Fourier Series Stuff

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[4]:
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from sympy import symbols,E,fourier_series, pi,Derivative,Eq,plot
# Example

# Computing the Fourier series of f(x)=e**x at n = 3 , a = -pi, b = pi:
x = symbols('x')
f = E**x
f
```

[4]:
$$e^{x}$$

[5]:
```
fs = fourier_series(f, (x, -pi, pi))
fst = fs.truncate(n=3)
fst
```

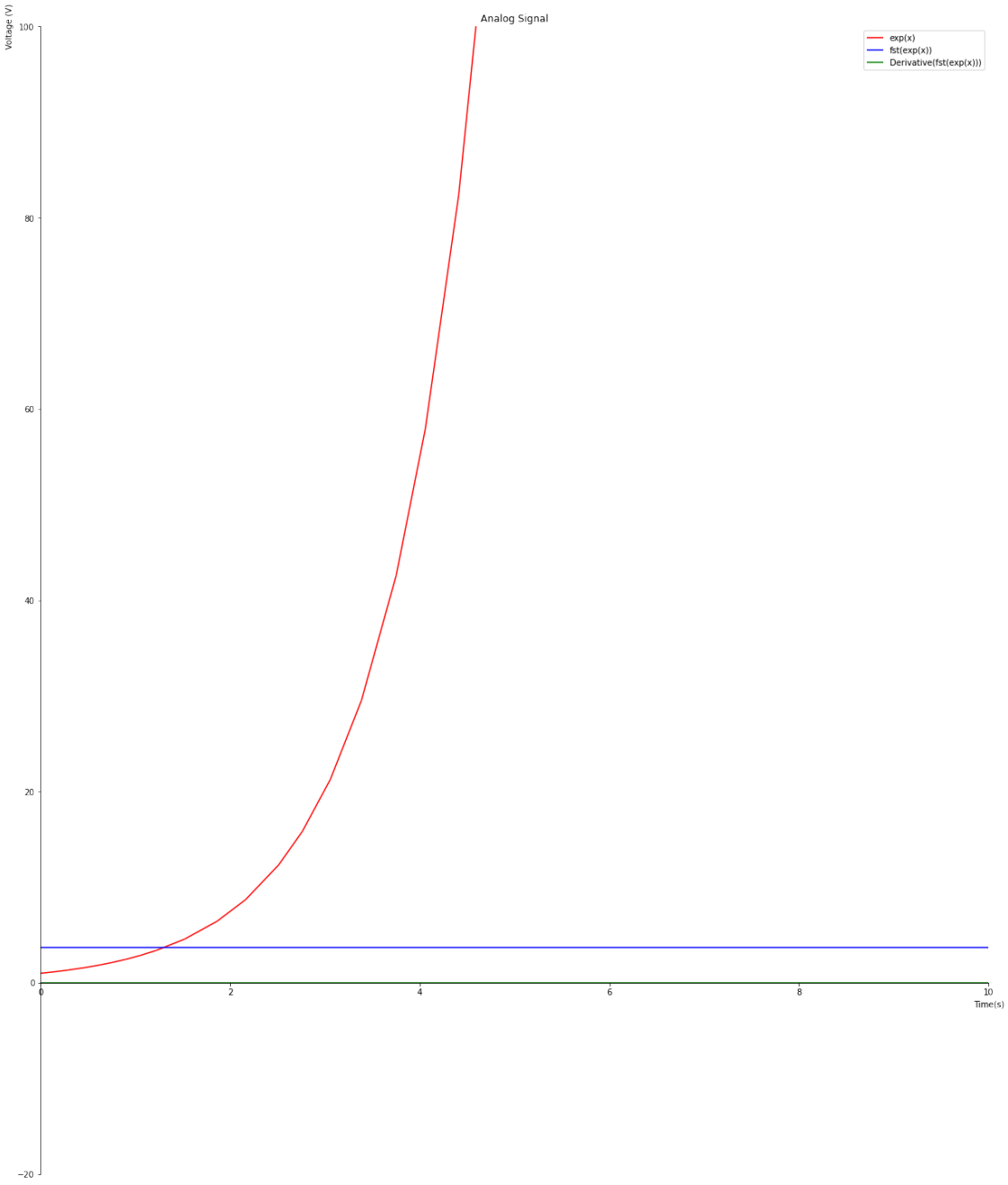
[5]:
$$\frac{\left(-\frac{1}{2e^{\pi}}+\frac{e^{\pi}}{2}\right)\sin\left(x\right)}{\pi}+\frac{\left(-\frac{2e^{\pi}}{5}+\frac{2}{5e^{\pi}}\right)\sin\left(2x\right)}{\pi}+\frac{\left(-\frac{e^{\pi}}{2}+\frac{1}{2e^{\pi}}\right)\cos\left(x\right)}{\pi}+\frac{\left(-\frac{1}{5e^{\pi}}+\frac{e^{\pi}}{5}\right)\cos\left(2x\right)}{\pi}+\frac{-\frac{1}{e^{\pi}}+e^{\pi}}{2\pi}$$

[6]:
```
# Compute the derivative of the Fourier series of e**x
e5 = Derivative(fst)
e6 = e5.doit()
Eq(e5,e6)
```

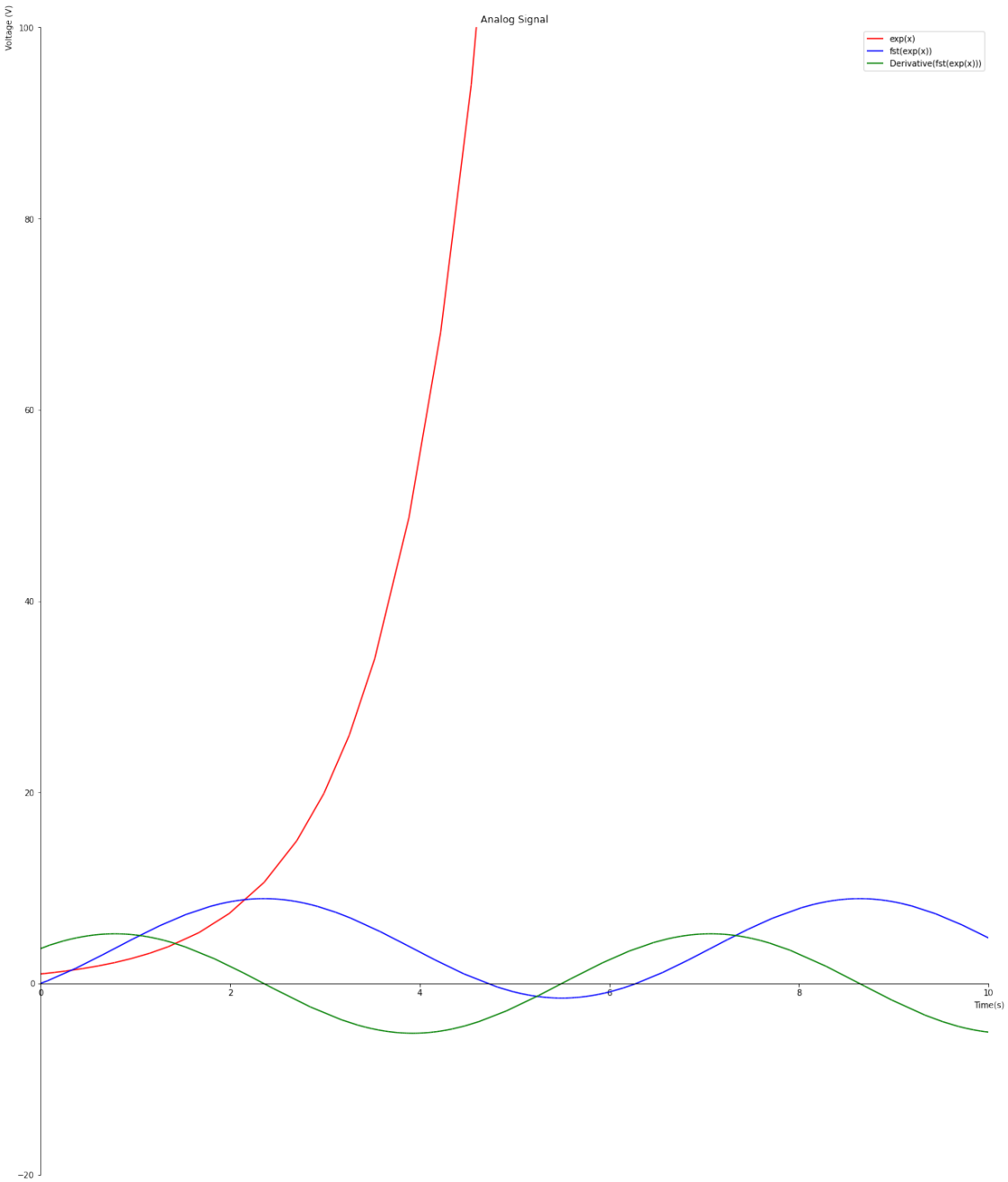
[6]:
$$\frac{d}{dx}\left(\frac{\left(-\frac{1}{2e^{\pi}}+\frac{e^{\pi}}{2}\right)\sin\left(x\right)}{\pi}+\frac{\left(-\frac{2e^{\pi}}{5}+\frac{2}{5e^{\pi}}\right)\sin\left(2x\right)}{\pi}+\frac{\left(-\frac{e^{\pi}}{2}+\frac{1}{2e^{\pi}}\right)\cos\left(x\right)}{\pi}+\frac{\left(-\frac{1}{5e^{\pi}}+\frac{e^{\pi}}{5}\right)\cos\left(2x\right)}{\pi}+\frac{-\frac{1}{e^{\pi}}+e^{\pi}}{2\pi}\right)=-\frac{\left(-\frac{e^{\pi}}{2}+\frac{1}{2e^{\pi}}\right)\sin\left(x\right)}{\pi}-\frac{2\left(-\frac{1}{5e^{\pi}}+\frac{e^{\pi}}{5}\right)\sin\left(2x\right)}{\pi}+\frac{\left(-\frac{1}{2e^{\pi}}+\frac{e^{\pi}}{2}\right)\cos\left(x\right)}{\pi}+\frac{2\left(-\frac{2e^{\pi}}{5}+\frac{2}{5e^{\pi}}\right)\cos\left(2x\right)}{\pi}$$

[8]:
```
u = 2
for i in ([u**q for q in range(8)]):
    p = plot(f,fs.truncate(n=i),Derivative(fs.truncate(n=i)).doit(),legend =␣
 ↪True,xlim = (0,10),ylim = (-20,100),title="Analog Signal", xlabel =␣
 ↪"Time(s)",ylabel = "Voltage (V)",size = (17,20),steps=1,show=False
