Rational Roots of Irrational Markets: Game Theory and Financial Bubbles

Arjun Maneesh Agarwal February 9, 2024

Chennai Mathematical Institute





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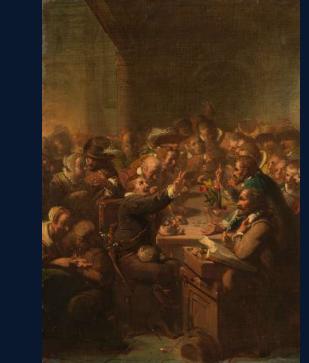
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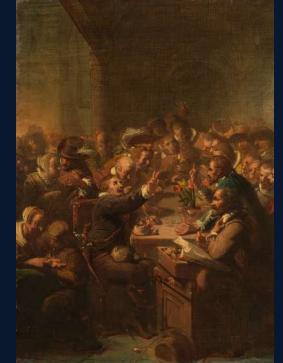
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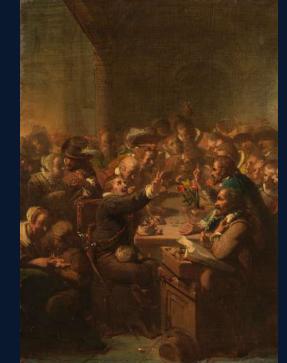
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- The Viceroy tulip became the most prized variety.
- At its peak, one Viceroy tulip was traded for:



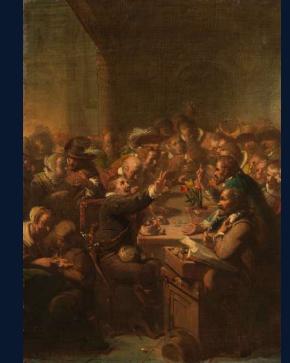
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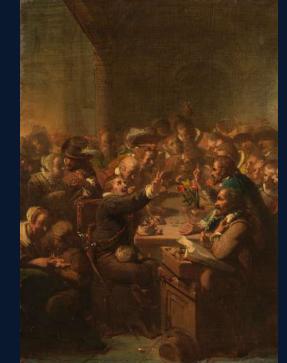
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- A silver cup, clothes, a bed with bedding, and even a ship!





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- The market collapsed, leaving many bankrupt.

Tulipomania Parararararararar Parararararararar Tulip Price Index_ Aec. 1. 1634 to Feb. 5, 1637 1634 1635 1636 1637



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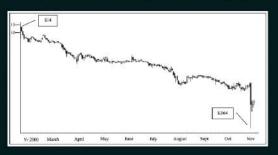
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 - The tipping point when rationality returns.





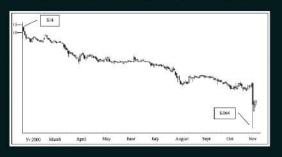


PETS.COM'S SHARE PRICE





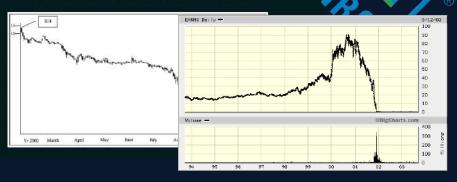
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\$4.50 -98% from IPO

2003

2010

2000

1998

<u>Rationality</u>





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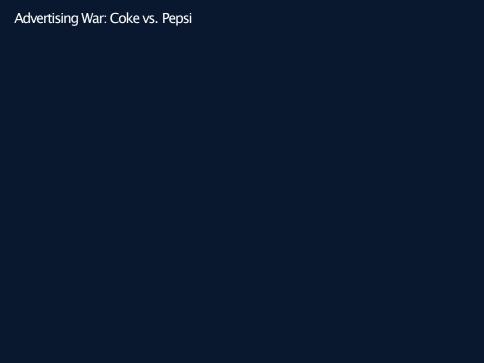
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- "Do the best you can given how you perceive the game and how you evaluate its various possible outcomes."
- In theory...but in practice?



