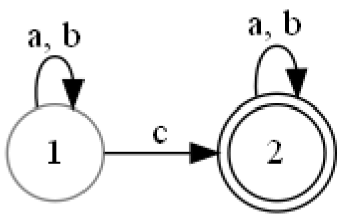
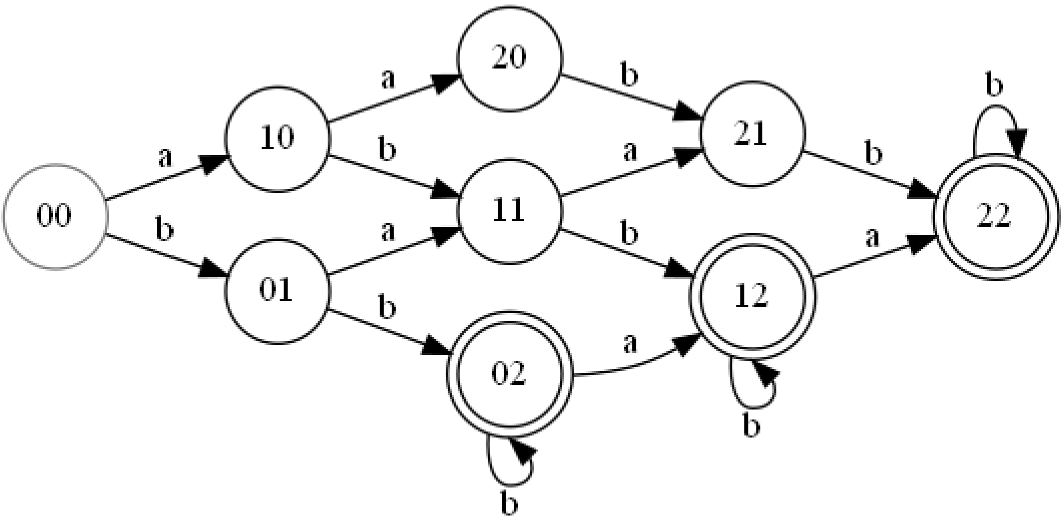
Задание 1

1. $$L = \{\omega \in \{a, b, c\}^* \mid |\omega|_c = 1\}$$



2. $$L = \{\omega \in \{a, b\}^* \mid |\omega|_a \leq 2, |\omega|_b \geq 2\}$$

вершины имеют названия ij, где i - количество встретившихся "a", j - количество встретившихся "b"



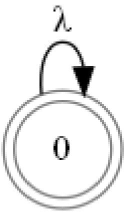
3. $$L = \{\omega \in \{a, b\}^* \mid |\omega|_a \neq |\omega|_b\}$$

т.к. $\overline{L} = \{\omega \in \{a, b\}^* \mid |\omega|_a = |\omega|_b\}$ не является регулярным, то и L не является регулярным языком, значит не возможно построить автомат, распознающий данный язык. Доказательство:

$$\begin{aligned} \omega &= a^n b^n \in \overline{L} \\ |\omega| &= 2n \geq n \\ xy &= a^i a^j, \quad i + j \leq n \\ \omega &= a^i a^j a^{n-i-j} b^n \\ \omega &= a^i a^{jk} a^{n-i-j} b^n \notin \overline{L} \quad, k > 1 \end{aligned}$$

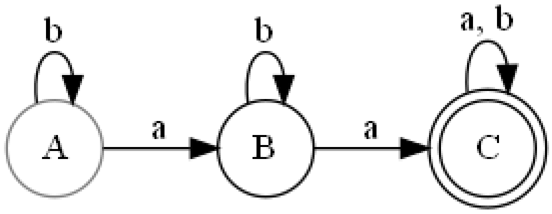
4. $$L = \{\omega \in \{a, b\}^* \mid \omega\omega = \omega\omega\omega\}$$

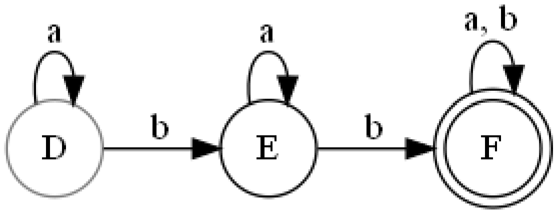
Если $|\omega| > 0$, то $\omega\omega \neq \omega\omega\omega$ значит язык состоит из пустого слова, $\lambda\lambda = \lambda\lambda\lambda = \lambda$



Задание 2

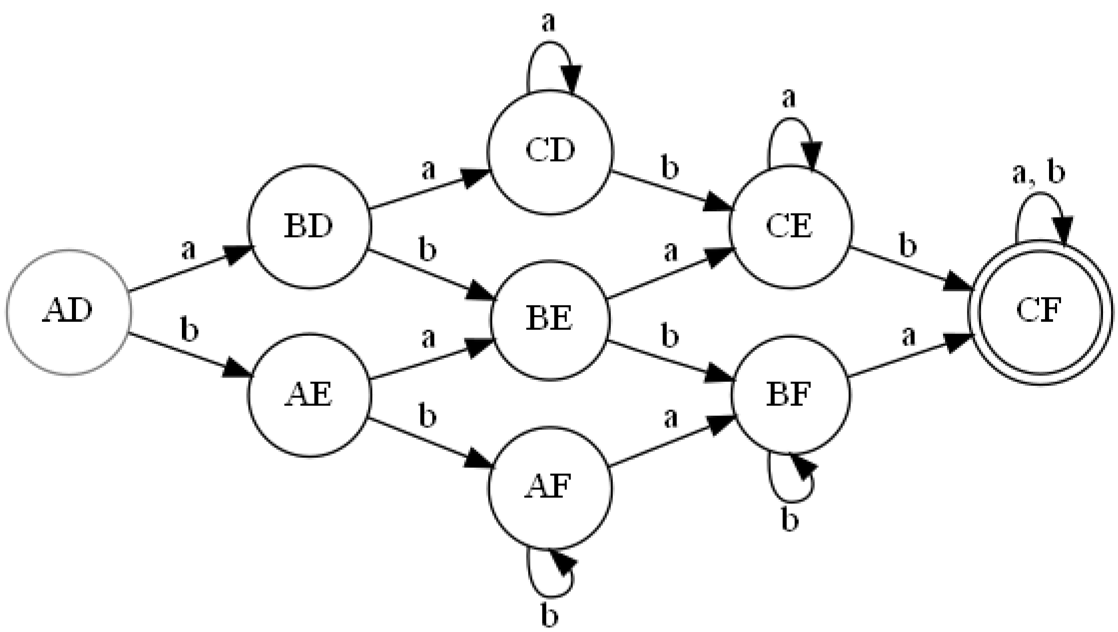
1. $L_1 = \{\omega \in \{a, b\}^* \mid |\omega|_a \geq 2 \wedge |\omega|_b \geq 2\}$
   $L_1 = \{\omega \in \{a, b\}^* \mid |\omega|_a \geq 2\} \cap \{\omega \in \{a, b\}^* \mid |\omega|_b \geq 2\}$



