ToC
HW-1
Regular languages and FA

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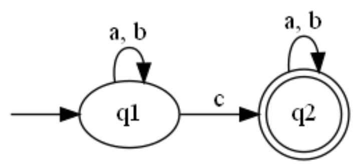
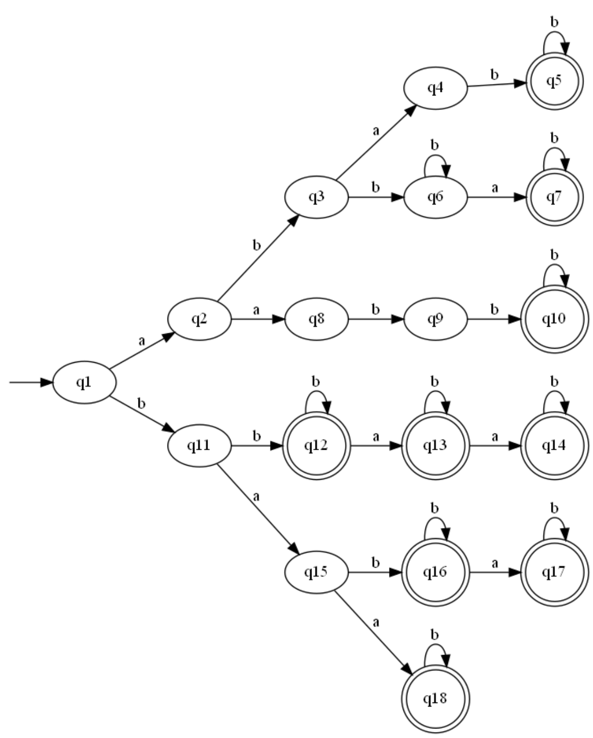
1 Task №1. Construct DFA recognizing language

1.1 $L = \{\omega \in \{a, b, c\}^* | |\omega|_c = 1\}$



1.2 $L = \{\omega \in \{a, b\}^* | |\omega|_a \leq 2, |\omega|_b \geq 2\}$

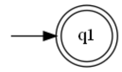


1.3 $L = \{\omega \in \{a, b\}^* | |\omega|_a \neq |\omega|_b\}$

It is impossible to construct DFA, since here we need to remember amount

1.4 $L = \{\omega \in \{a, b\}^* | \omega\omega = \omega\omega\omega\}$

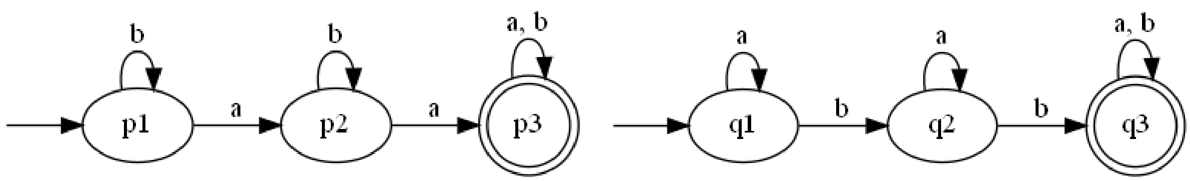
$\omega = \lambda$

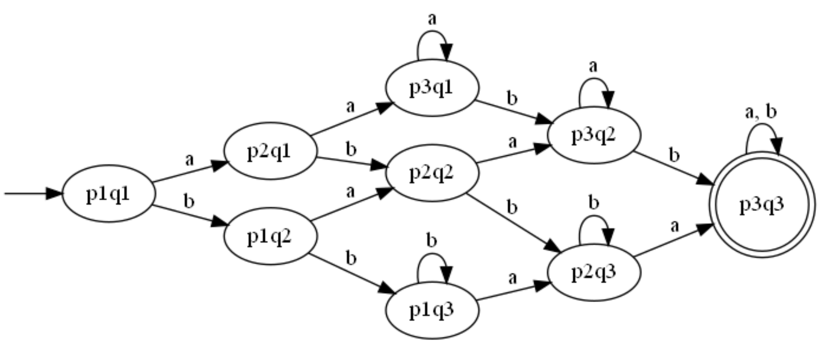


2 Task №2. Construct FA using Cartesian product

2.1 $L_1 = \{\omega \in \{a, b\} | |\omega|_a \geq 2 \wedge |\omega|_b \geq 2\}$

Let's construct 2 DFA:

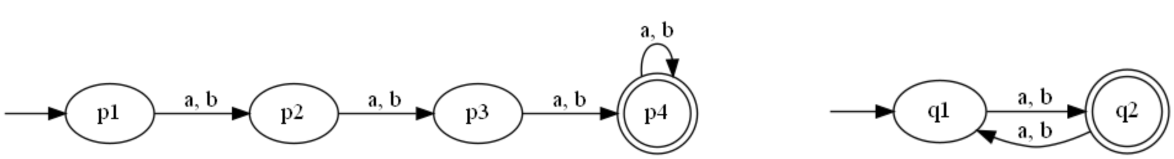


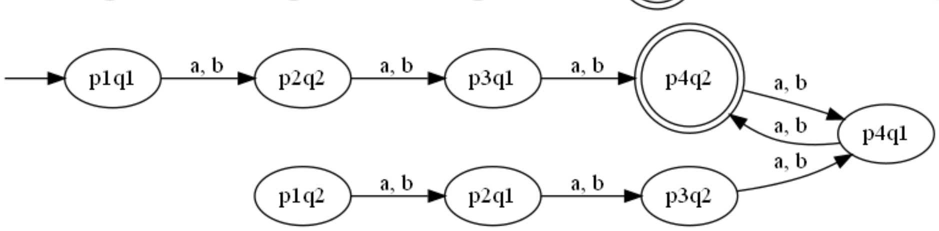


$\Sigma = \Sigma_1 \cup \Sigma_2 = \{a, b\}$
$Q = Q_1 \times Q_2 = \{p_1q_1, p_1q_2, p_1q_3, p_2q_1, p_2q_2, p_2q_3, p_3q_1, p_3q_2, p_3q_3, \}$
$s = (s_1, s_2) = (p_1, q_1)$
$T = T_1 \times T_2 = \{p_3q_3\}$
$\delta(p_1q_1, a) = p_2q_1, \delta(p_1q_1, b) = p_1q_2, \delta(p_1q_2, a) = p_2q_2, \delta(p_1q_2, b) = p_1q_3, \delta(p_1q_3, a) = p_2q_3, \delta(p_1q_3, b) = p_1q_3, \delta(p_2q_1, a) = p_3q_1, \delta(p_2q_1, b) = p_2q_2, \delta(p_2q_2, a) = p_3q_2, \delta(p_2q_2, b) = p_2q_3, \delta(p_2q_3, a) = p_3q_3, \delta(p_2q_3, b) = p_2q_3, \delta(p_3q_1, a) = p_3q_1, \delta(p_3q_1, b) = p_3q_2, \delta(p_3q_2, a) = p_3q_2, \delta(p_3q_2, b) = p_3q_3, \delta(p_3q_3, a) = p_3q_3, \delta(p_3q_3, b) = p_3q_3$

2.2 $L_2 = \{\omega \in \{a, b\}^* | |\omega| \geq 3 \wedge |\omega| \textbf{ odd}\}$

Let's construct 2 DFA:

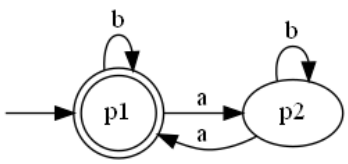


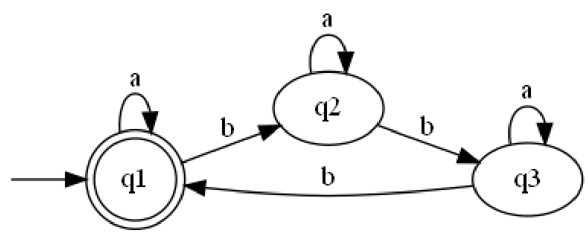


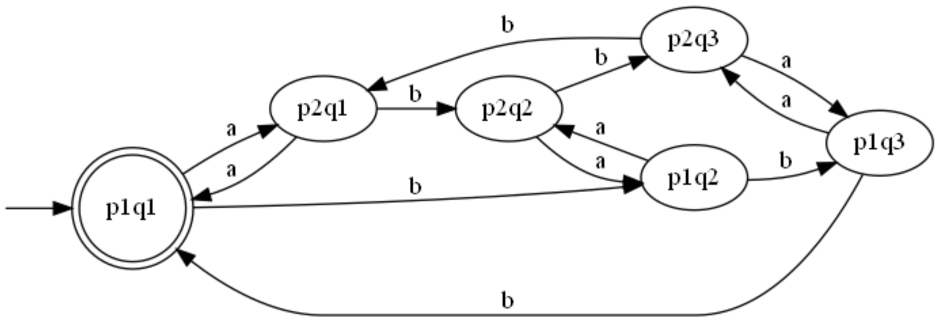
$\Sigma = \Sigma_1 \cup \Sigma_2 = \{a, b\}$
$Q = Q_1 \times Q_2 = \{p_1q_1, p_1q_2, p_2q_1, p_2q_2, p_3q_1, p_3q_2, p_4q_1, p_4q_2, \}$
$s = (s_1, s_2) = (p_1, q_1)$
$T = T_1 \times T_2 = \{p_4q_2\}$
$\delta(p_1q_1, a) = p_2q_2, \delta(p_1q_1, b) = p_2q_2, \delta(p_1q_2, a) = p_2q_1, \delta(p_1q_2, b) = p_2q_1, \delta(p_2q_1, a) = p_3q_2, \delta(p_2q_1, b) = p_3q_2, \delta(p_2q_2, a) = p_3q_1, \delta(p_2q_2, b) = p_3q_1, \delta(p_3q_1, a) = p_4q_2, \delta(p_3q_1, b) = p_4q_2, \delta(p_3q_2, a) = p_4q_1, \delta(p_3q_2, b) = p_4q_1, \delta(p_4q_1, a) = p_4q_2, \delta(p_4q_1, b) = p_4q_2, \delta(p_4q_2, a) = p_4q_1, \delta(p_4q_2, b) = p_4q_1$

2.3 $L_3 = \{\omega \in \{a, b\}^* | |\omega|_a$ **even** $\wedge |\omega|_b \vdots 3\}$

Let's construct 2 DFA:

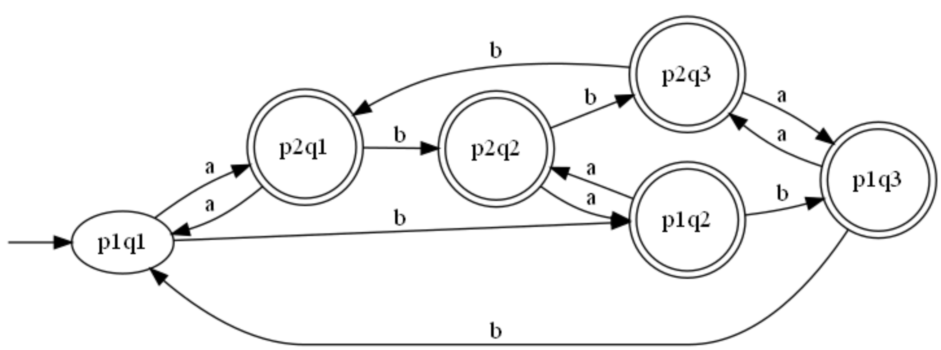






$\Sigma = \Sigma_1 \cup \Sigma_2 = \{a, b\}$
$Q = Q_1 \times Q_2 = \{p_1q_1, p_1q_2, p_1q_3, p_2q_1, p_2q_2, p_2q_3\}$
$s = (s_1, s_2) = (p_1, q_1)$
$T = T_1 \times T_2 = \{p_1q_1\}$
$\delta(p_1q_1, a) = p_2q_1, \delta(p_1q_1, b) = p_1q_2, \delta(p_1q_2, a) = p_2q_2, \delta(p_1q_2, b) = p_1q_3, \delta(p_1q_3, a) = p_2q_3, \delta(p_1q_3, b) = p_1q_1, \delta(p_2q_1, a) = p_1q_1, \delta(p_2q_1, b) = p_2q_2, \delta(p_2q_2, a) = p_1q_2, \delta(p_2q_2, b) = p_2q_3, \delta(p_2q_3, a) = p_1q_3, \delta(p_2q_3, b) = p_2q_1$

