ДЗ №1: Регулярные языки и конечные автоматы

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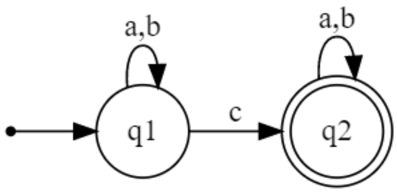
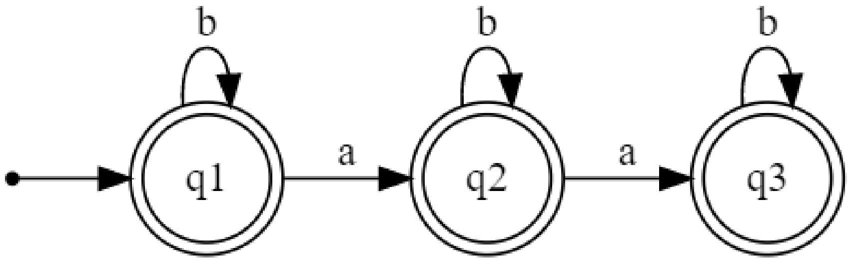
1 Задание №1. Построить конечный автомат, распознающий язык

Ответом на данное задание является конечный автомат, распознающий описанный язык. Автомат должен быть детерминированным.

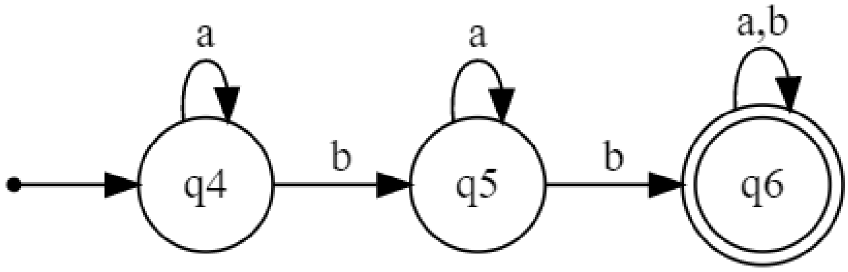
1. $L = \{\omega \in \{a, b, c\}^* | |\omega|_c = 1\}$



2. $L = \{\omega \in \{a, b\}^* | |\omega|_a \leq 2, |\omega|_b \geq 2\}$
   Рассмотрим L_1 и L_2 | $L = L_1 \cap L_2$:
   $L_1 = \{\omega \in \{a, b\}^* | |\omega|_a \leq 2\}$

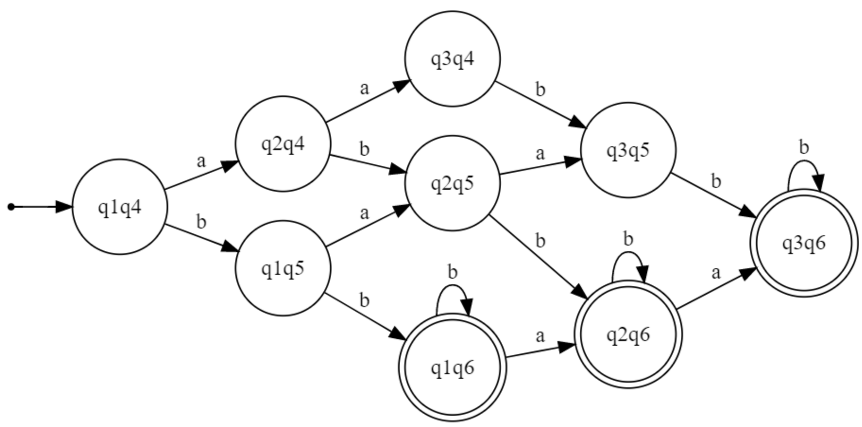


   $L_2 = \{\omega \in \{a, b\}^* | |\omega|_b \geq 2\}$



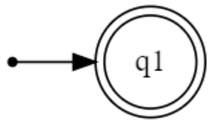
$A_1 = \{\Sigma_1 = \{a, b\}, Q_1 = \{q1, q2, q3\}, s_1 = q1, T_1 = \{q1, q2, q3\}, \delta_1\}$
$A_2 = \{\Sigma_2 = \{a, b\}, Q_2 = \{q4, q5, q6\}, s_2 = q4, T_2 = \{q6\}, \delta_2\}$
$A = \{\Sigma, Q, s, T, \delta\}$:

- $\Sigma = \Sigma_1 \cup \Sigma_2 = \{a, b\}$
- $Q = Q_1 \times Q_2 = \{\langle q1, q4\rangle, \langle q1, q5\rangle, \langle q1, q6\rangle, \langle q2, q4\rangle, \langle q2, q5\rangle, \langle q2, q6\rangle, \langle q3, q4\rangle, \langle q3, q5\rangle, \langle q3, q6\rangle\}$
- $s = \langle s1, s2\rangle = \langle q1, q4\rangle$
- $T = T_1 \times T_2 = \langle q1, q6\rangle, \langle q2, q6\rangle, \langle q3, q6\rangle$
- $\delta(\langle q_1, q_2\rangle, c) = (\delta_1(q_1, c), \delta_2(q_2, c))$:
  - $\delta(\langle q1, q4\rangle, a) = (\delta_1(q1, a), \delta_2(q4, a)) = \langle q2, q4\rangle$
  - $\delta(\langle q1, q4\rangle, b) = (\delta_1(q1, b), \delta_2(q4, b)) = \langle q1, q5\rangle$
  - $\delta(\langle q1, q5\rangle, a) = (\delta_1(q1, a), \delta_2(q5, a)) = \langle q2, q5\rangle$
  - $\delta(\langle q1, q5\rangle, b) = (\delta_1(q1, b), \delta_2(q5, b)) = \langle q1, q6\rangle$
  - $\delta(\langle q1, q6\rangle, a) = (\delta_1(q1, a), \delta_2(q6, a)) = \langle q2, q6\rangle$
  - $\delta(\langle q1, q6\rangle, b) = (\delta_1(q1, b), \delta_2(q6, b)) = \langle q1, q6\rangle$
  - $\delta(\langle q2, q4\rangle, a) = (\delta_1(q2, a), \delta_2(q4, a)) = \langle q3, q4\rangle$
  - $\delta(\langle q2, q4\rangle, b) = (\delta_1(q2, b), \delta_2(q4, b)) = \langle q2, q5\rangle$
  - $\delta(\langle q2, q5\rangle, a) = (\delta_1(q2, a), \delta_2(q5, a)) = \langle q3, q5\rangle$
  - $\delta(\langle q2, q5\rangle, b) = (\delta_1(q2, b), \delta_2(q5, b)) = \langle q2, q6\rangle$
  - $\delta(\langle q2, q6\rangle, a) = (\delta_1(q2, a), \delta_2(q6, a)) = \langle q3, q6\rangle$
  - $\delta(\langle q2, q6\rangle, b) = (\delta_1(q2, b), \delta_2(q6, b)) = \langle q2, q6\rangle$
  - $\delta(\langle q3, q4\rangle, a) = (\delta_1(q3, a), \delta_2(q4, a)) = \emptyset$
  - $\delta(\langle q3, q4\rangle, b) = (\delta_1(q3, b), \delta_2(q4, b)) = \langle q3, q5\rangle$
  - $\delta(\langle q3, q5\rangle, a) = (\delta_1(q3, a), \delta_2(q5, a)) = \emptyset$
  - $\delta(\langle q3, q5\rangle, b) = (\delta_1(q3, b), \delta_2(q5, b)) = \langle q3, q6\rangle$
  - $\delta(\langle q3, q6\rangle, a) = (\delta_1(q3, a), \delta_2(q6, a)) = \emptyset$
  - $\delta(\langle q3, q6\rangle, b) = (\delta_1(q3, b), \delta_2(q6, b)) = \langle q3, q6\rangle$



3. $L = \{\omega \in \{a, b\}^* || \omega|_a \neq |\omega|_b\}$
   Построить ДКА, распознающий данный язык, невозможно, так как нам понадобится запоминать количество одинаковых символов – это нельзя реализовать с помощью ДКА.

