Area Between 3 Exponential Curves

June 20, 2022

[1]:
```
from sympy import E,plot,Eq,nsolve,Integral,diff,Piecewise
from sympy.abc import x
f = E**x
f
```

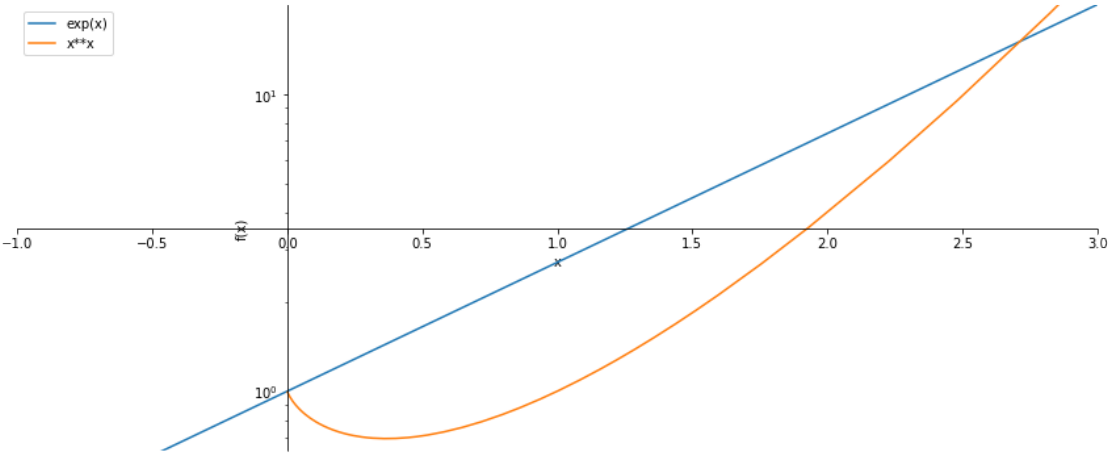
[1]: e^{x}

[9]:
```
g = x**x
g
```

[9]: x^{x}

[10]:
```
plot(f,g,yscale='log', xlim = (-1,3),ylim = (10**-0.2,10**1.3),legend = True,␣
↪size = (12,5))
```

```
<string>:1: RuntimeWarning: invalid value encountered in double_scalars
```



[10]: <sympy.plotting.plot.Plot at 0x7fc601eb5a90>

[11]:
```
# Step 1:
# Set f(x) = g(x) and solve to find points of intersection
eq2 = Eq(f,g)
eq2
```

[11]:
$$e^x = x^x$$

[12]:
```
# nsolve finds where both functions intersect
b1 = nsolve(eq2,2.7)
b1
```

[12]:
2.71828182845905

[13]:
```
# Step 2: Find the area between both curves
Integral(f-g,(x,0,b1))
```

[13]:
$$\int_0^{2.71828182845905} (-x^x + e^x)\, dx$$

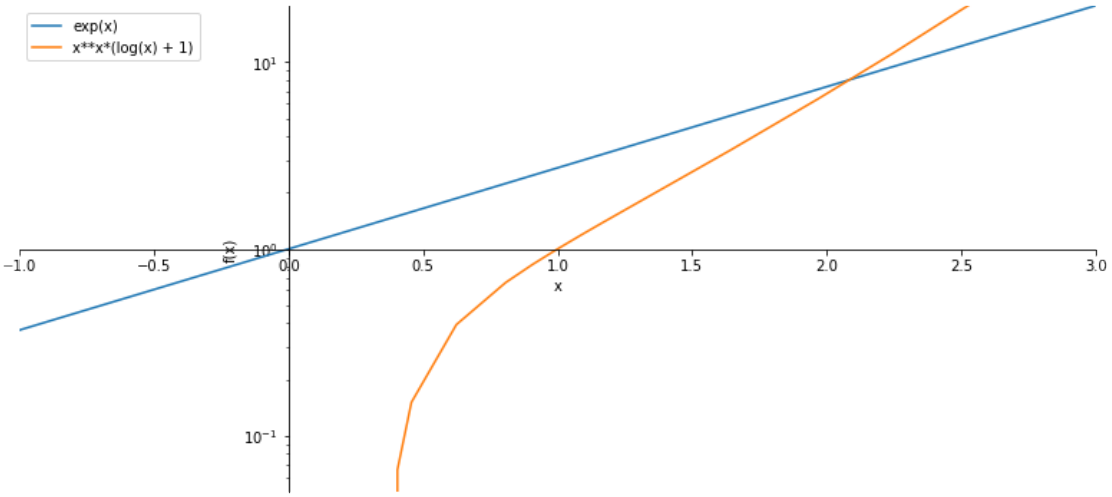
[14]:
```
total = Integral(f-g,(x,0,b1)).evalf()
total
```

