

# DFA 2

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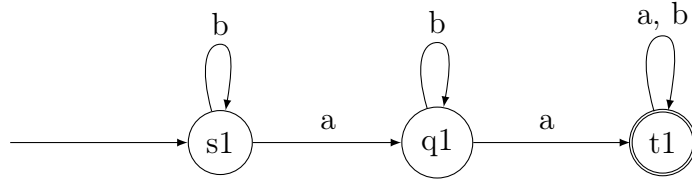
March 2022

## Task

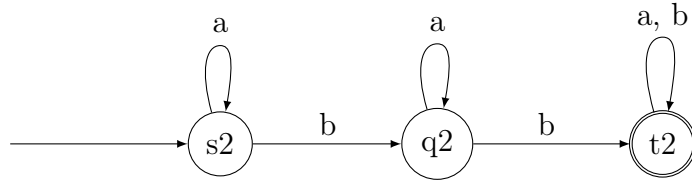
Ответом на данное задание является конечный автомат, распознающий описанный язык. Требуется, чтобы он был построен при помощи прямого произведения ДКА и его свойств.

1

$$L_1 = \{w \in \{a, b\}^* \mid |w|_a \geq 2 \wedge |w|_b \geq 2\}$$
$$L_{11} = \{w \in \{a, b\}^* \mid |w|_a \geq 2\}$$



$$L_{12} = \{w \in \{a, b\}^* \mid |w|_b \geq 2\}$$



$$L_1 = L_{11} \cap L_{12}$$

$$A_{11} = \{\Sigma = \{a, b\}, Q_1 = \{s_1, q_1, t_1\}, s_1, T_1 = \{t_1\}, \delta_1\}$$

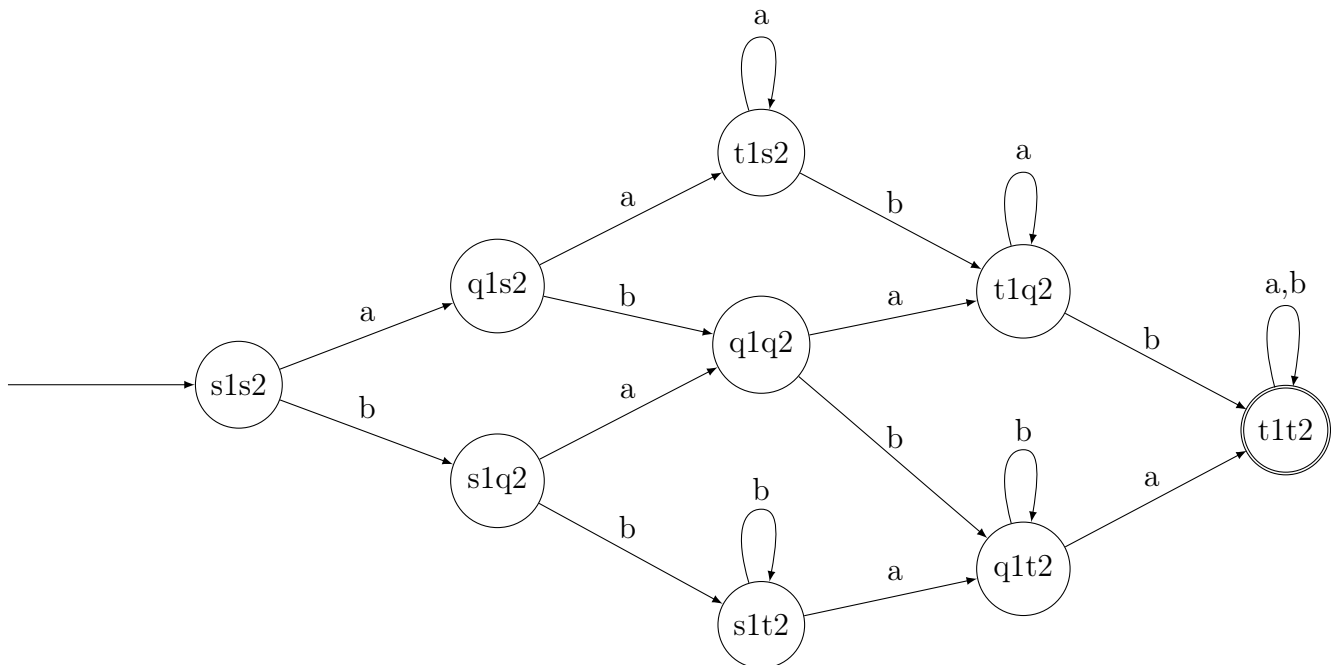
$$A_{12} = \{\Sigma = \{a, b\}, Q_2 = \{s_2, q_2, t_2\}, s_2, T_2 = \{t_2\}, \delta_2\}$$

