

Math 300 Homework 4

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1. Recall that in the textbook, a function $f : A \rightarrow B$ is said to be invertible if f is one-to-one and onto. (This is also called bijection in some literature.) Show that a function $f : A \rightarrow B$ is invertible if and only if there is a function $g : B \rightarrow A$ such that $g \circ f = \text{identity function on } A$ and $f \circ g = \text{identity function on } B$.

A function F is called the identity function given that $F(x) = x$. We are given that $f(g(x)) = x$ and $g(f(x)) = x$. We want to prove that f is both one-to-one and onto.

Let $f(g(x)) = f(g(y))$. Given that this is the identity function, we know that $f(g(x)) = x$ and $f(g(y)) = y$. As such, $x = y$. So, $f(g(x))$ is one-to-one.

We know that $f(g(x)) = x$. Let $y \in B$. We want to show that for some $x \in A$, $f(g(x)) = y$. Since $f(g(x))$ is the identity function, we know that $f(g(y)) = y$. As such, when $x = y$, $f(g(x)) = y$. So, $f(g(x))$ is onto.

2. Let $f : A \rightarrow B$ and $g : B \rightarrow C$ be functions. (a) Prove that f and g are injective, then so is $g \circ f$. Test
(b) Suppose $g \circ f$ is surjective. Does it follow that f is surjective?
(c) Suppose $g \circ f$ is surjective. Does it follow that g is surjective?