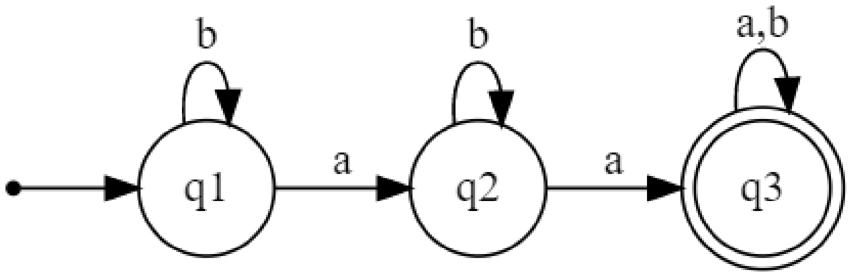
4. $L = \{\omega \in \{a, b\}^* | \omega\omega = \omega\omega\omega\}$
   Данному языку удовлетворяет только пустое слово:



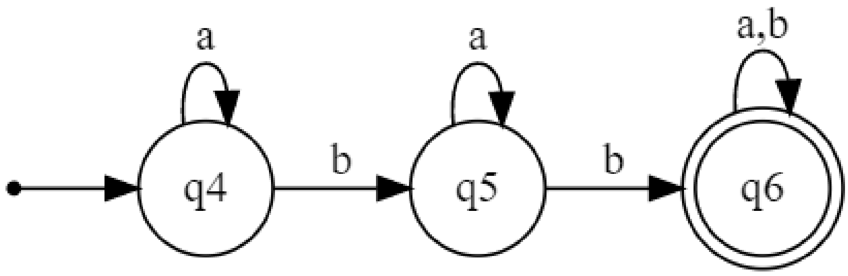
2 Задание №2. Построить конечный автомат, используя прямое произведение

Ответом на данное задание является конечный автомат, распознающий описанный язык. Требуется, чтобы он был построен при помощи прямого произведения ДКА и его свойств.

1. $L_1 = \{\omega \in \{a, b\}^* || \omega|_a \geq 2 \wedge |\omega|_b \geq 2\}$ $L_1 = L_{11} \cap L_{12}$
   $L_{11} = \{\omega \in \{a, b\}^* || \omega|_a \geq 2\}$



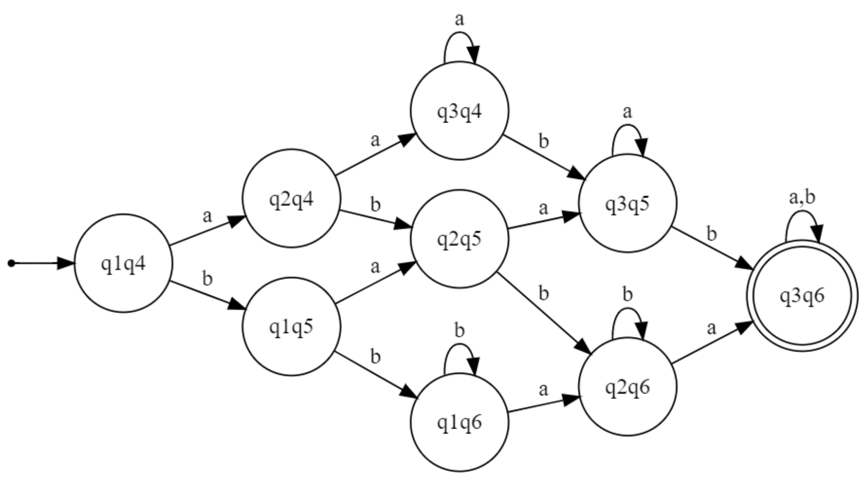
   $L_{12} = \{\omega \in \{a, b\}^* || \omega|_b \geq 2\}$



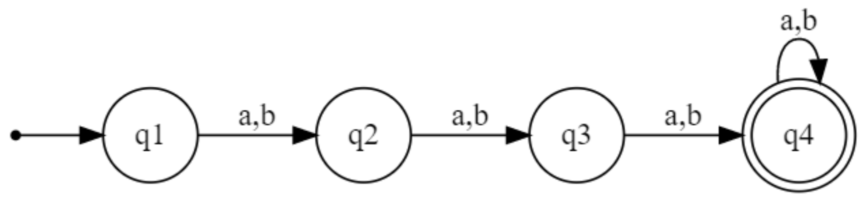
   $A_{11} = \{\Sigma_1 = \{a, b\}, Q_1 = \{q1, q2, q3\}, s_1 = q1, T_1 = \{q3\}, \delta_1\}$
   $A_{12} = \{\Sigma_2 = \{a, b\}, Q_2 = \{q4, q5, q6\}, s_2 = q4, T_2 = \{q6\}, \delta_2\}$
   $A_1 = \{\Sigma, Q, s, T, \delta\}$:

   - $\Sigma = \Sigma_1 \cup \Sigma_2 = \{a, b\}$
   - $Q = Q_1 \times Q_2 = \{\langle q1, q4\rangle, \langle q1, q5\rangle, \langle q1, q6\rangle, \langle q2, q4\rangle, \langle q2, q5\rangle, \langle q2, q6\rangle, \langle q3, q4\rangle, \langle q3, q5\rangle, \langle q3, q6\rangle\}$
   - $s = \langle s1, s2\rangle = \langle q1, q4\rangle$
   - $T = T_1 \times T_2 = \langle q3, q6\rangle$
   - $\delta(\langle q_1, q_2\rangle, c) = (\delta_1(q_1, c), \delta_2(q_2, c))$:
     - $\delta(\langle q1, q4\rangle, a) = (\delta_1(q1, a), \delta_2(q4, a)) = \langle q2, q4\rangle$
     - $\delta(\langle q1, q4\rangle, b) = (\delta_1(q1, b), \delta_2(q4, b)) = \langle q1, q5\rangle$

- $\delta(\langle q1, q5\rangle, a) = (\delta_1(q1, a), \delta_2(q5, a)) = \langle q2, q5\rangle$
- $\delta(\langle q1, q5\rangle, b) = (\delta_1(q1, b), \delta_2(q5, b)) = \langle q1, q6\rangle$
- $\delta(\langle q1, q6\rangle, a) = (\delta_1(q1, a), \delta_2(q6, a)) = \langle q2, q6\rangle$
- $\delta(\langle q1, q6\rangle, b) = (\delta_1(q1, b), \delta_2(q6, b)) = \langle q1, q6\rangle$
- $\delta(\langle q2, q4\rangle, a) = (\delta_1(q2, a), \delta_2(q4, a)) = \langle q3, q4\rangle$
- $\delta(\langle q2, q4\rangle, b) = (\delta_1(q2, b), \delta_2(q4, b)) = \langle q2, q5\rangle$
- $\delta(\langle q2, q5\rangle, a) = (\delta_1(q2, a), \delta_2(q5, a)) = \langle q3, q5\rangle$
- $\delta(\langle q2, q5\rangle, b) = (\delta_1(q2, b), \delta_2(q5, b)) = \langle q2, q6\rangle$
- $\delta(\langle q2, q6\rangle, a) = (\delta_1(q2, a), \delta_2(q6, a)) = \langle q3, q6\rangle$
- $\delta(\langle q2, q6\rangle, b) = (\delta_1(q2, b), \delta_2(q6, b)) = \langle q2, q6\rangle$
- $\delta(\langle q3, q4\rangle, a) = (\delta_1(q3, a), \delta_2(q4, a)) = \langle q3, q4\rangle$
- $\delta(\langle q3, q4\rangle, b) = (\delta_1(q3, b), \delta_2(q4, b)) = \langle q3, q5\rangle$
- $\delta(\langle q3, q5\rangle, a) = (\delta_1(q3, a), \delta_2(q5, a)) = \langle q3, q5\rangle$
- $\delta(\langle q3, q5\rangle, b) = (\delta_1(q3, b), \delta_2(q5, b)) = \langle q3, q6\rangle$
- $\delta(\langle q3, q6\rangle, a) = (\delta_1(q3, a), \delta_2(q6, a)) = \langle q3, q6\rangle$
- $\delta(\langle q3, q6\rangle, b) = (\delta_1(q3, b), \delta_2(q6, b)) = \langle q3, q6\rangle$