$$A_1 = \{\Sigma, Q, s, T, \delta\}$$

1.  $\Sigma = \{a, b\}$
2.  $Q = Q_1 \times Q_2 = \{\langle s_1, s_2 \rangle, \langle s_1, q_2 \rangle, \langle s_1, t_2 \rangle, \langle q_1, s_2 \rangle, \langle q_1, q_2 \rangle, \langle q_1, t_2 \rangle, \langle t_1, s_2 \rangle, \langle t_1, q_2 \rangle, \langle t_1, t_2 \rangle\}$
3.  $s = \langle s_1, s_2 \rangle$
4.  $T = T_1 \times T_2 = \{\langle t_1, t_2 \rangle\}$
5.  $\delta :$

- $\delta(\langle s_1, s_2 \rangle, a) = \langle \delta_1(s_1, a), \delta_2(s_2, a) \rangle = \langle q_1, s_2 \rangle$
- $\delta(\langle s_1, s_2 \rangle, b) = \langle \delta_1(s_1, b), \delta_2(s_2, b) \rangle = \langle s_1, q_2 \rangle$
- $\delta(\langle s_1, q_2 \rangle, a) = \langle \delta_1(s_1, a), \delta_2(q_2, a) \rangle = \langle q_1, q_2 \rangle$
- $\delta(\langle s_1, q_2 \rangle, b) = \langle \delta_1(s_1, b), \delta_2(q_2, b) \rangle = \langle s_1, t_2 \rangle$
- $\delta(\langle s_1, t_2 \rangle, a) = \langle \delta_1(s_1, a), \delta_2(t_2, a) \rangle = \langle q_1, t_2 \rangle$
- $\delta(\langle s_1, t_2 \rangle, b) = \langle \delta_1(s_1, b), \delta_2(t_2, b) \rangle = \langle s_1, s_2 \rangle$
- $\delta(\langle q_1, s_2 \rangle, a) = \langle \delta_1(q_1, a), \delta_2(s_2, a) \rangle = \langle t_1, s_2 \rangle$
- $\delta(\langle q_1, s_2 \rangle, b) = \langle \delta_1(q_1, b), \delta_2(s_2, b) \rangle = \langle q_1, q_2 \rangle$
- $\delta(\langle q_1, q_2 \rangle, a) = \langle \delta_1(q_1, a), \delta_2(q_2, a) \rangle = \langle t_1, q_2 \rangle$
- $\delta(\langle q_1, q_2 \rangle, b) = \langle \delta_1(q_1, b), \delta_2(q_2, b) \rangle = \langle q_1, t_2 \rangle$
- $\delta(\langle q_1, t_2 \rangle, a) = \langle \delta_1(q_1, a), \delta_2(t_2, a) \rangle = \langle t_1, t_2 \rangle$
- $\delta(\langle q_1, t_2 \rangle, b) = \langle \delta_1(q_1, b), \delta_2(t_2, b) \rangle = \langle q_1, s_2 \rangle$
- $\delta(\langle t_1, s_2 \rangle, a) = \langle \delta_1(t_1, a), \delta_2(s_2, a) \rangle = \langle t_1, s_2 \rangle$
- $\delta(\langle t_1, s_2 \rangle, b) = \langle \delta_1(t_1, b), \delta_2(s_2, b) \rangle = \langle t_1, q_2 \rangle$
- $\delta(\langle t_1, q_2 \rangle, a) = \langle \delta_1(t_1, a), \delta_2(q_2, a) \rangle = \langle t_1, q_2 \rangle$
- $\delta(\langle t_1, q_2 \rangle, b) = \langle \delta_1(t_1, b), \delta_2(q_2, b) \rangle = \langle t_1, t_2 \rangle$
- $\delta(\langle t_1, t_2 \rangle, a) = \langle \delta_1(t_1, a), \delta_2(t_2, a) \rangle = \langle t_1, t_2 \rangle$
- $\delta(\langle t_1, t_2 \rangle, b) = \langle \delta_1(t_1, b), \delta_2(t_2, b) \rangle = \langle t_1, s_2 \rangle$

