

# Math 327 Homework 1

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1. Recall that in the textbook, a function  $f : A \rightarrow B$  is said to be invertible if  $f$  is one-to-one and onto. (This is also called bijection in some literature.) Show that a function  $f : A \rightarrow B$  is invertible if and only if there is a function  $g : B \rightarrow A$  such that  $g \circ f = \text{identity function on } A$  and  $f \circ g = \text{identity function on } B$ .

A function  $F$  is called the identity function given that  $F(x) = x$ . We are given that  $f(g(x)) = x$  and  $g(f(x)) = x$ . We want to prove that  $f$  is both one-to-one and onto.

Let  $f(g(x)) = f(g(y))$ . Given that this is the identity function, we know that  $f(g(x)) = x$  and  $f(g(y)) = y$ . As such,  $x = y$ . So,  $f(g(x))$  is one-to-one.

We know that  $f(g(x)) = x$ . Let  $y \in B$ . We want to show that for some  $x \in A$ ,  $f(g(x)) = y$ . Since  $f(g(x))$  is the identity function, we know that  $f(g(y)) = y$ . As such, when  $x = y$ ,  $f(g(x)) = y$ . So,  $f(g(x))$  is onto.

2. Let  $f : A \rightarrow B$  and  $g : B \rightarrow C$  be functions.
  - (a) Prove that  $f$  and  $g$  are injective, then so is  $g \circ f$ .
  - (b) Suppose  $g \circ f$  is surjective. Does it follow that  $f$  is surjective?
  - (c) Suppose  $g \circ f$  is surjective. Does it follow that  $g$  is surjective?