2. $L_2 = \{\omega \in \{a, b\}^* | |\omega| \geq 3 \wedge |\omega| \text{ нечётное}\}$
$L_2 = L_{21} \cap L_{22}$
$L_{21} = \{\omega \in \{a, b\}^* | |\omega| \geq 3\}$

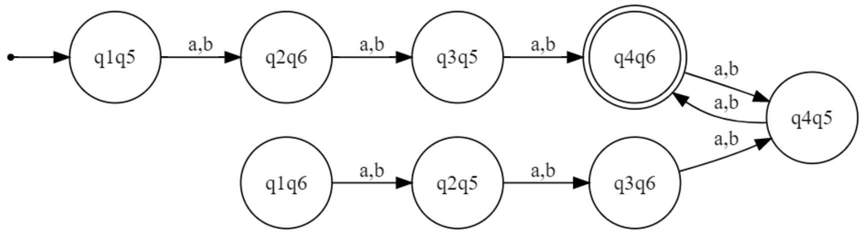


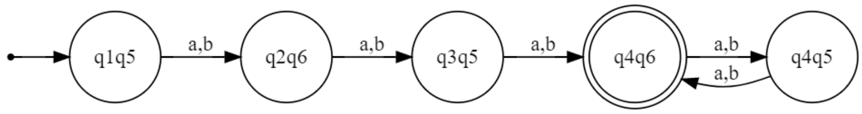
$L_{22} = \{\omega \in \{a, b\}^* | |\omega| \text{ нечётное}\}$

$A_{21} = \{\Sigma_1 = \{a, b\}, Q_1 = \{q1, q2, q3, q4\}, s_1 = q1, T_1 = \{q4\}, \delta_1\}$
$A_{22} = \{\Sigma_2 = \{a, b\}, Q_2 = \{q5, q6\}, s_2 = q5, T_2 = \{q6\}, \delta_2\}$
$A_2 = \{\Sigma, Q, s, T, \delta\}$:

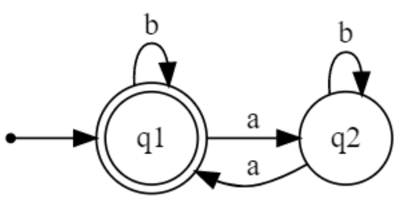
- $\Sigma = \Sigma_1 \cup \Sigma_2 = \{a, b\}$

- $Q = Q_1 \times Q_2 = \{\langle q1, q5\rangle, \langle q1, q6\rangle, \langle q2, q5\rangle, \langle q2, q6\rangle, \langle q3, q5\rangle, \langle q3, q6\rangle, \langle q4, q5\rangle, \langle q4, q6\rangle\}$
- $s = \langle s1, s2\rangle = \langle q1, q5\rangle$
- $T = T_1 \times T_2 = \langle q4, q6\rangle$
- $\delta(\langle q_1, q_2\rangle, c) = (\delta_1(q_1, c), \delta_2(q_2, c))$:
  - $\delta(\langle q1, q5\rangle, a) = (\delta_1(q1, a), \delta_2(q5, a)) = \langle q2, q6\rangle$
  - $\delta(\langle q1, q5\rangle, b) = (\delta_1(q1, b), \delta_2(q5, b)) = \langle q2, q6\rangle$
  - $\delta(\langle q1, q6\rangle, a) = (\delta_1(q1, a), \delta_2(q6, a)) = \langle q2, q5\rangle$
  - $\delta(\langle q1, q6\rangle, b) = (\delta_1(q1, b), \delta_2(q6, b)) = \langle q2, q5\rangle$
  - $\delta(\langle q2, q5\rangle, a) = (\delta_1(q2, a), \delta_2(q5, a)) = \langle q3, q6\rangle$
  - $\delta(\langle q2, q5\rangle, b) = (\delta_1(q2, b), \delta_2(q5, b)) = \langle q3, q6\rangle$
  - $\delta(\langle q2, q6\rangle, a) = (\delta_1(q2, a), \delta_2(q6, a)) = \langle q3, q5\rangle$
  - $\delta(\langle q2, q6\rangle, b) = (\delta_1(q2, b), \delta_2(q6, b)) = \langle q3, q5\rangle$
  - $\delta(\langle q3, q5\rangle, a) = (\delta_1(q3, a), \delta_2(q5, a)) = \langle q4, q6\rangle$
  - $\delta(\langle q3, q5\rangle, b) = (\delta_1(q3, b), \delta_2(q5, b)) = \langle q4, q6\rangle$
  - $\delta(\langle q3, q6\rangle, a) = (\delta_1(q3, a), \delta_2(q6, a)) = \langle q4, q5\rangle$
  - $\delta(\langle q3, q6\rangle, b) = (\delta_1(q3, b), \delta_2(q6, b)) = \langle q4, q5\rangle$
  - $\delta(\langle q4, q5\rangle, a) = (\delta_1(q4, a), \delta_2(q5, a)) = \langle q4, q6\rangle$
  - $\delta(\langle q4, q5\rangle, b) = (\delta_1(q4, b), \delta_2(q5, b)) = \langle q4, q6\rangle$
  - $\delta(\langle q4, q6\rangle, a) = (\delta_1(q4, a), \delta_2(q6, a)) = \langle q4, q5\rangle$
  - $\delta(\langle q4, q6\rangle, b) = (\delta_1(q4, b), \delta_2(q6, b)) = \langle q4, q5\rangle$

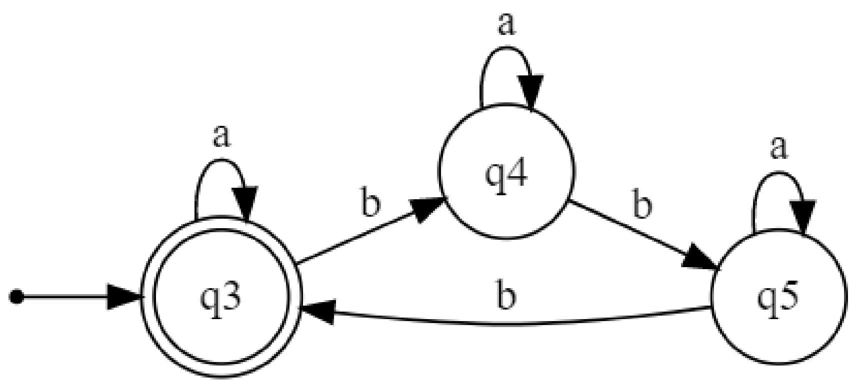




3. $L_3 = \{\omega \in \{a, b\}^* || \omega|_a$ четно $\wedge |\omega|_b$ кратно трем$\}$
   $L_3 = L_{31} \cap L_{32}$
   $L_{31} = \{\omega \in \{a, b\}^* || \omega|_a$ четно $\}$

