^m \omega | \omega \in \{a, b\}^*, 1 \leq |\omega|_b \leq m\}$

4.4 $L = \{a^k b^m a^n | k = n \vee m > 0\}$

4.5 $L = \{ucv | u \in \{a, b\}^*, v \in \{a, b\}^*, u \neq v^R\}$

5 Task №5. Implement algorithms