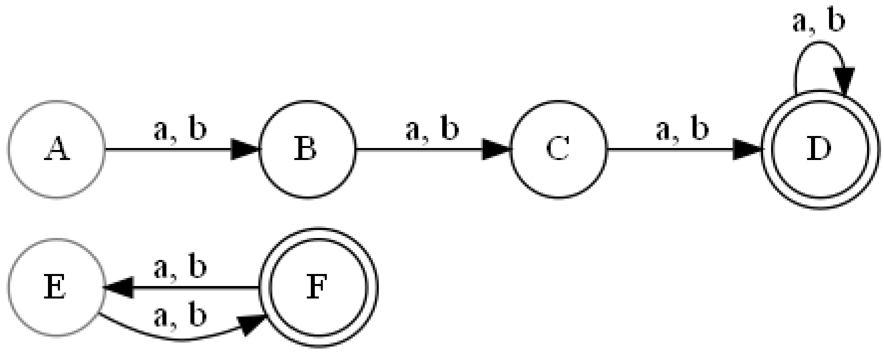


$$
\begin{aligned}
\Sigma &= a, b \\
Q &= \{AD, AE, AF, BD, BE, BF, CD, CE, CF\} \\
S &= AD \\
T &= CF
\end{aligned}
$$

$$
\begin{array}{lll}
\delta(AD, a) = BD & \delta(BD, a) = CD & \delta(CD, a) = CD \\
\delta(AD, b) = AE & \delta(BD, b) = BE & \delta(CD, b) = CE \\
\delta(AE, a) = BE & \delta(BE, a) = CE & \delta(CE, a) = CE \\
\delta(AE, b) = AF & \delta(BE, b) = BF & \delta(CE, b) = CF \\
\delta(AF, a) = BF & \delta(BF, a) = CF & \delta(CF, a) = CF \\
\delta(AF, b) = AF & \delta(BF, b) = BF & \delta(CF, b) = CF
\end{array}
$$

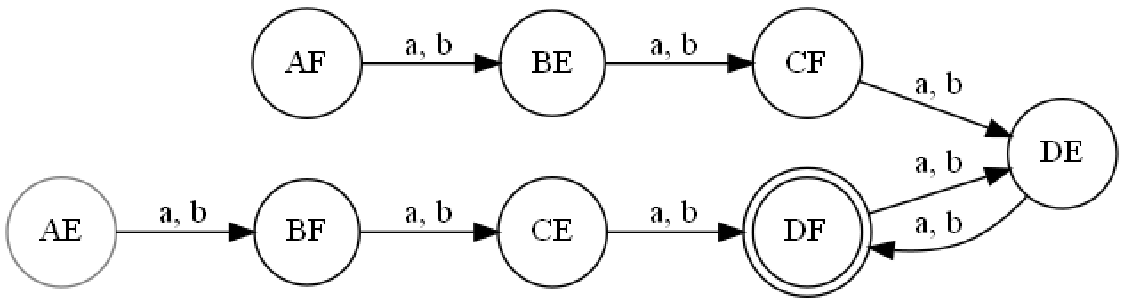


2. $L_2 = \{\omega \in \{a,b\}^* \mid |\omega| \geq 3 \wedge |\omega| \textit{нечетно}\}$
$L_2 = \{\omega \in \{a,b\}^* \mid |\omega| \geq 3\} \cap \{\omega \in \{a,b\}^* \mid |\omega| \textit{нечетно}\}$



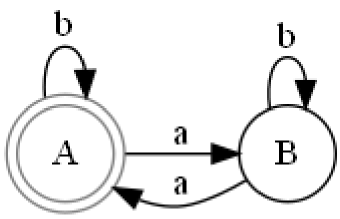
$$\begin{aligned}
\Sigma &= a, b \\
Q &= \{AE, AF, BE, BF, CE, CF, DE, DF\} \\
S &= AE \\
T &= DF
\end{aligned}$$

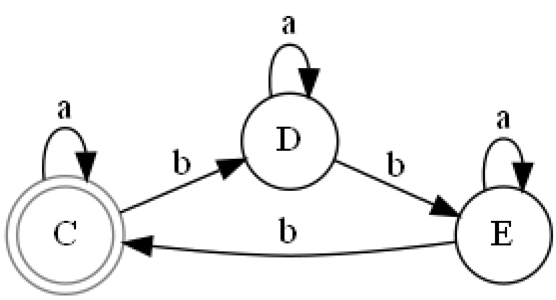
$$\begin{array}{ll}
\delta(AE,a) = BF & \delta(CE,a) = DF \\
\delta(AE,b) = BF & \delta(CE,b) = DF \\
\delta(AF,a) = BE & \delta(CF,a) = DE \\
\delta(AF,b) = BE & \delta(CF,b) = DE \\
\delta(BE,a) = CF & \delta(DE,a) = DF \\
\delta(BE,b) = CF & \delta(DE,b) = DF \\
\delta(BF,a) = CE & \delta(DF,a) = DE \\
\delta(BF,b) = CE & \delta(DF,b) = DE
\end{array}$$