Advertising War: Coke vs. Pepsi Without any advertising, each company earns \$5b/year

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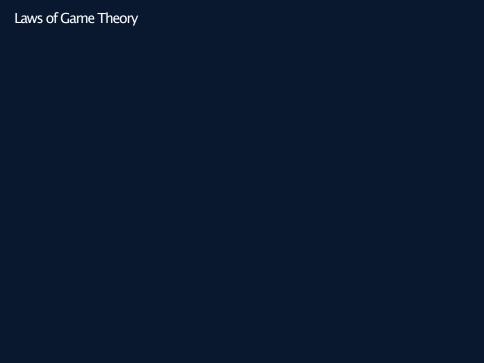
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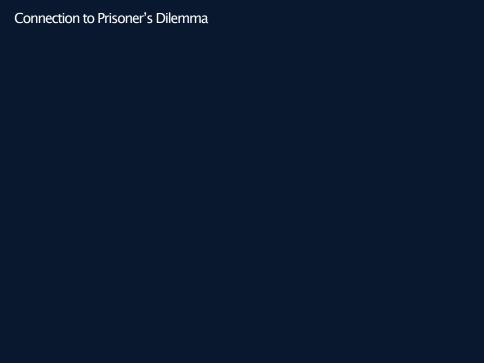
- What will the Cola companies do?
- Is there a better feasible outcome?



Laws of Game Theory

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- Second Law of Game Theory: Rational Choice may lead to outcomes which suck.



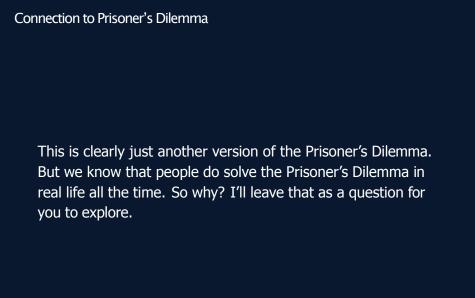












Hint: Look for repeated interactions!





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Cocaine Oligopoly

From one coke to another, cocaine is controlled by oligopolies. Let's work with a small case: two cartels have about 100 tons each. Let's say the cost per gram is $200 - q_1 - q_2$, where q_1 and q_2 are the amounts of cocaine the cartels(in tons) let into the market. Here, 200 is just a constant representing the maximum someone will pay per gram(in USD).

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- What does the price converge to?

Best Response Analysis

Let's say if my opponent is producing q_2 , my best response is to produce q'_1 . Using simple differentiation, for a given q_2 , cartel 1's best response is:

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This is called the Cournout Price!



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Example: Political Candidates

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- Play to win the election!

Median Voter Theorem

Proof.

- Suppose not.
- Without loss of generality, suppose that p₁ has more votes than p₂ and that p₁ < b_{median}.
- Then p_2 will deviate and instead choose $p_2 = p_1 + \varepsilon$, with ε small, so that $p_2 < b_{median}$.
- From single-peakedness, all the voters with ideal points in the interval $[p_2, \infty)$ prefer p_2 to p_1 .
- Since $p_2 < b_{median}$, this is more than half of the voters.
- So p₂ would win, and thus would prefer to deviate. So it is not an
 equilibrium for p₁ to win with p₁ < b_{median}.
- Thus the only equilibrium where there is no profitable deviation is
 p₁ = p₂ = b_{median}.





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- Why doesn't this always happen? That's deeper voting theory, which we won't cover today.



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Simple Retelling: Beach Vendors

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- Firms locate to maximize sales!

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Branding can solve this problem. Explore the candidate-voter model and branding on a line for more insights.