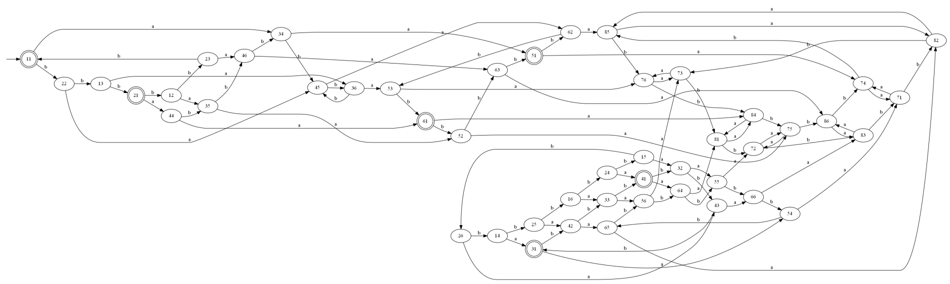
2.4 $L_4 = \overline{L_3}$



$T = \{p_1q_2, p_1q_3, p_2q_1, p_2q_2, p_2q_3\}$

2.5 $L_5 = L_2 \backslash L_3$

$L_5 = L_2 \backslash L_3 = L_2 \cap \overline{L_3} = L_2 \cap L_4$
$T = T_2 \times (Q_4 \backslash T_4) = \{p_4 q_2 r_1 t_1\}$
$\Sigma = \Sigma_2 \cup \Sigma_4 = \{a, b\}$
$s = (s_2, s_4) = (p_1, q_1, r_1, t_1)$
$Q = Q_2 \times Q_4 = \{...\}$
$|Q| = 48$



3 Task №3. Construct minimal DFA by regular expression

3.1 $(ab + aba)^* a$

Let's construct NFA with ε, then construct NFA without ε, then construct DFA using transition table

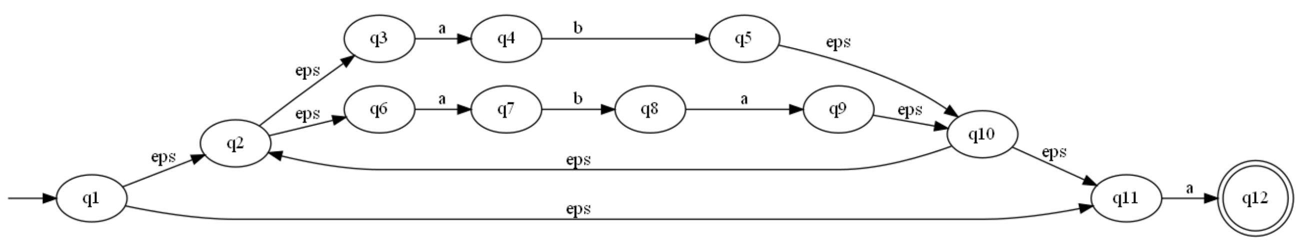


Figure 1: εNFA

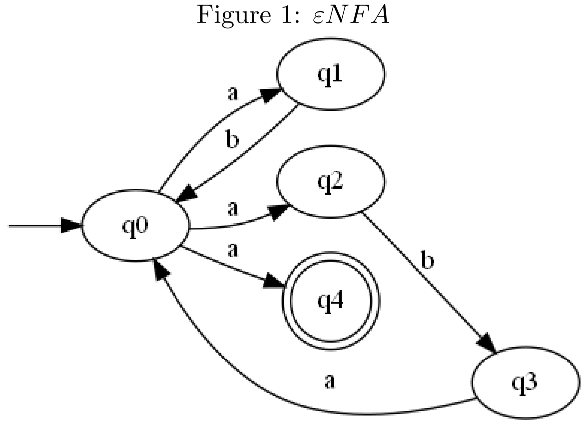


Figure 2: NFA

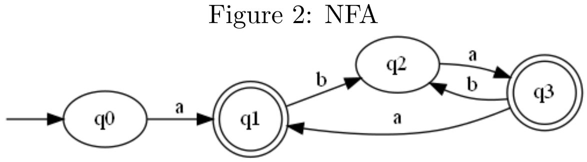


Figure 3: min DFA

NFA		
	a	b
q_0	$\{q_1, q_2, q_4\}$	$\emptyset$
q_1	$\emptyset$	q_0
q_2	$\emptyset$	q_3
q_3	q_0	$\emptyset$
q_4	$\emptyset$	$\emptyset$