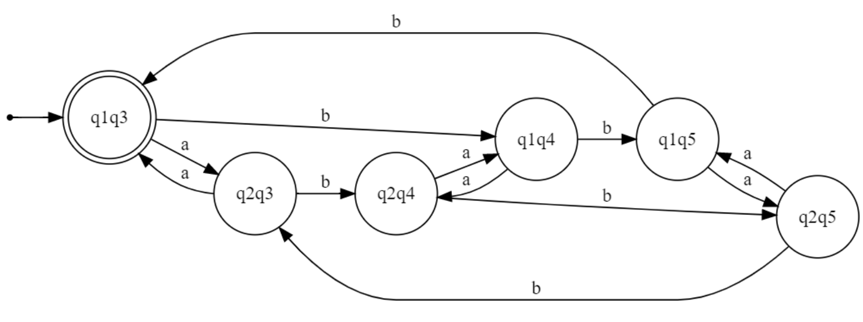


$L_{32} = \{\omega \in \{a, b\}^* || \omega|_b$ кратно трем$\}$

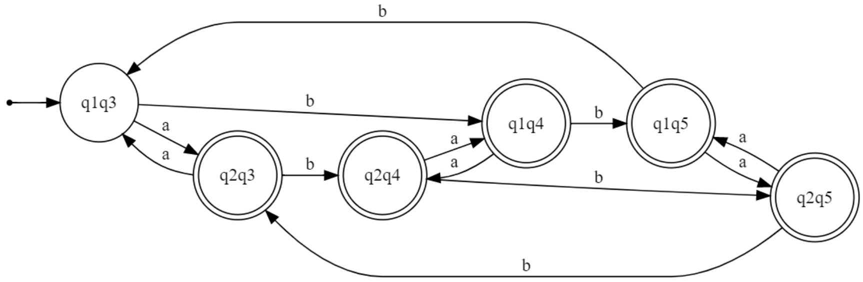


$A_{31} = \{\Sigma_1 = \{a, b\}, Q_1 = \{q1, q2\}, s_1 = q1, T_1 = \{q1\}, \delta_1\}$
$A_{32} = \{\Sigma_2 = \{a, b\}, Q_2 = \{q3, q4, q5\}, s_2 = q3, T_2 = \{q3\}, \delta_2\}$
$A_3 = \{\Sigma, Q, s, T, \delta\}$:

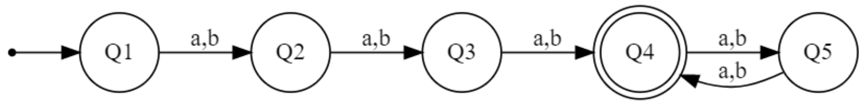
- $\Sigma = \Sigma_1 \cup \Sigma_2 = \{a, b\}$
- $Q = Q_1 \times Q_2 = \{\langle q1, q3\rangle, \langle q1, q4\rangle, \langle q1, q5\rangle, \langle q2, q3\rangle, \langle q2, q4\rangle, \langle q2, q5\rangle\}$
- $s = \langle s1, s2\rangle = \langle q1, q3\rangle$
- $T = T_1 \times T_2 = \langle q1, q3\rangle$
- $\delta(\langle q_1, q_2\rangle, c) = (\delta_1(q_1, c), \delta_2(q_2, c))$:
  - $\delta(\langle q1, q3\rangle, a) = (\delta_1(q1, a), \delta_2(q3, a)) = \langle q2, q3\rangle$
  - $\delta(\langle q1, q3\rangle, b) = (\delta_1(q1, b), \delta_2(q3, b)) = \langle q1, q4\rangle$
  - $\delta(\langle q1, q4\rangle, a) = (\delta_1(q1, a), \delta_2(q4, a)) = \langle q2, q4\rangle$
  - $\delta(\langle q1, q4\rangle, b) = (\delta_1(q1, b), \delta_2(q4, b)) = \langle q1, q5\rangle$
  - $\delta(\langle q1, q5\rangle, a) = (\delta_1(q1, a), \delta_2(q5, a)) = \langle q2, q5\rangle$
  - $\delta(\langle q1, q5\rangle, b) = (\delta_1(q1, b), \delta_2(q5, b)) = \langle q1, q3\rangle$
  - $\delta(\langle q2, q3\rangle, a) = (\delta_1(q2, a), \delta_2(q3, a)) = \langle q1, q3\rangle$
  - $\delta(\langle q2, q3\rangle, b) = (\delta_1(q2, b), \delta_2(q3, b)) = \langle q2, q4\rangle$
  - $\delta(\langle q2, q4\rangle, a) = (\delta_1(q2, a), \delta_2(q4, a)) = \langle q1, q4\rangle$
  - $\delta(\langle q2, q4\rangle, b) = (\delta_1(q2, b), \delta_2(q4, b)) = \langle q2, q5\rangle$
  - $\delta(\langle q2, q5\rangle, a) = (\delta_1(q2, a), \delta_2(q5, a)) = \langle q1, q5\rangle$
  - $\delta(\langle q2, q5\rangle, b) = (\delta_1(q2, b), \delta_2(q5, b)) = \langle q2, q3\rangle$

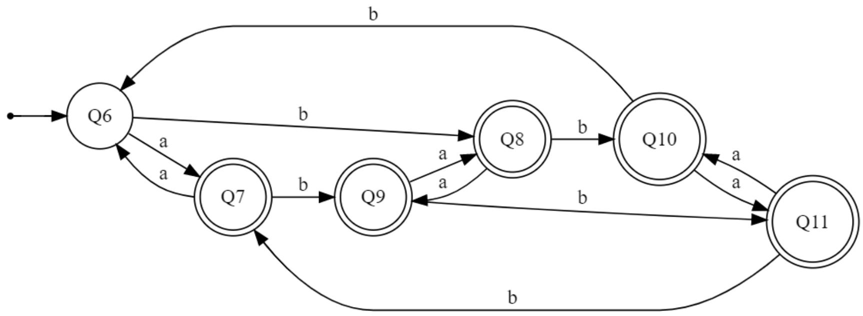


4. $L_4 = \overline{L_3}$
   $L_4 = \overline{L_3} = \{\Sigma_3, Q_3, s_3, T_3 = Q_3 \setminus T_3, \delta_3\}$
   $T_4 = Q_3 \backslash T_3 = \{\langle q1, q3\rangle, \langle q1, q4\rangle, \langle q1, q5\rangle, \langle q2, q3\rangle, \langle q2, q4\rangle, \langle q2, q5\rangle\} \backslash \{\langle q1, q3\rangle\} = \{\langle q1, q4\rangle, \langle q1, q5\rangle, \langle q2, q3\rangle$



5. $L_5 = L_2 \setminus L_3$
   $L_5 = L_2 \setminus L_3 = L_2 \cap \overline{L_3}$ Для удобства переобозначим вершины автоматов, распознающих языки L_2 и $\overline{L_3}$:



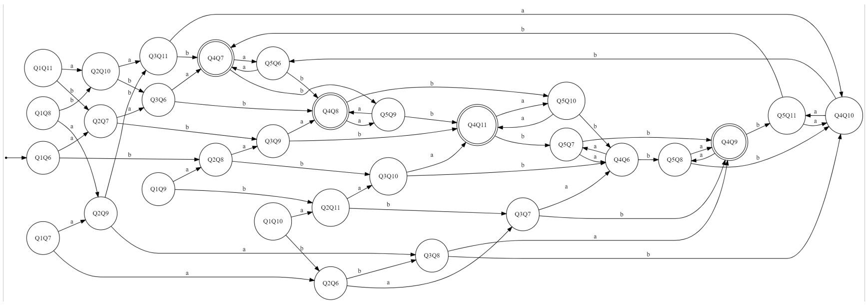


$A_3 = \{\Sigma, Q, s, T, \delta\}$:

- $\Sigma = \Sigma_2 \cup \Sigma_{\overline{3}} = \{a, b\}$
- $Q = Q_2 \times Q_{\overline{3}} = \{\langle Q1, Q6\rangle, \langle Q1, Q7\rangle, \langle Q1, Q8\rangle, \langle Q1, Q9\rangle, \langle Q1, Q10\rangle, \langle Q1, Q11\rangle,$
  $\langle Q2, Q6\rangle, \langle Q2, Q7\rangle, \langle Q2, Q8\rangle, \langle Q2, Q9\rangle, \langle Q2, Q10\rangle, \langle Q2, Q11\rangle,$
  $\langle Q3, Q6\rangle, \langle Q3, Q7\rangle, \langle Q3, Q8\rangle, \langle Q3, Q9\rangle, \langle Q3, Q10\rangle, \langle Q3, Q11\rangle,$
  $\langle Q4, Q6\rangle, \langle Q4, Q7\rangle, \langle Q4, Q8\rangle, \langle Q4, Q9\rangle, \langle Q4, Q10\rangle, \langle Q4, Q11\rangle,$
  $\langle Q5, Q6\rangle, \langle Q5, Q7\rangle, \langle Q5, Q8\rangle, \langle Q5, Q9\rangle, \langle Q5, Q10\rangle, \langle Q5, Q11\rangle\}$
- $s = \langle s_2, s_{\overline{3}}\rangle = \langle Q1, Q6\rangle$
- $T = T_2 \times T_{\overline{3}} = \langle Q4, Q7\rangle, \langle Q4, Q8\rangle, \langle Q4, Q9\rangle, \langle Q4, Q10\rangle, \langle Q4, Q11\rangle$
- $\delta(\langle q_2, q_{\overline{3}}\rangle, c) = (\delta_1(q_2, c), \delta_2(q_{\overline{3}}, c))$:
  - $\delta(\langle Q1, Q6\rangle, a) = (\delta_1(Q1, a), \delta_2(Q6, a)) = \langle Q2, Q7\rangle$
  - $\delta(\langle Q1, Q6\rangle, b) = (\delta_1(Q1, b), \delta_2(Q6, b)) = \langle Q2, Q8\rangle$
  - $\delta(\langle Q1, Q7\rangle, a) = (\delta_1(Q1, a), \delta_2(Q7, a)) = \langle Q2, Q6\rangle$
  - $\delta(\langle Q1, Q7\rangle, b) = (\delta_1(Q1, b), \delta_2(Q7, b)) = \langle Q2, Q9\rangle$
  - $\delta(\langle Q1, Q8\rangle, a) = (\delta_1(Q1, a), \delta_2(Q8, a)) = \langle Q2, Q9\rangle$
  - $\delta(\langle Q1, Q8\rangle, b) = (\delta_1(Q1, b), \delta_2(Q8, b)) = \langle Q2, Q10\rangle$
  - $\delta(\langle Q1, Q9\rangle, a) = (\delta_1(Q1, a), \delta_2(Q9, a)) = \langle Q2, Q8\rangle$
  - $\delta(\langle Q1, Q9\rangle, b) = (\delta_1(Q1, b), \delta_2(Q9, b)) = \langle Q2, Q11\rangle$
  - $\delta(\langle Q1, Q10\rangle, a) = (\delta_1(Q1, a), \delta_2(Q10, a)) = \langle Q2, Q11\rangle$

- $\delta(\langle Q1, Q10\rangle, b) = (\delta_1(Q1, b), \delta_2(Q10, b)) = \langle Q2, Q6\rangle$
- $\delta(\langle Q1, Q11\rangle, a) = (\delta_1(Q1, a), \delta_2(Q11, a)) = \langle Q2, Q10\rangle$
- $\delta(\langle Q1, Q11\rangle, b) = (\delta_1(Q1, b), \delta_2(Q11, b)) = \langle Q2, Q7\rangle$
- $\delta(\langle Q2, Q6\rangle, a) = (\delta_1(Q2, a), \delta_2(Q6, a)) = \langle Q3, Q7\rangle$
- $\delta(\langle Q2, Q6\rangle, b) = (\delta_1(Q2, b), \delta_2(Q6, b)) = \langle Q3, Q8\rangle$
- $\delta(\langle Q2, Q7\rangle, a) = (\delta_1(Q2, a), \delta_2(Q7, a)) = \langle Q3, Q6\rangle$
- $\delta(\langle Q2, Q7\rangle, b) = (\delta_1(Q2, b), \delta_2(Q7, b)) = \langle Q3, Q9\rangle$
- $\delta(\langle Q2, Q8\rangle, a) = (\delta_1(Q2, a), \delta_2(Q8, a)) = \langle Q3, Q9\rangle$
- $\delta(\langle Q2, Q8\rangle, b) = (\delta_1(Q2, b), \delta_2(Q8, b)) = \langle Q3, Q10\rangle$
- $\delta(\langle Q2, Q9\rangle, a) = (\delta_1(Q2, a), \delta_2(Q9, a)) = \langle Q3, Q8\rangle$
- $\delta(\langle Q2, Q9\rangle, b) = (\delta_1(Q2, b), \delta_2(Q9, b)) = \langle Q3, Q11\rangle$
- $\delta(\langle Q2, Q10\rangle, a) = (\delta_1(Q2, a), \delta_2(Q10, a)) = \langle Q3, Q11\rangle$
- $\delta(\langle Q2, Q10\rangle, b) = (\delta_1(Q2, b), \delta_2(Q10, b)) = \langle Q3, Q6\rangle$
- $\delta(\langle Q2, Q11\rangle, a) = (\delta_1(Q2, a), \delta_2(Q11, a)) = \langle Q3, Q10\rangle$
- $\delta(\langle Q2, Q11\rangle, b) = (\delta_1(Q2, b), \delta_2(Q11, b)) = \langle Q3, Q7\rangle$
- $\delta(\langle Q3, Q6\rangle, a) = (\delta_1(Q3, a), \delta_2(Q6, a)) = \langle Q4, Q7\rangle$
- $\delta(\langle Q3, Q6\rangle, b) = (\delta_1(Q3, b), \delta_2(Q6, b)) = \langle Q4, Q8\rangle$
- $\delta(\langle Q3, Q7\rangle, a) = (\delta_1(Q3, a), \delta_2(Q7, a)) = \langle Q4, Q6\rangle$
- $\delta(\langle Q3, Q7\rangle, b) = (\delta_1(Q3, b), \delta_2(Q7, b)) = \langle Q4, Q9\rangle$
- $\delta(\langle Q3, Q8\rangle, a) = (\delta_1(Q3, a), \delta_2(Q8, a)) = \langle Q4, Q9\rangle$
- $\delta(\langle Q3, Q8\rangle, b) = (\delta_1(Q3, b), \delta_2(Q8, b)) = \langle Q4, Q10\rangle$
- $\delta(\langle Q3, Q9\rangle, a) = (\delta_1(Q3, a), \delta_2(Q9, a)) = \langle Q4, Q8\rangle$
- $\delta(\langle Q3, Q9\rangle, b) = (\delta_1(Q3, b), \delta_2(Q9, b)) = \langle Q4, Q11\rangle$
- $\delta(\langle Q3, Q10\rangle, a) = (\delta_1(Q3, a), \delta_2(Q10, a)) = \langle Q4, Q11\rangle$
- $\delta(\langle Q3, Q10\rangle, b) = (\delta_1(Q3, b), \delta_2(Q10, b)) = \langle Q4, Q6\rangle$
- $\delta(\langle Q3, Q11\rangle, a) = (\delta_1(Q3, a), \delta_2(Q11, a)) = \langle Q4, Q10\rangle$
- $\delta(\langle Q3, Q11\rangle, b) = (\delta_1(Q3, b), \delta_2(Q11, b)) = \langle Q4, Q7\rangle$
- $\delta(\langle Q4, Q6\rangle, a) = (\delta_1(Q4, a), \delta_2(Q6, a)) = \langle Q5, Q7\rangle$
- $\delta(\langle Q4, Q6\rangle, b) = (\delta_1(Q4, b), \delta_2(Q6, b)) = \langle Q5, Q8\rangle$
- $\delta(\langle Q4, Q7\rangle, a) = (\delta_1(Q4, a), \delta_2(Q7, a)) = \langle Q5, Q6\rangle$
- $\delta(\langle Q4, Q7\rangle, b) = (\delta_1(Q4, b), \delta_2(Q7, b)) = \langle Q5, Q9\rangle$
- $\delta(\langle Q4, Q8\rangle, a) = (\delta_1(Q4, a), \delta_2(Q8, a)) = \langle Q5, Q9\rangle$
- $\delta(\langle Q4, Q8\rangle, b) = (\delta_1(Q4, b), \delta_2(Q8, b)) = \langle Q5, Q10\rangle$
- $\delta(\langle Q4, Q9\rangle, a) = (\delta_1(Q4, a), \delta_2(Q9, a)) = \langle Q5, Q8\rangle$
- $\delta(\langle Q4, Q9\rangle, b) = (\delta_1(Q4, b), \delta_2(Q9, b)) = \langle Q5, Q11\rangle$
- $\delta(\langle Q4, Q10\rangle, a) = (\delta_1(Q4, a), \delta_2(Q10, a)) = \langle Q5, Q11\rangle$
- $\delta(\langle Q4, Q10\rangle, b) = (\delta_1(Q4, b), \delta_2(Q10, b)) = \langle Q5, Q6\rangle$
- $\delta(\langle Q4, Q11\rangle, a) = (\delta_1(Q4, a), \delta_2(Q11, a)) = \langle Q5, Q10\rangle$
- $\delta(\langle Q4, Q11\rangle, b) = (\delta_1(Q4, b), \delta_2(Q11, b)) = \langle Q5, Q7\rangle$
- $\delta(\langle Q5, Q6\rangle, a) = (\delta_1(Q5, a), \delta_2(Q6, a)) = \langle Q4, Q7\rangle$
- $\delta(\langle Q5, Q6\rangle, b) = (\delta_1(Q5, b), \delta_2(Q6, b)) = \langle Q4, Q8\rangle$
- $\delta(\langle Q5, Q7\rangle, a) = (\delta_1(Q5, a), \delta_2(Q7, a)) = \langle Q4, Q6\rangle$
- $\delta(\langle Q5, Q7\rangle, b) = (\delta_1(Q5, b), \delta_2(Q7, b)) = \langle Q4, Q9\rangle$