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Nash Equilibrium



Introduction

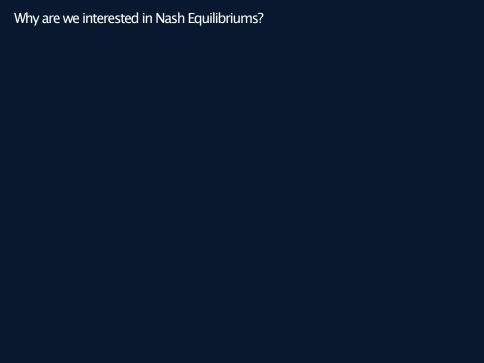
Coming back to our topic, Nash Equilibriums are these magical stalemate points. The formal definition is:

Definition Given a strategy profile $a_1, a_2, a_3, \ldots, a_n$; $b_1, b_2, b_3, \ldots, b_n$ and onwards for agents,

WLOG, $(a_1, b_1, c_1, ...)$ is a Nash equilibrium if and only if:

$$U(a_1\mid b_1,c_1,\dots)\geq U(a_i\mid b_1,c_1,\dots)\quad \forall i$$

and the same for b and c and so on.



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- They are self-fulfilling and self-adjusting.





• Equilibrium does not mean optimal!

Limitations of Nash Equilibrium

- Equilibrium does not mean optimal!
- Many interesting games have more than one Nash Equilibrium!



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There is no pure strategy Nash equilibrium. However, if A chooses PNT with probability p and MTP with probability 1 - p, then we can solve for p to find a mixed Nash equilibrium.



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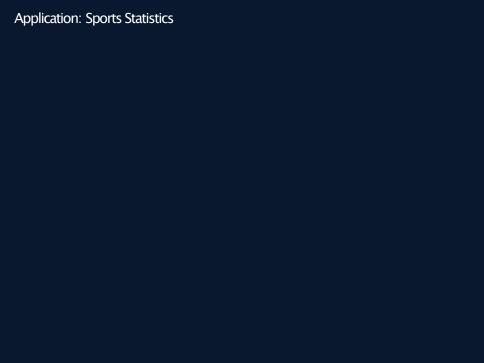
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- Although the algorithm to find it is much simpler, just assign probabilities to all non-dominated choices of A and solve so that B's choices are equal. It is a bunch of linear equations really.



Application: Sports Statistics
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- We'll see a poker theory example later down the line.



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- The first is Pareto dominant, but we often converge to the bad equilibrium.



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 - And we are still getting data on this one, Celsius's 12 billion dollar collapse(2024).













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 - It played a key role in providing liquidity to FTX and executing complex arbitrage strategies.



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 - In November 2022, a leaked Alameda balance sheet revealed that much of its assets were FTT-based, raising concerns about FTX's solvency.





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 - FTX had invested \$500 million in AI startup Anthropic, which later surged in value.
 - If SBF had managed to restore trust and survive the crisis, not declare bankruptcy, this investment could have been a financial lifeline.





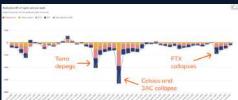


















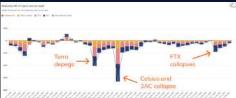


• What is the moral of this story?









Sequential Games





he hat game	
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Due to my inability to fit a tikz diagram in a slide, we will make the game tree on the board!



Morel Hazard

 Here is the problem, adding 50 each has the best payoff. But player 1 is afraid of player 2 getting greedy. Morel Hazard

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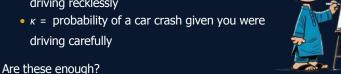
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- This is called a morel hazard.

Car Insurance

- The classic problem is car insurance. What should the the terms be so that you don't mistreat your car?
- This process is called incentive design. If we change the payoffs in such a way that it is no longer a good idea to mistreat your car, well people won't.

Let's look at what all we need to consider here

- p = insurance premium
- r = recovery if car crashes
- c = cost of car
- ρ = probability of a car crash given you were driving recklessly





Car Insurence Cont.	
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These kind of analysis are studied a lot in contract theory and policy design. You will also see them appear time and again in risk management and investment work.



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Backward induction procedure:

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- Note perfect rationality has been assumed.

Extended example: Boomer Snap



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First, some context. Marvel Snap employs poker-like betting mechanics. Players enter a match wagering 1 point and can "Snap!" on any turn to double their wager. Their opponent is given the option to either match the raise, or to retreat and forfeit the pot. Additionally, the game will always double the wager on the final turn. If you snap on this final turn, you are effectively re-raising to quadruple your wager. This is the notorious "boomer snap".



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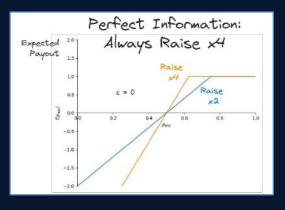
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Why is then boomer snap bad?



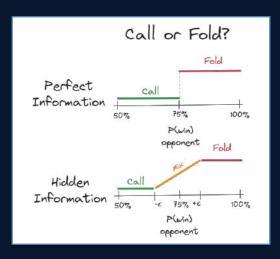
 Of course, Marvel Snap is not a perfect information game. Opponents cannot see the cards in your hand. Additionally, players are not perfect computers. They might misjudge the situation and miscalculate P(win), which can be interpreted as hidden information. Given the variety of cards and decks, Snap seems to have even more hidden information than Poker. Let's add this to our model.

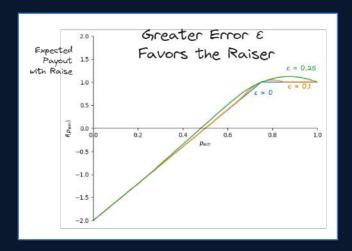
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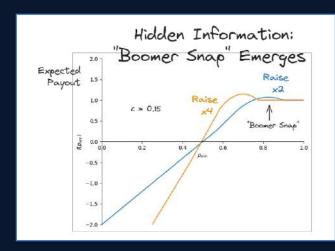
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 - Note that p̂(win) is unbiased. The expected value of p̂(win) is the true value P(win).
 Payouts are still calculated based on P(win)







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- But this is also not a complete model? What are some hidden assumptions and simplifications we are making?



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- Asymmetric information. We assumed that both players had the same error ε. However, if your opponent has more information than you, then that further muddles the decision. This scenario often arises, as one player may have played more cards to create a winning board state, but the other player may have more cards in hand or have been preparing a powerful combo for the final turn.
 'Market for Lemons' by Akerlof is a great start to talk about such asymmetric scenarios.



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Note: Spence and Akerlof along with Stiglitz got the 2001 Nobel for the mentioned works.

- Raise error. As we'll see going forward, people are not perfectly rational. They make irrational decisions for many reasons.
- Multiple turns. Our model was a single turn affair, but betting in Snap occurs over multiple turns. In Snap, you can only "snap!" once in the whole game. A boomer snapper held onto their one snap until the final turn. If they have a dominant lead, they likely should have snapped earlier. This is similar to poker, where strong hands will push repeated small raises rather than a single aggressive raise.
- Curiosity calls. In an extended affair like a poker tournament, it is sometimes be valuable to pay for information on your opponent's play style. Similarly, many competitive snap players do Boomer snap and take up boomer snaps just to understand their opponents better.

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Other than that, such analysis is part of an intersection field of Probability, combinatorics, behavioural psychology and Game theory called Poker Theory

<u>Bubbles</u>





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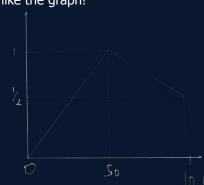
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- As we can see the people are not sign-ist.
- They just prefer to not be the minority which is justifiable.





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- What do we observe here?



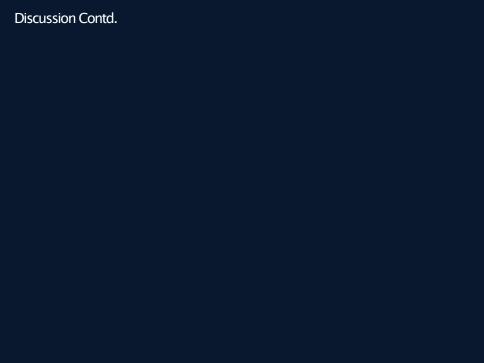
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- Coming back to the model, what are the Nash Equilibriums here?



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Note: The last one is not something that really happens.
 It is induced by our method of modelling. We should be aware of such things when modelling the world!



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In reality, we can only be sure about our neighbours.



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- An excellent resource for this and policy methods to solve this is Vi Hart and Nicky Case's polygons simulation.