Верхняя ветвь может быть удалена, т.к. ее вершины недостежимы

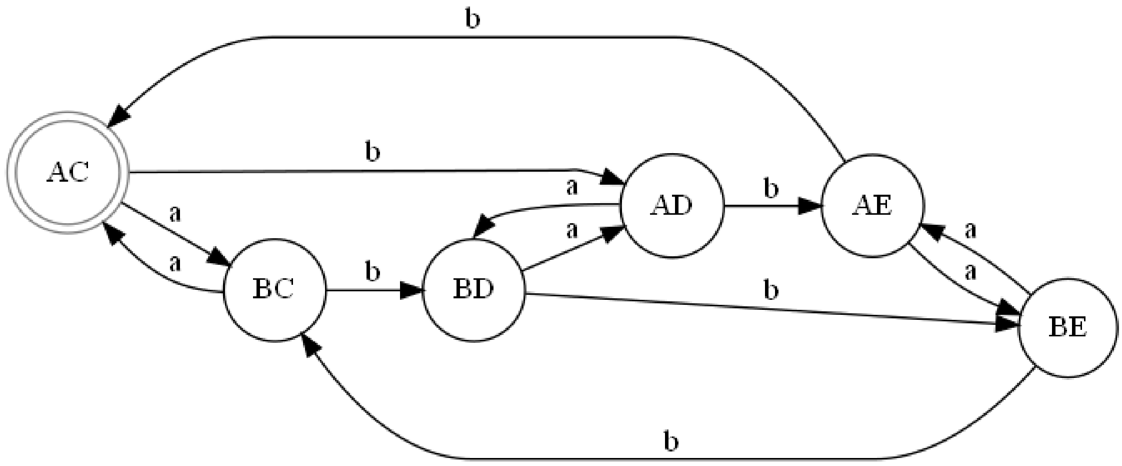
3. $L_1 = \{\omega \in \{a,b\}^* \mid |\omega|_a \textit{четно} \wedge |\omega|_b \textit{ кратно трем}\}$
$L_1 = \{\omega \in \{a,b\}^* \mid |\omega|_a \textit{четно}\} \cap \{\omega \in \{a,b\}^* \mid |\omega|_b \textit{ кратно трем}\}$



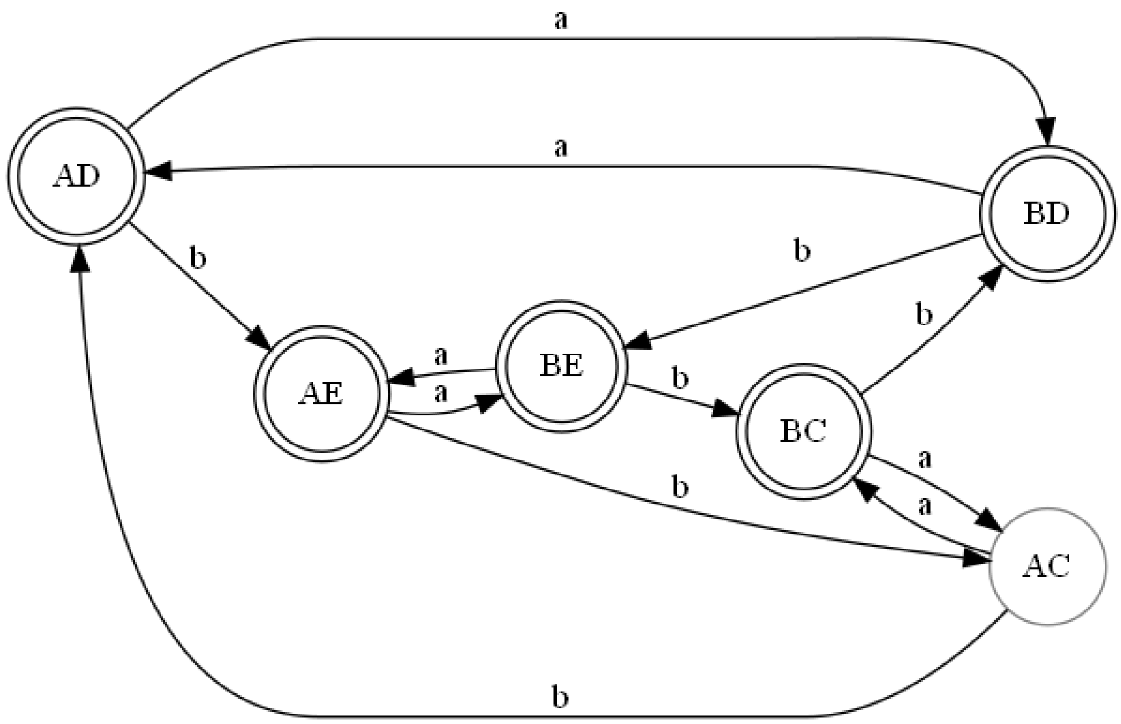


$$
\begin{aligned}
\Sigma &= a, b \\
Q &= \{AC, AD, AE, BC, BD, BE\} \\
S &= AC \\
T &= AC
\end{aligned}
$$