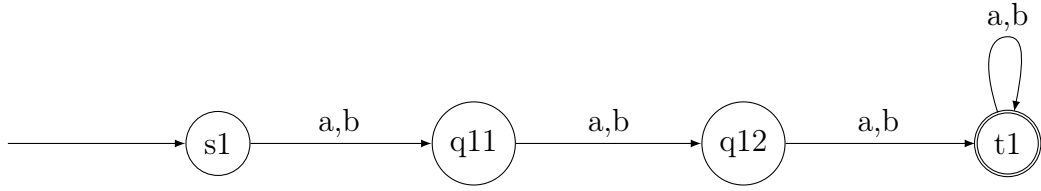
Теперь можем построить автомат:



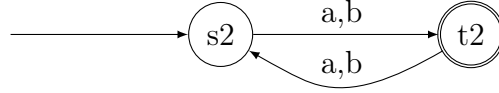
2

$$L_2 = \{w \in \{a, b\}^* \mid |w| \geq 3 \wedge |w| \text{ нечётное}\}$$

$$L_{21} = \{w \in \{a, b\}^* \mid |w| \geq 3\}$$



$$L_{22} = \{w \in \{a, b\}^* \mid |w| \text{ нечётное}\}$$



$$L_2 = L_{21} \cap L_{22}$$

$$A_{21} = \{\Sigma = \{a, b\}, Q_1 = \{s_1, q_{11}, q_{12}, t_1\}, s_1, T_1 = \{t_1\}, \delta_1\}$$

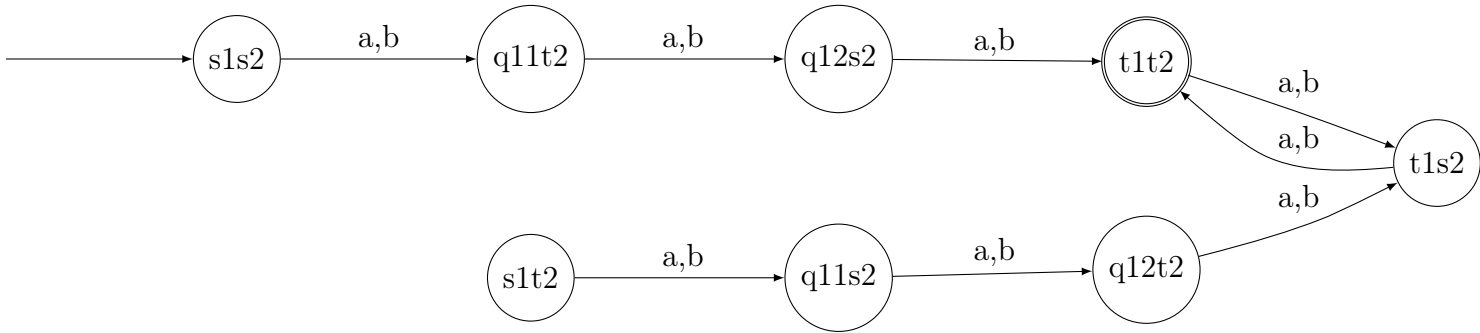
$$A_{22} = \{\Sigma = \{a, b\}, Q_2 = \{s_2, t_2\}, s_2, T_2 = \{t_2\}, \delta_2\}$$

$$A_2 = \{\Sigma, Q, s, T, \delta\}$$

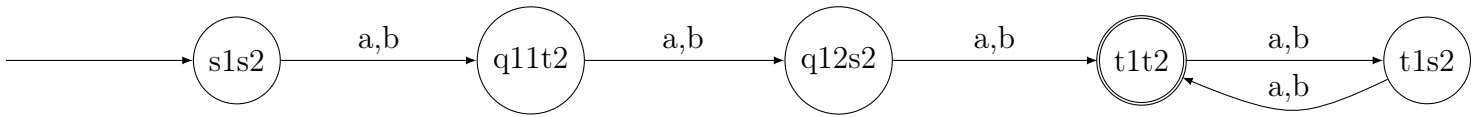
1.  $\Sigma = \{a, b\}$
2.  $Q = Q_1 \times Q_2 = \{\langle s_1, s_2 \rangle, \langle s_1, t_2 \rangle, \langle q_{11}, s_2 \rangle, \langle q_{11}, t_2 \rangle, \langle q_{12}, s_2 \rangle, \langle q_{12}, t_2 \rangle, \langle t_1, s_2 \rangle, \langle t_1, t_2 \rangle\}$
3.  $s = \langle s_1, s_2 \rangle$
4.  $T = T_1 \times T_2 = \{\langle t_1, t_2 \rangle\}$
5.  $\delta :$