[14]:
5.41053960613562

[15]:
```
##########################################################################################
```

[20]:
```
# Now finding the area between the differentiated curves
plot(diff(f),diff(g),yscale='log', xlim = (-1,3),ylim = (10**-1.3,10**1.
↪3),legend = True, size = (11,5))
```

```
<string>:1: RuntimeWarning: invalid value encountered in double_scalars
```



[20]:
```
<sympy.plotting.plot.Plot at 0x7fc6018a72e0>
```

[21]:
```
# Step 1:
# Set f'(x) = g'(x) to find points of intersection
eq3 = Eq(diff(f),diff(g))
eq3
```

[21]:
$$e^x = x^x \left(\log\left(x\right) + 1\right)$$

[22]:
```
# this is our upper bound for our upcoming Integral
b2 = nsolve(eq3,2)
b2
```

[22]:
$$2.08710845301751$$

[23]:
```
# Step 2: Find the area between both curves
Integral(diff(f)-diff(g),(x,0,b2))
```

[23]:
$$\int_{0}^{2.08710845301751} \left(-x^x \left(\log\left(x\right) + 1\right) + e^x\right)\, dx$$

[25]:
```
Integral(diff(f)-diff(g),(x,0,b2)).evalf(10)
```

[25]:
$$3.417219252$$

[14]:
```
##########################################################################################################
```

[26]:
```
# Finding the area for the left half between these 3 curves
plot(f,g,diff(g),yscale='log', xlim = (-1,3),ylim = (10**-0.2,10**1.3),legend =␣
↪True,size = (12,5))
```

```
<string>:1: RuntimeWarning: invalid value encountered in double_scalars
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```

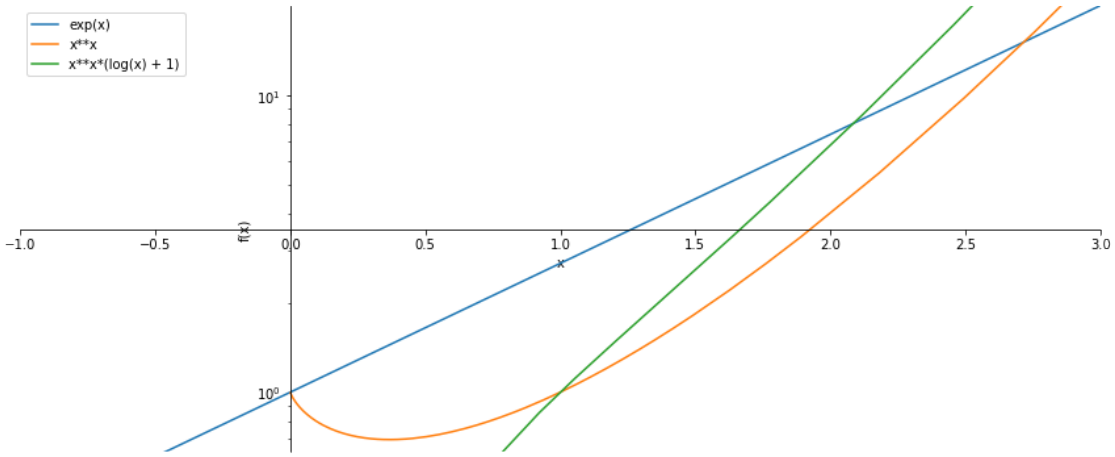

[26]:
```
<sympy.plotting.plot.Plot at 0x7fc60175d250>
```

[27]:
```
Integral(f-g,(x,0,b1))
```

[27]:

$$\int_{0}^{2.71828182845905} \left(-x^{x} + e^{x}\right)\, dx$$

[28]:
```
eq4 = Eq(g,diff(g))
nsolve(eq4,1)
```

[28]:

1.0

[29]:
```
p = Piecewise((g, x < 1), (diff(g), x > 1))
p
```

[29]:

$$\begin{cases} x^{x} & \text{for } x < 1 \\ x^{x} \left(\log\left(x\right) + 1\right) & \text{for } x > 1 \end{cases}$$

[30]:
```
Integral(f-p,(x,0,b2))
```

[30]:

$$\int_{0}^{2.08710845301751} \begin{cases} -x^{x} + e^{x} & \text{for } x < 1 \\ -x^{x} \left(\log\left(x\right) + 1\right) + e^{x} & \text{for } x > 1 \end{cases}\, dx$$

[31]:
```
left_half = Integral(f-p,(x,0,b2)).evalf()
left_half
```

[31]:

2.63379

[32]:
```
right_half = total-left_half
right_half
```

[32]:

2.77675102047277