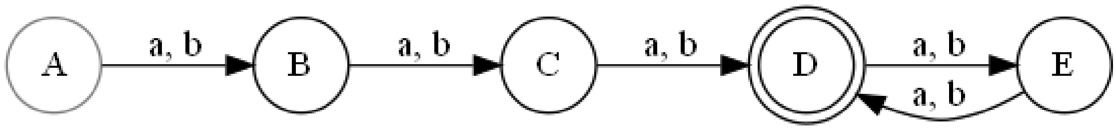
$$
\begin{array}{ll}
\delta(AC, a) = BC & \delta(BC, a) = AC \\
\delta(AC, b) = AD & \delta(BC, b) = BD \\
\delta(AD, a) = BD & \delta(BD, a) = AD \\
\delta(AD, b) = AE & \delta(BD, b) = BE \\
\delta(AE, a) = BE & \delta(BE, a) = AE \\
\delta(AE, b) = AC & \delta(BE, b) = BC
\end{array}
$$

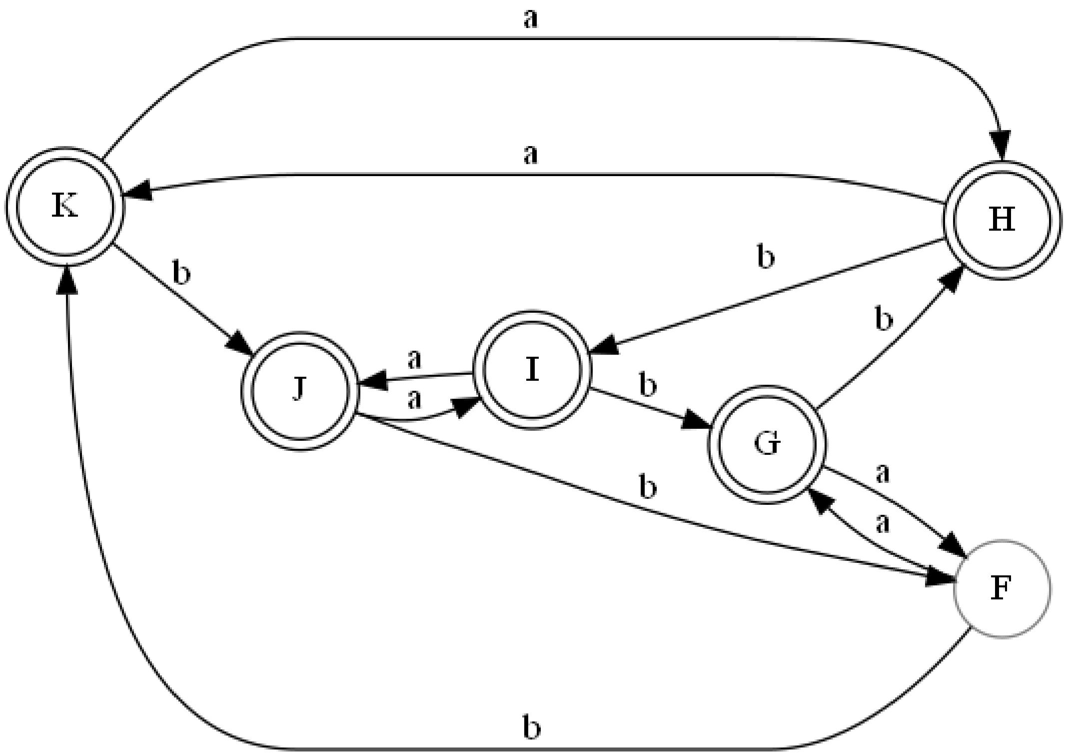


4. $L_4 = \overline{L}_3$



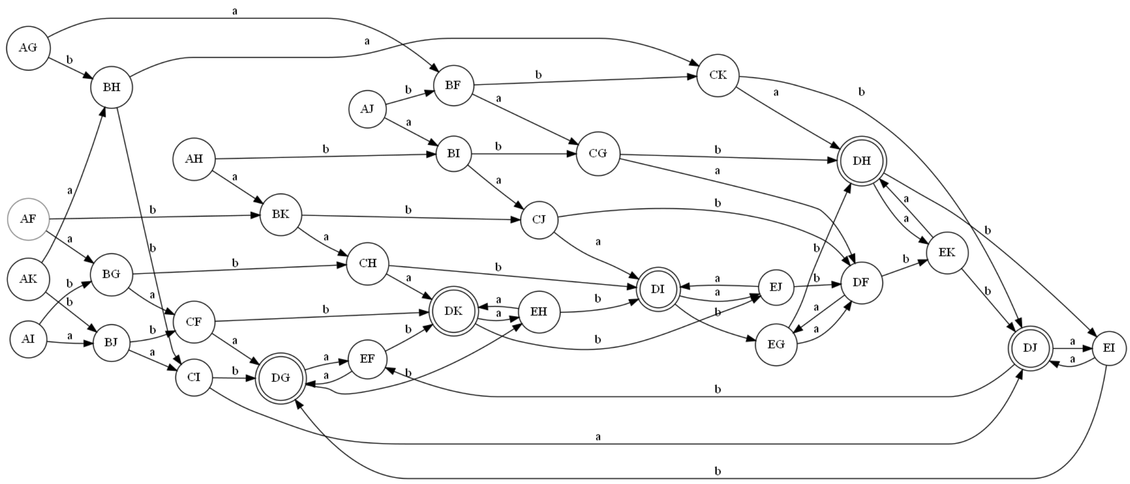
5. $L_5 = L_2 \setminus L_3 = L_2 \cap L_4$





$$
\begin{aligned}
\Sigma &= a, b \\
Q &= \{AF, AG, AH, AI, AJ, AK, \\
&\quad BF, BG, BH, BI, BJ, BK, \\
&\quad CF, CG, CH, CI, CJ, BK, \\
&\quad DF, DG, DH, DI, DJ, DK, \\
&\quad EF, EG, EH, EI, EJ, EK, \} \\
S &= AF \\
T &= \{DG, DH, DI, DJ, DK\}
\end{aligned}
$$