- $\delta(\langle s_1, s_2 \rangle, a) = \langle \delta_1(s_1, a), \delta_2(s_2, a) \rangle = \langle q_{11}, t_2 \rangle$
- $\delta(\langle s_1, s_2 \rangle, b) = \langle \delta_1(s_1, b), \delta_2(s_2, b) \rangle = \langle q_{11}, t_2 \rangle$
- $\delta(\langle s_1, t_2 \rangle, a) = \langle \delta_1(s_1, a), \delta_2(t_2, a) \rangle = \langle q_{11}, s_2 \rangle$
- $\delta(\langle s_1, t_2 \rangle, b) = \langle \delta_1(s_1, b), \delta_2(t_2, b) \rangle = \langle q_{11}, s_2 \rangle$
- $\delta(\langle q_{11}, s_2 \rangle, a) = \langle \delta_1(q_{11}, a), \delta_2(s_2, a) \rangle = \langle q_{12}, t_2 \rangle$
- $\delta(\langle q_{11}, s_2 \rangle, b) = \langle \delta_1(q_{11}, b), \delta_2(s_2, b) \rangle = \langle q_{12}, t_2 \rangle$
- $\delta(\langle q_{11}, t_2 \rangle, a) = \langle \delta_1(q_{11}, a), \delta_2(t_2, a) \rangle = \langle q_{12}, s_2 \rangle$
- $\delta(\langle q_{11}, t_2 \rangle, b) = \langle \delta_1(q_{11}, b), \delta_2(t_2, b) \rangle = \langle q_{12}, s_2 \rangle$
- $\delta(\langle q_{12}, s_2 \rangle, a) = \langle \delta_1(q_{12}, a), \delta_2(s_2, a) \rangle = \langle t_1, t_2 \rangle$
- $\delta(\langle q_{12}, s_2 \rangle, b) = \langle \delta_1(q_{12}, b), \delta_2(s_2, b) \rangle = \langle t_1, t_2 \rangle$
- $\delta(\langle q_{12}, t_2 \rangle, a) = \langle \delta_1(q_{12}, a), \delta_2(t_2, a) \rangle = \langle t_1, s_2 \rangle$
- $\delta(\langle q_{12}, t_2 \rangle, b) = \langle \delta_1(q_{12}, b), \delta_2(t_2, b) \rangle = \langle t_1, s_2 \rangle$
- $\delta(\langle t_1, s_2 \rangle, a) = \langle \delta_1(t_1, a), \delta_2(s_2, a) \rangle = \langle t_1, t_2 \rangle$
- $\delta(\langle t_1, s_2 \rangle, b) = \langle \delta_1(t_1, b), \delta_2(s_2, b) \rangle = \langle t_1, t_2 \rangle$
- $\delta(\langle t_1, t_2 \rangle, a) = \langle \delta_1(t_1, a), \delta_2(t_2, a) \rangle = \langle t_1, s_2 \rangle$
- $\delta(\langle t_1, t_2 \rangle, b) = \langle \delta_1(t_1, b), \delta_2(t_2, b) \rangle = \langle t_1, s_2 \rangle$

Строим автомат  $A_2$  :



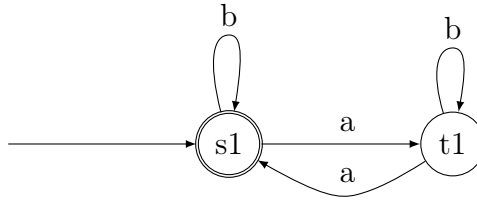
Минимизируем:



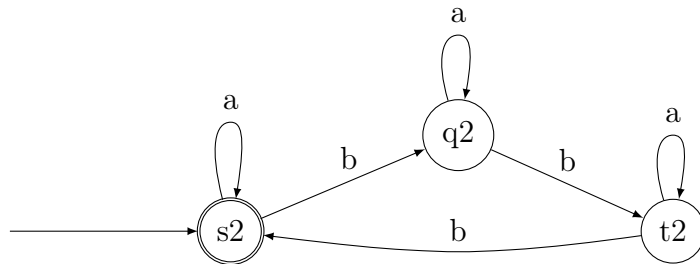
3

$$L_3 = \{w \in \{a, b\}^* \mid |w|_a \text{ чётно} \wedge |w|_b \text{ кратно трём} \}$$

$$L_{31} = \{w \in \{a, b\}^* \mid |w|_a \text{ чётно} \}$$



$$L_{32} = \{w \in \{a, b\}^* \mid |w|_b \text{ кратно трём} \}$$



$$L_3 = L_{31} \cap L_{32}$$

$$A_{31} = \{\Sigma = \{a, b\}, Q_1 = \{s_1, t_1\}, s_1, T_1 = \{s_1\}, \delta_1\}$$

$$A_{32} = \{\Sigma = \{a, b\}, Q_2 = \{s_2, q_2, t_2\}, s_2, T_2 = \{s_2\}, \delta_2\}$$

$$A_3 = \{\Sigma, Q, s, T, \delta\}$$

$$1. \Sigma = \{a, b\}$$

$$2. Q = Q_1 \times Q_2 = \{\langle s_1, s_2 \rangle, \langle s_1, q_2 \rangle, \langle s_1, t_2 \rangle, \langle t_1, s_2 \rangle, \langle t_1, q_2 \rangle, \langle t_1, t_2 \rangle\}$$

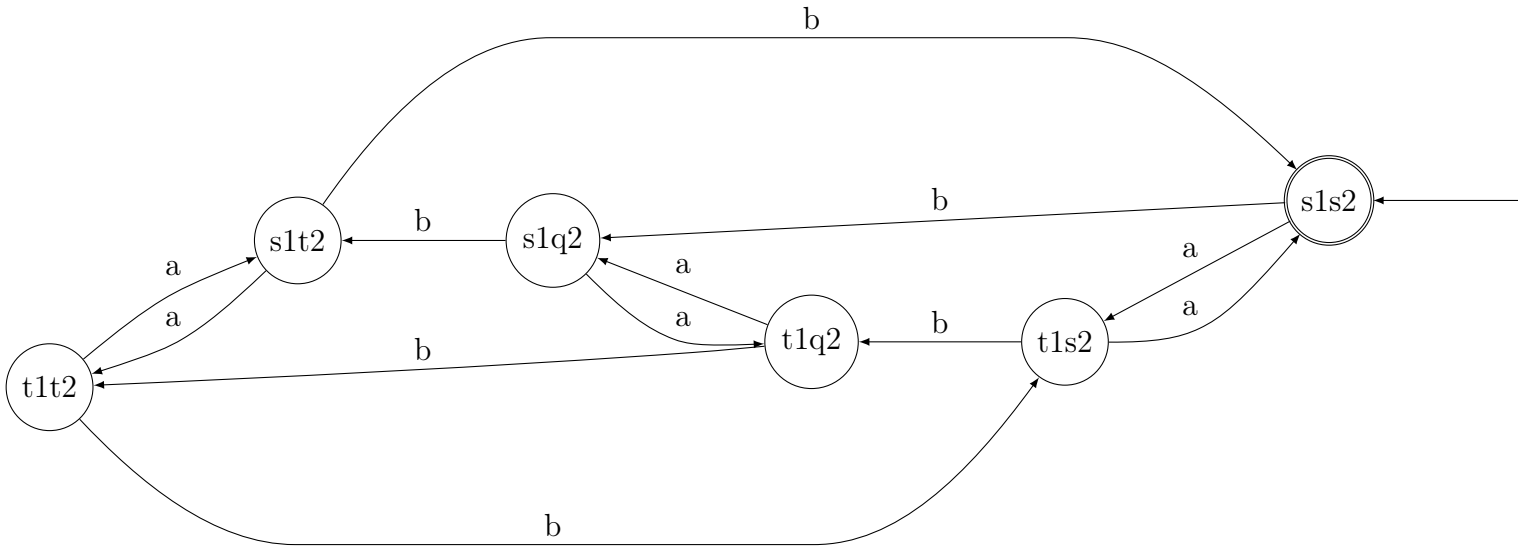
$$3. s = \langle s_1, s_2 \rangle$$

4.  $T = T_1 \times T_2 = \{\langle s_1, s_2 \rangle\}$

5.  $\delta$  :