DFA		
	a	b
q0	$q_1 q_2 q_4$	$\emptyset$
$q_1 q_2 q_4$	$\emptyset$	$q_0 q_3$
$q_0 q_3$	$q_0 q_1 q_2 q_4$	$\emptyset$
$q_0 q_1 q_2 q_4$	$q_1 q_2 q_4$	$q_0 q_3$

Finally let's rename nodes: $q_0 = q_0, q_1 = q_1 q_2 q_4, q_2 = q_0 q_3, q_3 = q_0 q_1 q_2 q_4$
Minimization:
eq 0: {q0, q2} {q1, q3}
eq 1: {q0, q2} {q1, q3}
Constructed DFA is already minimal

3.2 $a(a(ab)^*b)^*(ab)^*$

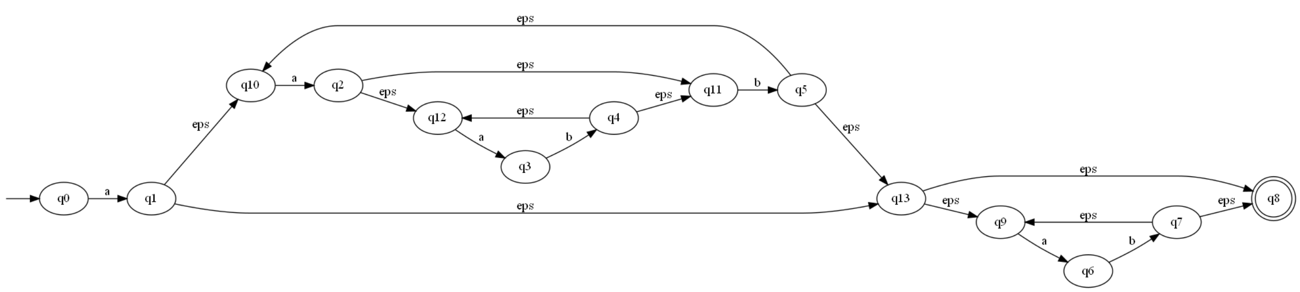


Figure 4: εNFA

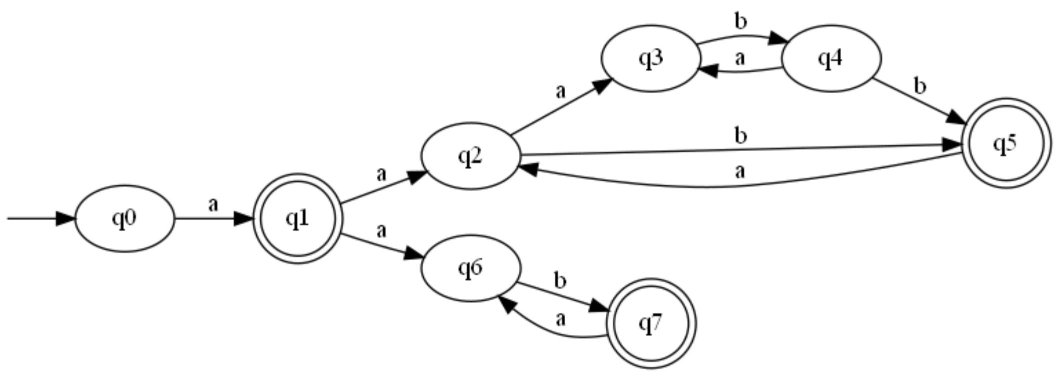


Figure 5: NFA

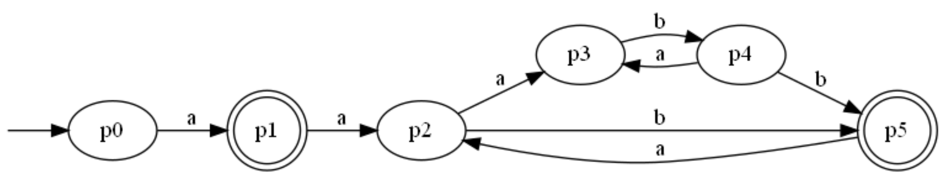


Figure 6: DFA

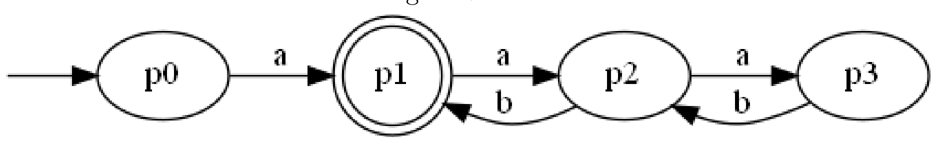


Figure 7: min DFA

NFA to DFA

NFA		
	a	b
q_0	q_1	$\emptyset$
q_1	$\{q_2, q_6\}$	$\emptyset$
$\{q_2, q_6\}$	q_3	$\{q_5, q_7\}$
q_3	$\emptyset$	q_4
$\{q_5, q_7\}$	$\{q_2, q_6\}$	$\emptyset$