        ) # here we are doing the same math above and plotting but with loops␣
 ↪varying our n values of fourier equation
    p[0].line_color = 'r'
```

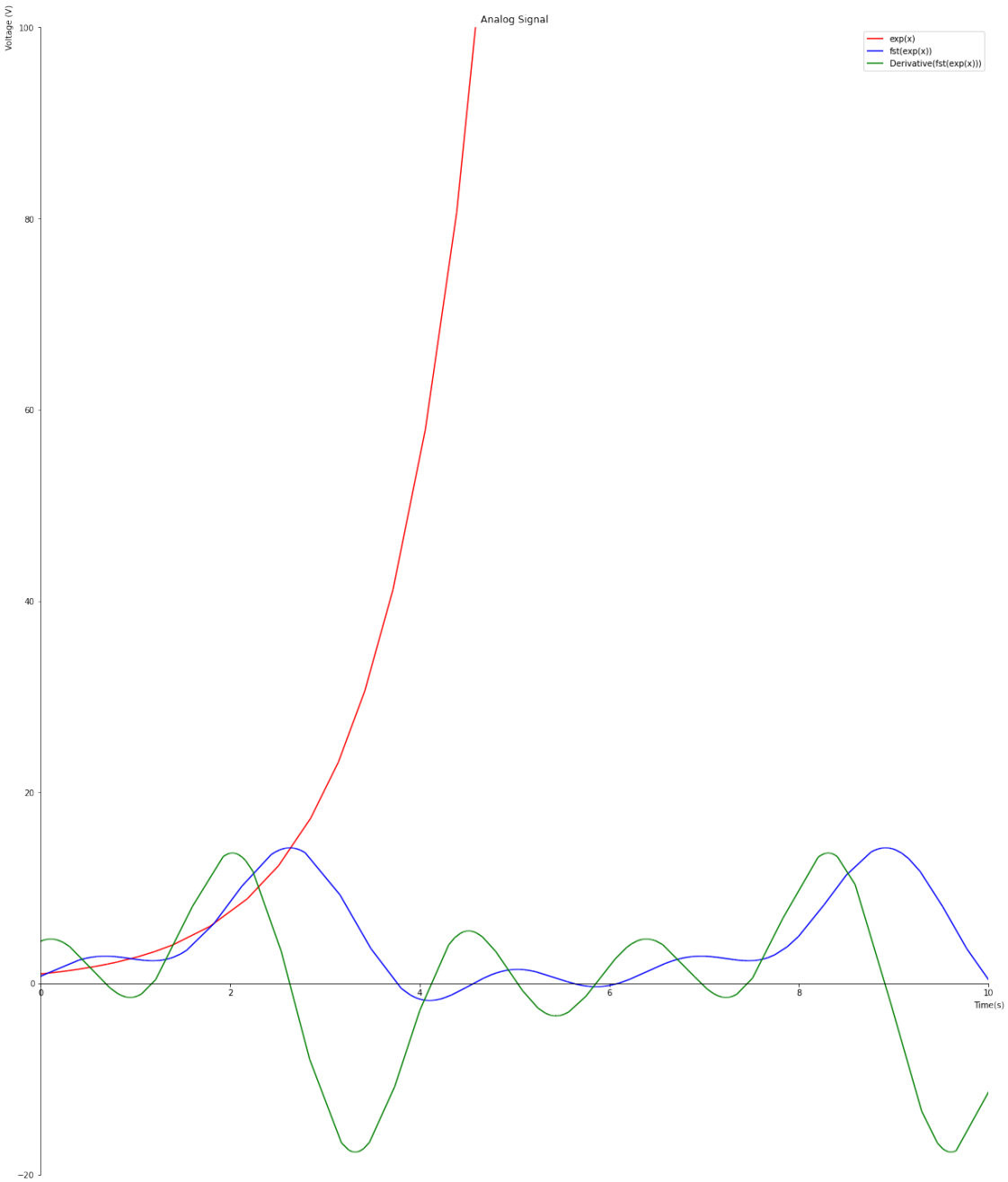
```
    p[1].line_color = 'b'
    p[1].label = 'fst(exp(x))'    # fst is fourie series truncated
    p[2].line_color = 'g'
    p[2].label = 'Derivative(fst(exp(x)))'
    p.show()
    print("                                                    i or n = ",i)
```



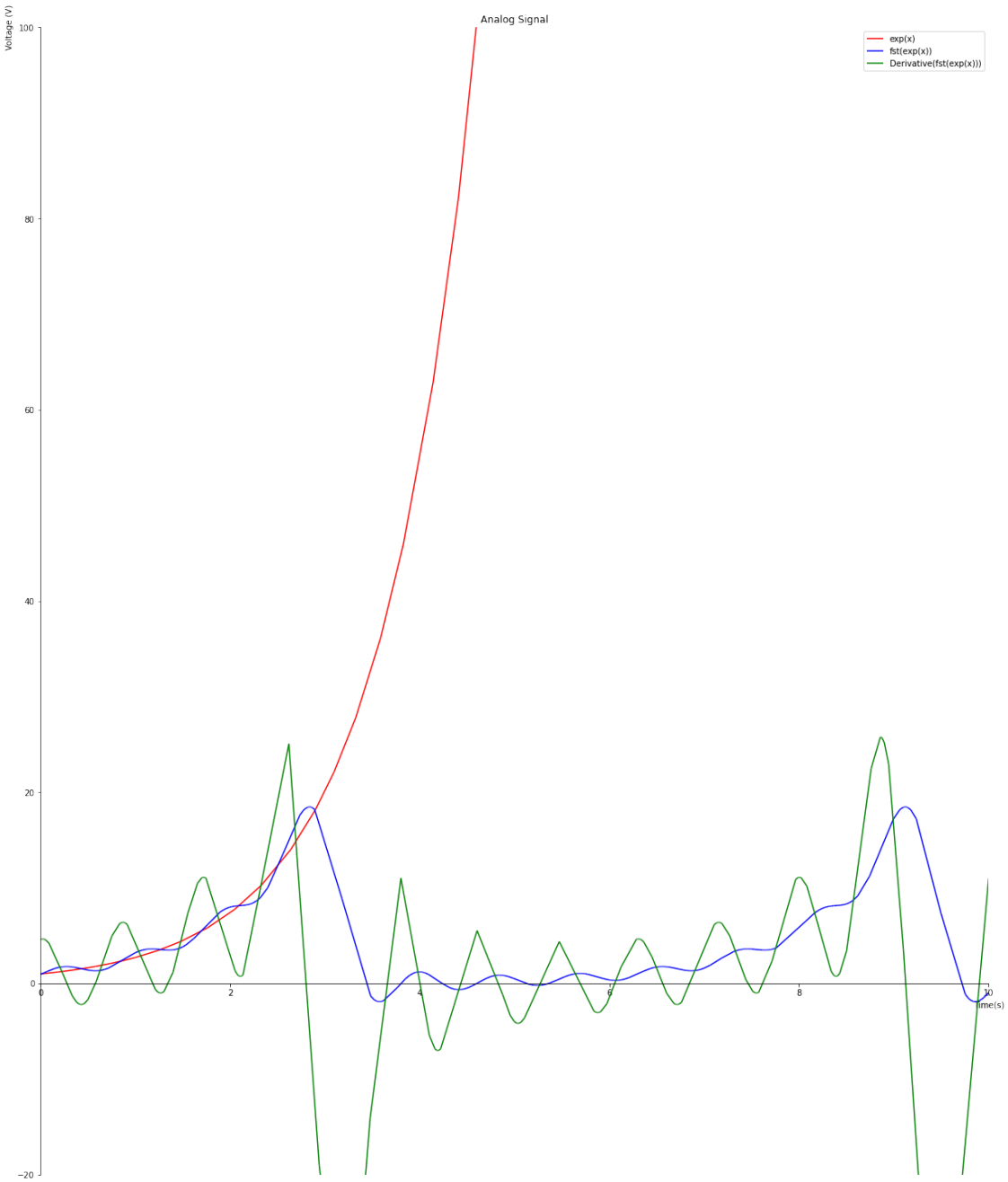

```
                                                    i or n =  1