- $\delta(\langle s_1, s_2 \rangle, a) = \langle \delta_1(s_1, a), \delta_2(s_2, a) \rangle = \langle t_1, s_2 \rangle$
- $\delta(\langle s_1, s_2 \rangle, b) = \langle \delta_1(s_1, b), \delta_2(s_2, b) \rangle = \langle s_1, q_2 \rangle$
- $\delta(\langle s_1, q_2 \rangle, a) = \langle \delta_1(s_1, a), \delta_2(q_2, a) \rangle = \langle t_1, q_2 \rangle$
- $\delta(\langle s_1, q_2 \rangle, b) = \langle \delta_1(s_1, b), \delta_2(q_2, b) \rangle = \langle s_1, t_2 \rangle$
- $\delta(\langle s_1, t_2 \rangle, a) = \langle \delta_1(s_1, a), \delta_2(t_2, a) \rangle = \langle t_1, t_2 \rangle$
- $\delta(\langle s_1, t_2 \rangle, b) = \langle \delta_1(s_1, b), \delta_2(t_2, b) \rangle = \langle s_1, s_2 \rangle$
- $\delta(\langle t_1, s_2 \rangle, a) = \langle \delta_1(t_1, a), \delta_2(s_2, a) \rangle = \langle s_1, s_2 \rangle$
- $\delta(\langle t_1, s_2 \rangle, b) = \langle \delta_1(t_1, b), \delta_2(s_2, b) \rangle = \langle t_1, q_2 \rangle$
- $\delta(\langle t_1, q_2 \rangle, a) = \langle \delta_1(t_1, a), \delta_2(q_2, a) \rangle = \langle s_1, q_2 \rangle$
- $\delta(\langle t_1, q_2 \rangle, b) = \langle \delta_1(t_1, b), \delta_2(q_2, b) \rangle = \langle t_1, t_2 \rangle$
- $\delta(\langle t_1, t_2 \rangle, a) = \langle \delta_1(t_1, a), \delta_2(t_2, a) \rangle = \langle s_1, t_2 \rangle$
- $\delta(\langle t_1, t_2 \rangle, b) = \langle \delta_1(t_1, b), \delta_2(t_2, b) \rangle = \langle t_1, s_2 \rangle$

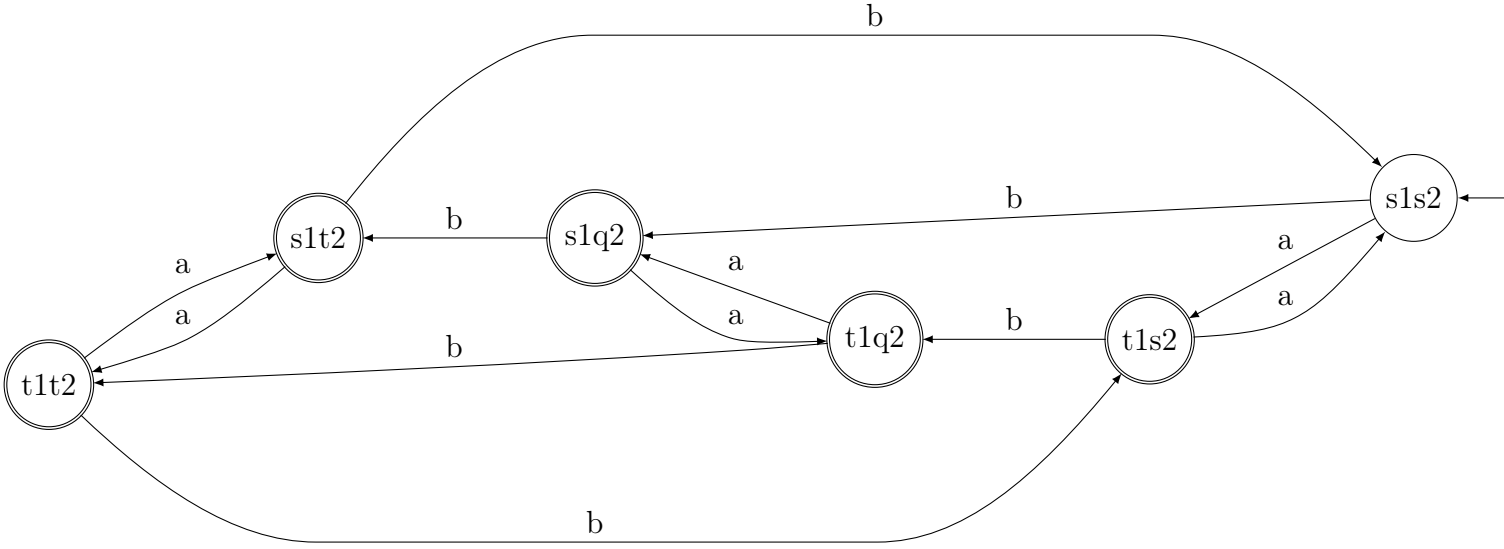
АВТОМАТ  $A_3$  :