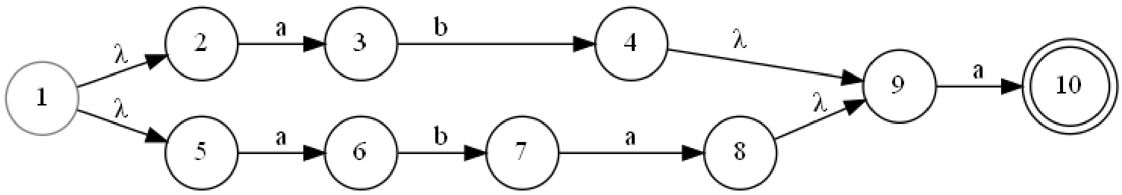
$\delta(AF,a) = BG$	$\delta(AF,b) = BK$	$\delta(AG,a) = BF$	$\delta(AG,b) = BH$
$\delta(AH,a) = BK$	$\delta(AH,b) = BI$	$\delta(AI,a) = BJ$	$\delta(AI,b) = BG$
$\delta(AJ,a) = BI$	$\delta(AJ,b) = BF$	$\delta(AK,a) = BH$	$\delta(AK,b) = BJ$
$\delta(BF,a) = CG$	$\delta(BF,b) = CK$	$\delta(BG,a) = CF$	$\delta(BG,b) = CH$
$\delta(BH,a) = CK$	$\delta(BH,b) = CI$	$\delta(BI,a) = CJ$	$\delta(BI,b) = CG$
$\delta(BJ,a) = CI$	$\delta(BJ,b) = CF$	$\delta(BK,a) = CH$	$\delta(BK,b) = CJ$
$\delta(CF,a) = DG$	$\delta(CF,b) = DK$	$\delta(CG,a) = DF$	$\delta(CG,b) = DH$
$\delta(CH,a) = DK$	$\delta(CH,b) = DI$	$\delta(CI,a) = DJ$	$\delta(CI,b) = DG$
$\delta(CJ,a) = DI$	$\delta(CJ,b) = DF$	$\delta(CK,a) = DH$	$\delta(CK,b) = DJ$
$\delta(DF,a) = EG$	$\delta(DF,b) = EK$	$\delta(DG,a) = EF$	$\delta(DG,b) = EH$
$\delta(DH,a) = EK$	$\delta(DH,b) = EI$	$\delta(DI,a) = EJ$	$\delta(DI,b) = EG$
$\delta(DJ,a) = EI$	$\delta(DJ,b) = EF$	$\delta(DK,a) = EH$	$\delta(DK,b) = EJ$
$\delta(EF,a) = DG$	$\delta(EF,b) = DK$	$\delta(EG,a) = DF$	$\delta(EG,b) = DH$
$\delta(EH,a) = DK$	$\delta(EH,b) = DI$	$\delta(EI,a) = DJ$	$\delta(EI,b) = DG$
$\delta(EJ,a) = DI$	$\delta(EJ,b) = DF$	$\delta(EK,a) = DH$	$\delta(EK,b) = DJ$



Задание 3

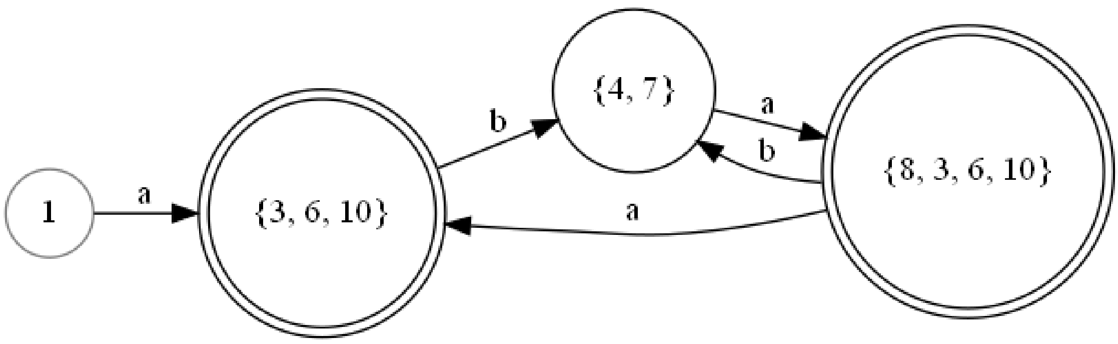
1. $(ab + aba)^* a$

   Построим НКА:



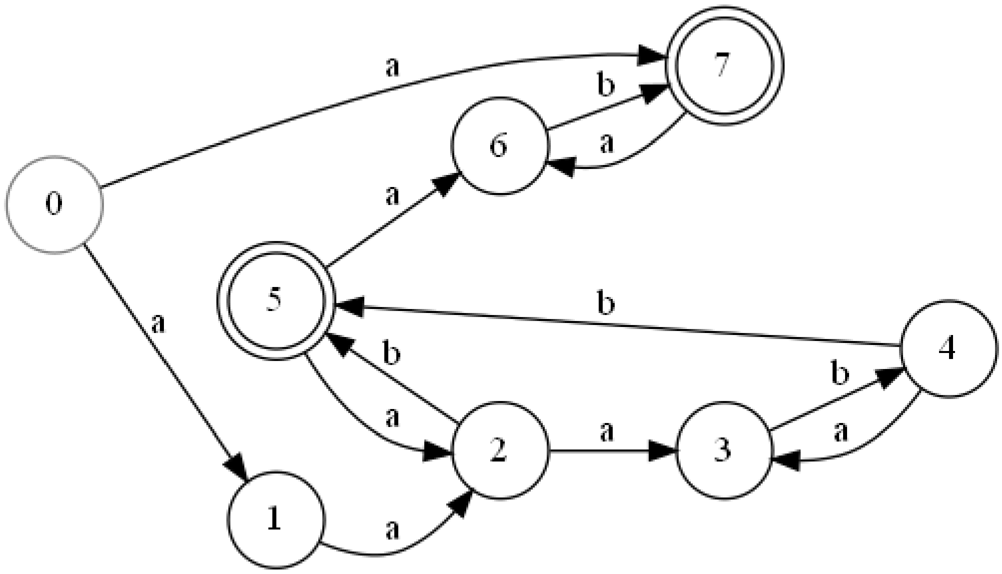
   По полученному НКА, построим ДКА. Данный автомат уже является минимальным