```


Analog Signal
Voltage (V)
Time(s)
exp(x)
fst(exp(x))
Derivative(fst(exp(x)))
100
80
60
40
20
0
−20
0
2
4
6
8
10

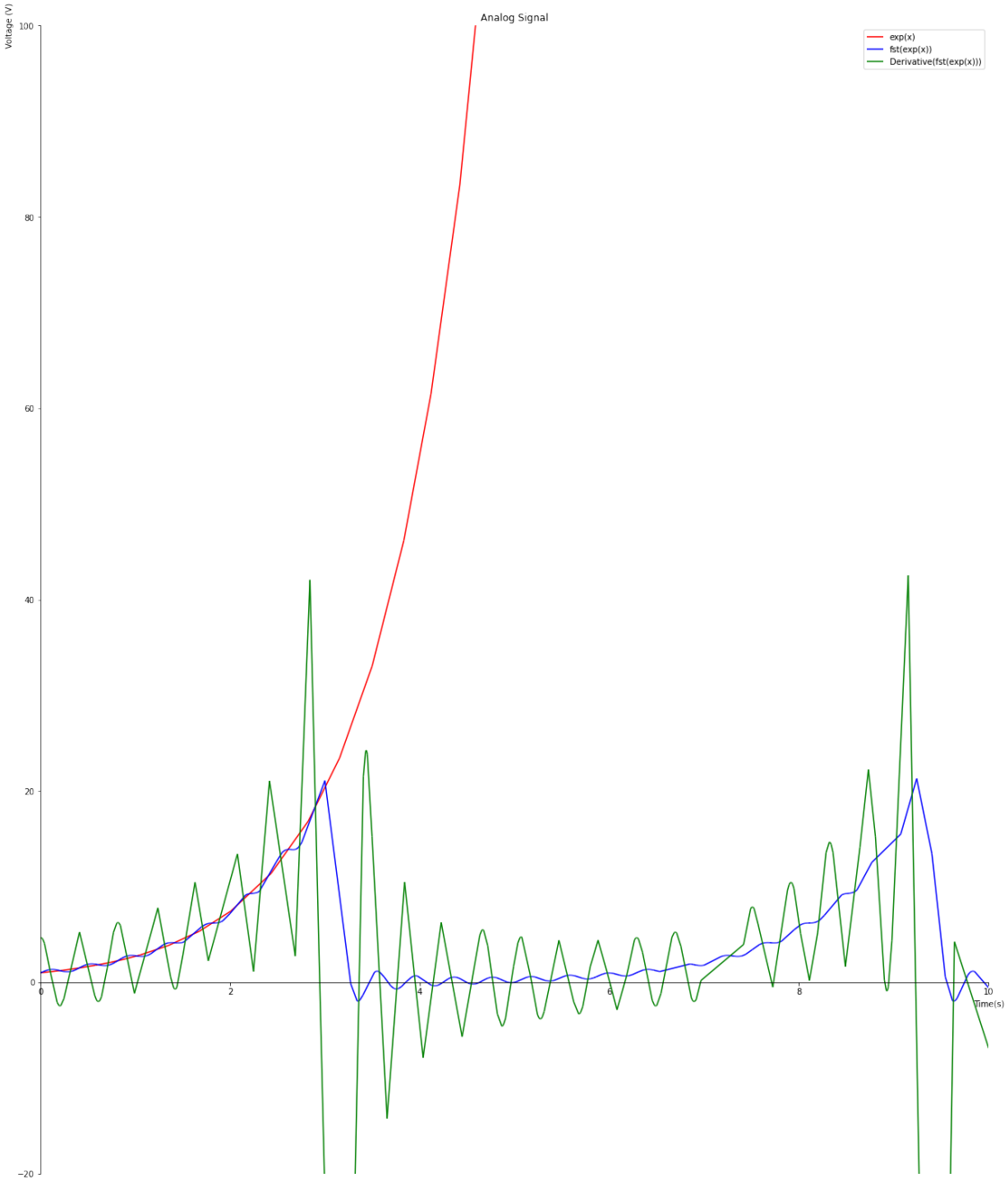
```
i or n =  2
```


Analog Signal
exp(x)
fst(exp(x))
Derivative(fst(exp(x)))
Voltage (V)
Time(s)
100
80
60
40
20
0
−20
0
2
4
6
8
10

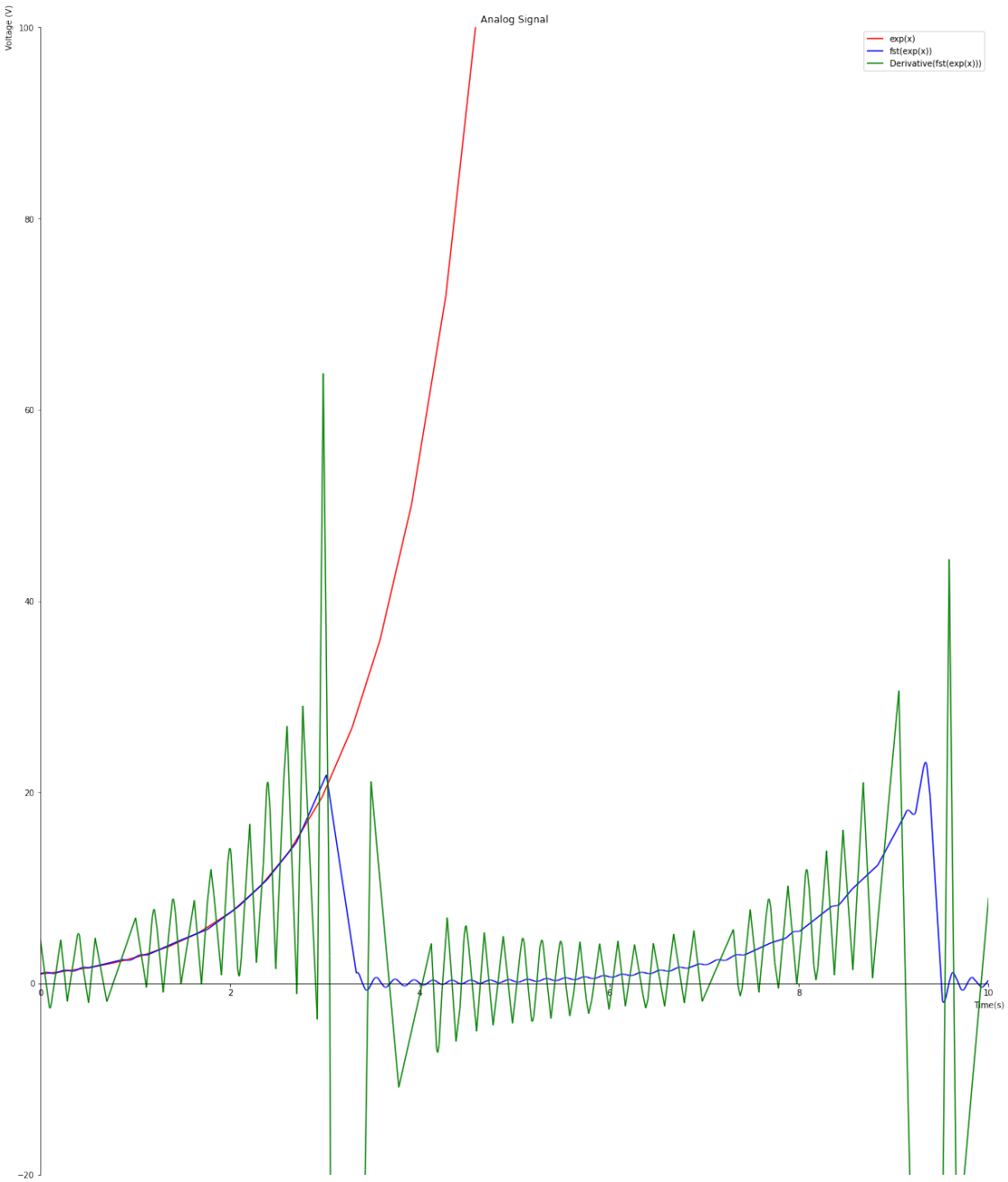
i or n = 4


Analog Signal
exp(x)
fst(exp(x))
Derivative(fst(exp(x)))
Voltage (V)
100
80
60
40
20
0
−20
0
2
4
6
8
10
Time(s)

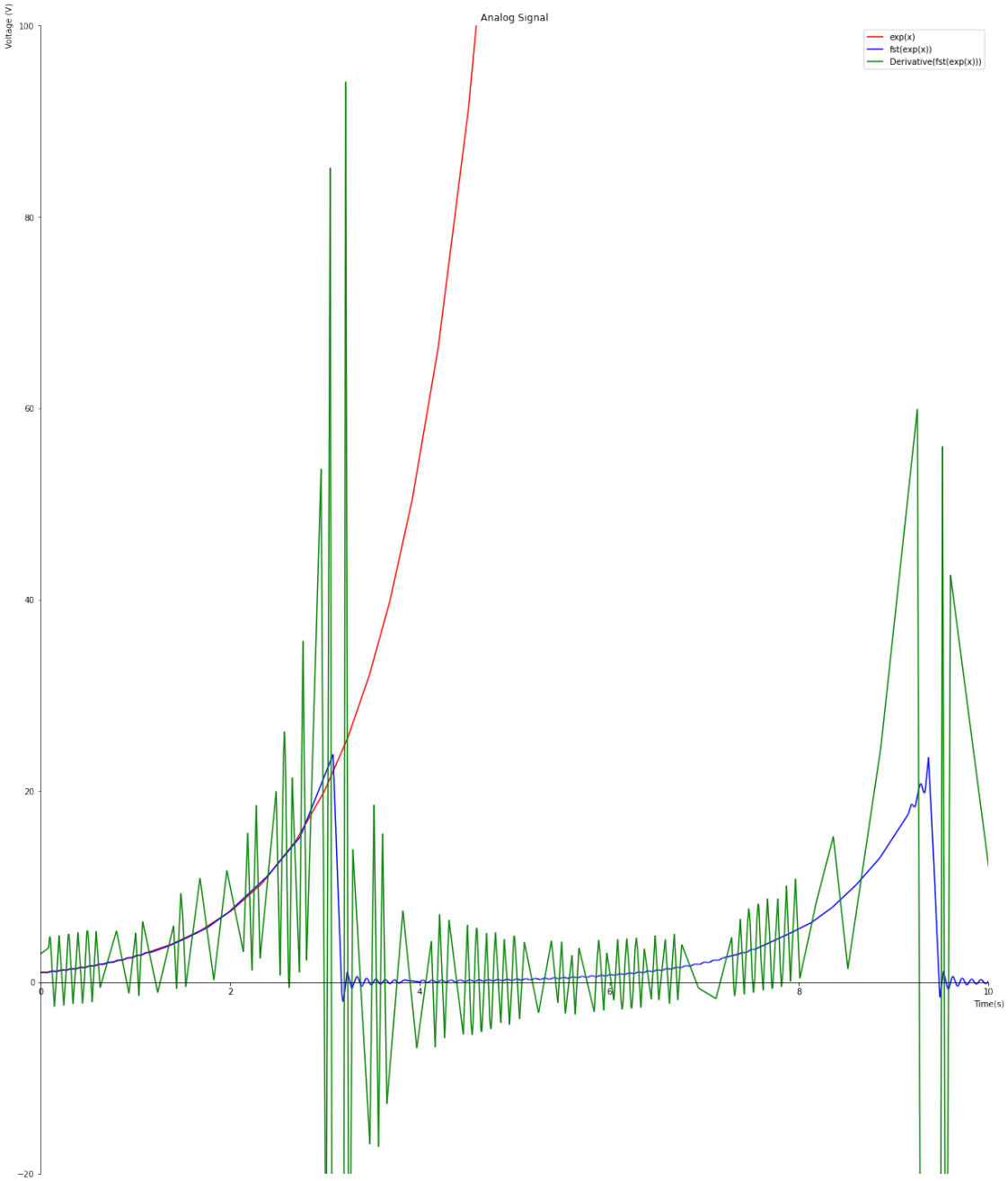
