

Book 2, Section 2, Lemma 2a

Where n , m , and c are constants. x , u , and v are variables.

$$\frac{d}{dx} x^{\frac{n}{m}} = \frac{n}{m} x^{\frac{n-m}{m}} \quad \text{can be easier to use vs} \quad \frac{d}{dx} x^n = nx^{n-1}$$

Ex. 1:

$$\frac{d}{dx} x^{\frac{2}{3}} = \frac{2}{3} x^{\frac{2-3}{3}} = \frac{2}{3} x^{-\frac{1}{3}} = \frac{2}{3\sqrt[3]{x}}$$

vs.

$$\frac{d}{dx} x^{\frac{2}{3}} = \frac{2}{3} x^{\frac{2}{3}-1} = \frac{2}{3} x^{\frac{2}{3}-\frac{3}{3}} = \frac{2}{3} x^{-\frac{1}{3}}$$

Then by extrapolation:

$$\int x^{\frac{n}{m}} = \frac{m}{n+m} x^{\frac{n+m}{m}} + c$$

Ex. 2:

$$\int x^{\frac{4}{5}} = \frac{5}{4+5} x^{\frac{4+5}{5}} + c = \frac{5}{9} x^{\frac{9}{5}} + c$$

And (with the product rule):

$$\frac{d}{dx} u^m v^n = mu^{m-1} v^n + nv^{n-1} u^m$$