AMATH 482/582: HOMEWORK 1

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ABSTRACT. Your report should contain a brief, 100 word abstract describing what is contained in the document and what you did. **Don't forget 6 pages max**.

1. Introduction and Overview

Imagine we are given the following problem: There is a submarine traveling thorugh Puget Sound. We've been tasked with finding the submarine and tracking it over a 24 hour period. We've been given a 64 x 64 x 64 grid of acoustic data points at 30 minute intervals, but the data is noisy. The submarine is also moving, so we need to determine its path if we want to find it.

Since we're dealing with acoustic frequencies, we should immediately be thinking of Fourier Transforms. These will be key in order for us to find the path of the submarine.

2. Theoretical Background

We know the data is noisy. As such, we can use an interesting property of the Fourier Transform in order to help get rid of the noise. Let the function be represented by the following:

$$f(x) = g(x) + \xi$$

Where ξ is the noise and $f(x)$ is the submarine data. The clean data should then be represented by $g(x)$, so the question is how do we get rid of the noise. If we assume the noise is random in distribution with mean 0, then taking the Fourier Transform of the random variable will still be some random variable. As such, we can write this as

$$f(\hat{x}) = g(\hat{x}) + \xi$$

We have a distribution over time (one point at every 30 mintues for 24 hours). If we imagine that there are n number of such points, then if we were to take the average of this fourier transform over time, it would become

$$\frac{1}{n}\sum_{n=0}^{n-1} f(\hat{x}) = \frac{1}{n}\sum_{n=0}^{n-1} g(\hat{x}) + \frac{1}{n}\sum_{n=0}^{n-1} \xi$$

However, since ξ is randomly distributed with mean 0, the noise should approach 0 as well. So, this should simplify to

$$\frac{1}{n}\sum_{n=0}^{n-1} f(\hat{x}) = \frac{1}{n}\sum_{n=0}^{n-1} g(\hat{x})$$

Effectively meaning that we have dealt with the noise. Once we have gotten rid of the noise, we should be able to scan through the fourier transform and find the highest frequency. Since there should be no other frequencies in the data, we can use this frequency to find the central frequency (and it's location).

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When we create a filter to denoise the data when finding the path, we can extend the Gaussian filter to three dimensions. In two dimensions, the filter is

$$G(x, y, \sigma) = e^{-\frac{x^2+y^2}{2\sigma^2}}$$

Where σ is chosen by us. We can extend this definition to three dimensions by just adding another dimension z. In this case, the Gaussian Filter would become

$$G(x, y, z, \sigma) = e^{-\frac{x^2+y^2+z^2}{2\sigma^2}}$$

We will use this filter to attempt to de-noise along the path that the submarine takes. We can also center this around the location of the max frequency in order to better reduce noise around the sub. Let (a, b, c) be the x, y, and z coordinates of the max frequency. Our Gaussian filter would then look like

$$G(x, y, z, \sigma) = e^{-\frac{(x-a)^2+(y-b)^2+(z-c)^2}{2\sigma^2}}$$

Also, the data would be given to us from range -32 to 32, but we want to start at 0. So, we will add 32 to each dimension as well to get the final Gaussian filter as

$$G(x, y, z, \sigma) = e^{-\frac{(x-a+32)^2+(y-b+32)^2+(z-c+32)^2}{2\sigma^2}}$$

3. Algorithm Implementation and Development

For this assignment, I used Python. I used NumPy to do all mathematical operations and the Fourier Transforms. I used Matplotlib to plot the data and the path of the submarine, as well as any other plots.

4. Computational Results

This is perhaps the most important section of your report. You want to dedicate more space here and present your numerical results in a clear, concise and meaningful way. Also include a discussion of your numerics. Think hard about how you can use your space most efficiently. For example, include subplots and multiple error curves on the same plot etc. Ask us for advice when the time comes.

You will most definitely need tables and figures. So here is an example.

row 1	column 1	column 2
row 2	column 1	column 2
row 3	column 1	column 2

Table 1. Don't forget to include a caption for your table. Say a few words about what is being shown.

Make sure your table is labeled and referenced withing the text using `\ref` as such Table 1. In fact, you can use `\ref` to cite anything else in the document such as sections (ex. Section 1). This will create hyperlinks in your pdf after compilation and automatically update the numbers and tags whenever you change anything.

Figures are very similar to tables. Here's an example:

You may also need to include multiple figures:

Once again, make sure all your figures are referenced like Figure **??** or Figure **??** in the text body of the report and discussed in detail. This is where you will make observations about your results and we will look at these very closely.

Also note, I am using PDF figures. These give you the best looking graphs but PNG works well too. I advise staying away from JPG as it always looks weird and low quality.] Both Python and MATLAB can output figures in PDF or PNG.

5. Summary and Conclusions

Wrap up your report with a brief summary of what you did and what you discovered. Finish with some conclusions and possibly future directions if any.

Acknowledgements

Make sure you you clearly state any help you received including collaborations with your peers. Help from TAs or other mentors, professors, etc that helped you with your assignment. Here's a formal example:

The author is thankful to Prof. X for useful discussions about the QR algorithm. We are also thankful to Dr. Strange for suggesting the JAX software package for automatic differentiation. Furthermore, our peer Jean Grey was helpful in implementation of spectral clustering in Python.