q_4	q_3	q_5

Minimization:
eq 0: {p0, p2, p3, p4} {p1, p5}
eq 1: {p0, p2, p4} {p3} {p1, p5}
eq 2: {p2, p4} {p0} {p3} {p1, p5}
eq 3: {p2, p4} {p0} {p3} {p1, p5}

3.3 $(a + (a + b)(a + b)b)^*$

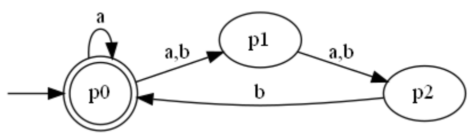


Figure 8: NFA

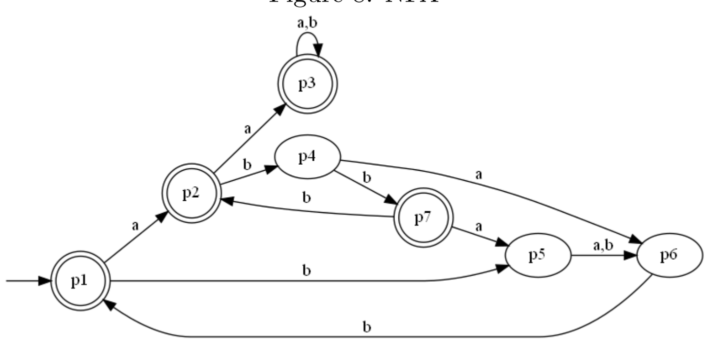


Figure 9: DFA, min DFA

NFA to DFA

NFA		
	a	b
q_1	$\{q_1, q_2\}$	q_2
$\{q_1, q_2\}$	$\{q_1, q_2, q_3\}$	$\{q_2, q_3\}$
$\{q_1, q_2, q_3\}$	$\{q_1, q_2, q_3\}$	$\{q_1, q_2, q_3\}$
q_2	q_3	q_3
$\{q_2, q_3\}$	q_3	$\{q_1, q_3\}$
$\{q_1, q_3\}$	q_2	$\{q_1, q_2\}$
q_3	$\emptyset$	q_1

Minimization:
eq 0: {p4, p5, p6} {p1, p2, p3, p7}
eq 1: {p4, p6} { p5} {p1, p2} {p3} {p7}
eq 2: {p1} {p2} {p3} {p4} {p5} {p6} {p7}
Constructed DFA is already minimal

3.4 $(b + c)((ab)^*c + (ba)^*)^*$

Minimization:

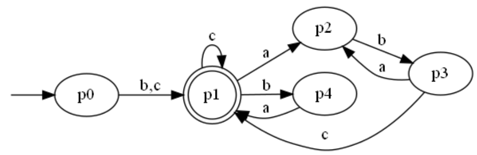


Figure 10: 3-4 min DFA

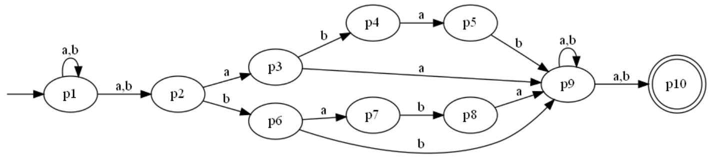


Figure 11: NFA,3-5

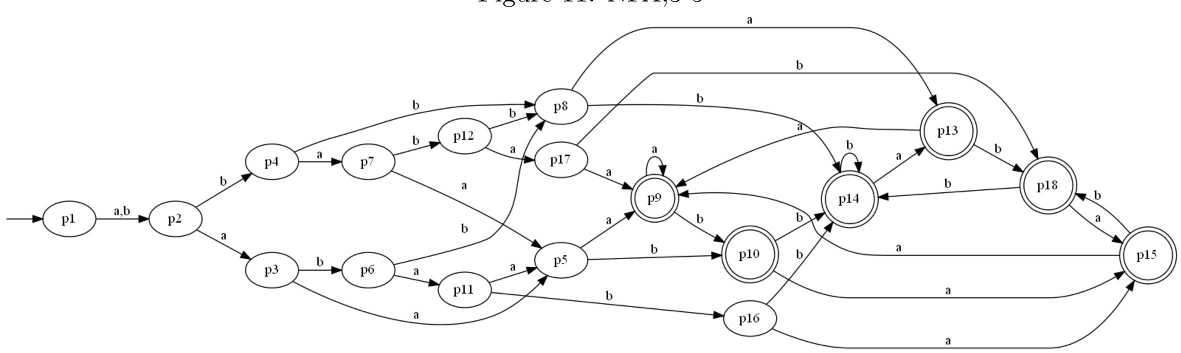


Figure 12: DFA,3-5

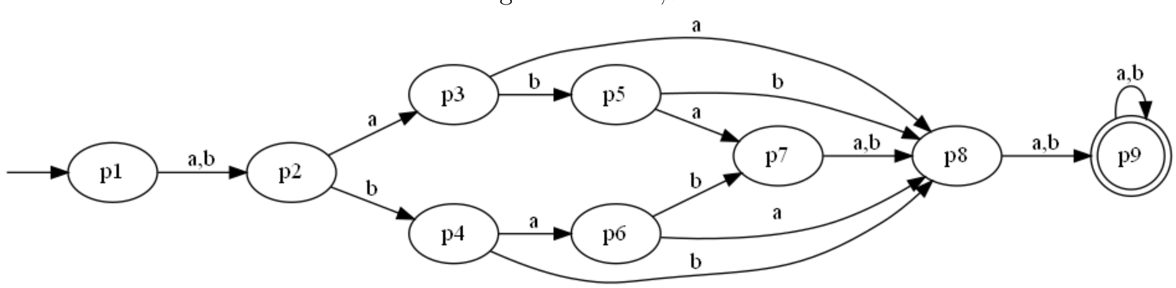


Figure 13: min DFA,3-5

eq0: {1,3,4,5} {2}
eq: {1}{2}{3}{4} {5}

3.5 $(a+b)^+(aa+bb+abab+baba)(a+b)^+$

Minimization:
eq0: {1,2,3,4,5,6,7,8,11,12,16,17} {9,10,13,14,15,18}
eq1: {1,2,3,4,6,7,11,12} {5,8,16,17} {9,10,13,14,15,18}
eq2: {1,2} {3,7} {4,6} {11,12} {5,8,16,17} {9,10,13,14,15,18}
eq3: {1} {2} {3} {4} {6} {7} {11,12} {5,8,16,17} {9,10,13,14,15,18}
eq4: {1} {2} {3} {4} {6} {7} {11,12} {5,8,16,17} {9,10,13,14,15,18}

