In [2]:
```
from sympy import *
from sympy.abc import x,n
```

In [15]:
```
# Evaluate the following integrals using a computer algebra system
```

In [16]:
```
a = Integral(ln(x))
a
```

Out[16]:

$$\int \log (x)\, dx$$

In [17]:
```
b = Integral(ln(x)*x)
b
```

Out[17]:

$$\int x \log (x)\, dx$$

In [18]:
```
c = Integral(ln(x)*x**2)
c
```

Out[18]:

$$\int x^2 \log (x)\, dx$$

In [19]:
```
d = Integral(ln(x)*x**3)
d
```

Out[19]:

$$\int x^3 \log (x)\, dx$$

In [20]:
```
e = Integral(ln(x)*x**7)
e
```

Out[20]:

$$\int x^7 \log (x)\, dx$$

In [21]:
```
# Solutions to the above equations:
```

In [22]:
```
Eq(a,a.doit())
```

Out[22]:

$$\int \log (x)\, dx = x \log (x) - x$$

In [23]:
```
Eq(b,b.doit())
```

Out[23]:

$$\int x \log (x)\, dx = \frac{x^2 \log (x)}{2} - \frac{x^2}{4}$$

In [24]:
```
Eq(c,c.doit())
```

Out[24]:

$$\int x^2 \log(x)\, dx = \frac{x^3 \log(x)}{3} - \frac{x^3}{9}$$

In [25]:
```
Eq(d,d.doit())
```

Out[25]:

$$\int x^3 \log(x)\, dx = \frac{x^4 \log(x)}{4} - \frac{x^4}{16}$$

In [26]:
```
Eq(e,e.doit())
```

Out[26]:

$$\int x^7 \log(x)\, dx = \frac{x^8 \log(x)}{8} - \frac{x^8}{64}$$

In [27]:
```
# Guess the value of the integral x**n * ln(x)
```

$$\int x^n \ln(x)\, dx = \frac{x^{n+1} \ln x}{n+1} - \frac{x^{n+1}}{(n+1)^2} + c$$

In [29]:
```
# Use integration by parts to prove this conjecture and what n values ar
```

In [14]:
```
((((x**n+1)/(n+1))**2)/2)
```

Out[14]:

$$\frac{(x^n + 1)^2}{2(n+1)^2}$$

The integration by parts formula states:

$$\int_a^b u(x)v'(x)\, dx = \Big[u(x)v(x)\Big]_a^b - \int_a^b u'(x)v(x)\, dx$$
$$= u(b)v(b) - u(a)v(a) - \int_a^b u'(x)v(x)\, dx.$$

Or, letting $u = u(x)$ and $du = u'(x)\, dx$ while $v = v(x)$ and $dv = v'(x)\, dx$, the formula can be written more compactly:

$$\int u\, dv = uv - \int v\, du.$$ — wiki

$$\int \underset{(dv)}{x^n} \underset{(u)}{\ln x}\, dx = uv - \int v\, du$$

$$u = \ln x \qquad dv = x^n$$

$$du = \frac{1}{x}dx \qquad v = \frac{x^{n+1}}{n+1}$$

$$= uv - \left(\frac{v^2}{2}\right) + c$$

$$= \frac{x^{n+1}\ln x}{n+1} - \frac{\left(\frac{x^{n+1}}{n+1}\right)^2}{2} + c$$

$$= \frac{x^{n+1}\ln x}{n+1} - \frac{\frac{x^{2n+2}}{(n+1)^2}}{2} + c$$

$$= \frac{x^{n+1}\ln x}{n+1} - \frac{x^{2n+2}}{2(n+1)^2} + c$$

$$= \frac{x^{n+1}\ln x}{n+1} - \frac{\left(\frac{x^{n+1}}{n+1}\right)^2}{2} + c$$

$$= \frac{x^{n+1}\ln x}{n+1} - \frac{\frac{2(x^{n+1})}{(n+1)^2}}{2} + c$$

$$= \frac{x^{n+1}\ln x}{n+1} - \frac{x^{n+1}}{(n+1)^2} + c$$