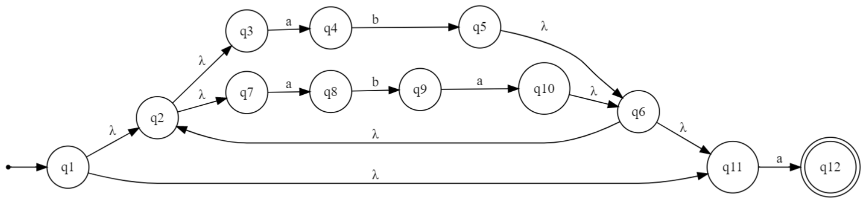
– $\delta(\langle Q5, Q8\rangle, a) = (\delta_1(Q5, a), \delta_2(Q8, a)) = \langle Q4, Q9\rangle$
– $\delta(\langle Q5, Q8\rangle, b) = (\delta_1(Q5, b), \delta_2(Q8, b)) = \langle Q4, Q10\rangle$
– $\delta(\langle Q5, Q9\rangle, a) = (\delta_1(Q5, a), \delta_2(Q9, a)) = \langle Q4, Q8\rangle$
– $\delta(\langle Q5, Q9\rangle, b) = (\delta_1(Q5, b), \delta_2(Q9, b)) = \langle Q4, Q11\rangle$
– $\delta(\langle Q5, Q10\rangle, a) = (\delta_1(Q5, a), \delta_2(Q10, a)) = \langle Q4, Q11\rangle$
– $\delta(\langle Q5, Q10\rangle, b) = (\delta_1(Q5, b), \delta_2(Q10, b)) = \langle Q4, Q6\rangle$
– $\delta(\langle Q5, Q11\rangle, a) = (\delta_1(Q5, a), \delta_2(Q11, a)) = \langle Q4, Q10\rangle$
– $\delta(\langle Q5, Q11\rangle, b) = (\delta_1(Q5, b), \delta_2(Q11, b)) = \langle Q4, Q7\rangle$



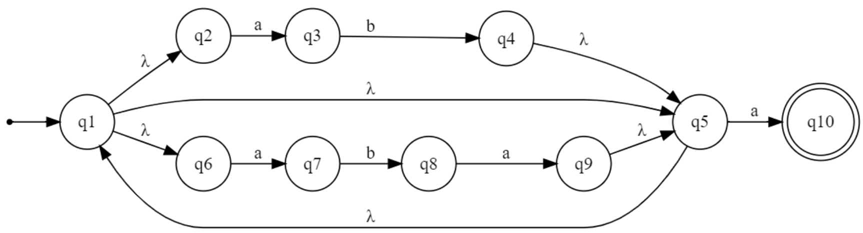
3 Задание №3. Построить минимальный ДКА по регулярному выражению.

Ответом на данное задание является минимальный ДКА, который допускает тот же язык, что описывается регулярным выражением.
Здесь мне уже не хватило сил на то, чтобы выписать все правила

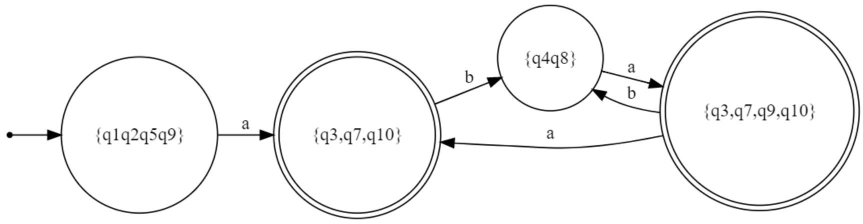
1. $(ab + aba)^*a$
   Строим НКА:



   Минимизируем λ и получим:

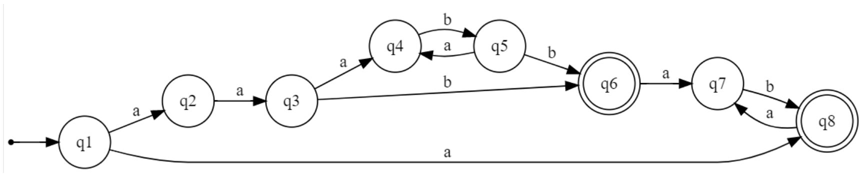


   Преобразуем в ДКА:

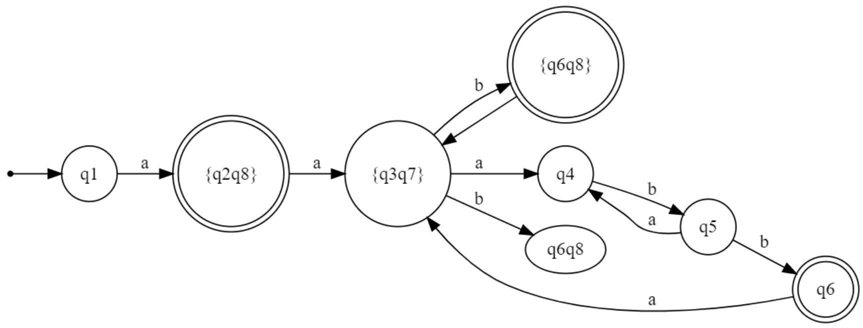


Данный ДКА уже является минимальным.