4

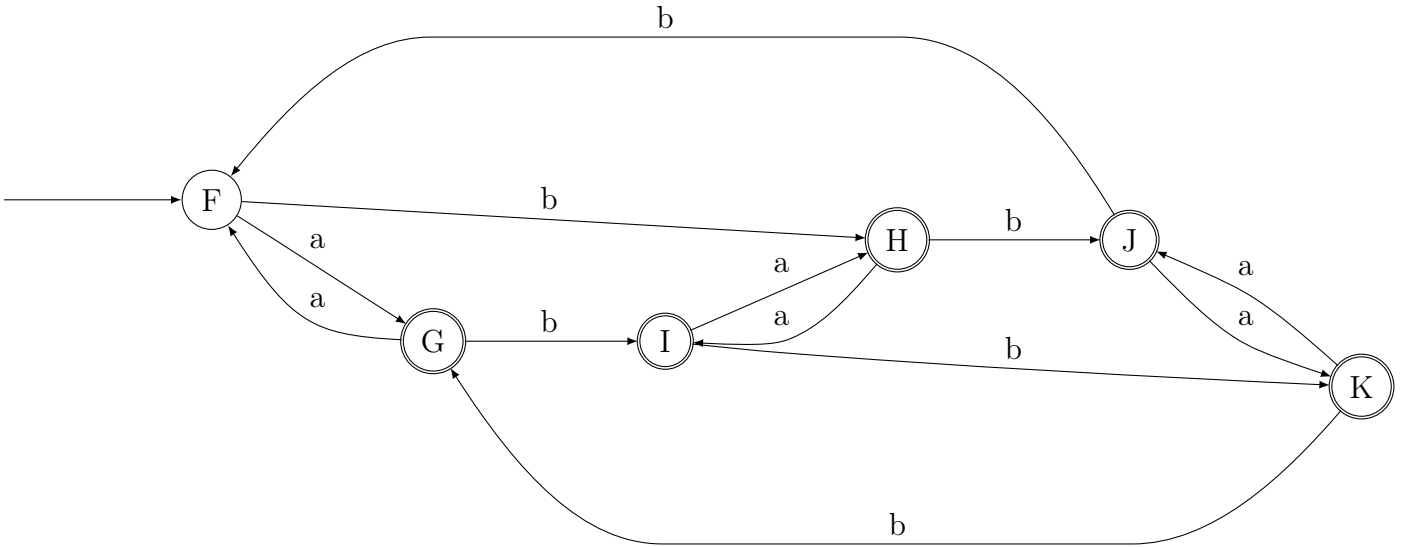
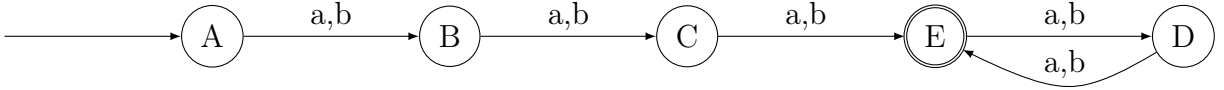
$$L_4 = \overline{L_3}$$

$$T_4 = Q_3 \setminus T_3 = \{\langle s_1, q_2 \rangle, \langle s_1, t_2 \rangle, \langle t_1, s_2 \rangle, \langle t_1, q_2 \rangle, \langle t_1, t_2 \rangle\}$$



