Math 327 Homework 1

Sathvik Chinta

October 7th, 2022

1. Recall that in the textbook, a function f : A → B is said to be invertible f is one-to-one and onto. (This is also called bijection in some literature.) Show that a function f : A → B is invertible if and only if there is a function g : B → A such that g ∘ f = identity function on A and f ∘ g = identity function on B.

   A function F is called the identity function given that $F(x) = x$. We are given that $f(g(x)) = x$ and $g(f(x)) = x$. We want to prove that f is both one-to-one and onto.

   Let $f(g(x)) = f(g(y))$. Given that this is the identity function, we know that $f(g(x)) = x$ and $f(g(y)) = y$. As such, $x = y$. So, $f(g(x))$ is one-to-one.

   We know that $f(g(x)) = x$. Let $y \in B$. We want to show that for some $x \in A$, $f(g(x)) = y$. Since $f(g(x))$ is the identity function, we know that $f(g(y)) = y$. As such, when $x = y$, $f(g(x)) = y$. So, $f(g(x))$ is onto.

2. Let f : A → B and g : B → C be functions.
   (a) Prove that f and g are injective, then so is g ∘ f. Test

   (b) Suppose g ∘ f is surjective. Does it follow that f is surjective?

   (c) Suppose g ∘ f is surjective. Does it follow that g is surjective?