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- Secondly, ever since social media has become more common, we are getting a resurgence in casteism and segregation. Well, social media does make our model seem less hypothetical as we do have information on what are the population statistics of the other regions and all.
- But one has to ask, what does this have to do with Bubbles?



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 - In trading: Exiting before the trend reverses or identifying undervalued assets gives a competitive edge.

The Fashion Game

$$u(x_i) = -\|x - x_i\| - \lambda n(x_i)$$

The Fashion Game

 We model the expression of social identity as a game played by a population of N individuals. Let us say there are d aspects (or dimensions) of identity. Each person i chooses an expression of his identity $xi \in \{a..b\}^d$, i.e., represented as a tuple of d integers from some interval. For example, in the case of choosing a colour to wear, three integers between 0 and 255 might correspond to shades of red, green, and blue that mix together to form any colour.

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 - Every player keeps changing their xi to get better payoffs.
 - As it turns out, we can prove that this game converges to a Nash equilibrium. But that would mean some people would always follow a trend and others never follow it. This is false from our experience in fashion and finance.

$$u(x_i) = -||x_i - x_{\eta(u)}|| - \lambda n_{\eta u}(x_i)$$

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- And this much, is enough for there to exist networks with no Nash equilibrium. The proof is by construction, can anyone get it?



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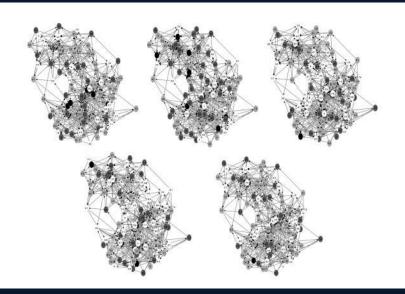
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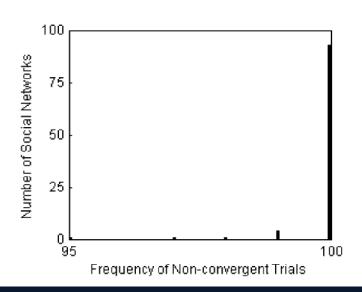
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- While obtaining a formal proof of this is much harder(and open), we can use computational modelling to get an idea. This is the same set of issues as Schelling's automata.
- We need to make some changes here to fit a financial market. I shall do them on the board.







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- This is true both sides. Despite many trends coming and going, people still wear Polo's.
- Similarly, despite market ups and downs, Tata, ITC etc are always dependable investments.
- Finally, We can now begin to understand the role of networks and local interaction. Popularity cycles(bubbles), perpetual change(bursts), and novel expressions(arbitrages) of social identity(makes) should be expected when people observe their neighbours in realistic, directed social networks and care about being unique(not being the last man holding the dollar) as well as fitting in(riding the momentum).





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So is the greatest book on investing just wrong in 21st century?



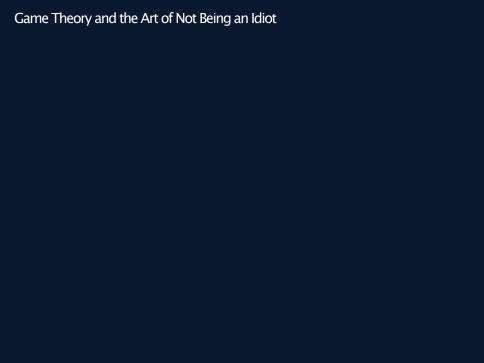
Conclusion

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So, what's the lesson here? Markets aren't rational. People aren't rational. But you? You can choose to be.



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- Understanding the Game is More Important Than Playing It –
 Everyone wants to win, but few understand why the game
 works the way it does. Study incentives. Study behavior. That's
 where the real power is.





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But if you feel this was all too much ado about nothing, then well I have a tulip to sell you!



PS: Keep Playing the Game

Game Theory, market design, poker theory, contract theory, incentive design, voting theory, mathematical economics, and behavioural finance are gold mines of useful insights. If today made you think, dive deeper. Because the more you understand the game, the better your odds of not getting played.

Thank You!



Questions? Arguments? Outrage? Bring it on.