NFA to DFA		
	a	b
p_1	p_2	p_2
p_2	p_3	p_4
p_3	p_5	p_6
p_4	p_7	p_8
p_5	p_9	p_{10}
p_6	p_{11}	p_8
p_7	p_5	p_{12}
p_8	p_{13}	p_{14}
p_9	p_9	p_{10}
p_{10}	p_{15}	p_{14}
p_{11}	p_5	p_{16}
p_{12}	p_{17}	p_8
p_{13}	p_9	p_{18}
p_{14}	p_{13}	p_{14}
p_{15}	p_9	p_{18}
p_{16}	p_{15}	p_{14}
p_{17}	p_9	p_{18}
p_{18}	p_{15}	p_{14}

Min		
	a	b
a_1	a_2	a_2
a_2	a_3	a_4
a_3	a_8	a_5
a_4	a_6	a_8
a_5	a_7	a_8
a_6	a_8	a_7
a_7	a_8	a_8
a_8	a_9	a_9
a_9	a_9	a_9

4 Task №4. Determine whether the language is regular or not

if it's regular construct FA else proof that language is irregular using pumping lemma

4.1 $L = \{(aab)^n b(aba)^m | n \geq 0, m \geq 0\}$

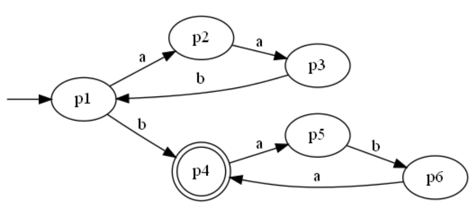


Figure 14: 4-1 DFA

4.2 $L = \{uaav | u \in \{a, b\}^*, v \in a, b^*, |u|_b \geq |v|_a\}$

Fix n.

$\omega = b^n aaa^n, |\omega| \geq n$

$\omega = xyz, |y| \geq 1, |xy| \leq n$

x=$b^f, y = b^l, z = b^{n-f-l} aaa^n$

f+l $\leq n, l \geq 1$

$\omega = xy^kz = b^fb^{kl}b^{n-f-l}aaa^n$
if k=0 then $\omega = b^fb^{n-f-l}aaa^n = b^{n-l}aaa^n$
$|u|_b \geq |v|_a$ isn't true
lemma not true
L is not regular

4.3 $L = \{a^m\omega|\omega \in \{a,b\}^*, 1 \leq |\omega|_b \leq m\}$

Fix n.
$\omega = a^nb^n, |\omega| \geq n$
$\omega = xyz, |y| \geq 1, |xy| \leq n$
x=a$^f, y = a^l, z = a^{n-f-l}b^n$
f+l $\leq n, l \geq 1$
$\omega = xy^kz = a^fa^{kl}a^{n-f-l}b^n$
if k=0 then $\omega = a^{n-l}b^n$
$1 \leq |\omega|_b \leq m$ isn't true
lemma not true
L is not regular

4.4 $L = \{a^kb^ma^n|k = n \vee m > 0\}$

Fix n.
$\omega = a^nba^n, |\omega| \geq n$
$\omega = xyz, |y| \geq 1, |xy| \leq n$
x=a$^f, y = a^l, z = a^{n-f-l}ba^n$
f+l $\leq n, l \geq 1$
$\omega = xy^kz = a^{n+k(l-1)}ba^n$
lemma not true
L is not regular

4.5 $L = \{ucv|u \in \{a,b\}^*, v \in \{a,b\}^*, u \neq v^R\}$

$\omega = (ab)^nc(ba)^n = \alpha_1...\alpha_{4n+1}$
$|\omega| \geq n$, $\omega = xyz$
$x = \alpha_1...\alpha_i$, $y = \alpha_{i+1}...\alpha_{i+j}$,
$z = \alpha_{i+j+1}...\alpha_{2n}c(ba)^n$
i+j $\leq n, j \geq 1$
$|xy| \leq n$
$|y| \geq 1$
$\omega = xy^kz = \alpha_1...\alpha_i(\alpha_{i+1}...\alpha_{i+j})^k\alpha_{i+j+1}...\alpha_{2n}c(ba)^n$
lemma not true
L is not regular

5 Task №5. Implement algorithms