	a	b
1	3,6,10	∅
3,6,10	∅	4, 7
4,7	8,3,610	∅
8,3,610	3,6,10	47



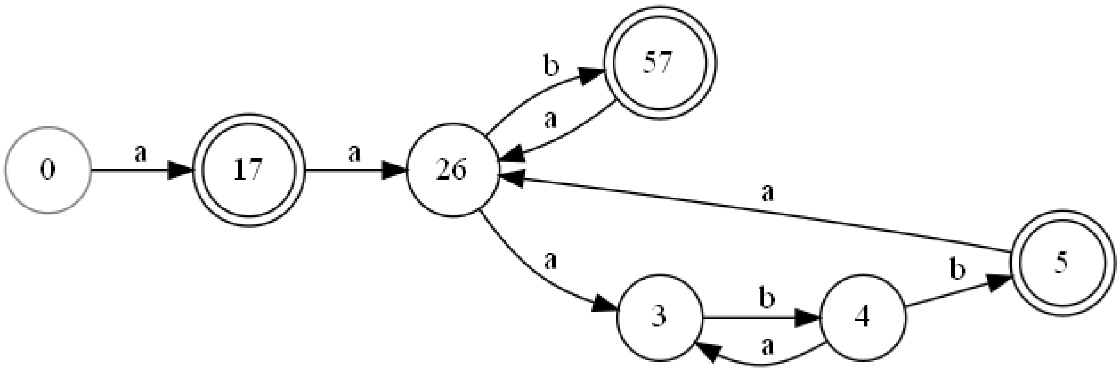
2. $a(a(ab)^*b)^*(ab)^*$

Построим НКА



Построим ДКА:

	a	b
0	17	∅
17	26	∅
26	3	57
3	∅	4
57	26	∅
4	3	5
5	26	∅

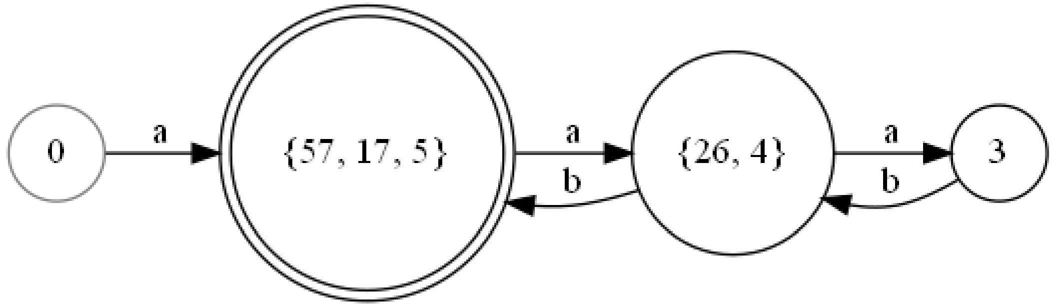


Минимизируем данный автомат

0 эквивалентность: (0, 26, 3, 4, ∅), (57, 17, 5)

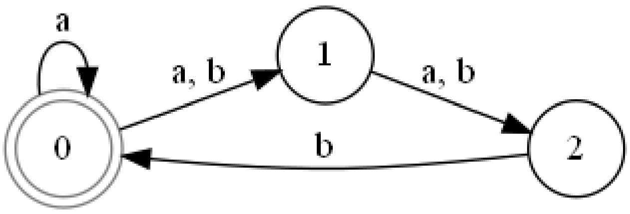
1 эквивалентность: (0), (26, 4), (3, ∅), (57, 17, 5)

2 эквивалентность: (0), (26, 4), (3), (∅), (57, 17, 5)



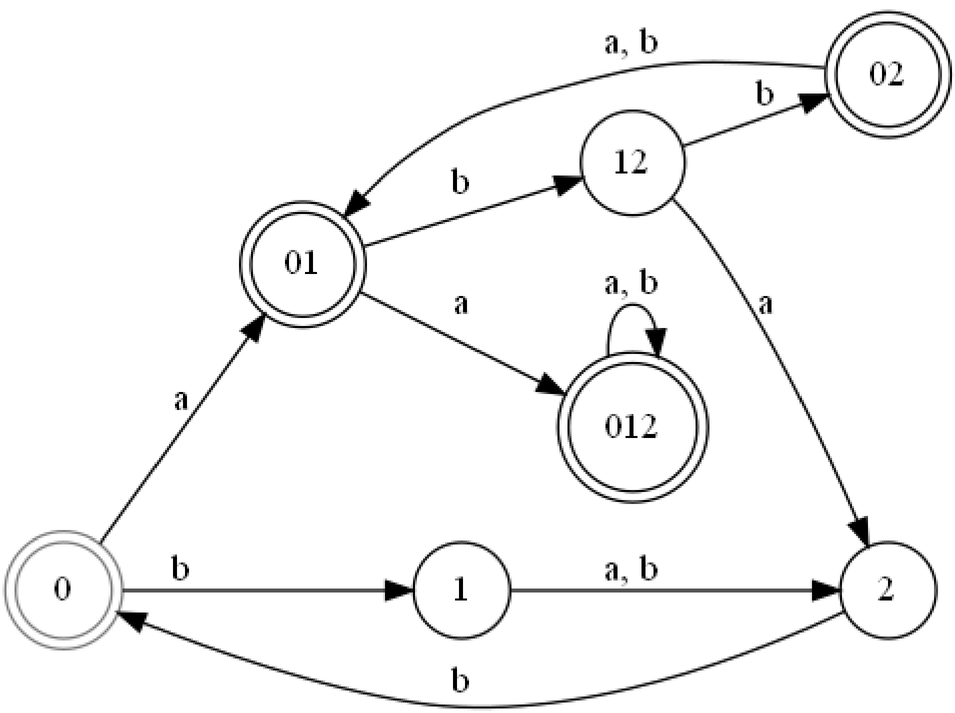
3. $(a + (a + b)(a + b)b)^*$

Построим НКА



Построим ДКА

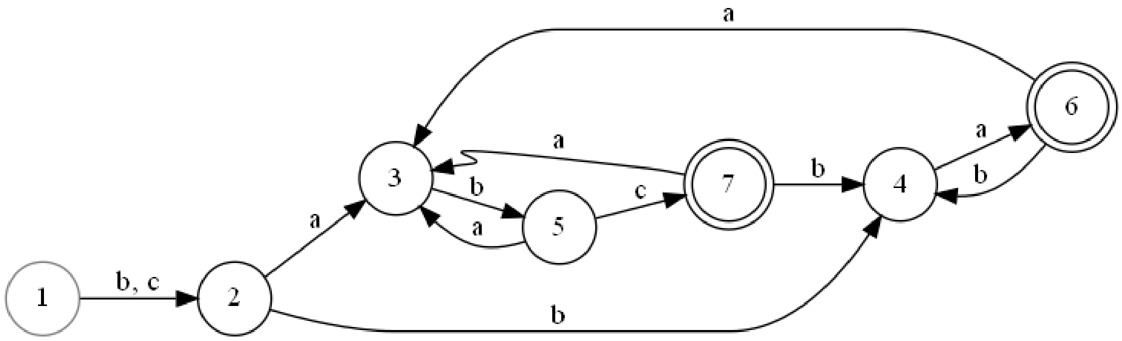
	a	b
0	01	1
01	012	12
1	2	2
012	012	012
12	2	02
2	∅	0
02	01	01



Данный автомат уже минимальный

4. $(b+c)((ab)^*c+(ba)^*)^*$

Построим ДКА



Минимизируем его

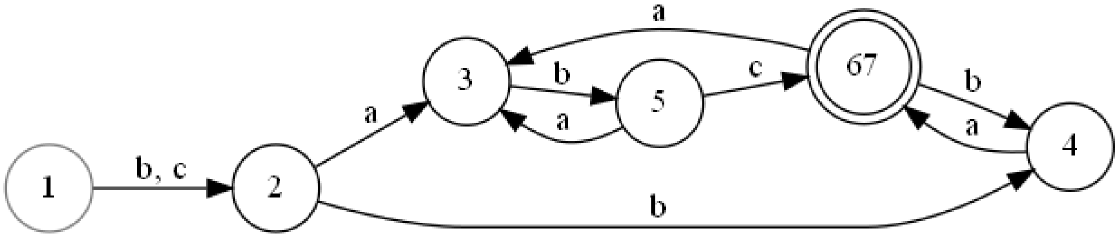
0 эквивалентность: (1, 2, 3, 4, 5, $\varnothing$), (6, 7)

1 эквивалентность: (1, 2, 3, $\varnothing$), (4), (5), (6, 7)

2 эквивалентность: (1, 2, 3, $\varnothing$), (4), (5), (6, 7)

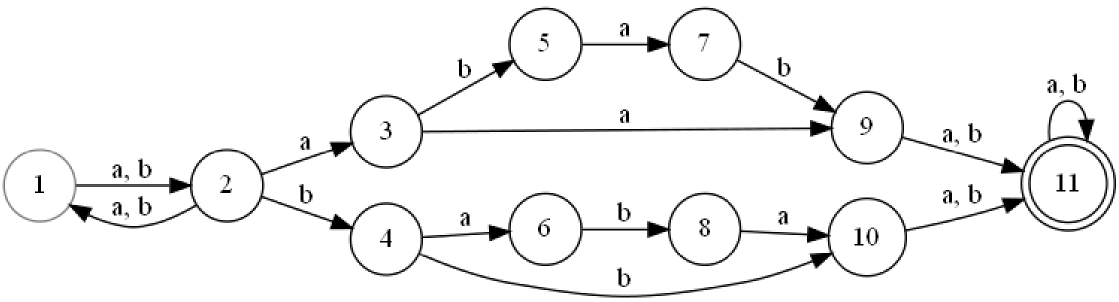
3 эквивалентность: (1, $\varnothing$), (2), (3), (4), (5), (6, 7)

4 эквивалентность: (1), ($\varnothing$), (2), (3), (4), (5), (6, 7)



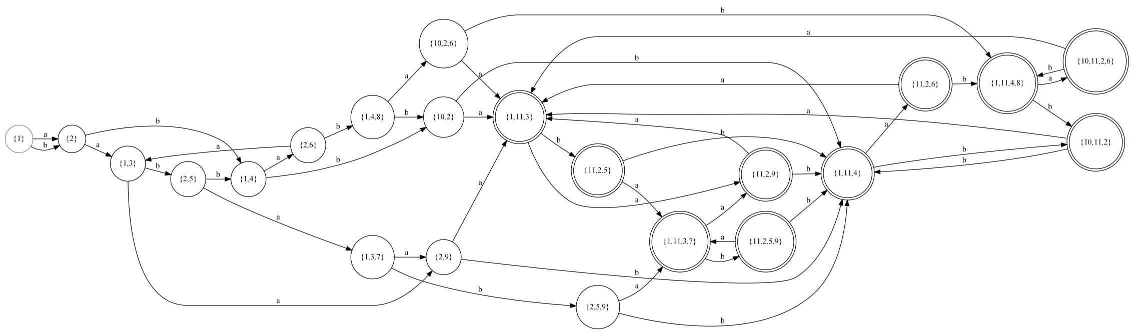
5. $(a+b)^+(aa+bb+abab+baba)(a+b)^+$

Построим НКА



Построим по нему ДКА

	a	b
1	2	2
2	13	14
13	29	25
14	26	2,10
29	1,11,3	1,11,4
25	137	14
26	13	148
2,10	1,11,3	1,11,4
1,11,3	11,2,9	11,2,5
1,11,4	11,2,6	10,11,2
137	29	259
148	10,2,6	10,2
11,2,9	1,11,3	1,11,4
11,2,5	1,11,3,7	1,11,4
11,2,6	1,11,3	1,11,4,8
10,11,2	1,11,3	1,11,4
259	1,11,3,7	1,11,4
10,2,6	1,11,3	1,11,4,8
1,11,3,7	11,2,9	11,2,5,9
1,11,4,8	10,11,2,6	10,11,2
11,2,5,9	1,11,3,7	1,11,4
10,11,2,6	1,11,3	1,11,4,8