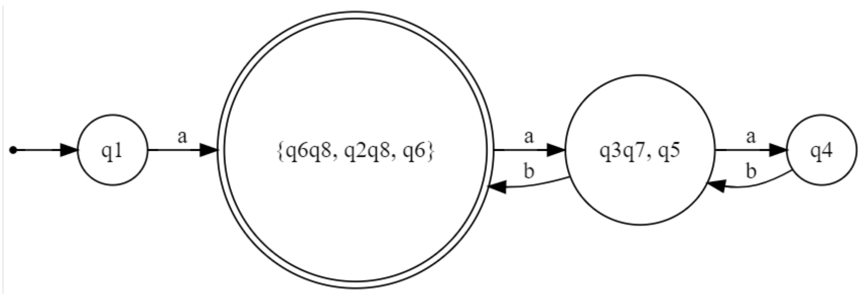
2. $a(a(ab)^*b)(ab)^*$
   Строим НКА:



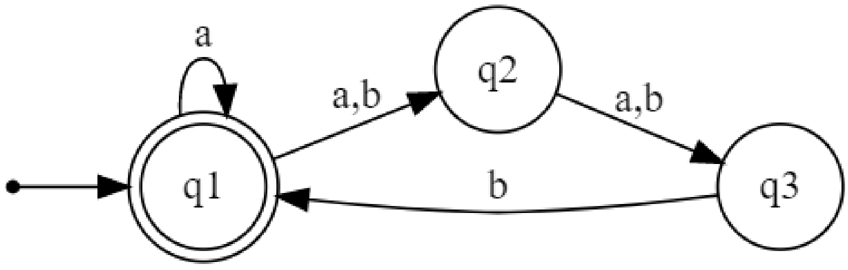
Преобразуем в ДКА:



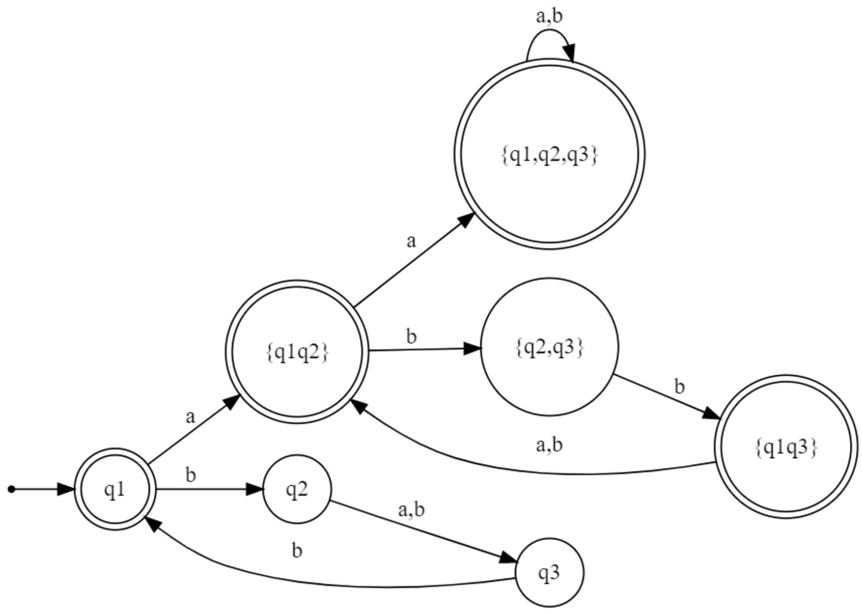
Минимизируем его:



3. $(a + (a + b)(a + b)b)^*$
   Строим НКА:

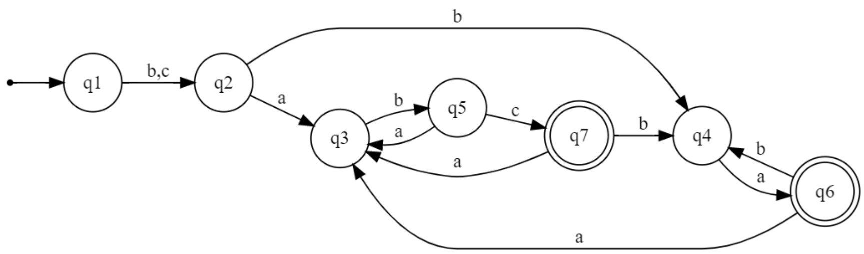


Преобразуем в ДКА:

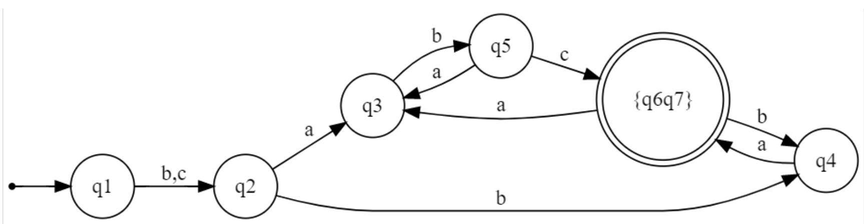


Данный ДКА уже является минимальным.

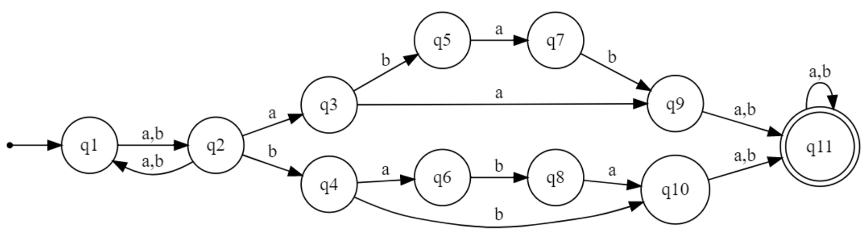
4. $(b+c)((ab)^*c+(ba)^*)^*$
Построим ДКА:



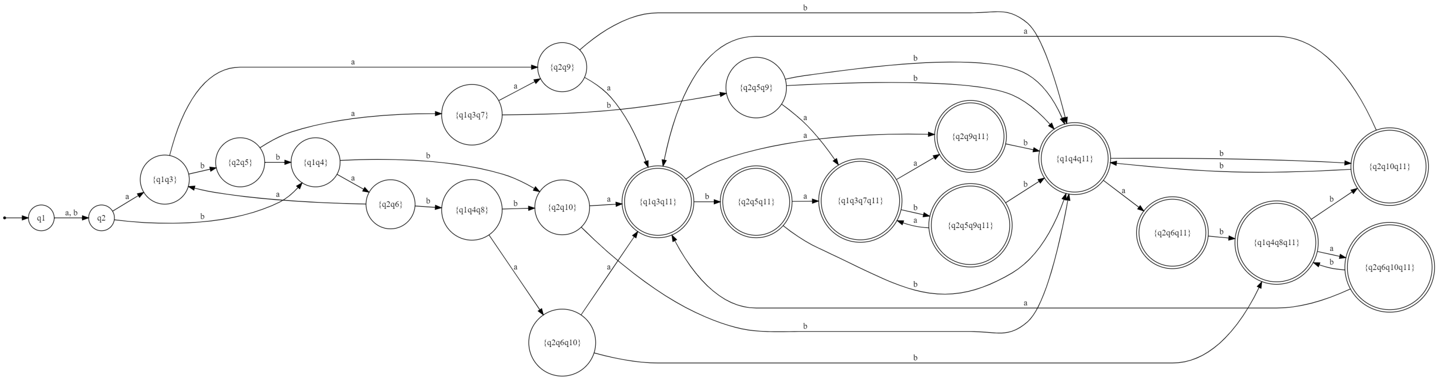
Минимизируем его:



5. $(a+b)^+(aa+bb+abab+baba)(a+b)^+$
Построим НКА:

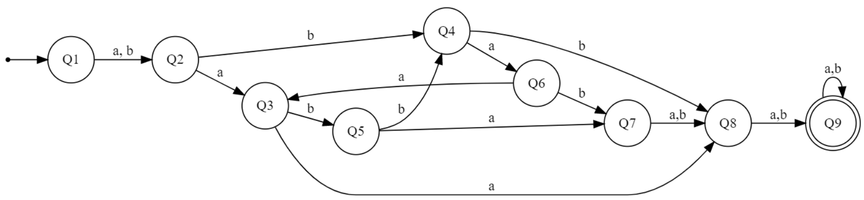


Преобразуем в ДКА:



Минимизируем ДКА и переобозначим эквивалентные вершины:

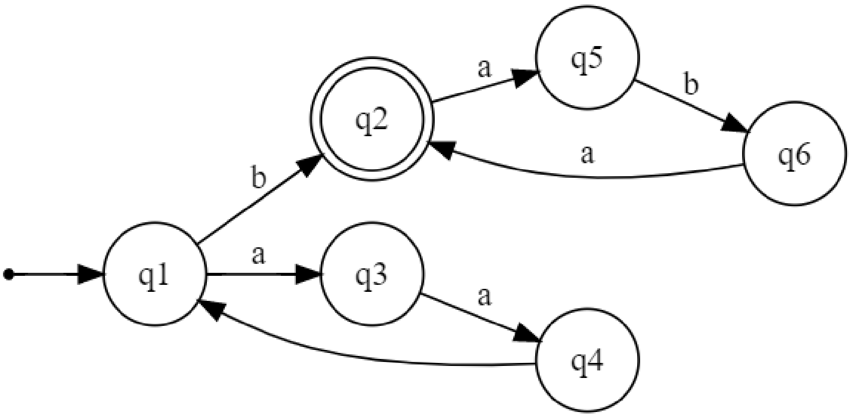
- q1 - Q1
- q2 - Q2
- q1q3 - Q3
- q1q4 - Q4
- q2q5 - Q5
- q2q6 - Q6
- q1q3q7, q1q4q8 - Q7
- q2q9, q2q6q10, q2q5q9, q2q10 - Q8
- q2q5q11, q1q3q11, q2q6q1011, q2q6q11, q1q4q8q11, q1q4q11, q2q10q11, q1q3q7q11, q2q9q11, q2q5q9q11 - Q9



4 Задание №4. Построить минимальный ДКА по регулярному выражению.

Ответом на данное задание является минимальный ДКА, который допускает тот же язык, что описывается регулярным выражением.

1. $L = \{(aab)^n(aba)^m | n \geq 0, m \geq 0\}$
   Конечный автомат, распознающий данный язык:



Соответственно, язык является регулярным.

2. $L = \{uaav | u \in \{a, b\}^*, v \in \{a, b\}^*, |u|_b \geq |v|_a\}$
   Воспользуемся леммой о разрастании.
   Зафиксируем $\forall n \in \mathbb{N}$ и рассмотрим слово $\omega = b^n aaa^n, |\omega| = 2n + 2 \geq n$.

$$\omega = xyz, |y| \neq 0, |xy| \leq n :$$
$$x = b^m, y = b^l, z = b^{n-m-l} aaa^n$$
$$m + l \leq n, l \neq 0$$
$$\omega = xy^k z = b^m b^{il} b^{n-m-l} aaa^n$$
$$\text{Пусть i=0, тогда: } \omega = b^{n-l} aaa^n \notin L, l \neq 0$$

   Лемма не выполняется. Делаем вывод, что язык не является регулярным.

3. $L = \{a^m \omega | \omega \in \{a, b\}^*, 1 \leq |\omega|_b \leq m\}$
   Воспользуемся леммой о разрастании.
   Зафиксируем $\forall n \in \mathbb{N}$ и рассмотрим слово $\omega = a^n b^n, |\omega| = 2n \geq n$.

$$\omega = xyz, |y| \neq 0, |xy| \leq n :$$
$$x = a^m, y = a^l, z = a^{n-m-l} b^n$$
$$m + l \leq n, l \neq 0$$
$$\omega = xy^k z = a^m a^{il} a^{n-m-l} b^n$$
$$\text{Пусть i=0, тогда: } \omega = a^{n-l} b^n \notin L, l \neq 0$$

   Лемма не выполняется. Делаем вывод, что язык не является регулярным.

4. $L = \{a^k b^m a^n | k = n \vee m > 0\}$
   Воспользуемся леммой о разрастании.
   Зафиксируем $\forall n \in \mathbb{N}$ и рассмотрим слово $\omega = a^n b a^n, |\omega| = 2n + 1 \geq n$.

$$\omega = xyz, |y| \neq 0, |xy| \leq n :$$
$$x = a^m, y = a^l, z = a^{n-m-l} ba^n$$
$$m + l \leq n, l \neq 0$$
$$\omega = xy^i z = a^m a^{il} a^{n-m-l} ba^n$$
$$\text{Пусть i=2, тогда: } \omega = a^{n+l} ba^n \notin L, l \neq 0$$

   Лемма не выполняется. Делаем вывод, что язык не является регулярным.

5. $L = \{ucv | u \in \{a,b\}^*, v \in \{a,b\}^*, u \neq v^R\}$
Воспользуемся леммой о разрастании.
Зафиксируем $\forall n \in \mathbb{N}$ и рассмотрим слово $\omega = (ab)^n c(ab)^n = \alpha_1\alpha_2...\alpha_n...\alpha_{2n}...\alpha_{4n}\alpha_{4n+1}, |\omega| = 4n+1 \geq n$.

$$\omega = xyz, |y| \neq 0, |xy| \leq n:$$
$$x = \alpha_1\alpha_2...\alpha_m, y = \alpha_{m+1}\alpha_{m+2}...\alpha_{m+l}, z = \alpha_{m+l+1}\alpha_{m+l+2}...\alpha_{4n+1}c(ab)^n$$
$$m + l \leq n, l \neq 0$$
$$\omega = xy^iz = (\alpha_1\alpha_2...\alpha_m)(\alpha_{m+1}\alpha_{m+2}...\alpha_{m+l})^i, (\alpha_{m+l+1}\alpha_{m+l+2}...\alpha_{4n+1}c(ab)^n)$$
Пусть i=2, тогда: $\omega = (\alpha_1\alpha_2...\alpha_m)(\alpha_{m+1}\alpha_{m+2}...\alpha_{m+l})^2, (\alpha_{m+l+1}\alpha_{m+l+2}...\alpha_{4n+1}c(ab)^n) \notin L, l \neq 0$

Лемма не выполняется. Делаем вывод, что язык не является регулярным.