5

$$L_5 = L_2 \setminus L_3 = L_2 \cap \overline{L_3}$$



$$1. \Sigma = \{a, b\}$$

$$Q = Q_2 \times Q_3 = \{\langle A, F \rangle, \langle A, G \rangle, \langle A, I \rangle, \langle A, H \rangle, \langle A, J \rangle, \langle A, K \rangle, \\ \langle B, F \rangle, \langle B, G \rangle, \langle B, I \rangle, \langle B, H \rangle, \langle B, J \rangle, \langle B, K \rangle, \\ 2. \quad \langle C, F \rangle, \langle C, G \rangle, \langle C, I \rangle, \langle C, H \rangle, \langle C, J \rangle, \langle C, K \rangle, \\ \langle D, F \rangle, \langle D, G \rangle, \langle D, I \rangle, \langle D, H \rangle, \langle D, J \rangle, \langle D, K \rangle, \\ \langle E, F \rangle, \langle E, G \rangle, \langle E, I \rangle, \langle E, H \rangle, \langle E, J \rangle, \langle E, K \rangle\}$$

$$3. s = \langle A, F \rangle$$

$$4. T = T_2 \times T_3 = \{\langle E, G \rangle, \langle E, I \rangle, \langle E, H \rangle, \langle E, J \rangle, \langle E, K \rangle\}$$

5.  $\delta$ :

node	a	b
$\langle A, F \rangle$	$\langle B, G \rangle$	$\langle B, H \rangle$
$\langle A, G \rangle$	$\langle B, F \rangle$	$\langle B, I \rangle$
$\langle A, I \rangle$	$\langle B, H \rangle$	$\langle B, K \rangle$
$\langle A, H \rangle$	$\langle B, I \rangle$	$\langle B, J \rangle$
$\langle A, J \rangle$	$\langle B, K \rangle$	$\langle B, F \rangle$
$\langle A, K \rangle$	$\langle B, J \rangle$	$\langle B, G \rangle$
$\langle B, F \rangle$	$\langle C, G \rangle$	$\langle C, H \rangle$
$\langle B, G \rangle$	$\langle C, F \rangle$	$\langle C, I \rangle$
$\langle B, I \rangle$	$\langle C, H \rangle$	$\langle C, K \rangle$
$\langle B, H \rangle$	$\langle C, I \rangle$	$\langle C, J \rangle$
$\langle B, J \rangle$	$\langle C, K \rangle$	$\langle C, F \rangle$
$\langle B, K \rangle$	$\langle C, J \rangle$	$\langle C, G \rangle$
$\langle C, F \rangle$	$\langle E, G \rangle$	$\langle E, H \rangle$
$\langle C, G \rangle$	$\langle E, F \rangle$	$\langle E, I \rangle$
$\langle C, I \rangle$	$\langle E, H \rangle$	$\langle E, K \rangle$
$\langle C, H \rangle$	$\langle E, I \rangle$	$\langle E, J \rangle$
$\langle C, J \rangle$	$\langle E, K \rangle$	$\langle E, F \rangle$
$\langle C, K \rangle$	$\langle E, J \rangle$	$\langle E, G \rangle$
$\langle D, F \rangle$	$\langle E, G \rangle$	$\langle E, H \rangle$
$\langle D, G \rangle$	$\langle E, F \rangle$	$\langle E, I \rangle$
$\langle D, I \rangle$	$\langle E, H \rangle$	$\langle E, K \rangle$
$\langle D, H \rangle$	$\langle E, I \rangle$	$\langle E, J \rangle$
$\langle D, J \rangle$	$\langle E, K \rangle$	$\langle E, F \rangle$
$\langle D, K \rangle$	$\langle E, J \rangle$	$\langle E, G \rangle$
$\langle E, F \rangle$	$\langle D, G \rangle$	$\langle D, H \rangle$
$\langle E, G \rangle$	$\langle D, F \rangle$	$\langle D, I \rangle$
$\langle E, I \rangle$	$\langle D, H \rangle$	$\langle D, K \rangle$
$\langle E, H \rangle$	$\langle D, I \rangle$	$\langle D, J \rangle$
$\langle E, J \rangle$	$\langle D, K \rangle$	$\langle D, F \rangle$
$\langle E, K \rangle$	$\langle D, J \rangle$	$\langle D, G \rangle$

Построим соответствующий автомат:

