Math 327 Homework 7

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November 29th, 2022

1. **3**

   Let $n = k + 1$. Thus, we can express

   $$\sum_{k=1}^{\infty} \frac{1}{(k+1)[ln(k+1)]^{\alpha}}$$

   As

   $$\sum_{k=2}^{\infty} \frac{1}{n[ln(n)]^{\alpha}}$$

   The sequence of $log(n)$ is monotonically increasing. Thus, we know that $\frac{1}{nlog(n)}$ must then decrease. We also know that since $n \geq 2$, $\frac{1}{nlog(n)} \geq 0$. Since this sequence is bounded and monotonically decreasing, from the Monotone Convergence Theorem, we know that the sequence $\frac{1}{nlog(n)}$ must thus converge. From theorem 3.27 in the textbook, we know that if $a_1 \geq a_2 \geq a_3 ... \geq 0$, then the series $\sum_{n=1}^{\infty} a_n$ converges if and only if the series

   $$\sum_{k=0}^{\infty} 2^k a_{2^k}$$

   Converges. Thus, we can plug this into our equation to get

   $$\begin{aligned} &\sum_{k=1}^{\infty} 2^k \frac{1}{2^k [log(2^k)]^{\alpha}} \\ =&\sum_{k=1}^{\infty} \frac{1}{(k \log(2))^{\alpha}} \\ =&\frac{1}{\log(2)^{\alpha}} \sum_{k=1}^{\infty} \frac{1}{k^{\alpha}} \end{aligned}$$

   Since $\frac{1}{\log(2)^{\alpha}}$ is just a value no matter what α is, convergence depends on $\sum_{k=1}^{\infty} \frac{1}{k^{\alpha}}$. Using the p-series test, we know that this series only converges if $\alpha > 1$. Thus, we know that the series $\sum_{k=1}^{\infty} \frac{1}{(k+1)[ln(k+1)]^{\alpha}}$ converges if and only if $\alpha > 1$.

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