i or n = 8


Analog Signal
exp(x)
fst(exp(x))
Derivative(fst(exp(x)))
Voltage (V)
Time(s)
100
80
60
40
20
0
−20
0
2
4
6
8
10

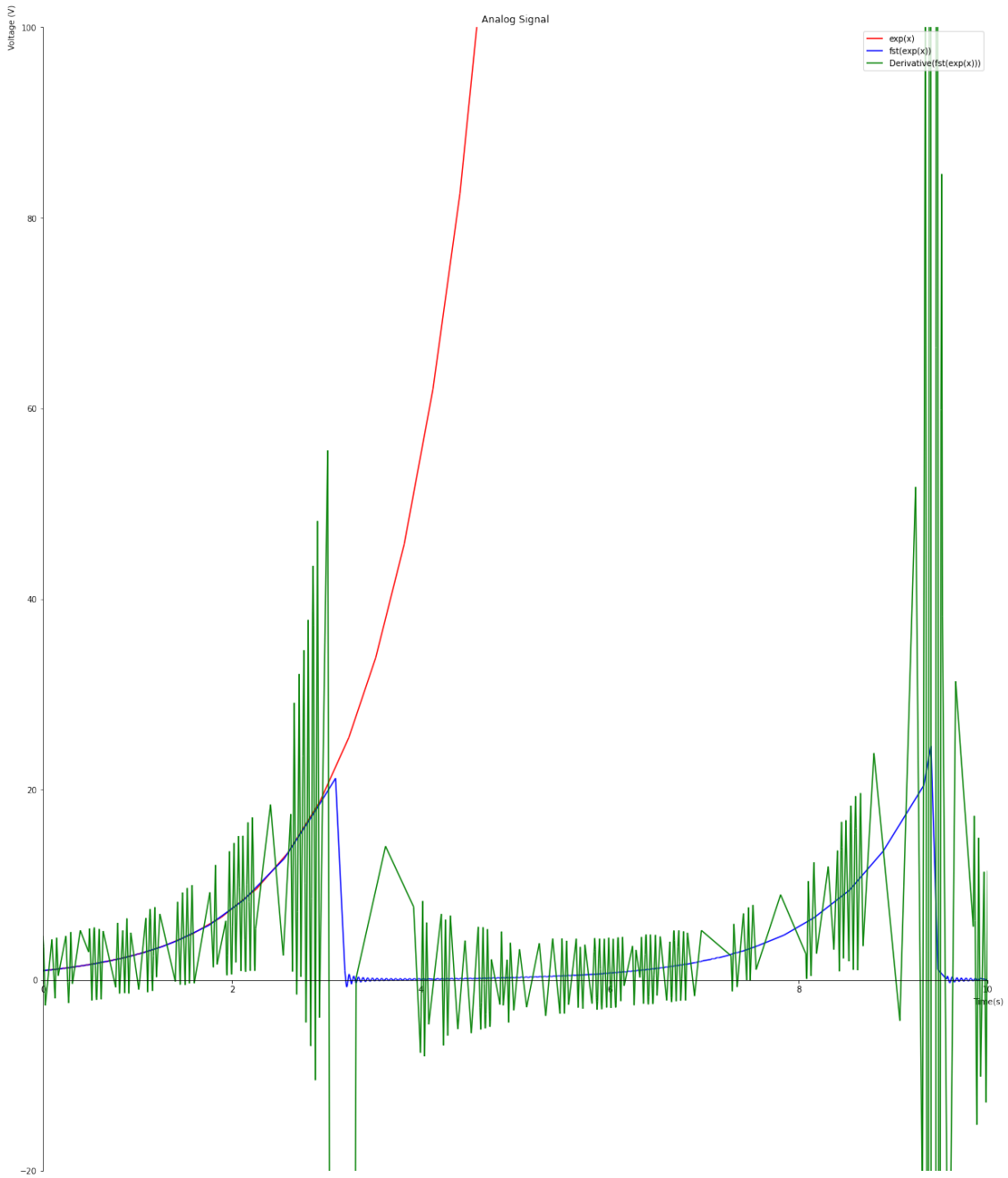
```
i or n =  16
```


Analog Signal
Voltage (V)
Time(s)
exp(x)
fst(exp(x))
Derivative(fst(exp(x)))
100
80
60
40
20
0
−20
0
2
4
6
8
10

```
i or n =  32
```


Analog Signal
exp(x)
fst(exp(x))
Derivative(fst(exp(x)))
Voltage (V)
Time(s)
100
80
60
40
20
0
−20
0
2
4
6
8
10

```
i or n =  64
```



i or n = 128

[9]:
```
print(f)
```

```
exp(x)
```

[10]:
```
print(fst) # at n = 3, a = -pi, b = pi
```

```
(-exp(-pi)/2 + exp(pi)/2)*sin(x)/pi + (-2*exp(pi)/5 + 2*exp(-pi)/5)*sin(2*x)/pi
```

```
+ (-exp(pi)/2 + exp(-pi)/2)*cos(x)/pi + (-exp(-pi)/5 + exp(pi)/5)*cos(2*x)/pi +
(-exp(-pi) + exp(pi))/(2*pi)
```

[11]:
```
print(Derivative(fst).doit())
```

```
-(-exp(pi)/2 + exp(-pi)/2)*sin(x)/pi - 2*(-exp(-pi)/5 + exp(pi)/5)*sin(2*x)/pi +
(-exp(-pi)/2 + exp(pi)/2)*cos(x)/pi + 2*(-2*exp(pi)/5 +
2*exp(-pi)/5)*cos(2*x)/pi
```