Минимизируем ДКА

Эквивалентные вершины:

{1} - 1

{2} - 2

{1,3} - 3

{1,4} - 4

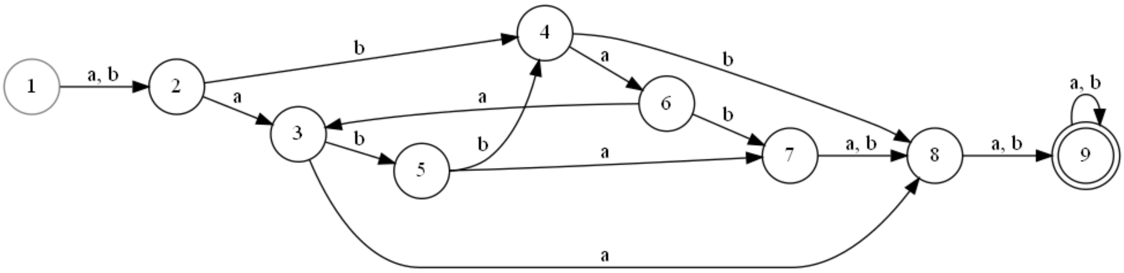
{2,5} - 5

{2,6} - 6

{1,3,7},{1,4,8} - 7

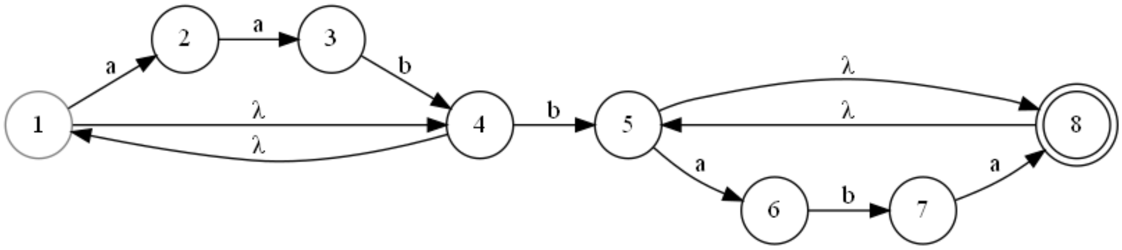
{2,9},{10,2,6},{2,5,9},{10,2} - 8

{11,2,5},{1,11,3},{10,11,2,6},{11,2,6},{1,11,4,8},{1,11,4},{10,11,2},{1,11,3,7},{11,2,9},{11,2,5,9} - 9



Задание 4

1. $L = \{(aab)^n b(aba)^m \mid n \geq 0, m \geq 0\}$



2. $L = \{uaav \mid u \in \{a,b\}^*, v \in \{a,b\}^*, |u|_b \geq |v|_a\}$

$$\omega = b^n aaa^n, |\omega| \geq n$$
$$\omega = xyz$$
$$x = b^i \quad y = b^j \quad i + j \leq n \quad j > 0$$
$$z = b^{n-i-j} aaa^n$$
$$|xy| \leq n \quad |y| > 0$$
$$xy^0 z = b^i b^{n-i-j} aaa^n = b^{n-j} aaa^n \notin L$$

3. $L = \{a^m w \mid w \in \{a,b\}^*, 1 \leq |w|_b \leq m\}$

$$\omega = a^n b^n, |\omega| \geq n$$
$$\omega = xyz$$
$$x = a^i \quad y = a^j \quad i + j \leq n \quad j > 0$$
$$z = a^{n-i-j} b^n$$
$$|xy| \leq n \quad |y| > 0$$
$$xy^0 z = a^i a^{n-i-j} b^n = a^{n-j} b^n \notin L$$

4. $L = \{a^k b^m a^n \mid k = n \vee m > 0\}$

$$\omega = a^n ba^n, |\omega| \geq n$$
$$\omega = xyz$$
$$x = a^i \quad y = a^j \quad i + j \leq n \quad j > 0$$
$$z = a^{n-i-j} ba^n$$
$$|xy| \leq n \quad |y| > 0$$
$$xy^k z = a^i a^{jk} a^{n-i-j} ba^n = a^{n-j(k-1)} ba^n \notin L \quad \forall k > 1$$

5. $L = \{ucv \mid u \in \{a,b\}^*, v \in \{a,b\}^*, u \neq v^R\}$

$$\omega = (ab)^n c(ba)^n = \alpha_1\alpha_2 \ldots \alpha_{4n+1}, |\omega| \geq n$$

$$\omega = xyz$$

$$x = \alpha_1\alpha_2 \ldots \alpha_i \quad y = \alpha_{i+1}\alpha_{i+2} \ldots \alpha_{i+j} \quad i+j \leq n \quad j > 0$$

$$z = \alpha_{i+j+1}\alpha_{i+j+2} \ldots \alpha_{2n} c(ba)^n$$

$$|xy| \leq n \quad |y| > 0$$

$$xy^k z = \alpha_1 \ldots \alpha_i (\alpha_{i+1} \ldots \alpha_{i+j})^k \alpha_{i+j+1} \ldots \alpha_{2n} c(ba)^n \notin L \quad \forall k > 1$$